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**Politician Preferences  
and  
Caps on Political Lobbying**

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# Politician Preferences and Caps on Political Lobbying

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## Abstract

This paper extends Che and Gale (1998) by allowing the incumbent politician to have a preference for the policy position of one of the lobbyists. The effect of a contribution cap is analyzed where two lobbyists contest for a political prize. The cap always helps the lobbyist whose policy position is preferred by the politician no matter whether it is the high-valuation or the low-valuation contestant. In contrast to Che and Gale, once the cap is binding a more restrictive cap always reduces expected aggregate contributions. However, the politician might support the legislation of a barely binding cap. When politician policy preferences perfectly reflect the will of the people, a more restrictive cap is always welfare increasing. When lobbyist's valuations completely internalize all social costs and benefits, a cap is welfare improving if and only if the politician favors the high-value policy. Even a barely binding cap can have significant welfare consequences.

Keywords: All-pay auction, campaign finance reform, explicit ceiling.  
JEL: D72, C72

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## **I. Introduction**

The concept of representative democracy is founded on the proposition that the actions of elected representatives will in some sense reflect the will of the people. Either the public elects people who's own views reflect their own, or the desire to be reelected leads politicians to try to act as though their views reflected the public's. In either case, it is likely that an incumbent politician will have preferences over policy alternatives.

There is growing concern that the need to raise money to finance election campaigns is diluting this fundamental premise of representative democracy. The total spending by presidential candidates tripled in the eight years from 1996 to 2004. Presidential and congressional elections cost \$4 billion in 2004<sup>1</sup>. The need to raise funds necessitates constant fund-raising which takes time from other duties, and invites the possibility of money driving legislative outcomes.<sup>2</sup>

There have been numerous attempts to regulate campaign financing by imposing contribution caps<sup>3</sup>. The most recent regulation on campaign financing is the Bipartisan Campaign Reform Act of 2002, also known as McCain-Feingold Bill. It sets clear limits on individuals' contributions to candidates and to political action committees.<sup>4</sup>

Natural intuition suggests that contribution caps would result in decreased aggregate contributions and hence spending. However Che and Gale (1998), henceforth CG, challenge this

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<sup>1</sup>Financial activity of the 2004 presidential elections exceeded \$1 billion. For information on campaign spending see <http://www.fec.gov> hosted by the Federal Election Commission, <http://www.campaignfinancesite.org> hosted by the Hoover Institute and <http://www.opensecrets.org> hosted by the Center for Responsive Politics.

<sup>2</sup>On the other hand, it is sometimes argued that the need to raise funds may serve as a device to help politicians learn about society's valuation of alternatives.

<sup>3</sup>In the U.S. expenditure limits have to be voluntary, since the 1976 Buckley v. Valeo Supreme Court ruling struck them down as limitations on free speech. The Supreme Court upheld limits on individual contributions.

<sup>4</sup>An individual can contribute a maximum of \$2000 to any candidate per election. An individual's contribution to a political action committee is limited to \$5000.

intuition in an all-pay auction setting. Aggregate contributions may go up with a decrease in the cap since in equilibrium the low-valuation bidder (lobbyists) becomes more aggressive<sup>5</sup>. This paper extends CG by allowing the politician to have a preference for the policy position of one of the lobbyists contesting for a political prize.

We show that if an incumbent politician has any preference over the possible policy alternatives, the equilibrium of the game with contribution caps is qualitatively different from the equilibrium when the politician is indifferent between policy alternatives. As long as a cap does not drive one of the bidders out altogether, it always helps the contestant who is favored by the politician no matter whether it is the high-valuation or the low-valuation bidder.

With any policy preference, however mild, making a binding cap more restrictive always decreases the expected contributions of both bidders. Nevertheless, the CG effect of a cap increasing the aggressiveness of the low-valuation bidder can still be seen at the point where the cap first becomes binding. Although a more restrictive cap makes the politician's preferred policy outcome more likely, in contrast to CG, no politician would support a cap that is more restrictive than barely binding. This suggests that it is likely to be quite difficult to get representatives to enact campaign finance legislation with more restrictive than just binding caps.

Two stylized welfare measures are provided. When politician policy preferences perfectly reflect the will of the people, a more restrictive cap is always welfare increasing. When lobbyist's valuations completely internalize all social costs and benefits, a cap is welfare improving if and only if the politician favors the policy of the high-valuation bidder. Even a barely binding contribution cap can have significant welfare effects.

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<sup>5</sup>Drazen, Limão and Stratmann (2005) find a related result in a very different framework. See the comment to CG of Kaplan and Wettstein (2006) for the case of non-rigid caps. And see Gale and Che (2006) for the reply. Gaviious, Moldovanu and Sela (2003) study an incomplete information environment and find that with convex cost, expected spending can go up when the cap is more restrictive. Amegashie (2003) analyzes caps on all-pay auctions when a committee awards the prize.

## II. The Framework

Two risk-neutral lobbyist compete for a political prize. The prize will be arising due to a policy choice of an incumbent politician. This may be a vote on impending legislation but may also be more subtle, such as attaching a rider to an upcoming bill creating a regulatory loophole, or pushing a particular wording in committee. The value of the political prize to lobbyist 1 is denoted by  $v_1$ , and the value of the prize to lobbyist 2 is  $v_2$ ,  $v_1 > v_2 > 0$ . The lobbyists make simultaneous contributions to the politician in power. In CG, the politician awards the prize to the highest contributor. The contributions are not returned to the lobbyist whose efforts fail. Since the contributions are sunk both for the winner and the loser, this political lobbying game is an all-pay auction.<sup>6</sup> If bidder  $i$  wins the prize, his payoff is  $v_i - b_i$ , if the rival wins bidder  $i$ 's payoff is  $-b_i$ ,  $i \in \{1, 2\}$ . CG study the effect of a cap on contributions in this framework.<sup>7</sup>

In this paper we modify the CG framework to allow for the politician to have a preference over possible policy alternatives supported by the two lobbyists. The politician's preference may be ideologically based or it may induced from the preference of the constituents who will be voting in the future. The interest groups lobby the politician and the politician awards the political prize based on the bids and his preference. The preferred lobbyist is in an advantageous position since he can win the prize with a lesser contribution than his rival's. The degree of the advantage depends on the intensity of the preference of the politician. The intensity of the preference for the policy position of bidder 2 will be put into monetary terms, denoted by  $\gamma \in (-\infty, \infty)$ . For example  $|\gamma|$  could represent the expected campaign costs required to offset the effect of taking a policy position that

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<sup>6</sup>The all-pay auction without a cap has been analyzed by Hillman and Riley (1989), and Baye et al. (1993, 1996). See Yildirim (2005) for a contest where players have the flexibility to add to their previous efforts and see Kaplan, Luski and Wettstein (2002) for a model where the size of the reward is a function of the bid. Prat (2002) has a model with multiple lobbyists contributing to competing politicians.

<sup>7</sup>In Banerjee and Chakraborty (2004) the effect of a cap is analyzed where bidders differ in information quality. Fang (2002) analyzes the effect of a cap for a lottery auction model.

is unpopular in the politician's district. If the politician favors bidder 2's position  $\gamma > 0$ . If the politician favors bidder 1's position  $\gamma < 0$ . It will be assumed that the politician awards the prize to bidder 1 if  $b_1 > b_2 + \gamma$  where  $b_1$  and  $b_2$  are the bids (contributions) of bidders 1 and 2. In case of a tie,  $b_1 = b_2 + \gamma$ , each contestant has an even chance to win the prize. The rules of the game, the valuations of the bidders and the preference of the politician are common knowledge.

Simple backward induction in the one-shot game that will be analyzed here would have the politician taking his preferred action regardless of bids since all contributions are sunk. Hence there would be no contributions. Thus implicitly we are assuming that this one-shot game is embedded in a repeated setting so that the politician has an incentive to reward high contributions in order to keep them coming in the future. However, as long as contributions, preferences and actions are common knowledge among lobbyists, the same lobbyists do not necessarily need to be involved in repeated contests.

In the literature on auctions with preferential treatment of bidders, preferential treatment in awarding the prize is modeled with multiplicative preference. The preferred bidder can win the prize with a smaller bid as long as the rival does not over-bid by a certain percentile. This is done to match the institutional setting being modeled such as FCC spectrum auctions where minority owned businesses have their bids boosted by a percentage. Corns and Schotter (1999) has a first-price sealed-bid auction framework<sup>8</sup> and Fu (2006) has an all-pay auction with affirmative action in college admissions where the examination scores are the bids.<sup>9</sup> However the preferential treatment in our political lobbying game is additive, not multiplicative. In our framework,  $\gamma$  represents the intensity of preference measured in monetary terms. This has the advantage that the politician's

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<sup>8</sup>First-price auctions with preferential treatment are used explicitly in Federal Communication Commission radio spectrum licenses to favor of minority and women-owned firms. "Buy-American" purchasing preference programs are another example.

<sup>9</sup>In Feess, Muehlheusser and Walzl (2004) bidders with incomplete information compete in an all-pay auction with multiplicative weights.

preference for a policy does not depend on the contributions he receives. The disadvantage is that we cannot directly use the results of the previous literature. Nevertheless, similar insights will be gained. Differences will be discussed in due course.

In the CG framework  $\gamma$  is zero and the only equilibrium without a cap is in mixed strategies. The low-valuation bidder never bids more than his valuation of the prize. Hence the high-valuation bidder is in an advantageous position. Bidder 1 wins the prize with probability  $1 - (v_2 / 2v_1) > 1/2$ . He has a strictly positive expected value of the game,  $v_1 - v_2$ . The expected value of the game to bidder 2 is zero. The politician's expected total revenue is  $v_2(v_1 + v_2)/2v_1$ , while expected spending of bidder 1 is  $v_2/2$  and the expected spending of bidder 2 is  $v_2^2/2v_1$ . We will first present the model with politician preferences and compare the result to the benchmark case where  $\gamma=0$ . As in CG, we then analyze the effect of a cap on political contributions.

### III. Equilibrium without a Cap

Too strong of a preference of the politician, either  $\gamma \geq v_1$  or  $\gamma \leq -v_2$  would curb competition between the contestants completely. Since a bidder will never bid more than his valuation, the preferred lobbyist could bid zero and still win the prize. Hence in equilibrium both lobbyists would contribute nothing. What follows will discuss the non-trivial cases where  $v_1 > \gamma > -v_2$ .

**Proposition 1:** *Without a contribution cap there is no pure-strategy Nash equilibrium.*

**Proof:** The case when  $\gamma=0$  is in CG. When  $\gamma < 0$ ,  $b_2 = b' \in [0, |\gamma|)$  cannot be sustained in a pure-strategy Nash equilibrium. Bidder 1's best response to  $b'$  would be 0. Bidder 2's best response to 0 would not be  $b'$  but to bid  $|\gamma| + \epsilon$ . A bid of  $b_2 = b' \in [|\gamma|, v_2)$  cannot be sustained in a pure-strategy equilibrium, either. Bidder 1's best response would be to bid  $b' + |\gamma| + \epsilon$ , and  $b'$  would no longer be optimal for bidder 2. Finally, a bid of  $b_2 = v_2$  cannot be sustained as a pure strategy equilibrium. Bidder 1's best response would be to bid  $v_2 + |\gamma| + \epsilon$ . Bidder 2's best response would have been to bid zero. Likewise, when  $\gamma > 0$ , bidders 2's best response to  $b_1 = b' \in [0, \gamma)$  would be 0. Bidder 1's best response to 0 would be  $\gamma + \epsilon$ .

Bidder 2's best response to  $b_1 = b' \in [\gamma, v_2 + \gamma)$  would be  $b' - \gamma + \epsilon$ , and  $b'$  would no longer be optimal for bidder 1. Finally, a bid of  $b_1 = b' \geq v_2 + \gamma$  cannot be sustained as a pure strategy equilibrium, either. Bidder 2's best response would be to bid zero since no bidder would ever bid higher than his valuation. Bidder 1's best response would then be  $\gamma + \epsilon$ .

□

**Lemma 1:** *Let  $i$  be the bidder that the politician does not favor: if  $\gamma > 0$  let  $i=1$ , if  $\gamma < 0$  let  $i=2$ . If  $\gamma \neq 0$  then bidder  $i$  will put zero probability on  $b_i \in (0, |\gamma|]$ .*

**Proof:** If bidder  $i$  contemplates  $b_i \in (0, |\gamma|)$  a bid of zero will win with the same probability as he must exceed his rival's bid by at least  $|\gamma|$  in order to win. If  $b_i = |\gamma|$  then he can win only if the other bidder bids zero, in which case there is an even chance of winning. So if that bid gave him a non-negative payoff he could double his chances of winning by a slight increase in his bid. And if it gave him a negative payoff he could get a zero payoff by dropping his bid to zero.

□

**Lemma 2:** *Without a contribution cap neither bidder will put any probability mass point on any bid greater than zero.*

**Proof:** Define a range  $B_1$  as  $(0, x)$  where  $x$  is an arbitrary number greater than  $v_1$ . Define  $b_1^*$  as the supremum of  $B_1$ . Suppose the lowest mass point of bidder 1 in  $B_1$  is given by  $b_1^* \in B_1$ . Then if  $\gamma > 0$  bidder 2 would not put any probability at  $b_2 = b_1^* - \gamma$  as a slight increase in his bid would result in a discrete increase in the probability of winning. As there is no probability of  $b_2 = b_1^* - \gamma$ , bidder 1 could lower his bid slightly without changing his probability of winning. A similar argument rules out mass points for bidder 2 on  $b_2 \in (0, b_2')$ .

□

When  $\gamma=0$  the politician is indifferent between the policy alternatives. This case is in CG. We study all other cases where the politician has a preference over policy alternatives. We will derive the equilibrium by looking at two cases. The first case is when  $\gamma \in (-v_2, 0)$  or when  $\gamma \in (v_1 - v_2, v_1)$ . Here the politician's preference is either in the direction of the high valuation bidder's policy or so strongly in the direction of the low valuation bidder's policy that it overwhelms bidder 1's advantage arising from his higher valuation. The second case is when  $\gamma \in (0, v_1 - v_2]$ , where the politician mildly favors the policy of the low-valuation bidder but not so much that it overwhelms bidder 1's advantage arising from his higher valuation. In either case, Proposition 1 shows that equilibrium is only in mixed-strategies.

***i) Politician Preferences That Give a Clear Advantage to One Bidder***

When  $\gamma \in (-v_2, 0) \cup (v_1 - v_2, v_1)$  the politician either favors bidder 1, who already has the advantage of having a higher valuation, or the politician so strongly favors bidder 2 that in equilibrium the strong preference overwhelms the valuation advantage of bidder 1. It will be shown that in either case, the expected value of the game to the bidder whose policy is not favored (bidder  $i$ ) is zero and the expected value of the game to the bidder whose policy is preferred (bidder  $j$ ) is strictly positive. Lemmas 1 and 2 above, Lemmas 3 and 4 below and Lemma 10 in Appendix B show that bidder  $i$  must be indifferent among all bids in  $\{0\} \cup (|\gamma|, v_i]$  and his rival must be indifferent among all bids in  $[0, v_i - |\gamma|]$ .

**Lemma 3:** *If there is no cap and  $\gamma \in (-v_2, 0) \cup (v_1 - v_2, v_1)$ , the bidder whose policy is not favored by the politician has an infimum bid of zero and expected value of the game to him is zero.*

**Proof:** Let  $i$  be the bidder that the politician does not favor: if  $\gamma > 0$  let  $i=1$ , if  $\gamma < 0$  let  $i=2$ . Let  $j$  be the other bidder. Let  $b_i' = v_i$ . Bidder  $i$ 's infimum bid must be less than  $b_i'$  since there can be no probability mass at  $v_i$  by Lemma 2. Suppose that bidder  $i$  has an infimum bid of  $b_i^{inf} \in (|\gamma|, b_i')$ . Then bidder  $j$  would never choose  $0 < b_j \leq b_i^{inf} - |\gamma|$ . If he did he would be paying a positive amount and would

lose for sure, since by Lemma 2, the probability of bidder i choosing exactly  $b_i^{\text{inf}}$  is zero in this range. Therefore bidder i could lower his infimum bid without changing the probability of winning.

Suppose that  $b_i^{\text{inf}} = |\gamma|$  where bidder i was mixing in the open interval above  $|\gamma|$  but not at  $|\gamma|$ , by Lemma 1. Then bidder j would never bid zero as this gives zero profit and he can win for sure with a bid of  $v_i - |\gamma| + \varepsilon$  yielding positive profit. Take a bid of  $b_i = b_i^{\text{inf}} + \varepsilon = |\gamma| + \varepsilon$ , the probability that bidder i wins with this bid is  $\int_{|\gamma|}^{|\gamma| + \varepsilon} f_2(x - |\gamma|) dx$ . Since bidder j has no mass points on  $(0, \varepsilon]$  by Lemma 2, this probability is close to zero for small  $\varepsilon$ , yielding a negative expected payoff for bidder i. Hence bidder i's infimum bid cannot be  $|\gamma|$ .

$b_i^{\text{inf}} \in (0, |\gamma|)$  is not possible by Lemma 1. Therefore a bid of zero is in the support of the mixed strategy of bidder i. At this bid he loses for sure, so the expected value of the strategy for bidder i must be zero. □

**Lemma 4:** *Let i be the bidder that the politician does not favor: if  $\gamma > 0$  let  $i=1$ , if  $\gamma < 0$  let  $i=2$ . Let j be the other bidder. If there is no cap and  $\gamma \in (-v_2, 0) \cup (v_1 - v_2, v_1)$ , bidder i has a supremum bid of  $v_i$ . Bidder j has a supremum bid of  $v_i - |\gamma|$ . The expected value of the game to bidder j is  $v_j + |\gamma| - v_i > 0$ .*

**Proof:** Suppose that bidder i had a supremum bid of  $b_i^{\text{sup}} < v_i$ . Then bidder j would never set  $b_j > \max[0, b_i^{\text{sup}} - |\gamma|]$  as he can win for sure with that bid since either  $b_i^{\text{sup}} < \gamma$  or by Lemma 2 the probability of bidder i choosing exactly  $b_i^{\text{sup}}$  is zero. Therefore bidder i could win for sure with  $b_i = b_i^{\text{sup}} + \varepsilon$  yielding a payoff greater than zero for small enough  $\varepsilon$ , a contradiction of Lemma 3.

Suppose that bidder j had a supremum bid of  $b_j^{\text{sup}} < v_i - |\gamma|$ . Then bidder i could win for sure with  $b_i = b_j^{\text{sup}} + |\gamma| + \varepsilon$  yielding a payoff greater than zero for small enough  $\varepsilon$ , a contradiction of Lemma 3. Bidder j can win for sure with a bid of  $v_i - |\gamma|$  since by Lemma 2 the probability of bidder i choosing exactly  $v_i$  is zero. Since  $v_i - \gamma$  is in the support of his mixed strategy and he wins for sure with that bid, the expected payoff for bidder j is  $v_j + |\gamma| - v_i$ . □

The expected value of the game to bidder  $i$ , whose policy is not favored, is zero by Lemma 3. And each bid in the set  $\{0\} \cup (|\gamma|, v_i]$  must yield the same expected payoff by Lemmas 2 and 3. His bid is sunk irrespective of the identity of the winner. He wins the prize  $v_i$  when he bids  $b \in (|\gamma|, v_i]$  with the probability that bidder  $j$  contributes less than  $b - |\gamma|$ . Hence,

$$v_i F_j(b - |\gamma|) - b = 0$$

the equilibrium cumulative density function of bidder  $j$  is  $F_j(b) = (b + |\gamma|) / v_i \quad \forall b \in [0, v_i - |\gamma|]$ . Bidder  $j$  has a probability mass equal to  $|\gamma| / v_i$  at zero.

The expected value of the game to bidder  $j$ , whose policy is favored, is  $v_j + |\gamma| - v_i$  by Lemma 4. Since bidder  $i$  would never bid higher than his valuation of the prize, bidder  $j$  could simply bid slightly above  $v_i - |\gamma|$  and win the contest. In equilibrium bidder  $j$  must be indifferent among all bids in  $[0, v_i - |\gamma|]$  by Lemmas 2 and 4. He wins the prize  $v_j$  when he bids  $b \in [0, v_i - |\gamma|]$  with the probability that bidder  $i$  does not exceed bidder  $j$ 's bid by more than  $|\gamma|$ :

$$v_j F_i(b + |\gamma|) - b = v_j - v_i + |\gamma|$$

The equilibrium commutative density function of bidder  $i$  is  $F_i(b) = (v_j - v_i + b) / v_j \quad \forall b \in (|\gamma|, v_i]$ . Bidder  $i$  has a probability mass equal to  $(v_j - v_i + |\gamma|) / v_j$  at zero. And he puts zero probability on  $(0, |\gamma|]$  from Lemma 1. Since his policy is not favored, bidder  $i$  would have a chance to win the prize only if he exceeds the rival's bid by more than  $|\gamma|$ . The equilibrium distribution functions are graphed in Figure 1.

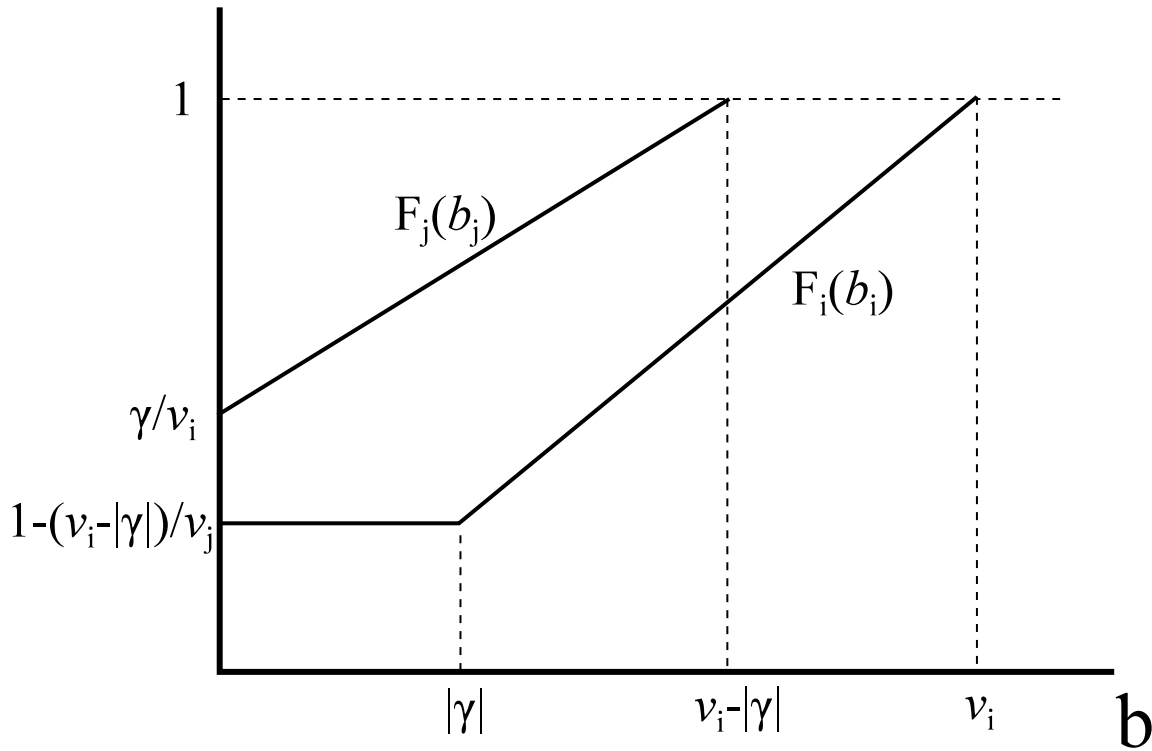
We can also calculate the probability that bidder  $i$ , who's policy is not favored wins the contest:

$$\int_{b=|\gamma|}^{v_i} F_j(x - |\gamma|) f_i(x) dx = \int_{b=|\gamma|}^{v_i} \frac{x}{v_i v_j} dx = (v_i^2 - \gamma^2) / 2v_i v_j$$

The stronger the preference for the favored lobbyist, the less likely it is that the rival wins the contest.

On  $b \in (|\gamma|, v_i]$  the p.d.f. of the bids of bidder  $i$  is  $f_i(b)=1/v_j$ . The expected contribution of bidder  $i$  is:

$$\int_{b_i=|\gamma|}^{v_i} x f_i(x) dx = \frac{v_i^2 - \gamma^2}{2v_j}$$



**Figure 1:** Equilibrium bid distributions without a cap and either favoring 1 ( $-v_2 < \gamma < 0$ ) or strongly favoring 2 ( $v_1 - v_2 < \gamma < v_1$ ). Bidder  $j$  is the bidder who's policy is favored, bidder  $i$  is the bidder who's policy is not favored.

On  $b \in (0, v_j - |\gamma|]$  the p.d.f. of the bids of bidder  $j$  is  $f_j(b)=1/v_i$ . The expected contribution of bidder  $j$  is:

$$\int_{b_j=0}^{v_j-|\gamma|} x f_j(x) dx = \frac{(v_j - |\gamma|)^2}{2v_i}$$

When the politician either strongly favors the policy position of the high-valuation bidder or strongly favors the policy position of the low-valuation bidder, the favored lobbyist is in an advantageous position. He could simply win for sure with bid of  $v_i - |\gamma|$ . The expected value of the game to that bidder  $j$  is positive while the lobbyist who's policy is not favored will have an expected value of zero. Competition over the political prize is lessened and the expected spending of both bidders goes down with an increase in the intensity of the politician's preference.

***ii) Mild Preference for the Policy Position of the Low-Valuation Bidder***

When  $\gamma \in (0, v_1 - v_2]$ , the politician has a mild preference for the policy position of the low-valuation bidder. Since bidder 2 would never bid higher than his valuation of the prize, bidder 1 could bid slightly above  $v_2 + \gamma$  and win the prize for sure. In equilibrium this will be enough to mitigate, but not eliminate the high-valuation bidder's advantage. Lemmas 1 and 2, Lemmas 8, 9, and 10 in the appendix show that bidder 1 is indifferent among all bids in  $(\gamma, v_2 + \gamma]$  and bidder 2 is indifferent among all bids in  $[0, v_2]$ .

The expected value of the game to bidder 1 is  $v_1 - v_2 - \gamma$ , by Lemma 9. He wins the prize  $v_1$  when he bids  $b \in (\gamma, v_2 + \gamma]$  only if bidder 2 bids less than  $b - \gamma$ .

$$v_1 F_2(b - \gamma) - b = v_1 - v_2 - \gamma$$

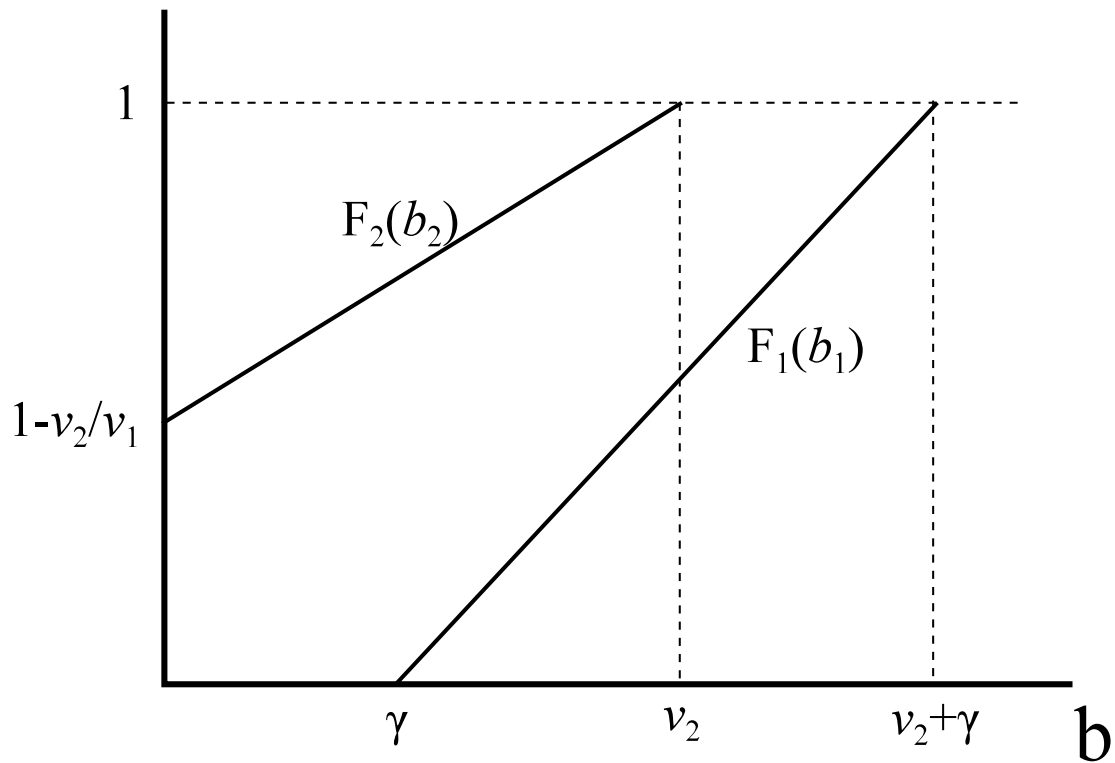
The equilibrium distribution function of bidder 2 is  $F_2(b) = (v_1 - v_2 + b) / v_1 \quad \forall b \in [0, v_2]$ . Bidder 2 has a probability mass of  $(v_1 - v_2) / v_1$  at zero.

The expected value of the auction to bidder 2 is zero by Lemma 8. All bids in  $[0, v_2]$  must yield the same expected payoff to bidder 2. He wins the prize  $v_2$  when he bids  $b \in [0, v_2]$ , only if bidder 1 bids less than  $b + \gamma$ .

$$v_2 F_1(b + \gamma) - b = 0$$

$F_1(b) = (b - \gamma) / v_2$  and  $f_1(b) = 1 / v_2 \quad \forall b \in (\gamma, v_2 + \gamma]$ . Bidder 1 puts zero probability on  $(0, \gamma]$  from Lemma

1. The equilibrium distribution functions are given in Figure 2.



**Figure 2:** Equilibrium bid distributions without a cap and mildly favoring the policy of bidder 2 ( $0 < \gamma \leq v_1 - v_2$ ).

The probability that bidder 1 wins, is the same as in the case of  $\gamma=0$ .

$$\int_{b=\gamma^+}^{v_2+\gamma} F_2(x-\gamma) f_1(x) dx = \int_{b=\gamma^+}^{v_2+\gamma} \frac{v_1 - v_2 + x - \gamma}{v_1 v_2} dx = 1 - (v_2 / 2v_1)$$

Compared to the case where the politician does not have a preference over policy choices, the expected spending of bidder 2 remains the same at  $v_2^2/2v_1$ , and the expected spending of bidder 1 goes up from  $v_2/2$  to:

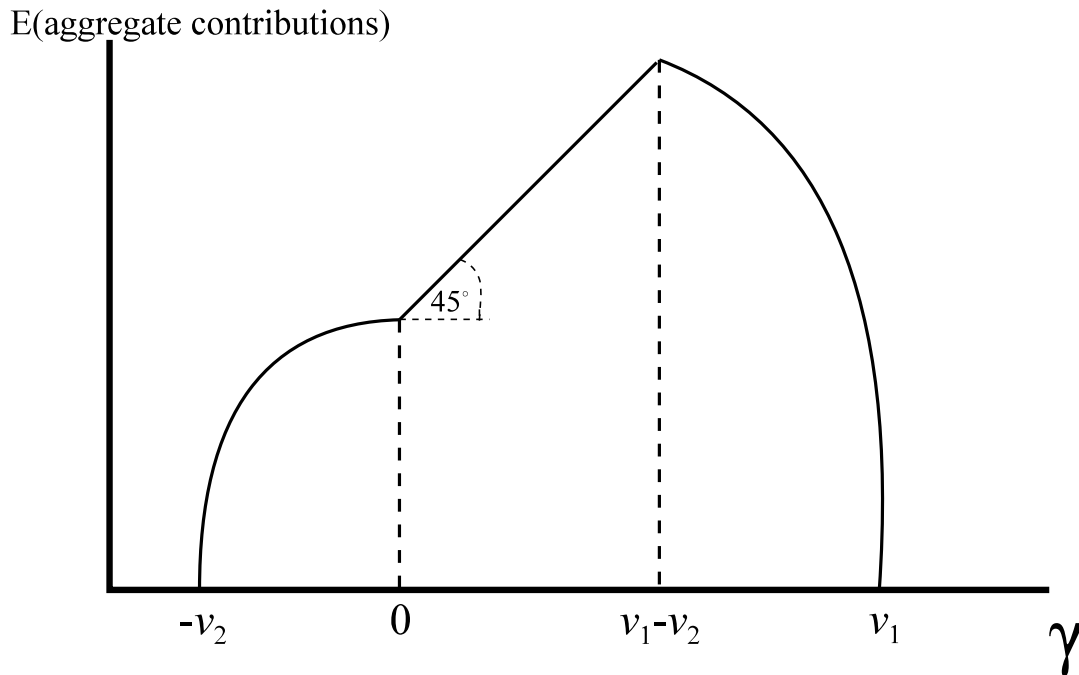
$$\int_{b=\gamma^+}^{v_2+\gamma} x f_1(x) dx = \int_{b=\gamma^+}^{v_2+\gamma} \frac{x}{v_2} dx = \frac{v_2 + 2\gamma}{2}$$

Mildly favoring the low-valuation bidder induces the high-valuation bidder to bid more aggressively. The probabilities of winning remain unchanged, since the greater effort of bidder 1 simply offsets the preference of the politician. Mildly favoring the low-valuation bidder ends up not helping the low-valuation bidder. The expected value of the game to the low-valuation bidder remains zero. It simply leads to an increase in the expected total revenue.

**Proposition 2:** *Without a cap, the expected aggregate contributions as a function of the preference parameter  $\gamma$  are concave, with a unique maximum at  $\gamma=v_1-v_2$ .*

**Proof:** The expected contributions of high-valuation and low-valuation bidders are demonstrated in the text for two possible ranges of  $\gamma$ . □

Figure 3 gives the expected aggregate contributions as a function of the preference parameter.



**Figure 3:** Expected aggregate contributions without a cap.

When the politician favors the policy of the high-valuation bidder,  $\gamma \in (-v_2, 0)$ , the expected aggregate contributions are lower than the case where  $\gamma = 0$ . With even preferences of the politician, the high-valuation bidder already has an advantage. When the politician also has a preference for his policy, this advantage increases. The low-valuation bidder is no longer as aggressive in bidding which also allows the high valuation bidder to decrease his expected contributions. Mildly favoring the policy of the low-valuation bidder,  $0 < \gamma \leq v_1 - v_2$ , decreases some of bidder 1's advantage. This induces the high-valuation bidder to bid more aggressively while the expected spending of the low-valuation bidder remains the same. The expected total efforts of the contributors is maximized when  $\gamma = v_1 - v_2$ . When the politician strongly favors the policy of the low-valuation bidder,  $v_1 - v_2 < \gamma < v_1$  competition over the political prize is lessened and the expected spending of both bidders goes down with an increase in  $\gamma$ .

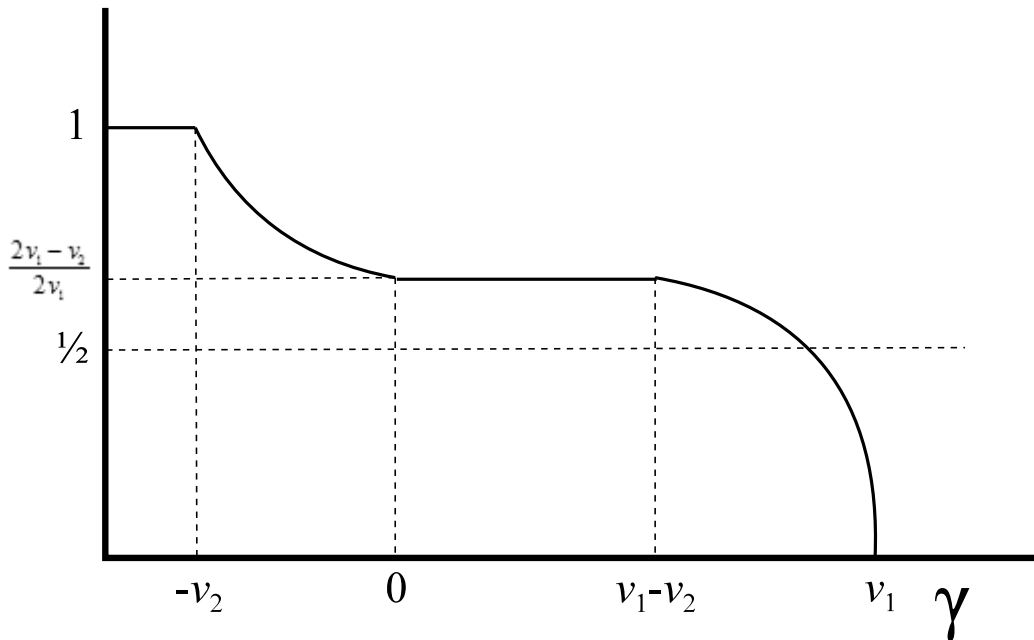
When the politician mildly favors the policy of the low-valuation bidder the results are different from Fu (2006) where the preferential treatment in college admission is modeled with a multiplicative weight for the minority candidate. With mild affirmative action Fu (2006) finds that the minority candidate (low-valuation bidder) does increase the expected effort level. Here the preference is additive rather than multiplicative. An increase in  $\gamma$  increases the aggressiveness of bidder 1 without changing the equilibrium strategy of bidder 2. The reason for this difference is that when the preference is multiplicative it changes the marginal cost of increasing the probability of victory for the high-valuation bidder.

Consider a high-value bidder contemplating increasing his bid to go from 25% chance of victory to 26% chance of victory. Holding constant the strategy of the other bidder, an increase in an additive preference does not change the cost of this marginal increase in probabilities. So if this marginal increase was beneficial before the change in the preference it still is afterward, and the high-valuation bidder simply becomes more aggressive and equilibrium probabilities remain unaltered. But with multiplicative preferences, holding the other bidder's strategy constant, an increase in the preference parameter increases the cost of going from 25% chance of victory to 26%. Hence while bidder 1 becomes more aggressive, it is not enough to completely offset the increase in the preference. With multiplicative preferences, in equilibrium bidder 2 exploits this by becoming more aggressive as well. Therefore with multiplicative preferences the spending of both players increases and the probability that bidder one wins decreases.

Thus unlike with multiplicative rules, with additive affirmative action rules in college admissions an increase in affirmative action would not increase the percentage of minorities being admitted but would just widen the gap in test scores between minority and non-minority students. For example an additive affirmative action rule is used by the College of Arts and Sciences at the University of Michigan where 20 points are added to the scores of minority applicants.

**Proposition 3:** *Without a cap, favoring the policy of one of the bidders changes the equilibrium probabilities of winning, except for the case where the policy of the low-valuation bidder is mildly favored,  $\gamma \in (0, v_1 - v_2]$ .*

**Proof:** The probability of winning of the high-valuation bidder is demonstrated in the text for three possible ranges of  $\gamma$ . □



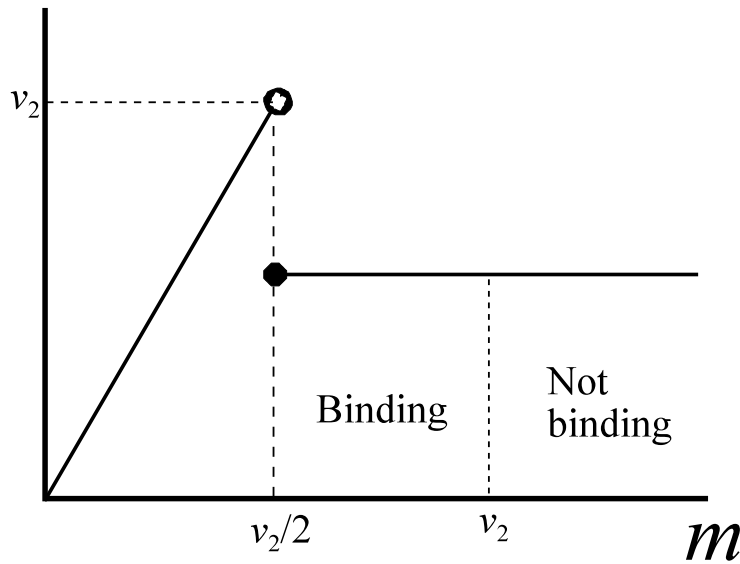
**Figure 4:** Probability that bidder 1 wins without a cap.

Figure 4 gives the probability that bidder 1 wins as a function of the preference parameter  $\gamma$ . Favoring the policy of the high-valuation bidder makes it more likely that the high-valuation bidder wins. And strongly favoring the policy of the low-valuation bidder makes it less likely that the high-valuation bidder wins. But there is no effect of the preference parameter on the equilibrium probabilities of winning when the politician mildly favors the policy of the low-valuation bidder. Favoring the low-valuation bidder's policy induces the high-valuation bidder to bid more aggressively. The greater effort of bidder 1 offsets the preference of the politician. Increasing  $\gamma$  simply leads to an increase in the expected aggregate contributions in this range.

#### IV. Equilibrium with a Cap

Before discussing the equilibrium with a cap, first define some terminology; denote  $m$  as the level of the contribution cap: Neither bidder can contribute more than  $m$ . A “*binding cap*” is a cap which is lower than the supremum of the support of the no-cap equilibrium bids of either bidder. As demonstrated in the previous section, when the politician favors the high-valuation bidder  $\gamma \in (-v_2, 0)$ , in the absence of a cap, the high-valuation bidder mixes in the range  $[0, v_2 - |\gamma|]$  and the low-valuation bidder mixes in the range  $\{0\} \cup (|\gamma|, v_2)$ . Hence a cap of  $m < v_2$  is binding when  $\gamma \in (-v_2, 0)$ . When  $\gamma \in (0, v_1 - v_2]$ , a cap  $m < v_2 + \gamma$  is binding. When  $\gamma \in (v_1 - v_2, v_1)$ , a cap  $m < v_1$  is binding. We will refer to a cap that is  $\epsilon$  less than the supremum of the support of the no-cap equilibrium bids as a “*barely binding*” cap. A more “*restrictive cap*” refers to a smaller  $m$ .

In the CG framework the politician is indifferent over policy alternatives,  $\gamma = 0$  and there is no pure-strategy Nash equilibrium in the absence of a contribution cap. However when a binding cap is introduced (in CG  $m < v_2$ ), the equilibrium can change qualitatively. For small caps,  $m \leq v_2/2$ , equilibrium does exist in pure-strategies. Both bidders bid by the amount of the cap exactly. And the expected aggregate contributions peak at  $m = v_2/2$ . See Figure 5 for expected aggregate contributions in CG, as a function of  $m$ . In the range of less restrictive caps  $m > v_2/2$ , equilibrium is once again only in mixed-strategies. Starting from a barely binding cap and letting the cap become more restrictive, at the point of  $m = v_2/2$  the nature of the equilibrium changes and there is a jump up in the expected aggregate contributions. Thus making a contribution cap more restrictive may increase aggregate contributions where both contributors bid by the amount of the cap.



**Figure 5:** Expected aggregate contributions with a cap when the politician has no policy preference ( $\gamma=0$ ).

Equilibrium with a contribution cap when the politician has a preference over policy alternatives is distinctly different from the equilibrium when politician is indifferent. As long as the cap does not completely suppress all contributions,  $m > |\gamma|$ , there is no pure-strategy Nash equilibrium no matter how restrictive the cap may be. Proposition 4 and the formal proof are below. When the politician has preferences over policy alternatives, with a contribution cap equilibrium exists only in mixed strategies.

**Proposition 4:** *If there is a binding contribution cap which does not completely suppress all contributions,  $m > |\gamma|$ , there is no pure-strategy Nash equilibrium when the politician has a preference favoring one of the contestants.*

**Proof:** Let  $i$  be the bidder that the politician does not favor: if  $\gamma > 0$  let  $i=1$ , if  $\gamma < 0$  let  $i=2$ . Let  $j$  be the other bidder.

Suppose bidder  $j$  bids  $b_j \in (m-|\gamma|, m]$  where  $m \leq v_j$ . When the preferred bidder bids up to the cap, bidder  $i$  cannot win the prize since he would be restricted by the cap and not possibly exceed  $b_j$  by  $|\gamma|$ . Hence it would be bidder  $i$ 's best response to bid zero. Bidder  $j$ 's best response to zero would be zero. Hence  $b_j \in (m-|\gamma|, m]$  cannot be sustained in a pure-strategy Nash equilibrium. If bidder  $j$  bids  $b_j = m-|\gamma|$ , bidder  $i$  would mildly prefer to bid  $m$  if  $m \leq v_i/2$  since he would then have an even chance to win the prize. However it would have been bidder  $j$ 's best response to bid  $m-|\gamma|+\epsilon$ . If  $m > v_i/2$ , bidder  $i$ 's best response to  $b_j = m-|\gamma|$  is zero. Bidder  $j$  would then have rather bid zero. Hence  $b_j = m-|\gamma|$  cannot be sustained in a pure-strategy Nash equilibrium. Since no player would bid higher than his own valuation,  $b_j = m$  where  $m > v_j$  cannot be sustained in a pure-strategy Nash equilibrium, either.

Suppose bidder  $i$  bids  $b_i = m$  where  $m \leq v_i$ . It would be bidder  $j$ 's best response to bid either  $m-|\gamma|+\epsilon$  if  $m-|\gamma| < v_j$  or zero if  $m-|\gamma| \geq v_j$ . It would then be bidder  $i$ 's best response to bid slightly more than  $m$  if  $m < v_j$  or to bid  $|\gamma|$ . Hence  $b_i = m$  where  $m \leq v_i$  cannot be sustained in a pure-strategy Nash equilibrium. Since no player would bid higher than his own valuation  $b_i = m$  where  $m > v_i$  cannot be sustained in a pure-strategy Nash equilibrium.

There can be no pure-strategy Nash equilibrium where bidder  $i$  bids  $m$ . And there can be no pure-strategy Nash equilibrium where bidder  $j$  bids  $b_j \in (m-|\gamma|, m]$ . For all other bids, the proof of Proposition 1 is still valid for the non-existence of pure-strategy Nash equilibrium.

□

**Lemma 5:** *If  $\gamma \neq 0$  and with a binding contribution cap, bidder  $i$  will not put a mass point on any bid  $b_i \in (0, m]$ , and bidder  $j$  will not put a mass point on any bid  $b_j \in (0, m - |\gamma|)$  where  $j$  is the bidder that is favored by the politician and  $i$  is the other bidder: if  $\gamma > 0$  then  $j=2$  and  $i=1$ , if  $\gamma < 0$  then  $j=1$  and  $i=2$ .*

**Proof:** Define  $B_i$  and  $B_j$  by  $B_i = (0, m]$ ,  $B_j = (0, m - \gamma)$ . The remainder of the proof of Lemma 2 applies directly.

□

**Lemma 6:** *With a binding contribution cap, if  $\gamma \neq 0$ , the bidder who's policy is not favored by the politician has an infimum bid of zero and the expected value of the game to him is zero.*

**Proof:** Let  $j$  be the bidder who's policy is favored by the politician and  $i$  the other bidder: if  $\gamma > 0$  then  $j=2$  and  $i=1$ , if  $\gamma < 0$  then  $j=1$  and  $i=2$ . If  $b_i^{\text{inf}} = m$  then bidder  $i$  would be playing a pure strategy and player  $j$ 's best response would be a pure strategy of  $b_j = m - |\gamma| + \varepsilon$ . By Proposition 4 there cannot be a pure-strategy equilibrium, so  $b_i^{\text{inf}} < m$ . Suppose that bidder  $i$  has an infimum bid of  $b_i^{\text{inf}} \in (|\gamma|, m)$ . Then bidder  $j$  would never choose  $0 < b_j \leq b_i^{\text{inf}} - |\gamma|$ . If he did he would be paying a positive amount and would lose for sure, since by Lemma 2, the probability of bidder  $i$  choosing exactly  $b_i^{\text{inf}}$  is zero in this range. Therefore bidder  $i$  could lower his infimum bid without changing the probability of winning.  $b_i^{\text{inf}} \in (0, |\gamma|)$  is not possible by Lemma 1.

Suppose that  $b_i^{\text{inf}} = \gamma$  where bidder  $i$  was mixing in the open interval above  $|\gamma|$  but not at  $|\gamma|$ , by Lemma 1. Then bidder  $j$  would never bid zero as this gives zero profit and he can win for sure with a bid of  $m$  yielding positive profit. Take a bid of  $b_1 = b_1^{\text{inf}} + \varepsilon = \gamma + \varepsilon$ , the probability that bidder  $i$  wins with this bid is  $\int_{|\gamma|}^{|\gamma|+\varepsilon} f_j(x - |\gamma|) dx$ . Since bidder  $j$  has no mass points on  $(0, \varepsilon]$  by Lemma 2, this probability is close to zero for small  $\varepsilon$ , yielding a negative expected payoff for bidder  $i$ . Hence bidder  $i$ 's infimum bid cannot be  $|\gamma|$ .

Therefore a bid of zero is in the support of the mixed strategy of bidder  $i$ . At this bid he loses for sure, so the expected value of the strategy for bidder  $i$  must be zero.

□

**Lemma 7:** *With a binding contribution cap, if  $\gamma \neq 0$ , the bidder who's policy is not favored by the politician has a supremum bid of  $m$ . The bidder who's policy is favored by the politician has a supremum bid of  $m - \gamma$  and expected value of the game to him is  $v_j + |\gamma| - m > 0$ , where  $v_j$  is his valuation.*

**Proof:** Let  $j$  be the bidder who's policy is favored by the politician and  $i$  the other bidder: if  $\gamma > 0$  then  $j=2$  and  $i=1$ , if  $\gamma < 0$  then  $j=1$  and  $i=2$ . Suppose that bidder  $i$  had a supremum bid of  $b_i^{\text{sup}} < m$ . Then bidder  $j$  would never set  $b_j > \max[0, b_i^{\text{sup}} - |\gamma|]$  as he can win for sure with that bid since either

$b_i^{\text{sup}} < \gamma$  or by Lemma 5 the probability of bidder  $i$  choosing exactly  $b_i^{\text{sup}}$  is zero. Therefore bidder  $i$  could win for sure with  $b_i = b_i^{\text{sup}} + \varepsilon$  yielding a payoff greater than zero for small enough  $\varepsilon$ , a contradiction of Lemma 6.

Suppose that bidder  $j$  had a supremum bid of  $b_j^{\text{sup}} < m - |\gamma|$ . Then bidder  $i$  could win for sure with  $b_i = b_j^{\text{sup}} + |\gamma| + \varepsilon$  yielding a payoff greater than zero for small enough  $\varepsilon$ , a contradiction of Lemma 6. Bidder  $j$  can win for sure with a bid of  $m - |\gamma|$  since by Lemma 5 the probability of bidder  $i$  choosing exactly  $m$  is zero. Since  $m - |\gamma|$  is in the support of his mixed strategy and he wins for sure with that bid, the expected payoff for bidder  $j$  is  $v_j + |\gamma| - m$ .  $\square$

The policy of bidder  $j$  is favored by the politician, and bidder  $i$  is the rival. Lemmas 1 and 5, Lemmas 6 and 7 below and Lemma 11 in Appendix B demonstrate that when the cap is binding bidder  $i$  must be indifferent among all bids in  $\{0\} \cup (|\gamma|, m]$  and bidder  $j$  must be indifferent among all bids in  $[0, m - |\gamma|]$ .

The expected value of the game to bidder  $i$  is zero by Lemma 6. And each bid in the set  $\{0\} \cup (|\gamma|, m]$  must yield the same expected payoff. He wins the prize  $v_i$  when he bids  $b \in (|\gamma|, m]$ , only if the favored bidder bids less than  $b - |\gamma|$ . Hence,

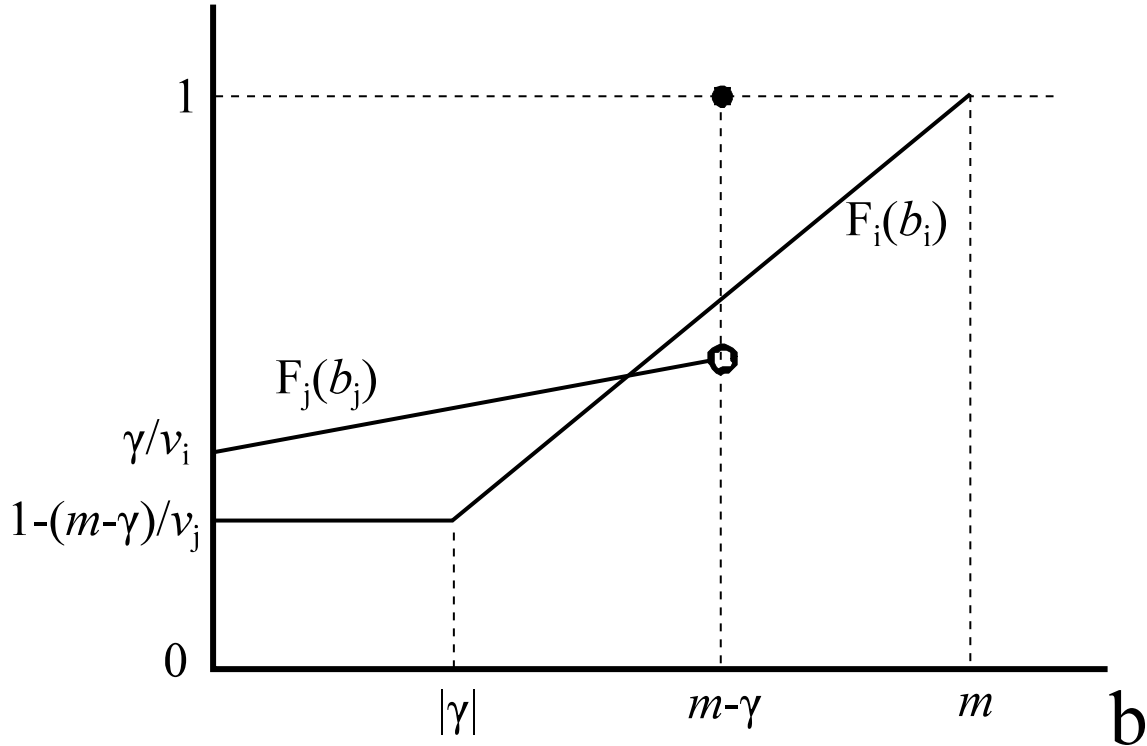
$$v_i F_j(b - |\gamma|) - b = 0$$

the equilibrium distribution function of the bidder who's policy is favored is  $F_j(b) = (b + |\gamma|) / v_i$ ,  $\forall b \in [0, m - |\gamma|]$ . The bidder who's policy is favored has a probability mass equal to  $|\gamma| / v_i$  at zero. The equilibrium distribution function is discontinuous. There is a probability mass equal to  $1 - F_j(m - |\gamma|) = 1 - m / v_i$  at  $m - |\gamma|$ .

The expected value of the game to the bidder who's policy is favored is  $v_j + |\gamma| - m$  by Lemma 7. Bidder  $j$  must be indifferent among all bids in  $[0, m - |\gamma|]$ . He wins the prize  $v_j$  when he bids  $b \in [0, m - |\gamma|]$  only with the probability that bidder  $i$  does not exceed bidder  $j$ 's bid by more than  $|\gamma|$ :

$$v_j F_i(b + |\gamma|) - b = v_j + |\gamma| - m$$

The equilibrium distribution function of bidder  $i$  is  $F_i(b) = (v_j - m + b)/v_j \forall b \in (|\gamma|, m]$ . Bidder  $i$  has a probability mass equal to  $(v_j - m + |\gamma|)/v_j$  at zero. There is a gap in the support of equilibrium bids. By Lemma 1 bidder  $i$  puts zero probability on  $(0, |\gamma|)$ . The equilibrium distribution functions of bidder  $i$  and bidder  $j$  are graphed in Figure 6.



**Figure 6:** Equilibrium distributions with a cap. Bidder  $j$  is the bidder whose policy is favored, bidder  $i$  is the bidder whose policy is not favored.

On  $b \in (|\gamma|, m]$  the p.d.f. of the bids of bidder  $i$  is  $f_i(b) = 1/v_j$ . The expected contribution of bidder  $i$  is:

$$\int_{b_i = |\gamma|^+}^m x f_i(x) dx = \frac{m^2 - \gamma^2}{2v_j}$$

On  $b \in (0, v_i - |\gamma|]$  the p.d.f. of the bids of bidder  $j$  is  $f_j(b) = 1/v_i$ . The expected contribution of bidder  $j$  is:

$$\int_{b_j=0}^{(m-|\gamma|)^-} x f_j(x) dx + (m - |\gamma|)(1 - m/v_i) = \frac{(m - \gamma)^2}{2v_i} + \frac{(m - \gamma)(v_i - m)}{v_i} = \frac{(m - \gamma)}{2v_i} (2v_i - m - \gamma)$$

We can also calculate the probability that the bidder  $i$  wins the contest. Since bidder  $j$ 's policy is preferred, bidder  $i$  has no chance to win the prize if he bids less than  $|\gamma|$ . He wins the prize when he bids  $b_i \in (|\gamma|, m]$  only with the probability that bidder  $j$  does not exceed  $b_i - |\gamma|$ :

$$\int_{b=|\gamma|^+}^m F_j(x - |\gamma|) f_i(x) dx = \int_{b=|\gamma|^+}^m \frac{x}{v_i v_j} dx = (m^2 - \gamma^2) / 2v_i v_j$$

The probability of winning the contest for  $i$  decreases in  $m$  when the cap is binding. In the case of mildly favoring the policy position of the low-valuation bidder there is a discontinuity in the probability of winning at the point where the cap is barely binding. This discontinuity will be discussed shortly.

**Proposition 5:** *The cap always helps the lobbyist whose policy position is preferred by the politician no matter whether it is the high-valuation or the low-valuation bidder.*

**Proof:** The expected values of the game to bidders in all possible scenarios are discussed in the text. When  $\gamma < 0$ , in the absence of a cap, the expected value of the game to the high-valuation bidder is  $(v_1 - v_2 - \gamma)$  where and the expected value of the game to the low-valuation bidder is zero. With a binding cap ( $m < v_2$ ), the expected value of the game to the high-valuation bidder goes up to  $v_1 - m - \gamma$  and the expected value to the low-valuation bidder remains the same. The cap helps the high-valuation bidder when its policy position is preferred.

When  $\gamma \in (0, v_1 - v_2]$ , in the absence of a cap, the expected value of the game to the low-valuation bidder is 0 and the expected value of the game to the high-valuation bidder is  $(v_1 - v_2 - \gamma)$ . With a binding cap ( $m < v_2 + \gamma$ ), the expected value of the game to the low-valuation bidder goes up from zero to  $(v_2 - m + \gamma)$  and the expected value to the high-valuation bidder goes down to zero. The cap helps the low-valuation bidder when his policy position is mildly preferred.

When  $\gamma \in (v_1 - v_2, v_1)$ , in the absence of a cap, the expected value of the game to the low-valuation bidder is  $(v_2 - v_1 + \gamma)$  and the expected value of the game to the high-valuation bidder is zero. With a binding cap ( $m < v_1$ ), the expected value of the game to the low-valuation bidder goes up to  $(v_2 - m + \gamma)$  and the expected value to the high-valuation bidder remains zero. The cap helps the low-valuation bidder when its policy is strongly preferred.  $\square$

Since the maximum the less preferred lobbyist can contribute is  $m$ , the favored lobbyist is in an advantageous position. The favored lobbyist has the choice of outbidding his rival for sure by bidding slightly above  $m - |\gamma|$ . So while the less preferred bidder is effectively constrained by the cap, the favored bidder is not. This allows the favored bidder to capture a higher positive expected payoff.

**Proposition 6:** *When the politician has a preference over policy alternatives, making a binding cap more restrictive always reduces expected aggregate contributions.*

**Proof:** When there is a binding contribution cap, the expected contributions of bidder 1 and of bidder 2 are given in the text when the politician favors the policy of the high-valuation lobbyist and where he favors the policy of the low-valuation lobbyist. When the politician favors the policy of the low-valuation bidder, the expected aggregate contributions are  $\frac{(m - \gamma)}{2v_1v_2} (m(v_1 - v_2) - \gamma(v_1 + v_2) + 2v_1v_2)$ . This is increasing in  $m$ . Hence making the cap more restrictive reduces expected aggregate contributions  $\forall \gamma \in (0, v_1)$ . When the politician favors the policy of the high-valuation bidder, the expected aggregate contributions are  $m + \gamma - (m^2 - \gamma^2) \frac{v_1 - v_2}{2v_1v_2}$ . The derivative of this expression

is  $1 - \frac{v_1 - v_2}{v_1 v_2} m > 0$  since for a binding cap  $m < v_2$ . Hence making the cap more restrictive reduces expected aggregate contributions  $\forall \gamma \in (-v_2, 0)$ .  $\square$

When the politician has no policy preference, making a binding cap more restrictive can increase expected aggregate expenditures. When  $m$  is very restrictive it is worthwhile for both bidders to pay it even though they have only a 50% chance of winning. In CG this switch from competition in mixed-strategies to pure strategies of both bidders paying the maximum leads to an increase in the expected aggregate contributions as  $m$  decreases past a threshold. When the politician has a policy preference, however mild, this type of pure-strategy equilibrium can no longer occur. If both bidders contribute the maximum amount the prize will go to the bidder who's policy is favored, so the bidder who's policy is not favored would not play such a pure strategy. Hence equilibrium is always in mixed-strategies as long as the cap does not curtail all contributions from both bidders.

Nevertheless, the Che and Gale effect of a cap increasing the aggressiveness of the low-valuation bidder still exists, but it occurs at the point where the cap just becomes binding.

**Proposition 7:** *When the politician has a preference over policy alternatives, imposition of a cap may lead to an increase in expected aggregate contributions if and only if the politician mildly favors the policy position of the low-valuation bidder.*

**Proof:** When there is no cap, the expected contributions of bidder 1 and bidder 2 are found in the text for each possible range of  $\gamma$ . When  $\gamma < 0$ , without a cap the expected aggregate contribution is  $\frac{(v_2 + \gamma)^2}{2v_2} + \frac{v_2^2 - \gamma^2}{2v_1}$  which is the same as the expected aggregate contribution with a cap evaluated at  $m=v_2$  where the cap just becomes binding for the case where the politician favors the policy of the high-valuation bidder. Hence there is no discontinuity going from a non-binding to a binding cap. When  $\gamma \in (v_1 - v_2, v_1)$ , without a cap the expected aggregate contributions are  $\frac{v_1^2 - \gamma^2}{2v_2} + \frac{(v_1 - \gamma)^2}{2v_1}$  which is the same as the expected aggregate contribution with a cap evaluated at  $m=v_1$  where the cap just

becomes binding. Hence once again, there is no discontinuity going from a non-binding to a binding cap. When  $\gamma \in (0, v_1 - v_2]$ , without a cap the expected aggregate contribution are  $\frac{v_2 + 2\gamma}{2} + \frac{v_2^2}{2v_1}$  which is lower than the expected aggregate contribution with a cap evaluated at  $m = v_2 + \gamma$  where the cap just becomes binding. Hence there is a discontinuity going from a non-binding to a binding cap. The expected aggregate contributions jump up by an amount of  $\frac{v_1(v_1 - v_2 - \gamma)}{v_2}$ .

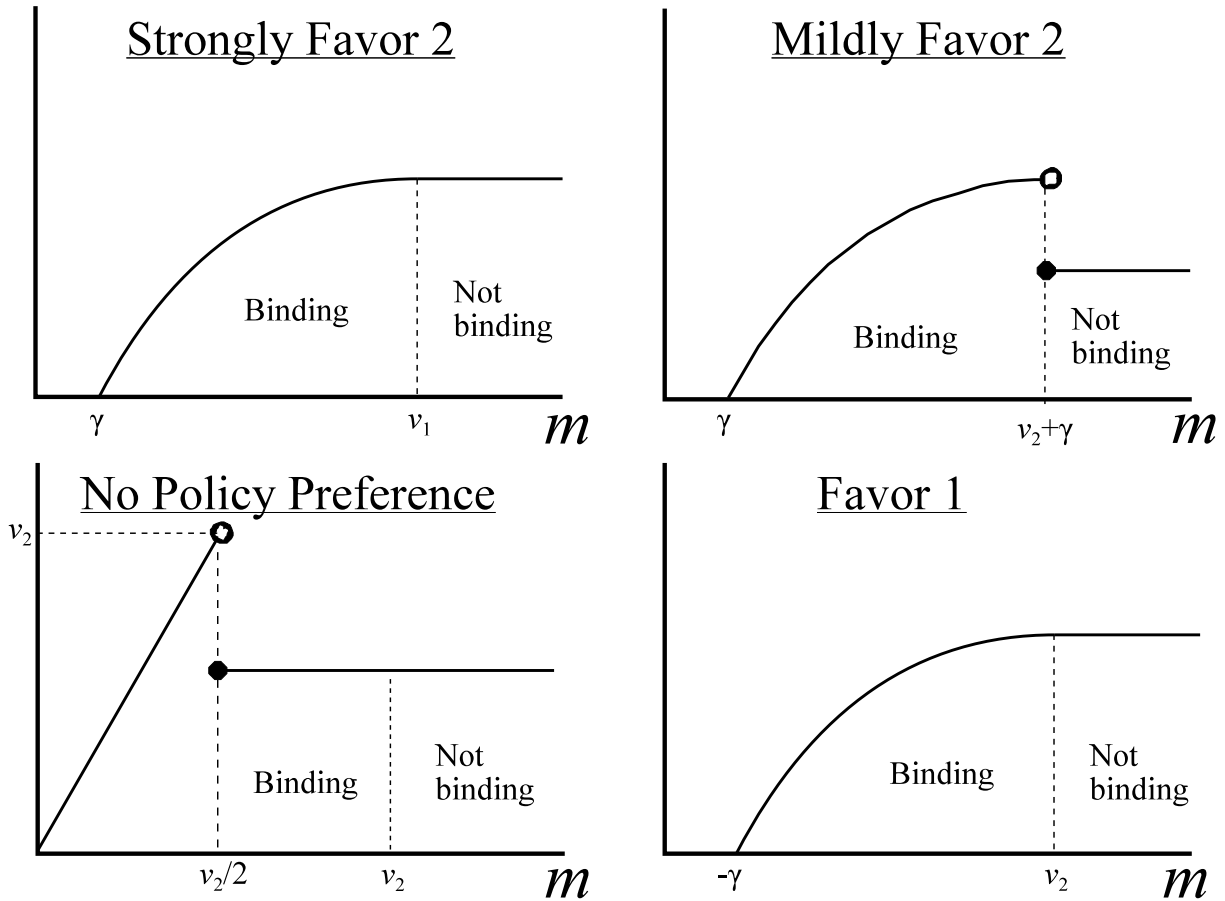
□

Figure 7 shows expected aggregate expenditures as a function of the level of the contribution cap. If the politician has a policy preference, making a binding cap more restrictive strictly decreases aggregate expenditure as long as the cap does not completely curtail all spending,  $m > |\gamma|$ . Likewise, making a binding cap more restrictive strictly increases the probability that the politician's preferred policy is enacted, Figure 8.

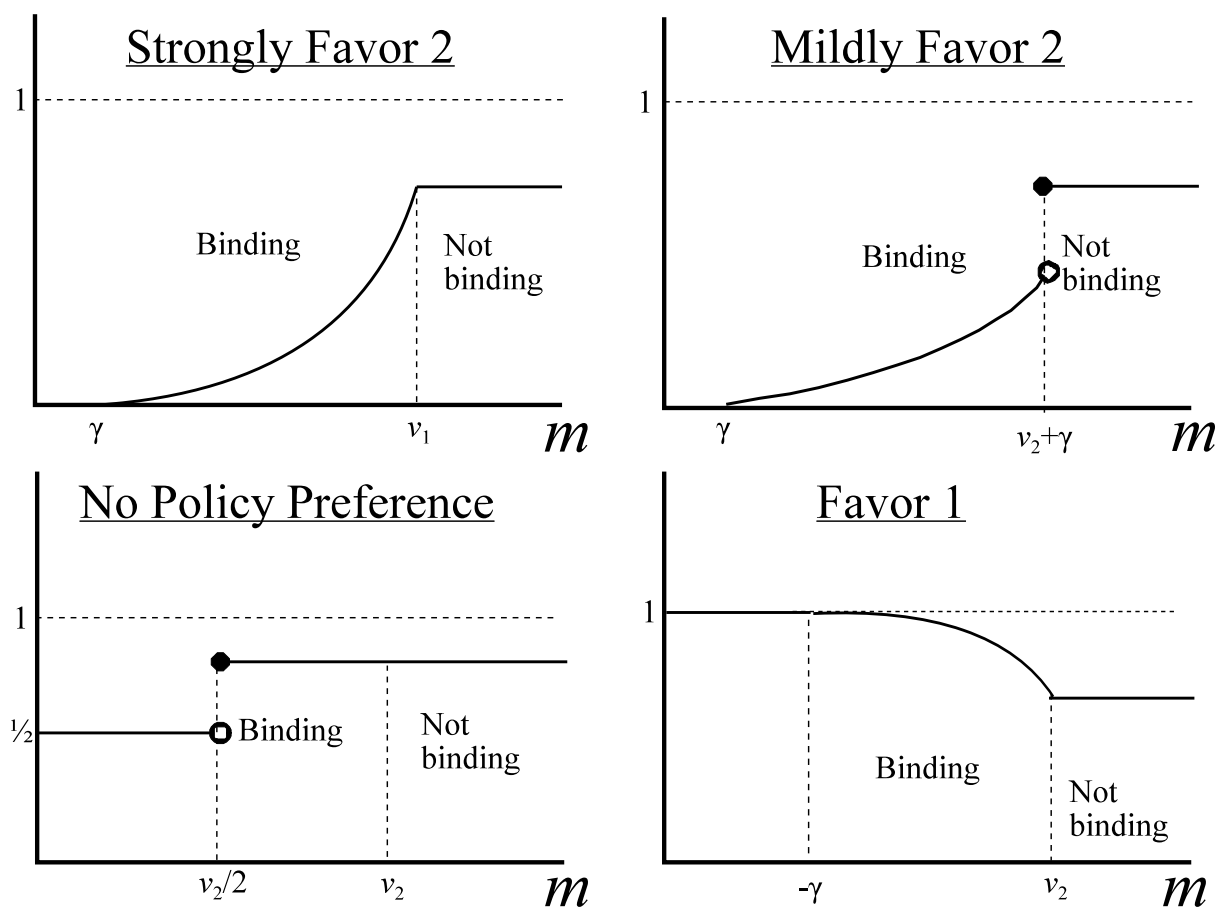
When the politician has a policy preference, the CG effect of increased expected aggregate contributions as a result of a more restrictive cap can be seen at the point where the cap first becomes binding, rather than at an interior point. In CG the discontinuity in the expected aggregate contributions arises because a more restrictive cap switched the equilibrium from mixed strategies to pure strategies of both players contributing the maximum amount. When the politician has a policy preference, however mild, the equilibrium is always in mixed strategies (for  $m > |\gamma|$ ). However, when the politician mildly favors the policy of the low valuation bidder, the imposition of a cap changes which player has the advantage in the mixed-strategy equilibrium.

When  $\gamma \in (0, v_1 - v_2)$  and  $m \geq v_2 + \gamma$ , bidder 1 has an advantage in the competition. He can bid more than  $v_2 + \gamma$  and win for sure with a positive payoff. In equilibrium he is able to use this advantage to compete away all of bidder 2's surplus and secure himself a positive expected payoff. However, when the contribution cap becomes binding, when  $m$  falls just below  $v_2 + \gamma$ , the situation is suddenly reversed. With a binding contribution cap bidder 2 can bid just above  $m - \gamma$ , guaranteeing victory and a positive payoff. This advantage induces him to bid more aggressively in equilibrium

and compete away all of bidder 1's surplus. The contribution cap does not change the basic nature of the competition (it is still in mixed strategies) but swings the advantage from the high value bidder to the bidder whose policy is favored by the politician. So at the point where the cap first becomes binding, a more restrictive cap leads to a sudden increase in expected aggregate contributions and a sudden decrease in the probability that the high value bidder wins the competition.



**Figure 7:** Expected aggregate contributions with a cap.



**Figure 8:** The probability that bidder 1 wins with a contribution cap.

## V. The Politicians' Choice of Contribution Cap

We have treated the contribution cap as an exogenous feature of the political system. However in a representative democracy it is the incumbent representatives themselves that must enact the legislation capping political contributions. So it is worth considering whether it is worthwhile for them to do so, and if so what level of cap they would choose.

There are two important caveats. The first is that no consideration will be taken of the possibility of direct political pressure for caps. If there is strong public desire for contribution caps they might be enacted even if the politicians themselves do not directly favor them. Secondly,

political competition is not modeled here so all results reflect what will happen to the incumbent politician's ability to raise funds, but not what will happen to his opponent's fund raising. Even if a cap decreases expected contributions to the incumbent politician, it could potentially have a greater negative impact on his opponent.

In practice the ability to raise funds is much greater for incumbents than for challengers. In the 2004 elections the average incumbent senator raised \$8,614,011 while the average challenger raised \$962,074. These of course include races with just a token challenger. Candidates for open seats, which generally include serious challengers, raised an average of \$2,949,264, substantially below the amount the average incumbent was able to raise.<sup>10</sup> There are many motives for making political contributions, ranging from the idealistic to the purely cynical self-interested desire to buy legislative favors that is modeled here. But clearly this type of contribution is easier for an incumbent to attract than a challenger. The donation is only of potential value if the politician is in office to pay back the favor. Over the past five election cycles from 1996 to 2004 96.8% of House incumbents and 88% of Senate incumbents were returned to office.<sup>11</sup> So suppose that, all else equal, the incumbent politician believes that restricting the ability to raise funds will hurt him at least as much as it will hurt his potential challenger and therefore wishes to maximize the ability to collect contributions.

If politician has no preference over policy alternatives CG shows that expected aggregate contributions are maximized with a contribution cap of  $m = v_2 / 2$ . So an incumbent politician will support a binding cap on political contributions of  $v_2 / 2$ . If politician does have a preference over

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<sup>10</sup>The figures for House races are similar, although the amounts are lower. The average incumbent raised \$1,122,505 while the average challenger raised \$192,964. Candidates for open seats raised an average of \$569,738.

<sup>11</sup>For comparison, if these probabilities were constant and equal for all members and there were no voluntary retirements or deaths, the expected average time in office would be roughly 32 years for Senators and 42 years for Representatives.

policy alternatives one needs to take this preference into account when evaluating the expected payoff of the politician:

$$E(b_1 + b_2) + \Pr(b_2 > b_1 - \gamma)\gamma.$$

A contribution cap increases the probability that the politicians preferred policy is enacted but can decrease the expected level of contributions.

**Proposition 8:** *When the politician has preferences over policy alternatives, the politician would strictly support relaxing any binding contribution cap. However, the politician might legislate a just binding contribution cap.*

**Proof:** If the politician favors the policy of bidder 1, his expected payoff with a binding cap is:

$$\frac{[\gamma^3 - (v_1 - v_2)\gamma^2] + 2v_1v_2m + (v_1 - v_2 - \gamma)m^2}{2v_1v_2}.$$

This is strictly increasing in  $m$  so the politician would prefer a marginal increase in the level of any binding cap. If the politician favors the policy of bidder 2 there are similar incentives. Increasing the level of a binding cap increases the probability of the preferred policy but decreases the expected contributions. The politician's expected payoff with a binding cap is:

$$\frac{[\gamma^3 - (v_1 - v_2)\gamma^2] + 2v_1v_2m + (v_1 - v_2 - \gamma)m^2}{2v_1v_2}$$

which is strictly increasing in  $m$  over the range where the cap is binding. So the politician would always support a marginal increase in an existing binding contribution cap.

Consider now a politician contemplating an increase in the cap that would move from a binding cap to a situation where the cap was not binding. If the politician either favors the policy of bidder 1 ( $\gamma < 0$ ) or strongly favors the policy of bidder 2 ( $\gamma \geq v_1 - v_2$ ) then there is no discontinuity at the point where the cap is just barely binding ( $m$  equal  $v_2$  or  $v_1$  respectively) so the politician would be willing to remove the cap. However, if the politician has a weak preference for the policy of

bidder 2,  $0 < \gamma < v_1 - v_2$ , then relaxing the level of the cap past the point where it is first binding,  $m = v_2 + \gamma$ , will cause a discrete decrease in both the expected contributions and the probability of the preferred policy being enacted. So in this case the politician would like a contribution cap to exist, but to be just barely binding. Without a cap bidder 1 has the advantage of being able to bid high enough to guarantee victory. In equilibrium this reduces the aggressiveness of bidder 2. When the cap just becomes binding this advantage disappears and bidder 2 bids more aggressively, increasing the expected contributions. However further reduction in the level of the cap always reduces contributions.  $\square$

When the politician has no preference over policy alternatives he will strictly favor restrictive caps that cut the maximum equilibrium bid in half. However, if politicians have any preference over policy alternatives, however mild, they might introduce a cap which is barely binding but would not support caps which restrict bids much below those that would occur without any such legislation. Hence it is not surprising that while there have been numerous regulations aimed at capping political contributions over the last thirty years, they tend to contained loopholes which kept them from being truly binding. The most recent example of this phenomenon is the unlimited contributions permitted to 527 groups which came into being with the otherwise quite restrictive Bipartisan Campaign Reform Act of 2002.<sup>12</sup>

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<sup>12</sup>527 groups, which are named after section 527 of the Internal Revenue Code, are exempt from the restrictions of federal campaign law. These organizations can collect unlimited donations from corporations, unions and individuals. In 2004 the top 10 donors alone gave \$105 million to these groups. 527 group activities do not instruct whom to vote for directly, but typically the advocacy group's view of the candidate's standing on their issue is clear. The never ending political discussions on campaign reform now focus on contribution limits to 527 groups.

## VI. Welfare

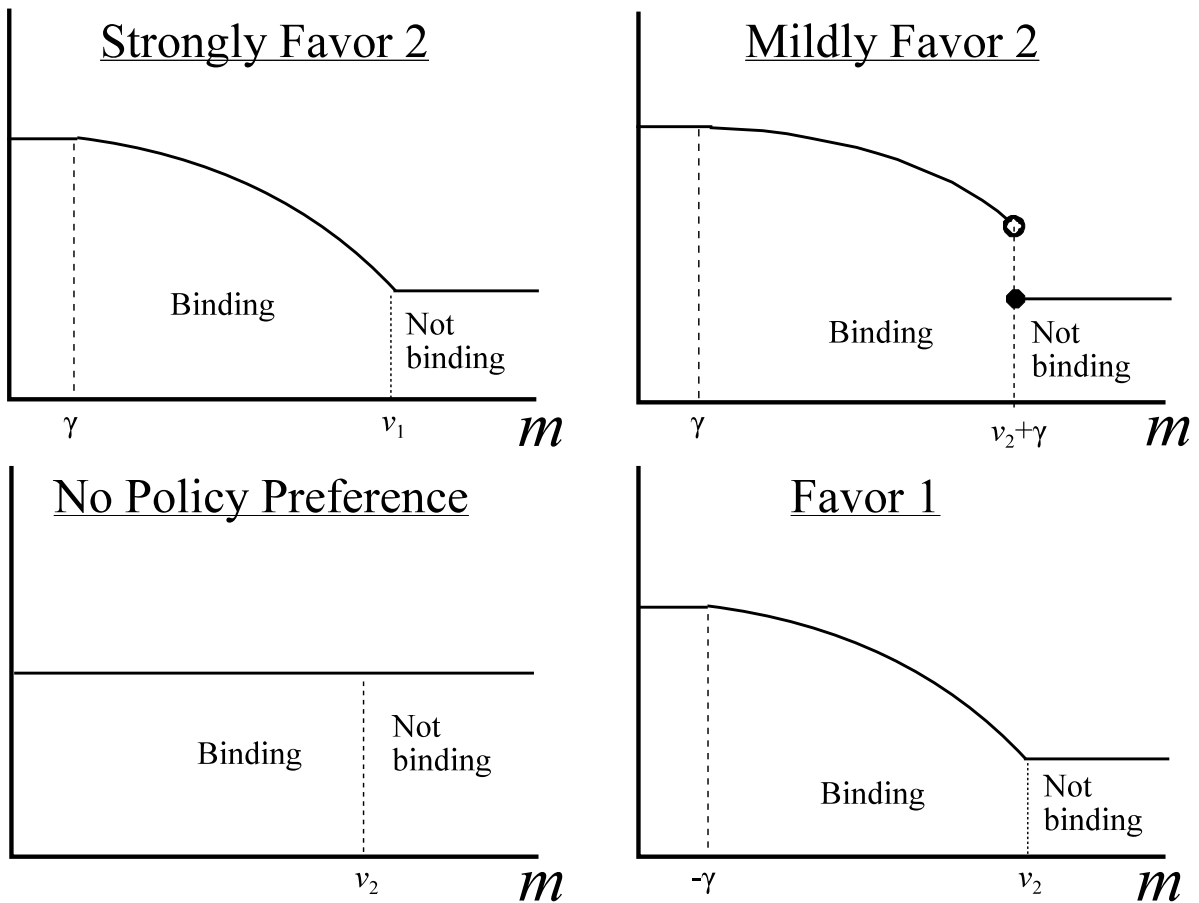
In order to examine the welfare effects of a cap on political contributions one needs to take a stand on how society's needs are reflected in the political process. The model has three parameters that might be associated with the welfare effects of the policy choice,  $\gamma$ ,  $v_1$  and  $v_2$ . Here we will analyze the welfare implications of a contributions cap under two extreme cases. On their own neither of these are particularly compelling, but taken together they give some idea of the welfare tradeoffs involved. They also demonstrate that even a barely binding contribution cap can have significant welfare consequences.

### *i) The democratic ideal: Politician policy preferences perfectly reflect the will of the people*

Representative democracy is predicated on the notion that the elected representatives will either reflect or internalize the will of the people. If this notion is correct, the parameter  $\gamma$  should reflect the will of the majority. In order to examine the implications of this, take the extreme case where it perfectly represents the welfare maximizing policy choice.

This should not be taken literally, however. Even if the electoral system is effective at selecting representatives whose views reflect the public's, it could not lead to a perfect match on every issue. More worryingly, majority rule systems are designed to count the number of people holding a view, but may be less adept at reflecting the intensity of that view. Nevertheless, consider the case where the sign of representative's preference correctly reflects the welfare maximizing policy choice.

Figure 9 plots welfare (the probability that the correct policy is taken) as a function of the level of the cap under this assumption:



**Figure 9:** Welfare under the democratic ideal – politician policy preferences represent the welfare maximizing choice.

When the politician mildly favors the low-valuation bidder, the probability that the low-valuation bidder wins is given by  $v_2/2v_1$  in the absence of a cap. When a binding cap is present the probability of winning is given by  $1 - \frac{m^2 - \gamma^2}{2v_1v_2}$ . When the cap just becomes binding  $m=v_2+\gamma$ , the probability of winning of the low-valuation bidder jumps up since  $\gamma \in (0, v_1 - v_2]$ . The binding cap gives an advantage to the favored low-valuation bidder, since the less-favored bidder is constrained by the cap  $m$ , whereas the favored bidder does not need to bid higher than  $m-\gamma$  to win. And hence he is not effectively constrained. This leads to an increase in the probability of winning. And in the expected payoff turns positive from zero. In all other cases, the favored bidder already has a positive expected

payoff before the introduction of the cap, and with a binding cap the expected value of the game increases continuously with a more restrictive  $m$ . Likewise the probability of winning of the favored bidder increases in a continuous fashion.

**Proposition 9:** *When politician policy preferences perfectly reflect the will of the people, a stricter cap is always welfare increasing.*

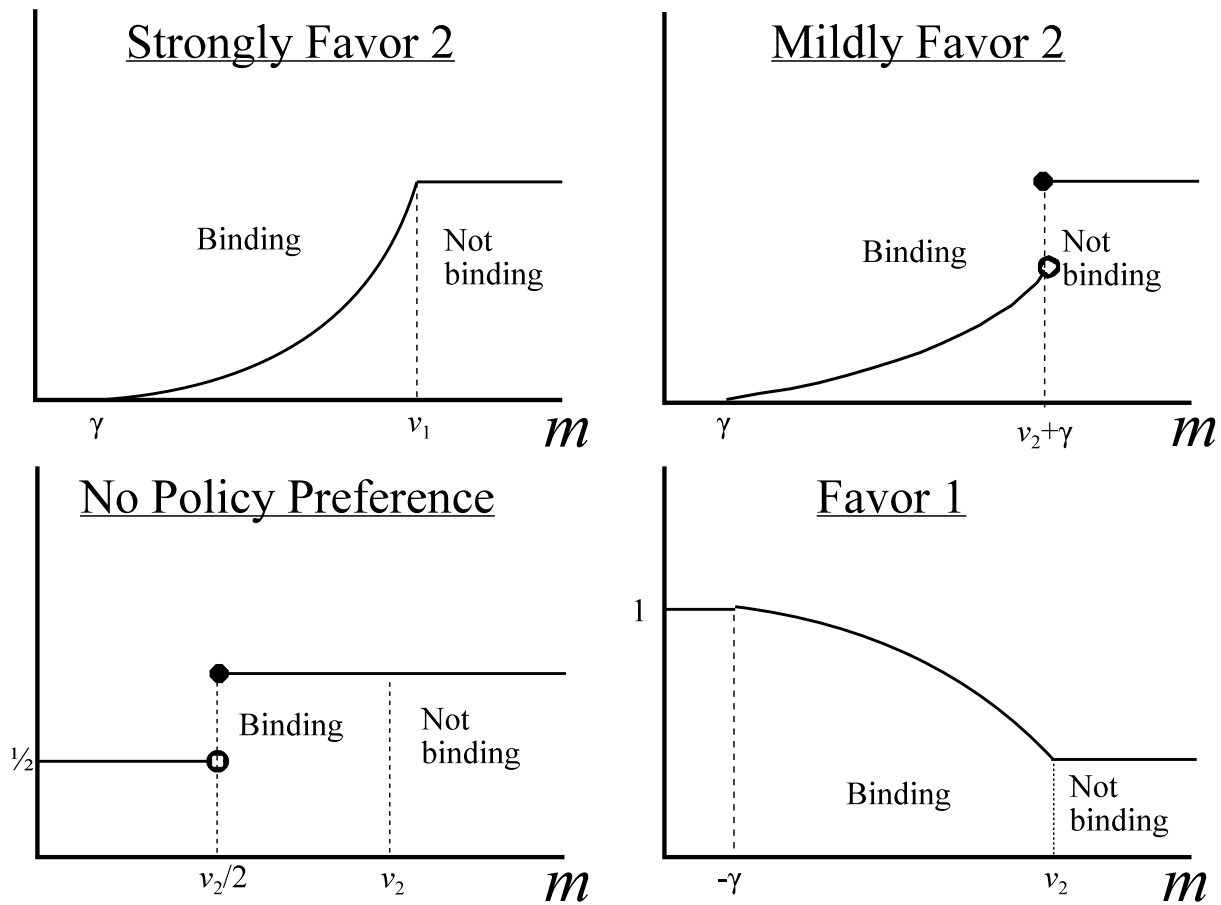
Proof: In text. □

Even a barely binding cap of the type likely to be enacted by self-interested politicians can lead to significant welfare gains. This is because of the CG effect where the existence of the cap causes the otherwise disadvantaged bidder to become more aggressive, which increases the expected spending, and also the probability that his preferred policy is enacted.

*ii) Perfect markets: Bidder valuations completely internalize all social costs and benefits*

Instead taking the democratic electoral system as the perfect aggregator of preferences, one could look to the market system. Suppose that the bidder's valuations  $v_1$  and  $v_2$  perfectly reflect the true value to society of the two policy actions.

There is of course reason to believe that the market system encompasses information about the value of the choices. However, to go from this to the assumption that the bidder valuations perfectly reflect social valuations is problematic. It requires that all information and free rider problems in lobbying have been solved, or at least that any difficulties in solving them are identical for both sides. Nevertheless, consider the case where the lobbyists' valuations perfectly reflect the social valuations of the two policies. In this case we would like to maximize the probability that policy 1 is enacted since  $v_1 > v_2$ . Figure 10 plots welfare under this assumption:



**Figure 10:** Welfare under the perfect markets assumption – bidder valuations internalize all social costs and benefits.

**Proposition 10:** *When bidder valuations completely internalize all social costs and benefits, a cap is welfare improving if and only if the politician favors the high-value policy.*

**Proof:** In text. □

Even a barely binding cap of the type likely to be chosen by politicians will have significant welfare costs if the politician mildly favors the low-value policy, and negligible benefits otherwise.

## **VII. Conclusion**

When the politician has a preference over policy alternatives, however mild, we have shown that the cap always helps the contestant whose policy preference is more aligned with the politician, no matter whether it is the high-valuation or the low-valuation bidder. While the imposition of a cap may lead to an increase in expected aggregate contributions, in contrast to Che and Gale once the cap is binding a more restrictive cap always reduces expected aggregate contributions. A politician who maximizes his own expected payoff which is a function of the aggregate contributions from the lobbyist and the policy outcome, would strictly support relaxing any binding contribution cap. However, he might legislate a barely binding contribution cap. When politician policy preferences perfectly reflect the will of the people, a stricter cap is always welfare increasing. When bidder valuations completely internalize all social costs and benefits, a cap is welfare improving if and only if the politician favors the high-value policy. Even a barely binding contribution cap can have significant welfare consequences.

This paper also contributes to auction theory in two ways. First, it characterizes the equilibrium of a preferential treatment all-pay auction where the degree of the preference of the seller is independent of the equilibrium bids. Previous models in the literature have the preferential treatment rule as a percentile bidding rule. While the percentile bidding rule might be more relevant in some settings, in others such as this political lobbying game an additive rather than a multiplicative measure for the preference might be more relevant. The two different specifications of the preferential treatment yield different predictions. When the preference measure is additive, we show that mildly favoring the low-valuation bidder does not improve his chances of winning, but it simply leads to a higher expected total revenue. Secondly, the paper characterizes the equilibrium of a preferential treatment all-pay auction with a cap. Che and Gale (1998) examine the effect of a cap on an all-pay auction with no preferential treatment. The existence of a preference of the seller however mild, is shown to change the equilibrium predictions. The pure-strategy equilibrium with

a cap no longer survives when the seller has a preference, however mild, over policy outcomes and the expected total revenue always goes down with a more restrictive binding cap.

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## APPENDIX A:

Conditions on the equilibrium when mildly favoring bidder 2,  $\gamma \in (0, v_1 - v_2]$  and no cap.

**Lemma 8:** *Bidder 1 will never bid more than  $v_2 + \gamma$ . Bidder 1 has an infimum bid of  $\gamma$ . Bidder 2 has an infimum bid of zero. The expected value of the game to bidder 2 is zero.*

**Proof:** Since bidder 2 has zero probability of bidding  $v_2$  or more, bidder 1 can win for sure with a bid of  $v_2 + \gamma$ , so he would never bid more than that.

Bidder 1 can win for sure with a bid of  $v_2 + \gamma$  yielding a profit of  $v_1 - v_2 - \gamma > 0$ . With a bid of zero he gets zero profit since he loses for sure so he will never give such a bid.  $b_1^{\text{inf}} \in (0, \gamma)$  is not possible by Lemma 1.  $b_1^{\text{inf}} = v_2 + \gamma$  would be a pure strategy with a pure strategy best reply by bidder 2, but pure-strategy equilibria are not possible by Proposition 1. Suppose that bidder 1 has an infimum bid of  $b_1^{\text{inf}} \in (\gamma, v_2 + \gamma)$ . Then bidder 2 would never choose  $0 < b_2 \leq b_1^{\text{inf}} - \gamma$ . If he did he would be paying a positive amount and would lose for sure, since by Lemma 2, the probability of bidder 1 choosing exactly  $b_1^{\text{inf}}$  is zero in this range. Therefore bidder 1 could lower his infimum bid without changing the probability of winning.

Suppose bidder 2 had an infimum bid of  $b_2^{\text{inf}} \in (0, v_2]$  then bidder 1 would never choose  $b_1 \leq b_2^{\text{inf}} + \gamma$ . If he did bidder 1 would lose for sure yielding a negative payoff, since by Lemma 2 the probability of bidder 2 choosing exactly  $v_2$  is zero and he can always guarantee a zero payoff of  $v_1 - v_2 - \gamma > 0$  with a bid of  $v_2 + \gamma$ . But then bidder 2 would prefer a bid of zero to  $b_2^{\text{inf}}$ . Therefore a bid of zero is in the support of the mixed strategy of bidder 2. At this bid he loses for sure, so the expected value of the strategy for bidder 2 must be zero.

□

**Lemma 9:** *Bidder 1 has a supremum bid of  $v_2 + \gamma$ . Bidder 2 has a supremum bid of  $v_2$ . The expected value of the game to bidder 1 is  $v_1 - v_2 - \gamma > 0$ .*

**Proof:** Suppose that bidder 1 had a supremum bid of  $b_1^{\text{sup}} < v_2 + \gamma$ . Then bidder 2 would never set  $b_2 > \max[0, b_1^{\text{sup}} - \gamma]$  as he can win for sure with that bid since either  $b_1^{\text{sup}} < \gamma$  or by Lemma 2 the

probability of bidder 1 choosing exactly  $b_1^{sup}$  is zero. Therefore bidder 2 could win for sure with  $b_2 = b_1^{sup} - \gamma + \varepsilon$  yielding a payoff greater than zero for small enough  $\varepsilon$ , a contradiction of Lemma 8.

Suppose that bidder 2 had a supremum bid of  $b_2^{sup} < v_2$ . Then bidder 1 would never set  $b_1 > b_2^{sup} + \gamma$  since he could win for sure with  $b_1 = b_2^{sup} + \gamma$ . Therefore bidder 2 could win for sure with  $b_2 = b_2^{sup} + \varepsilon$  yielding a payoff greater than zero for small enough  $\varepsilon$ , a contradiction of Lemma 8.

By Lemma 2, bidder 1 will win for sure with a bid of  $v_2 + \gamma$ , and it is in the support of his mixed strategy, so the expected value of the game to player 1 is  $v_1 - b_1^{sup} = v_1 - v_2 - \gamma > 0$ .

□

## Appendix B:

**Lemma 10:** *Without a contribution cap, for bidder 1, bids almost everywhere on  $b_1 \in [b_1', b_1'']$  and for bidder 2, bids almost everywhere on  $b_2 \in [b_2', b_2'']$ , must have positive probability, where*

$$\begin{aligned} \forall \gamma \in (v_1 - v_2, v_1) \quad & b_1' = \gamma, b_1'' = v_1 \quad \text{and} \quad b_2' = 0, b_2'' = v_1 - \gamma \\ \forall \gamma \in (0, v_1 - v_2] \quad & b_1' = \gamma, b_1'' = v_2 + \gamma \quad \text{and} \quad b_2' = 0, b_2'' = v_2 \\ \forall \gamma \in (-v_2, 0) \quad & b_1' = 0, b_1'' = v_2 + \gamma \quad \text{and} \quad b_2' = -\gamma, b_2'' = v_2 \end{aligned}$$

**Proof:** Suppose there were an interval  $(t, s)$  in  $[b_1', b_1'']$  where bidder 1 had zero probability of bidding. Then bidder 2 would have zero probability of bidding in  $(t - \gamma, s - \gamma)$  since he could lower his bid to  $t - \gamma$  and have the same chance of winning. But in this case bidder 1 would never bid  $s + \varepsilon$  as he could lower his bid to  $t$ , saving  $s + \varepsilon - t$  in bidding costs and losing only  $F_2(s + \varepsilon - \gamma) - F_2(t - \gamma)$  in probability. By Lemma 2 the loss in probability is negligible for small  $\varepsilon$ . So if there is an interval of zero probability it must go up to the top of the range, which depending on the level of  $\gamma$ ,

contradicts Lemma 4, 9 or 4. A symmetric argument rules out ranges of zero probability for bidder 2 in  $b_2 \in [b_2', b_2'']$ .

□

**Lemma 11:** *With a contribution cap, for bidder 1, bids almost everywhere on  $b_1 \in [b_1', b_1'']$  and for bidder 2, bids almost everywhere on  $b_2 \in [b_2', b_2'']$ , must have positive probability, where*

$$\begin{aligned} \forall \gamma \in (0, m) \quad & b_1' = \gamma, b_1'' = m \quad \text{and} \quad b_2' = 0, b_2'' = m - \gamma \\ \forall \gamma \in (-m, 0) \quad & b_1' = 0, b_1'' = m + \gamma \quad \text{and} \quad b_2' = -\gamma, b_2'' = m \end{aligned}$$

**Proof:** Suppose there were an interval  $(t, s)$  in  $[b_1', b_1'']$  where bidder 1 had zero probability of bidding. Then bidder 2 would have zero probability of bidding in  $(t - \gamma, s - \gamma)$  since he could lower his bid to  $t - \gamma$  and have the same chance of winning. But in this case bidder 1 would never bid  $s + \varepsilon$  as he could lower his bid to  $t$ , saving  $s + \varepsilon - t$  in bidding costs and losing only  $F_2(s + \varepsilon - \gamma) - F_2(t - \gamma)$  in probability. By Lemma 5 the loss in probability is negligible for small  $\varepsilon$ . So if there is an interval of zero probability it must go up to the top of the range, which, depending on the level of  $\gamma$ , contradicts Lemma 7. A symmetric argument rules out ranges of zero probability for bidder 2 in  $b_2 \in [b_2', b_2'']$ .

□