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Monopsony Power with Variable Effort

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Monopsony Power with Variable Effort

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Abstract

A monopsony model of the labour market is developed where wages and the effort level are chosen by the firm. Higher wages raise labour supply while higher effort reduces it. Wages will be below the socially optimal level while effort will be too high. Under a sufficient condition which is satisfied in many reasonable cases a minimum wage policy (with the effort level unrestricted) will lower worker utility and welfare. Under a sufficient condition a maximum effort level (with wages unrestricted) will raise employees utility but lower welfare. To be confident that regulatory policies improve welfare the government must be confident that it can choose and enforce the regulated levels of wages and effort correctly. By contrast an employment subsidy which depends only on the slope of the firm's labour supply curve can achieve the social optimum. The model can be thought of as a generic monopsony model where wage is input price, effort input quality and workers utility the input suppliers profit.

A simplified version of Bhaskar and To's (1999) model is used to illustrate. The cost of the employment subsidy which achieves the social optimum (and is equal to the transport costs of the marginal worker) is equal to monopsony profits.

I Introduction

Models where firms have upward sloping labour supply curves have become popular in recent years. These models have been rationalised in developed labour markets (where labour and capital are increasingly mobile) by appealing to labour market frictions. See for example Burdett and Mortensen (1998) for a search model where the firms labour supply curve slopes upwards, Bhaskar and To (1999) for a model of monopsonistic competition and Manning (1995) or Rebitzer and Taylor for efficiency wage models where higher wages are needed to increase employment. An alternative interpretation is one where in certain markets a small number of input buyers behave strategically. See Boal and Ransom (1997) for a survey on Monopsony in the labour market. An important policy implication of these models is that they provide a theoretical basis for positive employment effects from minimum wage increases.

In this paper I incorporate the firms choice of effort in a generic monopsony model and then in a simplified version of the model in Bhaskar and To (1999). In the model the firm chooses a wage effort combination which they offer to workers. This determines the number of workers they attract. It seems reasonable to assume that workers choice of where to work will depend on non-wage characteristics of the job which are part of the contract and which affect the productivity of the worker. These non-wage characteristics of the job may be how hard the worker is expected to work, how much the firm is expected to spend providing a safe or pleasant working environment, flexibility over when

the worker works or what tasks they perform etc. Incorporating this generalisation has important consequences for the policy implications of monopsony models.

II A General Monopsony Model

Price taking firms have the following profit function which is used to choose the optimal wage

$$\Pi = PF(L[w, x]x) - (ws + \mathbf{q})L[w, x] \quad (\text{II.1})$$

The product of effort (x) times employment (L) are efficiency units of labour (N). We assume that the labour supply function is separable in effort and wages. There is a percentage tax (subsidy) s on wages and a per unit tax (subsidy) of \mathbf{q} per worker. The first order condition for x is:

$$PF_N(w, x)[L(w, x) + L_x(w, x)x] = (ws + \mathbf{q})L_x(w, x) \quad (\text{II.2})$$

The first order condition for w implies

$$PF_N(w, x)L_w(w, x)x = sL + (ws + \mathbf{q})L_w(w, x) \quad (\text{II.3})$$

The first order conditions imply that :

$$PF_N(w, x) = \frac{-sL_x}{L_w} \quad (\text{II.4})$$

It is straightforward to show that if we change the above problem by making the subsidy

proportional to employment, that is if we set $s=1$ and $\mathbf{q} = -\frac{L^*}{L_w^*}$ (where $*$ denotes the

value at the socially optimal outcome) then both first order conditions will satisfy

$PF_N x = w$. The wage equals the value of the value of marginal product of the last worker. This is the social optimum and we will be illustrated this in more detail in an example later on. The result is stronger if the subsidy is proportional to employment, so that the profit function can be rewritten as:

$$\Pi = PF(L[w, x]) - (ws + \mathbf{q}L[w, x])L[w, x] \quad (\text{II.5})$$

In this case a per unit subsidy that is increasing in employment where $\mathbf{q} = -\frac{1}{L_w}$ will bring about the efficient outcome. In this case the government only needs to know the slope of the firm's labour supply curve to set the efficient subsidy. In the monopsonistic competition model analysed in the following section I show that the cost of the subsidy is equivalent to monopsony profits.

Note in the case where $s=1$ and $\mathbf{q}=0$ If $L_x \frac{x}{L} = \mathbf{e}$ and $L_w \frac{w}{L} = E$ the above first

order conditions imply:
$$\frac{PF_N x}{ws + \mathbf{q}} = \left(\frac{\mathbf{e}}{1 + \mathbf{e}} \right) = 1 + \frac{ws}{(ws + \mathbf{q})E} \quad (\text{II.6})$$

(II.6) implies that in equilibrium with no taxes or subsidies:

$$1 + E = -\mathbf{e} \quad (\text{II.7})$$

If $\mathbf{q} < 0$ then (II.7) implies:

$$1 + E < -\mathbf{e} \quad (\text{II.8})$$

If the subsidy is set at $\mathbf{q} = -\frac{L}{L_w}$ (II.8) implies that $-E = \mathbf{e}$ at the social optimum.

The second order conditions are derived in Appendix 1. These are all satisfied for the no subsidy case and for the subsidy case as long as $\mathbf{q} < 0$.

The cost of the optimal subsidy

If the optimal subsidy described above is given for each worker the cost per firm will be $\frac{wL}{E}$. Note that since this subsidy brings us to the social optimum where the wage equals the value of marginal product, this can be rewritten as:

$$\frac{PF_N xL}{E} \quad (\text{II.9})$$

From the profit function (II.1) we see that for monopsony profits to exceed the cost of the subsidy the following condition must hold:

$$\Pi = PF(N) - wL = PF(N) - PF_N xL > \mathbf{q}L = \frac{PF_N xL}{E} \quad (\text{II.10})$$

This condition can be rewritten as:

$$\frac{APL}{MPL} > 1 + \frac{1}{E} \quad (\text{II.11})$$

Where APL is the average product of labour at the social optimum and MPL is the marginal product of labour. The left hand side is the inverse of the elasticity of output with respect to employment and E is the firms labour supply elasticity.

The effect of a minimum wage on effort and employment

The effect of a minimum wage on effort would be:

$$\frac{dx}{dw} = -\frac{\mathbf{p}_{xw}}{\mathbf{p}_{xx}} > 0 \quad (\text{II.12})$$

It is clear from Appendix 1 that (II.12) is positive when the first order conditions hold.

Since we are assuming both first order conditions hold, the exercise is to assume that we

start at an equilibrium where both wages and effort are chosen freely and then impose a minimum wage slightly above the equilibrium level (see Manning (1995) for example).

The impact on employment $L(w,x[w])$ of a minimum wage (so that the optimal choice of effort now depends on the exogenously determined wage rate) would be

$$\frac{dL}{dw} = L_w + L_x \frac{dx}{dw} \quad (\text{II.13})$$

In Appendix 1 (b) I show that if the following condition is met then the employment effect of a minimum wage will be negative:

$$-PF_{NN}wL_w + \frac{L_{xx}}{L_w} > 0 \quad (\text{II.14})$$

For example if labour supply is linear in effort the employment effects are negative. We could think of a model where effort is fixed as a special case where L_{xx} is very large giving a vertical effort supply curve and implying positive employment effects.

The effect of a maximum effort level on wages and employment

In the same way given that the first order conditions hold, if a maximum effort level below the equilibrium level were imposed the optimal wage would fall in response and employment could increase:

$$-\frac{dw}{dx} = \frac{p_{xw}}{p_{ww}} < 0 \quad (\text{II.15})$$

The impact on employment would be

$$-\frac{dL}{dx} = -L_x - L_w \frac{dw}{dx} \quad (\text{II.16})$$

I show in Appendix 1 (c) that if the following condition is met then a maximum effort

requirement reduces employment:

$$-PF_{NN} \frac{L_w^3}{L_x} - L_{ww} > 0 \quad (\text{II.17})$$

The first term on the right hand side is negative, so for example if labour supply is linear ($L_{xx}=0$) then a restriction on maximum effort will increase employment.

Welfare analysis of a minimum wage or maximum effort level

Each potential worker has a utility function:

$$util_i = u(x, w, d_i) \quad (\text{II.18})$$

Unemployed workers get some reservation level of utility \bar{u} . Each worker i has an individual characteristic d_i . The differing values for are the basis for the upward sloping labour supply curve. In a traditional model of monopsony or oligopsony where firms have power in the local labour market we could think of d_i as representing different values for \bar{u} and thus different reservation wages amongst potential workers. In models where labour market frictions are the source of monopsony power d_i might represent distance to work or preference for a particular employer as in the model outlined in section III.

Alternatively it might represent the fact that workers have different information or search costs. The key point is that a firm that wishes to attract an additional worker must offer a wage effort combination which raises the utility of it's existing workers, while a firm which lowers employment can lower the utility of its remaining workers. If \mathbf{p} is profit per firm, we define the welfare function as:

$$Wf = Wf(\sum_{i=1}^k Util_i, \sum_{i=1}^n \mathbf{p}_i) \quad (\text{II.19})$$

Where welfare is increasing in the utility of any of the k potential workers, or in the profits of any of the n firms. We can see that if a binding minimum wage or maximum effort requirement leads to a fall in employment then welfare must fall. Each firm's profits must be lower since the regulated outcome could have been chosen in the absence of regulation but was not. Each worker who moves to unemployment has lower utility and since the firm is still on the labour supply curve after the regulation, but at a lower level of employment then each employed worker is worse off.

III An Example

The example I use is a simplified¹ version of the model of Bhaskar and To (1999). This is a model of horizontal job differentiation. “jobs are not inherently good or bad, but different workers have different preferences over non-wage characteristics”. A fixed number n firms are uniformly spaced around a circle of unit circumference. Workers travelling distance d to work face costs of td and $1/n$ is the distance between adjacent firms. There is a unit mass of homogeneous workers uniformly distributed around the circle. Each worker has a utility function:

$$U_t = w^a - x^b - td \quad (\text{III.1})$$

A worker accepts a job offering more than the reservation level of utility \bar{u} . The parameters satisfy the conditions $0 < a \leq 1$ and $1 < b$. The wage is w , x is effort and d is distance to work. The fact that utility is linear in transport costs but not in wages (unless

¹ The simplifications are that Bhaskar and To have the firm choose both capital and labour allowing them to analyse firm exit in response to minimum wage legislation while I assume labour is the only input, secondly I assume that workers are homogeneous.

$a=1$) is not intuitively appealing, but makes it easy to derive a labour supply function when $a < 1$. This case may be potentially interesting one although for most of the analysis we assume $a=1$ so this issue will not arise. Given this framework any worker within the distance $d^* = \frac{(w_i^a - x_i^b) - \bar{u}}{t}$ will get higher utility from accepting firm i's offer (where firm i is the closest firm to the worker) as long as $|(w_i^a - x_i^b) - (w_j^a - x_j^b)| < t/n$ for any other firm j (this will be true in equilibrium). This implies the labour supply curve for firm i of:

$$L_i = 2 \left[\frac{(w_i^a - x_i^b) - \bar{u}}{t} \right] \quad (\text{III.2})$$

$$(\text{III.2}) \text{ implies: } E = \frac{2aw_i^a}{tL_i} \text{ and } e = \frac{-2bx_i^b}{tL_i} \quad (\text{III.3})$$

Using (III.3) in equation (I.8) and setting $a=1$ we get² :

$$x^b = \frac{2w}{b+1} - \frac{\bar{u}}{b+1} \quad (\text{III.4})$$

Using the above term in the labour supply function gives equilibrium labour supply as:

$$L = \frac{(b-1)x^b - \bar{u}}{t} \quad (\text{III.5})$$

To solve the model explicitly we need to assume a production function and use (III.4) in the first order conditions.

² Appendix A3 derives the first order conditions for this example explicitly and solves for x, w and L .

An example;

In this section I assume a particular production function:

$$F(xL) = x^f L^f \quad (\text{III.6})$$

I show in Appendix A3 that given this technology the equilibrium expression for effort satisfies the following equation:

$$\frac{(b-1)}{\bar{u}} x^b - t \left(\frac{b}{pf} \right)^{\frac{1}{f-1}} x^{\frac{b-f}{f-1}} - 1 = ax^b - bx^{\frac{b-f}{f-1}} - 1 = 0 \quad (\text{III.7})$$

In the example given below in Table 1 we assume $f=0.5$, $\bar{u} = 1$ and $b=2$. In this case (III.7) can be written as:

$$ax^5 - b - x^3 = 0 \quad (\text{III.8})$$

The value for x that solves (III.8) can be used in (III.4) and (III.5) to solve for equilibrium wages and labour supply. It is easy to verify that the only real solution to this equation that gives positive labour supply is as in Table 1 for the given parameter values.

Using equations (II.6) to (II.9) I solve for the effects of minimum wages or maximum effort restrictions on effort, wages and employment. We see that the minimum wage leads to an increase in wages and effort and a small decrease in employment. A Maximum effort restriction leads to a fall in the wage and an increase in employment.

Table 1 Parameterised example of Monopsonistic competition model

	Value	Outcome variable	Value
Price	2	x	1.76
b	2	w	5.12
a	1	L	0.05
f	0.5	Π	0.33
n	15	% unemployed	0.30
t	45	$\frac{dx}{dw}$	0.30
s	1	$\frac{dL}{dw}$	-0.02
q	0	$-\frac{dw}{dx}$	-3.15
Firm output	0.57	$-\frac{dL}{dx}$	0.02

Welfare analysis of restrictions on wages or effort

The utility of an employed worker can be calculated from the utility function (III.1), while unemployed workers get the reservation level of utility implied by the parameter assumptions. If u is the unemployment rate and $Util$ is utility of an employed worker while E is total employment (remembering that the size of the labour force is unity), n is the number of firms and \mathbf{p} is profit per firm, we define the welfare function as:

$$Wf = \bar{u} + EUtil + n\mathbf{p} \quad (\text{III.4})$$

In Table 2 below I calculate the initial level of utility, profits and welfare. I also calculate the impact of the minimum wage on utility, profits and welfare using the following equations. The effect of a minimum wage on the distance the marginal worker travels to work is:

$$\frac{dd^*}{dw} = \frac{aw^{a-1} - bx^{b-1} \frac{dx}{dw}}{t} \quad (\text{III.5})$$

The impact of a maximum effort level on the distance of the marginal worker is:

$$\frac{dd^*}{dx} = \frac{-\frac{dw}{dx}aw^{a-1} + bx^{b-1}}{t} \quad (\text{III.6})$$

The impact on the utility of an average worker from a minimum wage is:

$$\frac{dUtil}{dw} = aw^{(a-1)} - bx^{b-1} \frac{dx}{dw} - \frac{t}{2} \frac{dd^*}{dw} = \frac{t}{2} \frac{dd^*}{dw} \quad (\text{III.7})$$

Note the workers are uniformly distributed so the change in distance travelled of the average worker is half the change in distance travelled of the marginal worker.

The impact on utility of a maximum effort requirement is:

$$\frac{dUtil}{dx} = -aw^{(a-1)} \frac{dw}{dx} + bx^{b-1} - \frac{t}{2} \frac{dd^*}{dx} = \frac{t}{2} \frac{dd^*}{dx} \quad (\text{III.8})$$

Before we can compute the impact of the policies on welfare we need to calculate the effect on profits. The change in profits from a minimum wage is:

$$\frac{d\mathbf{p}}{dw} = fx^{f-1}l^f \frac{dx}{dw} + fx^f l^{f-1} \frac{dl}{dw} - l - w \frac{dl}{dw} \quad (\text{III.9})$$

The effect on profits of a maximum effort requirement is:

$$-\frac{d\mathbf{p}}{dx} = -fx^{f-1}l^f - fx^f l^{f-1} \frac{dl}{dx} + w \frac{dl}{dx} + \frac{dw}{dx} l \quad (\text{III.10})$$

Next we use the above equations to derive the effect on welfare of a minimum wage:

$$\frac{dWf}{dw} = \frac{du}{dw} \bar{u} + \frac{dUtil}{dw} E + n \frac{dl}{dw} Util + n \frac{d\mathbf{p}}{dw} \quad (\text{III.11})$$

The effect on welfare of the maximum effort requirement is:

$$\frac{dWf}{dx} = -\frac{du}{dx} \bar{u} - \frac{dUtil}{dx} E - n \frac{dl}{dx} Util - n \frac{d\mathbf{p}}{dx} \quad (\text{III.12})$$

Table two gives the values of the above terms for the parameter values assumed in Table 1.

Table 2 Welfare analysis given parameter values assumed in Table 1

Util	1.52	Wf	6.35
\bar{u}	1.00	d^*	0.02
$\frac{dUt}{dw}$	-0.03	$-\frac{dUt}{dx}$	1.76
$\frac{d\mathbf{p}t}{dw}$	-0.02	$-\frac{d\mathbf{p}t}{dx}$	-0.73

$$\frac{dW_t}{dw} \quad -0.29 \quad -\frac{dW_t}{dx} \quad -10.3$$

Table 3 The Welfare Maximising outcome

x^*	1.56	L^*	0.06
w_f^*	6.49	Firm output	0.64

We see from Tables 1 and 2 that a minimum wage leads to increased effort and slightly lower employment. We see in our example that the minimum wage also lowers worker utility because of the deterioration in working conditions. Profits fall as we would expect and welfare falls. The minimum standard for effort (or working conditions) leads to lower wages being offered as we would expect. Employment increases as does worker utility but welfare falls as these gains are outweighed by the negative impact on profit. Table 3 shows that the optimal outcome has higher wages and lower effort than the market outcome. When both wages and effort are endogenous regulating one variable while the other can be freely chosen reduces welfare.

IV. Using Taxes and Subsidies to Achieve the First Best Outcome

in Appendix 2 I derive the first order conditions for the optimal choice of effort and employment. The first order conditions imply the following condition will be

satisfied³:

$$x^* = \left[\frac{\bar{u} + tL^*/2}{b-1} \right]^{\frac{1}{b}} \quad (\text{IV.1})$$

If we write this in terms of L we get:

$$L^* = \frac{2[(b-1)x^{*b} - \bar{u}]}{t} \quad (\text{IV.2})$$

When we use the production function assumed in section III. The first order conditions imply:

$$2t^{-1} \left(\frac{Pf}{b} \right)^{\frac{1}{f-1}} [(b-1)x^{*b} - \bar{u}] = x^{\frac{b-f}{f-1}} \quad (\text{IV.3}).$$

Appendix 3 on the other hand solves for the firms choice of effort and wages in the market for given levels of s (the percentage tax/subsidy on wages) and \mathbf{q} (the per worker tax/subsidy). We get the following relationship between employment and effort:

$$L = \frac{(b-1)x^b - \bar{u} - \mathbf{q}/s}{t} \quad (\text{IV.4})$$

Once again using the same production function and the firms first order conditions we can solve for effort:

$$t^{-1} \left(\frac{Pf}{bs} \right)^{\frac{1}{f-1}} [(b-1)x^b - (\bar{u} + \mathbf{q}/s)] = x^{\frac{b-f}{f-1}} \quad (\text{IV.5})$$

Notice the similarity between (IV.5) and (IV.3). In particular notice that if the government sets $s=1$ and $\mathbf{q} = \frac{-tL}{2}$ (the equivalent of $\mathbf{q} = -L/L_w$ used in section II.) then by substituting this value for \mathbf{q} into (IV.5) using (IV.4) for L we see that (IV.5) and (IV.3) are equivalent. The competitive firm will choose the socially optimal level of effort. Next taking the above value for \mathbf{q} and using it in (IV.4) we see that (IV.4) is now

³ A * indicates the first best outcome.

identical to (IV.2). This implies (since the firm will use the optimal level of effort in equation (IV.4)) that the firm will also choose the optimal level of employment. The above shows that a subsidy equal to the transport costs of the marginal worker employed at any firm will achieve the first best outcome. The cost of financing such a subsidy to any firm in equilibrium would be:

$$\text{cost} = qL = t \frac{L^2}{2} \quad (\text{IV.6})$$

This equals firms profits in equilibrium, so that a lump sum tax on profits used to subsidise employment can achieve the first best outcome without a deficit.

IV. Monopsony in the Product Market

The model developed above for the labour market can be thought of as a generic monopsony model where firms have market power over input suppliers. The wage is the price of the input, effort is the quality of the input and the utility function of firms is the profit function of input suppliers. Input suppliers face different transport costs to different firms. These costs may be actual transport costs or act as a proxy for any logistical advantage that makes it easier for an input supplier to supply to a particular firm. The results indicate that while output and the input price will be too low in a market equilibrium while quality will be too high regulating price will lower welfare in the absence of regulations on quality, as will regulations on quality in the absence of regulations on price. In practical terms a measure imposing a maximum level of quality might be difficult

to enforce in many cases, although it might be a case of the government preventing the monopsonist from forcing suppliers to meet particular product specifications. A per unit subsidy on the input achieves the socially desirable outcome.

V. Conclusion

Generalising standard monopsony models by incorporating effort into the labour supply function has important policy implications. Since the firm now chooses effort and wages fixing one of these variables while the other can be chosen freely leads to different results from a model where the wage alone is chosen. A minimum wage leads to a deterioration in working conditions and can lower employment, workers utility and welfare. Restrictions on effort lead to a lower wage and increase employment and utility, but lower welfare because of the impact on profit.

A policy that set both wages and effort could improve welfare in this framework but restricting one while the other is chosen freely lowers welfare while a per unit subsidy on employment financed by a profits tax leads to the socially optimal outcome.

The ability to regulate both wages and effort such that welfare increases assumes of course that the regulator knows the optimal solution and can enforce all the regulated variables equally well. The optimal subsidy on the other hand depends only on the slope of the firms labour supply curve and in the model of monopsonistic competition analysed analysed in section III. The cost of the subsidy could be recouped from a tax on monopsony profits.

Boal and Ransom (1997) conclude that Monopsony power is not quantitatively important in terms of its affect on wages. The model presented here opens up the

possibility that monopsony power could be more important quantitatively than had been thought. A firm with monopsony power might pay only slightly lower wages but workers utility could be much lower if working conditions are not as good as in other firms.

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Appendix 1.

(a) *Second order conditions*

The second order conditions for the firms problem are:

$$\mathbf{p}_{xx} = PF_{NN} [L + L_x x]^2 + (PF_N x - w^s) L_{xx} + 2PF_N L_x \quad (\text{A.1.1})$$

Note $w^s = ws + \mathbf{q}$

$$\mathbf{p}_{ww} = PF_{NN} [L_w x]^2 + (PF_N x - w^s) L_{ww} - (1 + s) L_w \quad (\text{A.1.2})$$

We see that for a firm satisfying the first order conditions and if $L_{ww} \leq 0$ and $L_{xx} \leq 0$, then the above derivatives will be negative.

The cross partial derivative is:

$$\mathbf{p}_{xw} = PF_{NN} [L_w x][L + L_x x] + (PF_N - w^s) L_{xw} + PF_N L_w - sL_x \quad (\text{A.1.3})$$

If we assume separability between effort and wages in the labour supply function this equation simplifies to.

$$\mathbf{p}_{xw} = PF_{NN} [L_w x][L + L_x x] + PF_N L_w - sL_x \quad (\text{A.1.4})$$

We also note at this stage that if the first order conditions hold then $[L + L_x x] < 0$.

Finally the determinant of the Hessian matrix $(\mathbf{p}_{ww}\mathbf{p}_{xx} - \mathbf{p}_{xw}^2)$ is

$$\begin{aligned}
|H| = & P^2 F_{NN}^2 [L + L_x x]^2 (L_w x)^2 + [(PF_N x - w^s) L_{ww} - (1+s)L_w] PF_{NN} [L + L_x x]^2 + \\
& (PF_N x - w^s) L_{xx} [PF_{NN} (L_w x)^2 + (PF_N x - w^s) L_{ww} - (1+s)L_w] \\
& + 2PF_N L_x [PF_{NN} (L_w x)^2 + (PF_N x - w^s) L_{ww} - (1+s)L_w] \\
& - P^2 F_{NN}^2 [L + L_x x]^2 (L_w x)^2 - 2PF_{NN} L_w x [L + L_x x] [PF_N L_w - sL_x] \\
& + (1+s)PF_N L_w L_x - F_N^2 L_w^2 + s^2 L_x^2
\end{aligned}$$

(A.1.5)

After we cancel out terms this becomes.

$$\begin{aligned}
|H| = & [(PF_N x - w^s) L_{ww} - (1+s)L_w] PF_{NN} [L + L_x x]^2 + \\
& (PF_N x - w^s) L_{xx} [PF_{NN} (L_w x)^2 + (PF_N x - w^s) L_{ww} - (1+s)L_w] \\
& + PF_N L_x [2PF_{NN} (L_w x)^2 + 2(PF_N x - w^s) L_{ww} - (1+s)L_w] \quad (A.1.6) \\
& - 2PF_{NN} L_w x [L + L_x x] [PF_N L_w - sL_x] - P^2 F_N^2 L_w^2 + s^2 L_x^2
\end{aligned}$$

We can verify easily that as long as the firm is satisfying the first order conditions and the following conditions hold: $L_x < 0$, $L_{xx} < 0$, $L_w > 0$, $L_{ww} < 0$ and $F_{NN} < 0$ then all terms on the first three lines of (A.1.6) are unambiguously positive. Using equation (II.4) we see the second term on the last line can be rewritten: $(PF_N)^2 L_w^2 = -s^2 L_x^2$ so that the last two terms on the last line cancel. The only ambiguity in (A.1.6) comes from the first term on the last line which is negative. We will show that this term is dominated by positive terms in the Hessian for the case when $s=1$ and $q < 0$. I show in the paper that these values for the tax variables will give the first best outcome. As long as the subsidy is not too big (A.1.6) will be positive.

Using the result from the first order condition that $PF_n = -\frac{L_x}{L_w}$ the first term on the last line of (A.1.6) can be rewritten:

$$- 2PF_{NN} L_w x [L + L_x x] [PF_N L_w - sL_x] = 4PF_{NN} L_w x L_x (L + L_x) \quad (A.1.7)$$

Using the fact that $PF_n = -\frac{L_x}{L_w}$ again the first term on the second last line can be rewritten as:

$$2PF_N L_x [PF_{NN} (L_w x)^2] = -2PF_{NN} L_w x L_x L_x x \quad (A.1.8)$$

The last term on the first line can be rewritten as:

$$-2L_w PF_{NN} [L + L_x x]^2 = -2L_w PF_{NN} L_w x L_x \frac{(L + L_x x)^2}{x L_x} \quad (\text{A.1.9})$$

If we add the right hand side of (A.1.7) to (A.1.9) and the sum is positive we know the determinant is positive and we are at a maximum:

$$4(L + L_x x) - 2L_x x - 2 \frac{(L + L_x x)^2}{x L_x} > 0 \quad (\text{A.1.10})$$

Dividing across by $(L + L_x x)$ (which we can see from the first order conditions is negative) (A.1.10) can be rewritten as:

$$4 - 2 \frac{1}{1 + \frac{1}{e}} - 2 \left(1 + \frac{1}{e}\right) < 0 \quad (\text{A.1.11})$$

If $-e > 1$ the condition in (A.1.11) is unambiguously satisfied. From the first order conditions in section II. we can show that

$$\frac{PF_{N_x}}{ws + q} = \left(\frac{e}{1 + e}\right) = 1 + \frac{ws}{(ws + q)E} \quad (\text{A.1.12})$$

(A.1.12) implies that in equilibrium with no taxes or subsidies:

$$1 + E = -e$$

If $q < 0$ then (A.1.12) implies:

$$1 + E < -e$$

In either case inequality (A.1.11) is satisfied and we are at a maximum.

(b) Employment effect of a minimum wage

Using equations (II.7) and (II.8) we see that the employment effect of a minimum wage will be negative if the following condition is met:

$$-\frac{L_x}{L_w} \frac{dx}{dw} = \frac{L_x}{L_w} \frac{\mathbf{p}_{xw}}{\mathbf{p}_{xx}} > 1 \quad (\text{A.1.12})$$

Using the fact that $PF_n = -\frac{L_x}{L_w}$ from the first order conditions and equations (A.1.1) and

(A.1.3) inequality (A.1.11) can be rewritten (we will set $s=1$ here):

$$\frac{PF_{NN} L_x X [L + L_x x] - \frac{2L_x^2}{L_w}}{PF_{NN} [L + L_x X] [L + L_x x] + (PF_N x - w^s) L_{xx} - \frac{2L_x^2}{L_w}} > 1 \quad (\text{A.1.12})$$

We see that all terms in the numerator and denominator are negative. The numerator is a bigger negative number than the denominator if the following term is positive:

$$PF_{NN} L [L + L_x x] + (PF_N x - w^s) L_{xx} > 0 \quad (\text{A.1.13})$$

If (A.1.13) is positive inequality (A.1.12) holds. Using the first order conditions in (A.1.13)

$$L [L + L_x x] = -w^s L_w L \quad \text{and} \quad (PF_N x - w^s) = \frac{L}{L_w}.$$

If inequality (A.1.12) holds a minimum wage slightly above the market level will reduce employment, that is if:

$$-PF_{NN} w L_w + \frac{L_{xx}}{L_w} > 0 \quad (\text{A.1.14})$$

(c) *Employment effects of a maximum effort requirement*

Using equation (II.13) we see that a maximum effort requirement will reduce employment if the following condition holds:

$$\frac{L_w p_{xw}}{L_x p_{ww}} > 1 \quad (\text{A.1.15})$$

Using (A.1.2), (A.1.4) and the fact that $PF_n = -\frac{L_x}{L_w}$ (A.1.15) can be written as:

$$\frac{PF_{NN} L_w^2 x [\frac{L}{L_x} + x] - 2L_w}{PF_{NN} L_w^2 x [x] + (PF_N x - w) L_{ww} - 2L_w} > 1 \quad (\text{A.1.16})$$

All terms in the numerator and denominator are non-positive. We see that if the following inequality holds then inequality (A.1.16):

$$-PF_{NN} L_x^2 \frac{L}{L_x} - (PF_N x - w) L_{ww} > 0 \quad \text{Using the first order conditions again this can be}$$

rewritten as:

$$-PF_{NN} \frac{L_w^3}{L_x} - L_{ww} > 0 \quad (\text{A.1.17})$$

If (A.1.17) holds we can say that employment falls as a result of the maximum effort requirement.

Appendix 2

In this appendix I look for the socially optimal outcome from the monopolistic competition model discussed in section 3. We will continue to assume there are n firms which we take as given (In reality the number of firms would be determined by entry and transport costs). The social welfare function is

$$Wf = \bar{u} + EUt + n\mathbf{p} \quad (\text{A.2.1})$$

We impose the constraint that wage payments in the workers utility function plus transport costs equal output. We assume $a=1$ so profit have the same weight as wages. Given that the production function has diminishing returns and the firms are symmetric we impose the condition that workers consume the output of their own firm only and pay the transport costs from this output also. The welfare function can be rewritten as:

$$n[PF(x, L) - x^b L - \frac{tL^2}{4}] + [1 - nL]\bar{u} \quad (\text{A.2.2})$$

Where $L \leq 1/n$. The expression for transport costs is $tL/4$. $L/2$ is the distance of the marginal worker from the firm given employment level L . This is divided by two to get the average distance since workers are uniformly distributed. We now choose the levels of effort and employment that maximise welfare. The first order condition for L implies:

$$P \frac{\partial F(xL)}{\partial N} x = x^b + \frac{tL}{2} + \bar{u} \quad (\text{A.2.3})$$

and for x :

$$P \frac{\partial F(xL)}{\partial N} = bx^{b-1} \quad (\text{A.2.4})$$

These can be used to solve for the optimal level of effort:

$$x^* = \left[\frac{\bar{u} + tL^*/2}{b-1} \right]^{\frac{1}{b}} \quad (\text{A.2.5})$$

If we write this in terms of L we get:

$$L^* = \frac{2[(b-1)x^{*b} - \bar{u}]}{t} \quad (\text{A.2.6})$$

For any given production function (A.2.6) can be used in the first order condition to solve for x . For example if we use the production function used in (III.3) in (A.2.4) we get:

$$2t^{-1} \left(\frac{Pf}{b}\right)^{\frac{1}{f-1}} [(b-1)x^b - \bar{u}] = x^{\frac{b-f}{f-1}} \quad (\text{A.2.7})$$

We can solve this equation for optimal effort for given parameter values and use the value for effort in (A.2.6) to solve for optimal employment.

Appendix 3

In this appendix we solve the first order conditions for the model of monopsonistic competition outlined in section III.

The first order condition on x implies:

$$\left(p \frac{\mathcal{F}(x, w)}{\mathcal{N}} x - w^s\right) = p \frac{\mathcal{F}(x, w)}{\mathcal{N}} \frac{[(w - x^b) - \bar{u}]}{bx^{b-1}} \quad (\text{A.3.1})$$

$$\left(p \frac{\mathcal{F}(x, w)}{\mathcal{N}} x - w^s\right) = s[(w - x^b) - \bar{u}] \quad (\text{A.3.2})$$

These imply that:

$$p \frac{\mathcal{F}(x, w)}{\mathcal{N}} = sbx^{b-1} \quad (\text{A.3.3})$$

Notice that (A.3.3) corresponds to the first order condition for effort in the social planners problem (A.2.4). If we use (A.3.4) in equation (A.3.2) we get:

$$w = \frac{(b+1)}{2} x^b + \frac{\bar{su} - \mathbf{q}}{s2} \quad (\text{A.3.5})$$

Taking the above term for w we get the following expression for equilibrium labour supply:

$$L = \frac{(b-1)x^b - \bar{u} - \mathbf{q}/s}{t} \quad (\text{A.3.6})$$

Its easy to verify that this holds in Table 1 when b=2.

To proceed beyond this point we need to assume a production function. If we use the production function in equation (III.3), equations (A.3.4) and (A.3.6) imply

$$t^{-1} \left(\frac{Pf}{bs} \right)^{\frac{1}{f-1}} [(b-1)x^b - (\bar{u} + \mathbf{q}/s)] = x^{\frac{b-f}{f-1}} \quad (\text{A.3.7})$$

In Table 1 where f=0.5, $\bar{u} = 1$ and b=2 this amounts to

$\mathbf{h}x^5 - \mathbf{b}x^3 - 1 = 0$ (A.3.8) Again its easy to verify that the only real solution to this equation that gives positive labour supply is as in Table 1 for the given parameter values.