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Staggered Price Contracts and Inflation Persistence: Some General Results

by

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Abstract

Despite their popularity as theoretical tools for illustrating the effects of nominal rigidities, some have questioned whether models based on Taylor-style staggered contracts can match the persistence of the empirical inflation process. This paper presents some general theoretical results about Taylor-style models. It is shown that these models do not have a problem matching high autocorrelations for inflation. However, they fail to explain a key feature of reduced-form Phillips-curve regressions: The positive dependence of inflation on its own lags. It is shown that staggered price contracting models instead predict that the coefficients on these lag terms should be negative.

1 Introduction

The staggered contracting specification introduced by John Taylor (1979) is commonly used to illustrate the macroeconomic effects of nominal rigidities. Most macroeconomists agree that nominal rigidities play an important role in influencing real-world pricing behavior, and Taylor's formulation of this idea is considered by many to be more realistic than some other popular formulations such as Calvo pricing. There have, however, been questions about the ability of models based on staggered price contracts to match important aspects of macroeconomic data. Chari, Kehoe, and McGrattan (2000) have argued that such models cannot resolve the "persistence problem" underlying empirical business cycle dynamics for output. In addition, there has been some debate about whether staggered contracting models can match the persistence of the empirical inflation process. In particular, Fuhrer and Moore (1995) have questioned whether these models can match the observed high autocorrelation of inflation.

This paper has two goals. The first is to establish some general theoretical results that (to my knowledge) have not been presented before, concerning the dynamics of the relationship between inflation and real activity under Taylor-style staggered pricing. The second is to clarify the dimensions along which these models do and do not match the inflation persistence seen in the data. It is shown that staggered price contracting can reproduce high autocorrelations for inflation. However, it is argued that this is not a particularly useful definition of inflation persistence. Conversely, staggered contracting models fail to explain a statistical regularity that, it can reasonably be argued, provides a more useful definition of inflation persistence: The positive dependence of inflation on its own lagged values in reduced-form "Phillips curve" regressions. It is shown that, in general, these models instead predict that these lagged dependent variable coefficients should be *negative.* This result is particularly noteworthy given that Taylor-style contracts are commonly cited as potentially providing an explanation for the empirical pattern of positive coefficients on lagged inflation in Phillips curve regressions.[1]

The contents are as follows. Section 2 reports some facts about inflation autocorrelations and reduced-form inflation regressions for the US and Euro area. Section 3 presents the theoretical results for the standard staggered price contracting specification. It is shown that inflation depends negatively on its own lagged values once one has conditioned on economic fundamentals (i.e. past and expected future economic activity). Section 4 extends

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[1]See, for instance, Dotsey (2002) or page 3 of Eller and Gordon (2003).

these results to a framework incorporating a mixture of contract lengths.

Sections 5 and 6 then discuss various testable predictions of the staggered contracting models based on different assumptions about the determination of output. Section 5 uses a simple model with an exogenous output gap to illustrate how Taylor-style contracts can match high autocorrelations while failing to match the evidence in reduced-form regressions. Section 6 derives the solution for the reduced-form process for inflation for the standard monetary model described in Chari, Kehoe, and McGrattan (2000) in which the output gap is determined by real money balances and money growth follows an AR(1) process. Finally, Section 7 discusses the models' problems with matching the evidence on inflation persistence in some more detail.

2   Evidence on Inflation Persistence

The concept of inflation persistence can be interpreted in different ways. However, probably the most common statistic cited to illustrate the persistence of inflation is the high value of its first-order autocorrelation coefficient. Table 1 reports these autocorrelations for quarterly GDP price inflation for the US and for the Euro Area.[2] They show first-order autocorrelations of almost 0.9 for both the US and the Euro area. Clearly, by this definition, inflation is indeed a persistent series.

A question worth posing about this fact, however, is whether it is in any way surprising. For instance, a wide range of theories about inflation, ranging from the simple to the sophisticated, suppose that inflationary pressures are determined by measures of economic slack such as the output gap or the unemployment rate. Table 1 shows that these measures are also quite persistent, with both output gaps having first-order autocorrelations of about 0.85. Indeed, for both the US and the Euro Area, the unemployment rate has a far higher autocorrelation coefficient than inflation.[3] The table also reports autocorrelations for the labor share: Galí and Gertler (1999) have proposed this as an alternative driving variable for inflation.[4] Again, these series are more autocorrelated than the corresponding inflation

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[2] The US GDP deflator was downloaded from the BEA's website, and the sample used was 1960:Q1 to 2003:Q2. The Euro-Area data are taken from the ECB's Area Wide Model database, described in Fagan, Henry, and Mestre (2001) and the sample used for this series was 1970:Q2 to 2002:Q4.

[3] The output gaps are defined by applying a Hodrick-Prescott filter to the log of real GDP.

[4] For the US, the labor share series was downloaded from the BLS website (www.bls.gov). For the Euro area, I follow Galí, Gertler and Lopez-Salido (2001) in defining this series as the ratio of wage compensation of employees to nominal GDP, where these variables are measured as *WIN* and *YEN* from the AWM

series.

In light of these results, it is hardly surprising that inflation autocorrelations are quite high, and matching this fact should not be considered too high a bar for a theoretical model. Table 1, however, still leaves open the question of the *source* of the high autocorrelation for inflation. Is this high autocorrelation simply driven by the autocorrelation imparted by the underlying driving variables, or does the persistence have some independent source? To address this question, Tables 2 and 3 report results for the US and Euro Area from regressions of the form

$$\pi_t = \alpha + \rho(1)\pi_{t-1} + \sum_{k=1}^{3} \psi_k \Delta\pi_{t-k} + \sum_{k=0}^{3} \gamma_i y_{t-i} + \epsilon_t, \tag{1}$$

where y is either the output gap, the unemployment rate or the labor share. If the persistence of inflation came simply from the autocorrelations in the driving variables, then we would expect to find a low value of the parameter $\rho(1)$. However, these regressions each report large and extremely statistically significant values of $\rho(1)$ for both the US and Euro area, and for each of the selected driving variables.

Some researchers, such as Cogley and Sargent (2001), have argued that the lagged dependent variable effect has weakened over time in the US. This is verified to some extent in Table 2, which shows that estimates of $\rho(1)$ for the post-1983 sample are lower for each of the specifications than for the previous period, and lower again for the sample beginning in 1991. The Euro area results, in contrast, show little systematic tendency for lower values of $\rho(1)$ for the later samples, consistent with the results of O'Reilly and Whelan (2004). The point relevant here for our analysis is merely that while there may be some evidence for changes over time in the size of the lagged dependent variable effect, the effect is always estimated to be positive and highly statistically significant.

These results show that inflation appears to have an *intrinsic* persistence or inertia that would result in a pattern of highly positively autocorrelated inflation, even if its driving variables were themselves only weakly autocorrelated. Indeed, one could argue that the pattern of positive dependence of inflation on its own lags documented in these regressions provides a useful definition of the concept of "inflation persistence" because it documents a phenomenon that is specific to the behavior of inflation, and does not depend solely on the exogenous *deus ex machina* of an autocorrelated driving variable.

database.

The rest of this paper will show that models based on staggered price contracts, while consistent with positively autocorrelated inflation, are completely inconsistent with the pattern of intrinsic inflation persistence described by these reduced-form regressions.

3   Inflation Under Staggered Contracts: A General Solution

This section introduces a standard model of staggered price setting, derives a general analytical solution for the form of the aggregate inflation process, and presents numerical calculations for a four-period-contract example.

3.1   The Optimal Contract Price

Following standard practice in recent literature on the modelling of sticky prices, such as Chari, Kehoe, and McGrattan (2000) and Woodford (2003), it is assumed that the economy consists of imperfectly competitive firms who have demand functions derived from Dixit-Stiglitz-style preferences. In other words, we assume an economy with n different types of firms, such that firm i is assumed to have a demand function

$$Y_{it} = Y_t \left(\frac{P_{it}}{P_t}\right)^{-\theta}, \tag{2}$$

where Y_t is total output, P_{it} is firm i's price, and P_t is the aggregate price level defined as

$$P_t = \left(\frac{1}{n}\sum_{i=0}^{n} P_{it}^{1-\theta}\right)^{\frac{1}{1-\theta}}. \tag{3}$$

The staggering of price setting is assumed to take the standard form: All price contracts last for n periods, and a fraction $\frac{1}{n}$ of firms reset their price each period. Restricted by this form of contracting, those firms that set a price at time t choose a price X_t to maximize the discounted sum of expected profits over the life of the contract. Formally, this problem consists of maximizing

$$\Pi_t = E_t \left[\sum_{k=0}^{n-1} \beta^k \left(Y_{t+k} P_{t+k}^\theta X_t^{1-\theta} - C_{t+k}\left(Y_{t+k} P_{t+k}^\theta X_t^{-\theta}\right)\right)\right], \tag{4}$$

where β is the firm's discount rate and C_t is its cost function at time t. Solving this problem yields the following formula for the optimal contract price

$$X_t = \frac{\theta}{\theta-1} \frac{E_t\left(\sum_{k=0}^{n-1} \beta^k Y_{t+k} P_{t+k}^\theta MC_{t+k}\right)}{E_t\left(\sum_{k=0}^{n-1} \beta^k Y_{t+k} P_{t+k}^\theta\right)}. \tag{5}$$

where MC_t stands for the firm's marginal cost at time t. Log-linearizing this expression around a constant output level and a zero inflation rate, and normalizing the desired markup to one, this becomes

$$x_t = \frac{E_t \left[\sum_{k=0}^{n-1} \beta^k mc_{t+k} \right]}{\sum_{k=0}^{n-1} \beta^k} \tag{6}$$

where lower-case symbols corresponds to logged variables. Finally, defining real marginal cost as

$$MC_t^r = \frac{MC_t}{P_t}, \tag{7}$$

and assuming a simple relationship between the log of real marginal cost and the output gap as derived, for instance, in Chapter 3 of Woodford (2003):

$$mc_t^r = \gamma y_t \tag{8}$$

the optimal contract price becomes

$$x_t = \frac{E_t \left[\sum_{k=0}^{n-1} \beta^k \left(p_{t+k} + \gamma y_{t+k} \right) \right]}{\sum_{k=0}^{n-1} \beta^k}. \tag{9}$$

Worth noting here is that, while this expression has been derived as the result of an optimal price-setting procedure, if we set $\beta = 1$, then this equation has the same algebraic format as the traditional Taylor (1979) staggered wage model. Taylor assumed that contract prices were a fixed markup over wages, and interpreted equations of the form of (9) as being the outcome of bargaining process in which workers were concerned about the expected real wage over the life of the contract, with the outcome depending on expected labor market conditions, represented here by the $E_t y_{t+k}$.

3.2 Solving for The Contract Price Process

The price level equation (3) can be log-linearized to give

$$p_t = \frac{1}{n} \sum_{k=0}^{n-1} x_{t-k}. \tag{10}$$

One obvious point that can immediately be drawn from this equation is that aggregate inflation is a moving average of the rate of change of the contract price:

$$\pi_t = \frac{1}{n} \sum_{k=0}^{n-1} \Delta x_{t-k}. \tag{11}$$

In light of this result, our strategy for deriving a solution for price inflation will involve first characterizing the behavior of the contract price.

The first step in solving for the process for the contract price is to substitute out the expected future price levels in terms of future and past contract prices to get

$$x_t = \frac{E_t \left[\sum_{k=0}^{n-1} \beta^k \left(\frac{1}{n} \sum_{r=0}^{n-1} x_{t+k-r} + \gamma y_{t+k} \right) \right]}{\sum_{k=0}^{n-1} \beta^k}. \tag{12}$$

This is a $2(n-1)$th-order stochastic difference equation in x_t and the properties of its solution underlie the properties of aggregate price inflation in this model. The equation can be re-arranged to give

$$n \left(\sum_{k=0}^{n-1} \beta^k \right) x_t = \sum_{k=0}^{n-1} \beta^k \sum_{r=0}^{n-1} E_t x_{t+k-r} + \gamma n Z_t, \tag{13}$$

where

$$Z_t = \sum_{k=0}^{n-1} \beta^k E_t y_{t+k}. \tag{14}$$

The form of this difference equation can be simplified somewhat by making use of the following equality:

$$\sum_{k=0}^{n-1} \beta^k \sum_{r=0}^{n-1} E_t x_{t+k-r} = \left(\sum_{k=0}^{n-1} \beta^k \right) x_t + \left(\sum_{k=0}^{n-2} \beta^k \right) (x_{t-1} + \beta E_t x_{t+1}) + \left(\sum_{k=0}^{n-3} \beta^k \right) \left(x_{t-2} + \beta^2 E_t x_{t+2} \right)$$
$$+ .... + \left(x_{t-n+1} + \beta^{n-1} E_t x_{t+n-1} \right).$$

In particular, defining the following polynomial

$$\sigma(x) = \sum_{k=1}^{n-1} \left(\sum_{m=0}^{n-k-1} \beta^m \right) x^k, \tag{15}$$

the contract price process can be re-written in terms of lag and forward operators as

$$E_t \left[\sigma(\beta F) - (n-1) \left(\sum_{k=0}^{n-1} \beta^k \right) + \sigma(L) \right] x_t = -\gamma n Z_t. \tag{16}$$

The key properties of this process can be then derived from the following results.

Proposition: The $2(n-1)$th-order polynomial equation

$$\left[\sigma(\beta\lambda) - (n-1) \left(\sum_{k=0}^{n-1} \beta^k \right) + \sigma(\lambda^{-1}) \right] \lambda^{n-1} = 0 \tag{17}$$

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has the following properties

(a) If λ_i is a solution, then $(\beta\lambda_i)^{-1}$ is also a solution.

(b) One and β^{-1} are both solutions.

(c) The other $2(n-2)$ solutions all have negative real components.

Proof: (a) The fact that all of the coefficients of the polynomial (including the intercept) are positive rules out zero solutions. Equation (17) thus holds when the term inside the square brackets in this equation is zero. The required result comes from noting that the term inside the square brackets is unchanged when λ is replaced with $(\beta\lambda)^{-1}$.

(b) Note from equation (13) that this polynomial can also be written as

$$\left[n \left(\sum_{k=0}^{n-1} \beta^k \right) \lambda - \sum_{k=0}^{n-1} \beta^k \sum_{r=0}^{n-1} \lambda^{k-r} \right] \lambda^{n-1} = 0 \tag{18}$$

and $\lambda = 1$ is clearly a solution to this equation. That β^{-1} is also a root follows directly from part (a).

(c) This property stems from Descartes' Rule of Signs, which states that the maximum number of roots of a polynomial with positive real components is given by the number of sign changes in the coefficients of the polynomial as one goes up in order of ascending power. Because each of the coefficients of σ are positive, there are two sign changes in this equation. Thus, the equation has at most two roots with positive real components. The previous result that one and β^{-1} are both roots thus ensures that there are no other roots with positive real components. $\square$

As long as there are no complex roots with absolute value between one and β^{-1}—and numerical calculations with a range of values for β and n confirm that this case does not arise—then part (a) of this proposition guarantees that the contract price process has a unique non-explosive solution. To see this, first note that the proposition implies that the polynomial has exactly $n-1$ roots outside the unit circle, and another $n-1$ roots on or inside the unit circle. This comes from the fact that part (a) implies that any root λ on or inside the unit circle has a corresponding root $(\beta\lambda)^{-1}$ that is outside the unit circle. In addition, for any root λ outside the unit circle with absolute value greater than or equal to β^{-1}, there is a corresponding root $(\beta\lambda)^{-1}$ that is on or inside the unit circle.

That these properties guarantee the existence of a unique non-explosive solution can be derived as follows. Let $\lambda_1, \lambda_2, \ldots \lambda_{n-2}$ represent the $n-2$ roots on or inside the unit circle

in addition to one. The contract price process can now be written as

$$E_t \left[\beta^{n-1}(F-1)(F-\beta^{-1}) \left\{ \prod_{i=1}^{n-2} (F-\lambda_i) \left(F-\frac{1}{\beta\lambda_i}\right) \right\} L^{n-1} x_t \right] = -\gamma n Z_t. \qquad (19)$$

Using the general principle of solving stable roots backwards and unstable roots forward, we need to apply the $n-1$ lag operators L^{n-1} to the roots on or inside the unit circle to leave only one possible non-explosive solution. Letting $\lambda_{n-1} = 1$, this solution can be written as:

$$\beta^{n-1} \left\{ \prod_{i=1}^{n-2} (1-\lambda_i L) \right\} \Delta x_t = -\gamma n E_t \left[\left\{ \prod_{i=1}^{n-1} \left(F-\frac{1}{\beta\lambda_i}\right) \right\}^{-1} Z_t \right]. \qquad (20)$$

Note also that

$$\frac{1}{F-(\beta\lambda_i)^{-1}} = -\frac{\beta\lambda_i}{1-\beta\lambda_i F} = -\beta\lambda_i \sum_{k=0}^{\infty} \beta^k \lambda_i^k F^k. \qquad (21)$$

Thus, the contract price process is

$$\left\{ \prod_{i=1}^{n-2} (1-\lambda_i L) \right\} \Delta x_t = \gamma n E_t \left[\left(\prod_{i=1}^{n-2} (-\lambda_i) \right) \left(\sum_{k=0}^{\infty} \beta^k \lambda_1^k F^k \right) ..... \left(\sum_{k=0}^{\infty} \beta^k \lambda_{n-1}^k F^k \right) Z_t \right]. \qquad (22)$$

Letting

$$\delta(L) = \left\{ \prod_{i=1}^{n-2} (1-\lambda_i L) \right\}, \qquad (23)$$

we obtain the solution for the rate of change of the new contract price as

$$\delta(L)\Delta x_t = \gamma n \sum_{k=0}^{\infty} \kappa_k E_t Z_{t+k}. \qquad (24)$$

3.3   Aggregate Price Inflation

Turning now from contract prices to the aggregate price level, let

$$\alpha(L) = \left(\sum_{j=0}^{n-1} L^j \right), \qquad (25)$$

be the n-period moving sum operator. Aggregate price inflation can then be written as

$$\pi_t = \frac{1}{n} \alpha(L) \Delta x_t, \qquad (26)$$

and equation (24) can be re-written as

$$\delta(L)\alpha(L)\Delta x_t = \gamma n\alpha(L)\left[\sum_{k=0}^{\infty}\kappa_k E_t Z_{t+k}\right]. \tag{27}$$

This results in the following expression for price inflation:

$$\delta(L)\pi_t = \gamma\alpha(L)\left[\sum_{k=0}^{\infty}\kappa_k E_t Z_{t+k}\right]. \tag{28}$$

Aggregate price inflation is a function of two factors. The first factor is current and past expectations about the future paths of the driving variable y_t. The second factor is inflation's own lagged values: From equation (23), we see that as long as contracts are longer than two periods in length, $(n > 2)$, then inflation will be directly affected by its own lags. The following result shows also that our results concerning the roots of the contract price equation pin down the nature of inflation's dependence on its own lags.

Proposition: All of the coefficients in the lag polynomial, $\delta(L)$, defined in equation (23) are positive.

Proof: Each of the terms $\lambda_1, \lambda_2, ..., \lambda_{n-2}$ have negative real components. Thus each of the coefficients on L in the $(1 - \lambda_i L)$ terms in $\delta(L)$ have positive real components. This implies that each of the coefficients on the various powers of L in $\delta(L)$ must also all be positive, which is the required result. □

This result has an important implication. It implies that the inflation process can be written as

$$\pi_t = \psi(L)\pi_{t-1} + \gamma\alpha(L)\sum_{k=0}^{\infty}\kappa_k E_t Z_{t+k}. \tag{29}$$

where all of the coefficients in the lag polynomial $\psi(L)$ are negative. In other words, staggered price contracts imply that once we condition on the effects of fundamentals (expectations of real marginal cost in the model of Section 2; expected labor market conditions in the traditional Taylor model), then either there is no intrinsic persistence (the case $n = 2$) or there are lagged dependent variable effects with negative signs.

3.4 Example: Four-Period Contracts

To provide a concrete example of these results, consider the case in which $\beta = 1$ and each firm in the economy sets fixed four-period contracts. This is an obvious benchmark case

because it is consistent with quarterly data and price contracts that last a year. It is also consistent with firms marking up wages that are fixed for a year, as in Taylor's original formulation. In this case, the polynomial equation determining the roots of the contract process is

$$\lambda^6 + 2\lambda^5 + 3\lambda^4 - 12\lambda^3 + 3\lambda^2 + 2\lambda + 1 = 0 \tag{30}$$

The six roots of this equation are

$$
\begin{aligned}
\lambda_1 &= -0.214 - 0.272i \\
\lambda_2 &= -0.214 + 0.272i \\
\lambda_3 &= 1
\end{aligned}
$$

and their inverses. These calculations imply the following lag polynomial

$$\delta(L) = 1 + 0.43L + 0.12L^2. \tag{31}$$

Thus, four-period contracts imply an inflation process of the form

$$\pi_t = -0.43\pi_{t-1} - 0.12\pi_{t-2} + \gamma\alpha(L)\sum_{k=0}^{\infty}\kappa_k E_t Z_{t+k}. \tag{32}$$

These calculations show that the prediction of negative coefficients on the lagged inflation terms is not just a theoretical curiosity: A realistic calibration of the model predicts quite large negative coefficients on the lagged inflation terms.

4 A Model with Multiple Contract Lengths

With an additional simplifying assumption, the results of the previous section can be extended to a case in which, instead of all firms having contracts of the same length, contracts of different lengths exist. In other words, the results can be extended to an economy in which there are different types of firms, with some having one-period contracts, some having two-period contracts, and so on. This section derives this extension and presents a numerical example.

4.1 Solution

We will denote the share of firms that set contracts of length k by θ_k, and as before a fraction $\frac{1}{k}$ of these firms reset their contracts each period. The maximum contract length

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is assumed to be n periods. To keep the algebraic derivations as simple as possible, I will restrict the analysis to the case of no discounting ($\beta = 1$). However, one can show that the relevant results generalize to the discounting case exactly as in the previous section. With this in mind, we assume that firms setting a k-period contract today set their price equal to

$$x_t^k = \frac{1}{k} \sum_{j=0}^{n-1} E_t p_{t+j} + \frac{\gamma}{k} \sum_{j=0}^{n-1} E_t y_{t+j}. \tag{33}$$

Thus, the average contract price set in period t is given by

$$\begin{aligned}
\bar{x}_t &= \theta_1 (p_t + \gamma y_t) + \frac{\theta_2}{2} [p_t + E_t p_{t+1} + \gamma (y_t + E_t y_{t+1})] \\
&\quad + \ldots\ldots + \frac{\theta_n}{n} [p_t + E_t p_{t+1} + \ldots + E_t p_{t+n-1} + \gamma (y_t + E_t y_{t+1} + E_t y_{t+n-1})]
\end{aligned} \tag{34}$$

This can be expressed more compactly as

$$\bar{x}_t = \sum_{k=0}^{n-1} f_k E_t p_{t+k} + \gamma \sum_{k=0}^{n-1} f_k E_t y_{t+k}, \tag{35}$$

where

$$f_k = \sum_{m=k+1}^{n} \frac{\theta_m}{m}. \tag{36}$$

Note that the f_k weights sum to one.

The aggregate price level in this economy is given by

$$p_t = \theta_1 x_t^1 + \frac{\theta_2}{2} \left(x_t^2 + x_{t-1}^2 \right) + \ldots\ldots + \frac{\theta_n}{n} \left(x_t^n + x_{t-1}^n + \ldots x_{t-n+1}^n \right). \tag{37}$$

To obtain an analytical solution, we make a simplifying assumption and follow Taylor (1993) in assuming that the price variations across each of the contracts set at date t are negligible, i.e. that $x_t^k \approx \bar{x}_t$. In this case, the price equation becomes

$$p_t = \theta_1 \bar{x}_t + \frac{\theta_2}{2} \left(\bar{x}_t + \bar{x}_{t-1} \right) + \ldots\ldots + \frac{\theta_n}{n} \left(\bar{x}_t + \bar{x}_{t-1} + \ldots \bar{x}_{t-n+1} \right), \tag{38}$$

which can re-written as

$$p_t = \sum_{k=0}^{n-1} f_k \bar{x}_{t-k}, \tag{39}$$

where the f_k weights are the same as in equation (36). Thus, aggregate price inflation can again be defined as a simple function of current and past rates of change of the average new contract price. This can be expressed as

$$\pi_t = \sum_{k=0}^{n-1} f_k \Delta \bar{x}_{t-k}, \tag{40}$$

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or, alternatively, defining

$$\eta(L) = \sum_{k=0}^{n-1} f_k L^k, \tag{41}$$

we can write

$$\pi_t = \eta(L)\Delta\bar{x}_t. \tag{42}$$

The same solution method as before can be employed to solve for the aggregate inflation process. Inserting equation (39) into equation (35), we get the following expression for the process for the average contract price.

$$\bar{x}_t = \sum_{i=0}^{n-1} f_i \sum_{k=0}^{n-1} f_k E_t \bar{x}_{t+i-k} + \gamma \sum_{i=0}^{n-1} f_i E_t y_{t+i}. \tag{43}$$

One can now use the same arguments as in the previous section to show that this contract price process also has the required characteristics to produce the same result as before, namely that lagged inflation has a negative effect on current inflation as long as $n > 2$.

First, note that the coefficients of this difference equation display a symmetric pattern, with the coefficients on x_{t-k} and $E_t x_{t+k}$ being the same. Specifically, the equation has the form

$$\begin{aligned}
\left(1 - f_0^2 - f_1^2 - \dots - f_{n-1}^2\right) x_t &= (f_0 f_{n-1})\left(x_{t-n+1} + E_t x_{t+n-1}\right) \\
&\quad + (f_0 f_{n-2} + f_1 f_{n-1})\left(x_{t-n+2} + E_t x_{t+n-2}\right) \\
&\quad + \dots + (f_0 f_1 + f_1 f_2 + \dots + f_{n-1} f_{n-2})\left(x_{t-1} + E_t x_{t+1}\right) \\
&\quad + \gamma \sum_{i=0}^{n-1} f_i E_t y_{t+i}
\end{aligned} \tag{44}$$

Letting

$$\tilde{Z}_t = \sum_{i=0}^{n-1} f_i E_t y_{t+i}, \tag{45}$$

the average contract price process can be written in terms of lag and forward operators as

$$E_t\left[\left\{\omega(F) - \left(1 - \sum_{k=0}^{n-1} f_k^2\right) + \omega(L)\right\} \bar{x}_t\right] = -\gamma\tilde{Z}_t. \tag{46}$$

where

$$\omega(x) = \sum_{k=0}^{n-1} \left(\sum_{r=1}^{k} f_{r-1} f_{n-k+r-1}\right) x^{n-k}. \tag{47}$$

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Again the properties of the contract price process stem from the properties of this polynomial equation.

Proposition: The $2(n-1)$th-order polynomial equation

$$\left[\omega(\lambda) - \left(1 - \sum_{k=0}^{n-1} f_k^2\right) + \omega(\lambda^{-1})\right]\lambda^{n-1} = 0 \tag{48}$$

where $\omega(x)$ is defined in equation (47), has the following properties
(a) If λ_i is a solution, then λ_i^{-1} is also a solution.
(b) There are two unit root solutions.
(c) The other $2(n-2)$ solutions all have negative real components.

Proof: (a) Again there are no zero solutions, and the term inside the bracket is unchanged when λ is replaced with λ^{-1}. This is sufficient to prove the result.

(b) Note from equation (43) that this polynomial can also be written as

$$\left[\lambda - \sum_{k=0}^{n-1} f_k \sum_{r=0}^{n-1} f_r \lambda^{k-r}\right]\lambda^{n-1} = 0.$$

One is a solution to this equation because $\sum_{k=0}^{n-1} f_k = 1$.

(c) Again, the required result is implied by Descartes' Rule of Signs. Because each of the f_i terms are less than one, we have $f_i^2 < f_i$. Thus

$$\sum_{k=0}^{n-1} f_k^2 < \sum_{k=0}^{n-1} f_k = 1.$$

So, the middle term is negative and there are two sign changes. $\square$

As before, these properties are sufficient to ensure that the aggregate price inflation process takes the form of equation (29) with the coefficients on lagged inflation being negative. The only differences being the technicalities that $\alpha(L)$ is replaced by $\eta(L)$ and Z_t is replaced by $\tilde{Z}_t$.

4.2   Example: Mix of Contracts up to Four Periods

To illustrate these results, consider the case in which there is an equal mix of one, two, three, and four-period contracts. In terms of the terminology above, this implies

$$\theta_1 = \theta_2 = \theta_3 = \theta_4 = \frac{1}{4} \tag{49}$$

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while the weights that determine the contract price equation become

$$
\begin{aligned}
f_0 &= \frac{1}{4}\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) = \frac{25}{48} \\
f_1 &= \frac{1}{4}\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) = \frac{13}{48} \\
f_2 &= \frac{1}{4}\left(\frac{1}{3} + \frac{1}{4}\right) = \frac{7}{48} \\
f_3 &= \frac{1}{4}\frac{1}{4} = \frac{3}{48}
\end{aligned}
$$

After some calculations, one can show that the polynomial equation associated with the roots of this contract price process is

$$
75\lambda^6 + 214\lambda^5 + 437\lambda^4 - 1452\lambda^3 + 437\lambda^2 + 214\lambda + 75 = 0 \tag{50}
$$

The six roots of this equation are

$$
\begin{aligned}
\lambda_1 &= -0.175 - 0.217i \\
\lambda_2 &= -0.175 + 0.217i \\
\lambda_3 &= 1
\end{aligned}
$$

and their inverses. These calculations imply the following lag polynomial

$$
\delta\left(L\right) = 1 + 0.350L + 0.077L^2. \tag{51}
$$

So, the process for inflation is of the form

$$
\pi_t = -0.35\pi_{t-1} - 0.08\pi_{t-2} + \gamma\eta(L)\sum_{k=0}^{\infty}\kappa_k E_t \tilde{Z}_{t+k}. \tag{52}
$$

Again, the size of the negative lagged dependent variables effect is quite large.

5 Autocorrelations versus Intrinsic Persistence

In Section 2 we noted that, while related, there were conceptual differences between the idea of inflation persistence as high autocorrelations and the idea of intrinsic persistence generated by a positive lagged dependent variable effect. Here, we use a simple example to illustrate how Taylor-style staggered contract models can match high autocorrelations for inflation, while failing to match the empirical evidence on intrinsic persistence.

The example is based on the assumption that the output gap is determined by an AR(1) process

$$y_t = \rho y_{t-1} + \epsilon_t, \tag{53}$$

where ϵ_t is assumed to be white noise. In the standard n-period contract model, this assumption allows us to simplify the Z_t variable to

$$Z_t = \sum_{k=0}^{n-1} \beta^k E_t y_{t+k} = \frac{1 - (\beta\rho)^n}{1 - \beta\rho} y_t. \tag{54}$$

In this case, all of the expectational variables, $E_t Z_{t+k}$, reduce to being multiples of y_t. This simplification means that there is no connection between lags of inflation and the expectational terms in equation (29), so that the coefficients on the lagged terms in the reduced-form representation are the same as in the structural representation.

Calculations left to Appendix A show that for $n = 4$, $\beta = 1$, and $\rho = 0.9$, the standard contracting model's solution reduces to[5]

$$\pi_t = -0.43\pi_{t-1} - 0.12\pi_{t-2} + 2.78\gamma\alpha(L) y_t. \tag{55}$$

Simulating this process, the first-order autocorrelation coefficient for inflation is 0.977. Thus, the model produces an inflation series that is more autocorrelated than its driving variable.[6] This may be a little surprising given the negative coefficients on the lagged dependent variables. This can be explained, however, by noting that the model predicts inflation is an $ARMA(2,3)$ series, with driving variable y_t. While, ceteris paribus, the AR component acts to make π_t less autocorrelated than y_t, the MA component tends to make it more so.

One formal way to explain this result is to compute the spectral properties of the filter that transforms y_t into π_t. In other words, we can analyze how the application of the filter

$$f(L) = \frac{(2.78)\gamma\left(1 + L + L^2 + L^3\right)}{1 + 0.43L + 0.12L^2}, \tag{56}$$

tends to promote the role of certain frequencies over others. Normalizing γ as $\frac{1}{2.78}$ for convenience, the spectral transformation of y_t implied by this filter is

$$f\left(e^{i\omega}\right) f\left(e^{-i\omega}\right) = \frac{4 + 6\cos\omega + 4\cos 2\omega + 2\cos 3\omega}{1.20 + 0.96\cos\omega + 0.12\cos 2\omega}, \tag{57}$$

---

[5]All of the examples of theoretical reduced-form processes reported in this paper were first calculated using the analytical methods described in the appendix, and then checked using the numerical solution algorithm for rational expectations models of Binder and Pesaran (1995).

[6]This result is not affected by the value of γ chosen.

where the numerator here describes the effect of the MA component and the denominator describes the effect of the AR component. As Figure 1 shows, on its own the effect of the AR component of the filter is to increase the role of higher-frequency cycles (the left panel), but the effect of the MA component is to increase the role of lower-frequency cycles (the middle panel). When the two components are put together (the right panel), we see that the combined effect produces a downward-sloping spectral transformation, implying that the inflation series will exhibit more low-frequency variation, and thus higher autocorrelations, than the driving variable.

These examples show that the Taylor-style staggered contracting does not have difficulty generating high autocorrelations for inflation, in contrast to the claims of Fuhrer and Moore (1995). Thus, these results support the findings of Guerrieri (2002) that this type of contracting model can match the high inflation autocorrelations seen in the data. However, at the same time, they also show that it is possible for the models to completely fail to capture a key element of the empirical inflation process that perhaps better describes what is meant by inflation persistence, i.e. the positive dependence of inflation on its own lagged values.

6   A Simple Monetary Model of Output

6.1   From Structural-Form to Reduced-Forms Relationships

We have shown that Taylor-style models imply a structural relationship of the form

$$\pi_t = \psi(L)\pi_{t-1} + \gamma\alpha(L)\sum_{k=0}^{\infty}\kappa_k E_t Z_{t+k}.$$

in which the coefficients in the $\psi(L)$ lag polynomial are all negative. On the face of it, this seems to strongly contradict the evidence from the regressions reported in Section 2. However, an important caveat to this interpretation is that the negative coefficients in this representation depend on the inclusion of unobservable expectational variables, while the evidence in Section 2 relates to reduced-form regressions relating inflation to its own lags and to current and lagged values of the relevant driving variable.

The example of an autoregressive output gap in the previous section got around this problem by assuming that lagged values of inflation contain no information about future output beyond what is already contained in current or lagged values of output, i.e. that

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there was no Granger causality going from inflation to output. In this case, the reduced form and structural coefficients on the lagged inflation terms are identical. In reality, however, this lack of causality may not be a reasonable assumption.

This suggests one potential route for reconciling the contracting models with the evidence in Section 2. If lags of inflation acted as positive leading indicators for the driving variable y_t, then this relationship could still potentially be consistent with positive coefficients on lagged inflation in a *reduced-form* regressions. Put formally, suppose this positive leading indicator role took the form of

$$\sum_{k=0}^{\infty} \kappa_k E_t Z_{t+k} = \upsilon(L)\, \pi_{t-1} + \zeta(L)\, y_t, \tag{58}$$

where the coefficients in the $\upsilon(L)$ polynomial were positive. In this case, the reduced-form relationship would be

$$\pi_t = [\psi(L) + \gamma\alpha(L)\upsilon(L)]\, \pi_{t-1} + \gamma\alpha(L)\zeta(L)\, y_t, \tag{59}$$

and it is possible that the positive coefficients in the $\gamma\alpha(L)\upsilon(L)$ polynomial could sufficiently outweigh the negative coefficients in the $\psi(L)$ polynomial to produce the positive coefficients seen in the estimated reduced-form relationships.

We now consider a standard monetary model with endogenously-determined output in which this positive causality is present, and examine whether such a model is likely to be consistent with the reduced-form evidence.

6.2   The Model

Here we consider the case in which the output gap is determined by real money balances

$$y_t = m_t - p_t, \tag{60}$$

and money growth evolves according to an AR(1) process:

$$\Delta m_t = \rho_m \Delta m_{t-1} + \epsilon_t^m. \tag{61}$$

These assumptions have previously been considered in conjunction with a staggered contracting model in the work of Chari, Kehoe, and McGrattan (2000).[7] Concerning the model

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[7] Adding a positive intercept to the money growth equation so that inflation is positive on average does not change the analysis here.

of pricing, we will restrict ourselves here to examining the pure n-period contracting model. Also, again for convenience we examine the case of the model with no discounting. However, all of the analytical results can be generalized to the case with discounting and the numerical calculations reported here are little changed by allowing for non-unit values for the discount parameter β.

Before deriving the implications for the reduced-form characterization of inflation in this case, we first note that this model contains exactly the positive causality linkages that could, potentially, imply positive coefficients on lagged inflation in a reduced-form regression. To see this, note that output growth in this model is determined by

$$\Delta y_t = \rho_m \Delta m_{t-1} - \pi_t + \epsilon_t. \tag{62}$$

Substituting in $\Delta m_{t-1} = \Delta y_{t-1} + \pi_{t-1}$ and equation (29)'s structural representation for inflation, we obtain

$$\Delta y_t = \rho_m \Delta y_{t-1} + (\rho_m - \psi(L)) \pi_{t-1} - \gamma\alpha(L) \sum_{k=0}^{\infty} \kappa_k E_t Z_{t+k}. \tag{63}$$

Because all of the coefficients in the $\psi(L)$ polynomial are negative, this implies that there will be positive causality from lagged inflation to output growth in this model: High lagged inflation tends to reduce inflation today and thus boost real money growth. It turns out, however, that this effect does not appear to be enough to reconcile this model with the reduced-form evidence.

An analytical solution for the reduced-form inflation process for this model can be obtained as follows. The contract price is set according to

$$x_t = \frac{1}{n} \sum_{k=0}^{n-1} E_t p_{t+k} + \frac{\gamma}{n} \sum_{k=0}^{n-1} E_t \left(m_{t+k} - p_{t+k} \right). \tag{64}$$

Applying the same techniques as before, it is shown in Appendix B that when money growth follows an AR(1) process the contract price is

$$x_t = \sum_{k=1}^{n-1} \mu_k x_{t-k} + \left(1 - \sum_{k=1}^{n-1} \mu_k \right) m_t + \varphi \Delta m_t. \tag{65}$$

where $0 < \sum_{k=1}^{n-1} \mu_k < 1$, the μ_k's are independent of the value of ρ, while φ depends on both ρ and γ. This implies a solution for the price level of form

$$p_t = \sum_{k=1}^{n-1} \mu_k p_{t-k} + \left(1 - \sum_{k=1}^{n-1} \mu_k \right) \alpha(L) m_t + \varphi\alpha(L) \Delta m_t \tag{66}$$

Finally, substituting $m_t = y_t + p_t$ and re-arranging, we obtain a reduced-form Phillips curve in terms of inflation and the output gap of the form

$$\pi_t = \sum_{k=1}^{n-1} \lambda_k \pi_{t-k} + \sum_{k=0}^{n} \delta_k y_{t+k} \tag{67}$$

In this reduced-form representation, the coefficients on lagged inflation are different from those in the structural representation of the same model (that is, from the coefficients in equation 29) and in theory they can be positive. However, numerical calculations show that these theoretical reduced-form inflation processes do not come close to matching those obtained from regressions.

For example, setting $\gamma = 0.50$, $n = 4$, and $\rho_m = 0.66$ (the value consistent with a quarterly AR(1) regression for M1 growth), one obtains the following inflation process:[8]

$$\pi_t = -0.50\pi_{t-1} - 0.08\pi_{t-2} + 0.28\pi_{t-3} + 0.58y_t + 0.30\left(y_{t-1} + y_{t-2} + y_{t-3}\right) - 0.28y_{t-4} \tag{68}$$

Though the sum of the lagged inflation coefficients in this case is slightly less negative than in the structural representation for this model (-0.30 relative to -0.55), it is clear that this process does not look anything like the pattern of large positive coefficients reported in Section 2. Again, though, the model does succeed in generating an inflation series that is autocorrelated, and more so than the output gap: In this case, inflation has an autocorrelation coefficient of 0.88, compared with 0.83 for the output gap.

Table 4 reports the reduced-form lagged inflation coefficients obtained under a range of different values of γ and ρ_m. The γ parameter in these calculations varies from 0.1 to 3.0, representing a range in which real marginal cost can be either far less variable or far more variable than the output gap. Our estimate of the money growth autocorrelation coefficient of 0.66 has a standard error of 0.056, so this suggests 0.5 to 0.8 as endpoints of a wide range of reasonable values for this parameter. The results show that the sums of the lag coefficients are almost all negative and none come close to matching even the smallest of the values on Table 2. In addition, the first lag coefficients are always highly negative, which fails to match the empirical pattern that this tends to be the most positive coefficient. Consider, for example, the full-sample regression for US GDP price inflation

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[8]This estimate of ρ is based on a sample of 1959:3 to 2004:2. The data were downloaded from the Federal Reserve Board's website. Chari, Kehoe, and McGrattan (2000) also estimate a regression for M1 growth and report a similar coefficient value of 0.57.

featuring the output gap. In this case, the sum of the coefficients is 0.94, and the first lag coefficient is 0.51 with a standard error of 0.09.

In addition, calculations reported in Appendix B show that the reduced-form lagged inflation coefficients reported here are not changed by generalizing the model by adding a stochastic monetary velocity shock.

6.3 Estimates of Calvo-Style Models

The approach taken in this paper has been to compare the reduced-form inflation processes implied by theoretical models with the evidence from empirical regressions for such specifications. An advantage of this approach is that it provides a relatively transparent way to illustrate the empirical shortcomings of Taylor-style contracting models. It is worth noting, however, that some other recent papers have discussed the implications of Taylor-style models for another type of regression estimation, namely GMM estimation of the so-called "hybrid" Calvo model proposed by Galí and Gertler (1999):

$$\pi_t = \gamma_b \pi_{t-1} + \gamma_f E_t \pi_{t-1} + \psi y_t. \tag{69}$$

This equation is consistent with a Calvo-style model in which a fraction of firms adopt backward-looking rules of thumb when setting prices. In the context of this model, positive estimates of γ_b are considered evidence for the existence of backward-looking agents. However, using simulated data from variants on the Taylor-style specifications considered here, Dotsey (2002) and Bakhshi, Burriel-Llombart, Khan, and Rudolf (2003) both show that one can obtain positive values of γ_b from GMM estimation of this equation. Thus, they warn against interpreting significant positive estimates of γ_b as evidence for backward-looking price-setters, since the Taylor-style models do not incorporate such behavior.

The findings of Dotsey and Bakhshi et al can be replicated using our model. For instance, simulating the model with $\gamma = 0.50$, $n = 4$, and $\rho = 0.66$ (the values that generate inflation equation 68) and estimating the equation via GMM using four lags of both inflation and the output gap, we obtain estimates of $\hat{\gamma}_b = 0.48, \hat{\gamma}_f = 0.64$, and $\hat{\psi} = -0.05$.[9] These estimates of γ_b and γ_f are close to those reported in a number of empirical studies, and the finding of a negative coefficient on the driving variable is also reported by Bakhshi et al. While the specific estimates obtained depend on the values of the underlying parameters chosen, these exercises do invariably produce positive estimates for the γ_b coefficient.

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[9]These estimates were based on taking the average of 10000 simulations, each based on a sample of 10000.

These results confirm the cautionary warnings Dotsey and Bakhshi et al concerning the interpetation of tests of the hybrid Calvo model. However, one should be cautious in interpreting the estimates of γ_b generated by these simulated data as an important piece of evidence in favor of the Taylor contracting approach. For example, in the case of the estimates just reported, the behavior of inflation in the underlying model is fully described by (68), and this equation's implications for inflation dynamics are strongly contradicted by the evidence from reduced-form regressions.

In addition, it is worth keeping in mind that the estimates of the hybrid Calvo equation in these simulation exercises are driven purely by the fact that, in the simulated economies, this equation badly mis-specifies the dynamics of inflation, so the estimated coefficients are driven by the correlations with the omitted variables such as the additional lags of output and inflation. And as one moves closer to the correct underlying specification of the model's dynamics, one can overturn the positive estimates on the lagged inflation term as well as on the $E_t\pi_{t+1}$ term. For example, consider the case of GMM estimation of

$$\pi_t = \gamma_b\pi_{t-1} + \gamma_f E_t\pi_{t-1} + \sum_{k=0}^{4} \psi_k y_{t-k}. \tag{70}$$

This specification adds in the additional lags of the output gap that belong in the correct model specification. Again simulating the case with $\gamma = 0.50$, $n = 4$, and $\rho = 0.66$, and estimating using $(\pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \pi_{t-4}, y_t, y_{t-1}, y_{t-2}, y_{t-3}, y_{t-4})$ as instruments, one now obtains $\hat{\gamma}_b = -0.95$ and $\hat{\gamma}_f = -2.66$. Overall, it could be argued that the complex interpretational issues raised by these exercises help to underscore the advantages of the simpler assessment procedure adopted in this paper based on deriving predictions for the properties of reduced-form equations.

7   Causality Tests

The results in the last section tell us that the causal linkages between inflation and output in a standard monetary model do not lead to an overturning of the prediction that Taylor contracting should imply negative coefficients on lagged inflation in reduced-form regressions. However, this cannot rule out the possibility that, in reality, these linkages are strong enough to overturn this prediction, and thus the theoretical results of the preceding section are misleading. This suggests a final route to checking whether the contracting models may be consistent with the evidence, which is to assess whether the relevant positive Granger-causality patterns from inflation to output are evident in the data.

From an a priori perspective it is, of course, also possible that the correct model implies a *negative* causal relationship from inflation to the output gap, and—if Taylor contracting were the correct model of pricing—then this would imply reduced-form lag coefficients that should be more negative than those calculated in the Section 3. This point is worth noting because realistic structural models embedding a staggered contracting specification for pricing often contain a policy rule in which the central bank targets a particular value of inflation. And a policy rule of this form implies that high values of inflation trigger higher interest rates and thus will tend to dampen future output gaps, suggesting a Granger causality relationship with the wrong sign for reconciling Taylor-style models with the reduced-form evidence.

With these considerations in mind, one can see from the results reported on Tables 5 and 6 that the positive causality argument does not appear to work well in practice. These tables report results from a series of Granger Causality tests, which test for causality running from inflation to each of the three driving variables discussed earlier (the output gap, the unemployment rate, and the labor share) for both the US and the Euro area. These results show little evidence of causal relationships of the correct signs to allow for reconciliation of the staggered contracting models with the reduced-form evidence.

First looking at the US results on Table 5, we see that the full-sample tests reject the hypothesis that inflation Granger causes the output gap or the labor share. There is evidence of causation running from inflation to the unemployment rate, but this relationship has the wrong sign for reconciling the Taylor contracting models with the evidence: Inflation appears to positively cause the unemployment rate, so a high lagged inflation rate should have an even more negative effect on current inflation than is indicated by the negative "intrinsic persistence" described by the $\psi(L)$ polynomial. Because of the possible (or perhaps likely) changes over time in the reduced-form relationships between inflation and other macroeconomic variables, the table also reports results for the other samples reported for the earlier reduced-form regressions. The findings of no causal relationships from inflation to the output gap or labor share, and an incorrectly-signed relationship from inflation to the unemployment rate, turn out to be robust across each of the sub-samples.

The results for the Euro area also point against the Granger causality argument. For the output gap and the unemployment rate, the results always indicate either the non-existence of a causal relationship running from inflation, or the existence of a relationship with the wrong sign. For the full sample, the tests do point to inflation Granger causing the labor

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share, and with the correct sign. However, Table 3 shows that there is no evidence for a statistically significant role for the labor share in reduced-form regressions for Euro area inflation for this sample, so this is of little help in illustrating how the staggered contracting approach could be reconciled with the widespread evidence of a positive lagged dependent variable effect.

8 Conclusions

The staggered price contracting specification introduced by John Taylor (1979) is commonly used to illustrate the macroeconomic effects of nominal rigidities. This paper has focused on the ability of this approach to match the empirical evidence on inflation persistence. Some of the previous research on this issue has focused on whether the model can capture the high autocorrelations seen in the inflation data. We have shown here that staggered contracting models have no problem matching these autocorrelations: These models generally produce an inflation series whose autocorrelations are higher than those of the already-highly-autocorrelated driving variables, such as the output gap.

More importantly, though, the paper presents new results that illustrate staggered contracting's implications for an alternative aspect of inflation persistence or inertia, namely the positive dependence of inflation on its own lags. This feature of inflation, while closely related to high autocorrelations, represents a distinct definition of inflation persistence or inertia, and it is possible for a model to match one version of inflation persistence and not the other.

It is quite commonly assumed that staggered contracting models can provide a microfoundation for the type of inflation inertia implied by the positive dependence on lag terms seen in inflation regressions. However, this paper shows that staggered contracting models actually imply that these lag coefficients should be negative. This appears to present a serious problem for matching the contracting approach with the data. For while there are ongoing debates about the magnitude and stability of the lagged dependent variable effects on inflation, there is no evidence in favor of the predictions derived here of a pattern of negative coefficients on these variables.

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A   Solution for AR(1) Model for y

This appendix derives the solution for the model discussed in Section 5 with four-period contracts, no discounting ($\beta = 1$) and AR(1) output growth with autoregressive parameter $\rho = 0.9$. Recall from Section 3.4, that the three non-explosive roots in this case are

$$
\begin{aligned}
\lambda_1 &= -0.21385 - 0.27202i \\
\lambda_2 &= -0.21385 + 0.27202i \\
\lambda_3 &= 1
\end{aligned}
$$

The easiest route to a concrete solution here is to let $a = -0.21385, b = 0.27202$, and note from equation (22) that the inflation process can be written as

$$
\delta(L)\pi_t = \gamma\alpha(L)\frac{1-\rho^4}{1-\rho}\left(a+bi\right)\left(a-bi\right)E_t\left[\left(\sum_{k=0}^{\infty}\left(a+bi\right)^k F^k\right)\left(\sum_{k=0}^{\infty}\left(a-bi\right)^k F^k\right)\left(\sum_{k=0}^{\infty}F^k\right)y_t\right].
$$

This can be combined with the fact that

$$
E_t F^k y_t = \rho^k y_t,
$$

to give the solution

$$
\begin{aligned}
\delta(L)\pi_t &= \gamma\alpha(L)\frac{1-\rho^4}{(1-\rho)^2}\frac{a^2+b^2}{(1-\rho(a+bi))\left(1-\rho(a-bi)\right)}y_t \\
&= \gamma\alpha(L)\frac{\left(1-\rho^4\right)\left(a^2+b^2\right)}{(1-\rho)^2\left[(1-\rho a)^2+(\rho b)^2\right]}y_t.
\end{aligned}
$$

For a value of $\rho = 0.9$, this implies

$$
\pi_t = -0.43\pi_{t-1} - 0.12\pi_{t-2} + 2.78\gamma\alpha\left(L\right)y_t.
$$

B   Solution for Money Growth Model

Here, we derive the analytical solution for the money growth model discussed in Section 6. In particular, the solution is derived for the more general case of the model in which, in addition to the money growth shock, there is also a stochastic shock to monetary velocity:

$$
\begin{aligned}
y_t &= m_t - p_t + v_t, \\
v_t &= \rho_v v_{t-1} + \epsilon_t^v, \\
\Delta m_t &= \rho_m \Delta m_{t-1} + \epsilon_t^m.
\end{aligned}
$$

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The contract price process for this model can be written as

$$x_t = \frac{1-\gamma}{n} \sum_{k=0}^{n-1} E_t p_{t+k} + \frac{\gamma}{n} \sum_{k=0}^{n-1} E_t \left(m_{t+k} + v_{t+k} \right).$$

Following the same substitutions as in Section 3, this becomes

$$E_t \left[\left\{ \eta(F) - n \left(\frac{n+\gamma-1}{1-\gamma} \right) + \eta(L) \right\} x_t \right] = -\frac{n\gamma}{1-\gamma} \left(X_t^m + X_t^v \right),$$

where

$$\eta(x) = \sum_{k=1}^{n-1} (n-k) x^k,$$

and

$$X_t^m = \sum_{k=0}^{n-1} E_t m_{t+k}$$

$$X_t^v = \sum_{k=0}^{n-1} E_t v_{t+k}$$

The characteristic equation for this process has $n-1$ roots inside the unit circle, and $n-1$ other roots that are the inverses of these stable roots. Letting $\lambda_1, \lambda_2, ..., \lambda_{n-1}$ represent the $n-1$ roots inside the unit circle, the solution can be re-written as

$$E_t \left[\left\{ \prod_{i=1}^{n-1} (F-\lambda_i)(F-\frac{1}{\lambda_i}) \right\} L^{n-1} x_t \right] = -\frac{n\gamma}{1-\gamma} X_t. \tag{71}$$

Again, there is only one non-explosive solution and it takes the form

$$\left\{ \prod_{i=1}^{n-1} (1-\lambda_i L) \right\} x_t = -\frac{n\gamma}{1-\gamma} E_t \left[\left\{ \prod_{i=1}^{n-1} (F-\frac{1}{\lambda_i}) \right\}^{-1} X_t \right],$$

$$= -\frac{n\gamma}{1-\gamma} E_t \left[\left(\prod_{i=1}^{n-1} (-\lambda_i) \right) \left(\sum_{k=0}^{\infty} \lambda_1^k F^k \right) ..... \left(\sum_{k=0}^{\infty} \lambda_{n-1}^k F^k \right) \left(X_t^m + X_t^v \right) \right].$$

Note that the roots of the contract process polynomial—and thus the lag coefficients in the contract price solution—are not affected by the parameters determining the two stochastic shocks in the model, but depend only on n and γ.

The AR(1) process for velocity implies

$$X_t^v = \left(\sum_{k=0}^{n-1} \rho_v^k \right) v_t = \frac{1-\rho_v^n}{1-\rho_v} v_t.$$

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Thus, the velocity related expectational term is

$$-\frac{n\gamma}{1-\gamma}\left(\prod_{i=1}^{n-1}(-\lambda_i)\right)\left(\frac{1-\rho_v^n}{1-\rho_v}\right)\left(\frac{1}{\prod_{i=1}^{n-1}(1-\rho_v\lambda_i)}\right)v_t = \psi v_t$$

An AR(1) process for money growth implies

$$X_t^m = nm_t + \left(\sum_{k=1}^{n-1}(n-k)\rho_m^k\right)\Delta m_t$$

In light of this calculation, the forward-looking component of the solution can be computed by noting that

$$\sum_{k=0}^{\infty}\lambda^k E_t m_{t+k} = \frac{m_t}{1-\lambda} + \frac{1}{1-\lambda}\sum_{k=1}^{\infty}\lambda^k E_t \Delta m_{t+k}$$

$$= \frac{m_t}{1-\lambda} + \frac{1}{1-\lambda}\frac{\rho_m\lambda}{1-\rho\lambda}\Delta m_t,$$

and thus that one can calculate a discounted expected sum of any linear combination of m_t and Δm_t as follows

$$\sum_{k=0}^{\infty}\lambda^k E_t\left[am_{t+k} + b\Delta m_{t+k}\right] = a\sum_{k=0}^{\infty}\lambda^k E_t m_{t+k} + b\sum_{k=0}^{\infty}\lambda^k E_t \Delta m_{t+k}$$

$$= \frac{a}{1-\lambda}m_t + \left[\frac{a}{1-\lambda}\frac{\rho_m\lambda}{1-\rho_m\lambda} + \frac{b}{1-\lambda}\right]\Delta m_t.$$

One can repeatedly apply these calculations to obtain a solution for the contract price process of the form

$$x_t = \sum_{k=1}^{n-1}\mu_k x_{t-k} + \nu m_t + \varphi\Delta m_t + \psi v_t.$$

Finally, the coefficient on m_t is pinned down by the fact that if the money supply were to be constant at m^*, this economy must have $x_t = m^*$ as its long-run steady-state solution for the contract price. This requires that $\nu = 1 - \sum_{k=1}^{n-1}\mu_k$. This implies the solution given as equation (65) in the text.

$$x_t = \sum_{k=1}^{n-1}\mu_k x_{t-k} + \left(1 - \sum_{k=1}^{n-1}\mu_k\right)m_t + \varphi\Delta m_t + \psi v_t.$$

The lagged dependent variable coefficients in this specification depend on the roots of the characteristic polynomial, which in turn depend only on the values of n and γ. For the

case in which $n = 4$ and $\gamma = 0.5$ discussed in Section 6, the three roots inside the unit circle are

$$
\begin{aligned}
\lambda_1 &= -0.16975 - 0.22434i \\
\lambda_2 &= -0.16975 + 0.22434i \\
\lambda_3 &= 0.46127
\end{aligned}
$$

These values imply a contract price process of the form.

$$
x_t = .12x_{t-1} + .08x_{t-2} + .04x_{t-3} + .76m_t + \varphi\Delta m_t + \psi v_t
$$

The value of φ depends on the money growth autocorrelation parameter. The value of $\rho_m = 0.66$ used as the baseline in the text implies a contract price process of the form.

$$
x_t = .12x_{t-1} + .08x_{t-2} + .04x_{t-3} + .76m_t + 0.71\Delta m_t + \psi v_t
$$

where ψ depends on the autocorrelation coefficient for velocity. These calculations were arrived at by following the analytical steps described here and then checked using the numerical solution method of Binder and Pesaran (1995).

Note also that following the same steps as in Section 6.2, one arrives at a reduced-form inflation process of the form

$$
\pi_t = \sum_{k=1}^{n-1} \lambda_k \pi_{t-k} + \sum_{k=0}^{n} \delta_k^y y_{t+k} + \sum_{k=0}^{n} \delta_k^v v_{t+k}
$$

where the reduced-form coefficients on inflation are the same whether the velocity shock is included in the model are not.

Table 1: First-Order Autocorrelations

	United States	*Euro Area*
Inflation	0.892	0.872
Output Gap	0.862	0.856
Unemployment Rate	0.975	0.998
Labor Share	0.912	0.993

Notes: Sample for US results in 1960:1-2003:2, for the Euro Area results is 1970:2-2002:4

Table 2: Reduced-Form Regressions for US GDP Price Inflation

	Driving Variables			
	None	Output Gap	Unemployment	Labor Share
1960:1-2003:2				
Estimated $\rho(1)$	0.940	0.938	1.033	0.927
	(0.046)	(0.040)	(0.045)	(0.044)
Driving Variable p-value	*NA*	0.000	0.000	0.040
1960:1-1983:4				
Estimated $\rho(1)$	0.920	0.900	1.021	0.919
	(0.051)	(0.046)	(0.049)	(0.048)
Driving Variable p-value	*NA*	0.000	0.000	0.049
1984:1-2003:2				
Estimated $\rho(1)$	0.819	0.781	0.941	0.704
	(0.092)	(0.091)	(0.117)	(0.102)
Driving Variable p-value	*NA*	0.042	0.047	0.010
1991:1-2003:2				
Estimated $\rho(1)$	0.582	0.714	0.817	0.580
	(0.165)	(0.171)	(0.242)	(0.159)
Driving Variable p-value	*NA*	0.014	0.516	0.501

Notes: These results relate to regressions of the form $\pi_t = \alpha + \rho(1)\pi_{t-1} + \sum_{k=1}^{3} \psi_k \Delta\pi_{t-k} + \sum_{k=0}^{3} \gamma_i y_{t-k} + \epsilon_t$, where y_t is the driving variable listed in the column headings. Figures in brackets are Newey-West standard errors.

Table 3: Reduced-Form Regressions for Euro Area GDP Price Inflation

| | Driving Variables | | | |
	None	Output Gap	Unemployment	Labor Share
1970:2-2002:4				
Estimated $\rho(1)$	0.960	0.976	0.884	0.891
	(0.038)	(0.035)	(0.066)	(0.111)
Driving Variable p-value	*NA*	0.000	0.038	0.502
1970:2-1983:4				
Estimated $\rho(1)$	0.675	0.853	0.800	0.939
	(0.156)	(0.123)	(0.147)	(0.261)
Driving Variable p-value	*NA*	0.000	0.333	0.174
1984:1-2002:4				
Estimated $\rho(1)$	0.832	0.877	0.832	0.486
	(0.062)	(0.057)	(0.077)	(0.133)
Driving Variable p-value	*NA*	0.180	0.078	0.010
1991:1-2002:4				
Estimated $\rho(1)$	0.836	0.754	0.914	0.515
	(0.131)	(0.129)	(0.198)	(0.270)
Driving Variable p-value	*NA*	0.067	0.130	0.012

Notes: These results relate to regressions of the form $\pi_t = \alpha + \rho(1)\pi_{t-1} + \sum_{k=1}^{3} \psi_k \Delta \pi_{t-k} + \sum_{k=0}^{3} \gamma_i y_{t-k} + \epsilon_t$, where y_t is the driving variable listed in the column headings. Figures in brackets are Newey-West standard errors.

Table 4: Reduced-Form Inflation Coefficients for Money Growth Model

	λ_1	λ_2	λ_3	$\sum_{k=1}^{3} \lambda_k$
$\gamma = 0.1$				
$\rho_m = 0.50$	-0.48	-0.13	0.09	-0.52
$\rho_m = 0.66$	-0.45	-0.07	0.16	-0.36
$\rho_m = 0.80$	-0.40	0.02	0.27	-0.11
$\gamma = 0.2$				
$\rho_m = 0.50$	-0.50	-0.14	0.12	-0.52
$\rho_m = 0.66$	-0.47	-0.07	0.20	-0.33
$\rho_m = 0.80$	-0.41	0.03	0.33	-0.04
$\gamma = 0.3$				
$\rho_m = 0.50$	-0.52	-0.15	0.14	-0.53
$\rho_m = 0.66$	-0.48	-0.07	0.24	-0.31
$\rho_m = 0.80$	-0.42	0.03	0.38	-0.01
$\gamma = 0.5$				
$\rho_m = 0.50$	-0.55	-0.16	0.17	-0.54
$\rho_m = 0.66$	-0.50	-0.08	0.28	-0.30
$\rho_m = 0.80$	-0.44	0.04	0.44	-0.03
$\gamma = 1.0$				
$\rho_m = 0.50$	-0.59	-0.19	0.21	-0.56
$\rho_m = 0.66$	-0.55	-0.09	0.35	-0.29
$\rho_m = 0.80$	-0.49	0.02	0.54	-0.07
$\gamma = 3.0$				
$\rho_m = 0.50$	-0.70	-0.27	0.31	-0.65
$\rho_m = 0.66$	-0.65	-0.16	0.50	-0.32
$\rho_m = 0.80$	-0.60	-0.03	0.72	0.10

Notes: Refers to coefficients in equation (67) for various values of γ (elasticity of real marginal cost with respect to output) and ρ_m (autocorrelation of money growth).

Table 5: Granger Causality Tests for US GDP Price Inflation

	Driving Variables:		
	Output Gap	Unemployment	Labor Share
1960:1-2003:2			
Estimated $\beta_1 + \beta_2 + \beta_3 + \beta_4$	-0.021	0.031	0.024
	(0.032)	(0.011)	(0.022)
p-value for $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$	0.618	0.007	0.623
1960:1-1983:4			
Estimated $\beta_1 + \beta_2 + \beta_3 + \beta_4$	-0.018	0.038	0.011
	(0.039)	(0.014)	(0.024)
p-value for $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$	0.569	0.011	0.773
1984:1-2003:2			
Estimated $\beta_1 + \beta_2 + \beta_3 + \beta_4$	0.007	0.072	0.024
	(0.063)	(0.026)	(0.076)
p-value for $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$	0.952	0.039	0.958
1991:1-2003:2			
Estimated $\beta_1 + \beta_2 + \beta_3 + \beta_4$	-0.198	0.111	-0.043
	(0.108)	(0.043)	(0.093)
p-value for $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$	0.249	0.018	0.938

Notes: These results relate to regressions of the form $y_t = \alpha + \sum_{k=1}^{4} \rho_k y_{t-k} + \sum_{k=1}^{4} \beta_k \pi_{t-k} + \epsilon_t$, where y_t is the driving variable listed in the column headings and π_t is inflation. Figures in brackets are Newey-West standard errors

Table 6: Granger Causality Tests for Euro Area GDP Price Inflation

	Driving Variables:		
	Output Gap	Unemployment	Labor Share
1970:1-2002:4			
Estimated $\beta_1 + \beta_2 + \beta_3 + \beta_4$	-0.006	0.012	0.105
	(0.012)	(0.005)	(0.022)
p-value for $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$	0.042	0.186	0.000
1970:1-1983:4			
Estimated $\beta_1 + \beta_2 + \beta_3 + \beta_4$	-0.113	0.027	0.178
	(0.041)	(0.010)	(0.037)
p-value for $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$	0.000	0.063	0.000
1984:1-2002:4			
Estimated $\beta_1 + \beta_2 + \beta_3 + \beta_4$	0.004	0.001	0.109
	(0.033)	(0.006)	(0.040)
p-value for $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$	0.176	0.610	0.063
1991:1-2002:4			
Estimated $\beta_1 + \beta_2 + \beta_3 + \beta_4$	0.033	0.041	0.193
	(0.065)	(0.015)	(0.071)
p-value for $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$	0.495	0.040	0.104

Notes: These results relate to regressions of the form $y_t = \alpha + \sum_{k=1}^{4} \rho_k y_{t-k} + \sum_{k=1}^{4} \beta_k \pi_{t-k} + \epsilon_t$, where y_t is the driving variable listed in the column headings and π_t is inflation. Figures in brackets are Newey-West standard errors

Figure 1

Why Taylor Contract Inflation is More Autocorrelated Than Its Driving Variable