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Semi-Analytical Method for the Extraction of the System Parameters in Application to Kinetic Energy Harvesters

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***Abstract*—In this paper, we propose a technique to extract system parameters of nonlinear MEMS devices using a combination of model reduction and nonlinear optimization. The model is tested on a MEMS energy harvesting device employing magnetic actuation and piezoelectric energy conversion.**

***Index Terms*—Energy harvesting, nonlinear oscillators, piezoelectric, harmonic balance**

I. INTRODUCTION

The extraction of the system parameters of nonlinear MEMS devices can be quite a challenging problem. Often, the only way to obtain these parameters is the fitting of experimentally obtained resonance curves (in the form of amplitude-frequency or voltage-frequency characteristics) with some pre-assumed lumped linear or nonlinear models (in the form of ordinary differential equations). This approach is usually implemented numerically and requires one to integrate differential equations many times with different parameters until a good fit is achieved. However, if a MEMS device is indeed nonlinear and there are many parameters to fit, the problem becomes very time and resource consuming. In this paper, we show a semi-analytical technique that reduces numerical integration to a nonlinear optimisation problem and allows a parameter extraction for nonlinear MEMS. As an example, we test this approach on a piezoelectric MEMS energy harvester and show very good agreement with its experimental characterisation.

II. STATEMENT OF THE PROBLEM AND METHOD

The most recent generation of microscale kinetic energy harvesters (KEHs) that convert mechanical vibrations to electricity is nonlinear [1], [2], and the modelling of such devices is a challenging task. Different physical mechanisms are used in these devices to facilitate energy conversion, and, hence, one generally distinguishes electrostatic kinetic energy harvesters (eKEHs) [3], [4], electromagnetic kinetic energy harvesters (emKEHs) [5], [6] and piezoelectric kinetic energy harvesters (pKEHs) [7], [8]. In this paper, we consider a kinetic energy harvester that is actuated through a magnetic force generated by an AC current acting on a magnetic mass. In addition, the force causes a deformation of a cantilever with a piezoelectric layer as shown in Fig. 1, and, for this reason, this harvester can be seen as a kind of piezoelectric kinetic harvesters.

One typical problem arising in the modelling of microscale resonators and, in particular, KEHs is the extraction of the parameters responsible for the dynamical behaviour of the system: nonlinear spring coefficients, air damping coefficient, etc. Usually, one has access to electrical signals in the system (current and voltage) while the extraction of the aforemen-

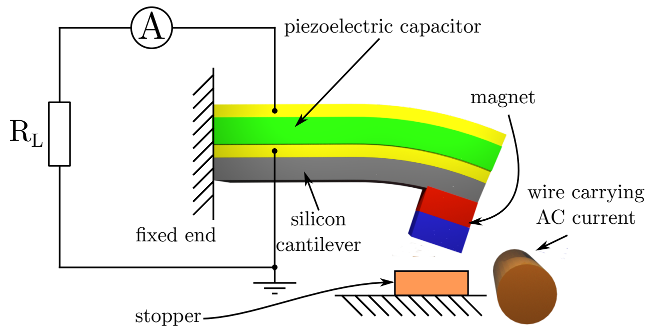


Fig. 1: The schematic diagram of the kinetic energy harvester used for characterisation in this study.

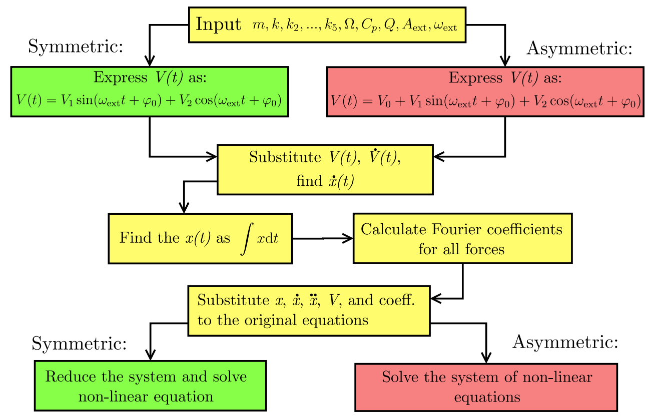


Fig. 2: The algorithm demonstrating the proposed semi-analytical method for kinetic energy harvesters with symmetrical and asymmetrical forces.

tioned parameters requires access to mechanical signals. The typical lumped model of a KEH can be described in terms of the Newton's second law:

$$m\ddot{x}+c\dot{x}+k(x)\cdot x+f_{\text{stop}}(x)+\Omega V-mA_{\text{ext}}\cos(\omega_{\text{ext}}t)=0, \quad (1)$$

where x is the displacement of the mobile mass or cantilever, V is the voltage generated in its piezoelectric layer, m is the movable mass of a harvester, Ω is the coupling factor, f_{stop} is the force of the impact between the movable mass of the device and stoppers, $k(x)$ is the nonlinear spring force, c is the damping coefficient, A_{ext} is the amplitude of external vibrations and ω_{ext} is their angular frequency.

The force due to stoppers is set to zero if the amplitude of KEH oscillations is small and the movable mass does not reach the stoppers. In the case when the movable mass hits the stopper, the impact is modelled as a stiff spring:

$$f_{\text{stop}}=\begin{cases} k_{\text{stop}}\cdot x, & \text{if mass hits the stopper} \\ 0, & \text{if mass does not hit the stopper} \end{cases} \quad (2)$$

In the most general case, the presence of a piecewise stopper force causes strong nonlinear behaviour. The equation in the electrical domain, completing the formulation of the system, is obtained from Kirchhoff's voltage law:

$$C_{\text{p}}\frac{\mathrm{d}V}{\mathrm{d}t}=\Omega\dot{x}-\frac{V}{R_{\text{L}}}, \quad (3)$$

where C_{p} is the capacitance of the piezoelectric layer and R_{L} is the load resistance.

The dynamics of the system can be explored through numerical simulations of the system of nonlinear coupled differential equations (1)–(3). A reliable result can be obtained by the use of any higher-order integration technique. The use of a lower-order integration method with a fixed step also provides an acceptable result in this case (see Fig. 3a) even though the integration of piecewise equations is involved.

However, to extract the parameters of the system, we propose a semi-analytical method based on the harmonic balance method. The main idea is to represent the dynamical variables as a combination of multiple harmonics. For instance, if, for simplicity, we write the signals in terms of the first harmonic, we obtain:

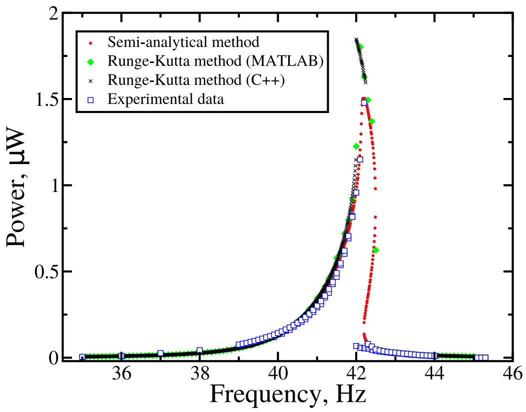
$$V(t)=V_0+V_1\sin(\omega_{\text{ext}}t)+V_2\cos(\omega_{\text{ext}}t). \quad (4)$$

We note that in the conventional implementation of the method, the decomposition is applied to the displacement x and velocity v. Knowing V_0, V_1 and V_2, one can obtain the voltage and the converted power P.

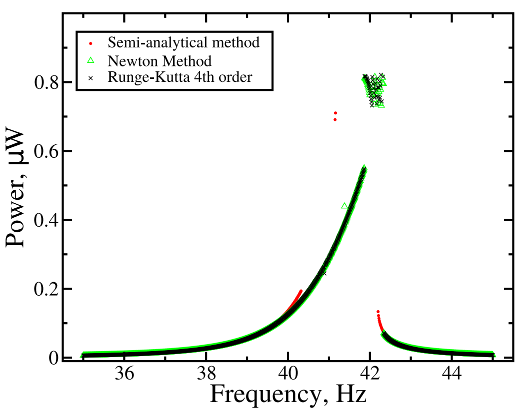
The implementation of the method for systems with and without impacts are very similar:

1) Represent the voltage using (4). For systems with symmetrical impacts and springs, the DC voltage component is zero: $V_0=0$.
2) Find the derivative $\dot{V}(t)$ and substitute V and $\dot{V}$ into equation (3) to get $\dot{x}(V_1,V_2,t)$. The resulting function is clearly a combination of $\sin(\omega_{\text{ext}}t)$ and $\cos(\omega_{\text{ext}}t)$ multiplied by time-independent coefficients.
3) Find $x(V_1,V_2,t)$ by integrating $\int\dot{x}(V_1,V_2,t)\mathrm{d}t$. The integration constant is equal to zero since we assume that the coordinate of the cantilever is zero in equilibrium.
4) Represent all forces acting on the system as a Fourier series. For example, a representative nonlinear spring force can be approximated as follows [9]:

$$k(x)\cdot x=kx+k_2x\cdot|x|+k_3x^3+k_4x^3|x|+k_5x^5. \quad (5)$$



(a)



(b)

Fig. 3: The converted power generated in the harvester: (a) without impact with the stopper and (b) with impact. The graphs show the experimental characterisation of the devices compared with the semi-analytical modelling and numerical simulations with the extracted parameters from Table I.

TABLE I: Parameters of the piezoelectric KEH

Parameter	Value
Proof mass (m)	7.36×10^{-5} kg
Quality factor (Q)	63.55
Coupling factor (Ω)	4.34×10^{-6} N/V
Piezoelectric capacitance (C_p)	2.1×10^{-9} F
Load resistance (R_{L})	1.9×10^{6} Ω
External acceleration (A_{ext})	$0.18g \simeq 1.76$ m/s^2
Spring parameters:	
k	4.65 N/m
k_2	6.53×10^{2} N/m^2
k_3	4.14×10^{4} N/m^3
k_4	-1.56×10^{8} N/m^4
k_5	-4.01×10^{9} N/m^5

In this expression, for the second term we can write:

$$k_2 x|x| \simeq A_{k2}(V_1, V_2)\sin(\omega_{\text{ext}}t) + B_{k2}(V_1, V_2)\cos(\omega_{\text{ext}}t), \tag{6}$$

and the coefficients can be calculated as:

$$A_{K2}(V_1, V_2) = \int_0^{2\pi/\omega_{\text{ext}}} x(V_1, V_2, t)\cdot |x(V_1, V_2, t)|\sin(\omega_{\text{ext}}t)\mathrm{d}t, \tag{7}$$

$$B_{K2}(V_1, V_2) = \int_0^{2\pi/\omega_{\text{ext}}} x(V_1, V_2, t)\cdot |x(V_1, V_2, t)|\cos(\omega_{\text{ext}}t)\mathrm{d}t. \tag{8}$$

It may be challenging to calculate these expressions for piecewise continuous functions; however, this can be done analytically for many functions.

5) The obtained expressions are substituted into equation (1) and grouped as containing the sin and cos functions:

$$\xi(V_1, V_2)\sin(\omega_{\text{ext}}t + \varphi_0) + \chi(V_1, V_2)\cos(\omega_{\text{ext}}t + \varphi_0) + mA_{\text{ext}}\cos(\omega_{\text{ext}}t) = 0, \tag{9}$$

which results in a system of two nonlinear equations. In the case of symmetrical forces, they are further reduced to one nonlinear equation:

$$\xi(V_1, V_2)^2 + \chi(V_1, V_2)^2 = (mA_{\text{ext}})^2. \tag{10}$$

III. Results and Discussion:

This algorithm allows one to reduce the system of differential equations (1)–(3) to one nonlinear algebraic equation (10), which is significantly easier to solve numerically and to optimise (if one solves the problem of parameter extraction). Moreover, this method allows one to avoid problems of instability of differential schemes for strongly nonlinear systems. Semi-analytical methods are well-developed for MEMS and are known to be very useful tools [10]. This algorithm has been applied to extract the parameters of the nonlinear harvester described in [8], and the extracted parameters are listed in Table I. They are very closed to the ones that have been found in [9] using purely numerical simulations.

The comparison between the Newton and Runge-Kutta integration methods applied to the dynamical (differential) equations and the proposed semi-analytical method shows a very good agreement with the experimental characterisation of the device in non-impact mode (see Fig. 3a). In the case of impact-mode operation where the mobile mass experience collisions with the stopper, the methods (in particular the semi-analytical one) may provide somewhat different results due to the issues with determining the gradient and the Hessian of the equivalent optimization problem (see Fig. 3b). As a conclusion, the described semi-analytical method allows one to extract the parameters of KEHs in a very fast and accurate manner in the cases of symmetrical forces, but improvements of the method are required when one deals with asymmetrical spring and stopper forces.

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