

Urban spatial location advantage: the dual of the transportation problem and its implications for land-use and transport planning

Introduction

Recent debates in the academic literature relating to transport and land-use have (among other things) been heavily concerned with issues surrounding spatial interaction, land-use arrangements and the efficiency of the transport network. In particular, research has tended to concentrate on major issues such as jobs-housing balance, excess commuting and accessibility issues (see Horner, 2004). Even though journey-to-work trips generally account for a minority of overall daily trip making (Horner, 2004), much of the focus of the research has been on commuting and particularly on the location of jobs and housing and the increased efficiency and sustainability of travel patterns which may result from a greater juxtaposition of jobs and housing. This issue, which ties in directly with the concept of excess commuting, are the most relevant for the current study.

The evidence demonstrating the nexus between jobs-housing balance and more efficient commuting is variable. Some scholars (Giuliano and Small, 1993; Cervero, 1989; Horner, 2002; Sultana, 2002; Zhao et al, 2010) have found that a better jobs-housing balance does in fact produce more efficient commuting patterns but others (Wachs et al, 1993; Peng, 1997) have found little evidence to support the assertion. In particular, Cervero (1989) argues that if urban areas are designed with a high jobs-housing imbalance then commuting will be greater than it could be under alternative (more efficient) jobs-housing arrangements. Nevertheless, the effectiveness of policies aimed at promoting a greater jobs-housing balance in urban areas continues to be unclear despite the fact that they are high on the policy agenda in many European and US cities.

Central to the jobs-housing debate has been the concept of excess commuting where a linear programming method is used to optimise spatial interaction between home and work locations within the context of a cost-minimising objective. These cost minimising solutions have been used as a proxy for the level of jobs-housing balance across metropolitan areas. They have been compared with observed

commuting patterns to determine the 'efficiency' of trip making whereby any deviation from the cost-minimising solution is considered to be excessive (Murphy, 2009). Excess commuting is a term which refers to the empirical difference between the average minimum commute necessitated by the existing distribution of jobs and housing in urban areas and the observed commute (White, 1988; Frost et al, 1998; Horner, 2002; Loo and Chow, 2010; Murphy and Killen, 2011). The greater the deviation between observed commuting patterns and the minimum required by the morphology of jobs-housing arrangements, the more 'excessive' the observed commuting patterns are in urban areas. In excess commuting studies, the average minimum commuting cost is usually calculated using the transportation problem of linear programming (TPLP) which is a specific case of the linear programming problem (see Killen, 1983).

While the TPLP has been used extensively in the literature for transportation planning issues, little attention has been paid to the companion dual solution. This is unfortunate because as will be shown here, the dual has a specifically spatial interpretation. In particular, from a spatial planning perspective, dual solutions provide an indicator of the relative spatial advantage of individual zonal units which can be mapped. Moreover, the dual can provide additional information about where additions to the transport network would be most or least efficient; it is possible also to determine where new trips entering the system should originate and terminate in order to maintain optimum efficiency for spatial interaction purposes. The current paper demonstrates the use for planning purposes of dual solutions of the TPLP within the context of Dublin, Ireland.

The Dual of the Transportation Problem of Linear Programming (TPLP)

Description

Linear programming problems have two companion solutions: the *primal* solution and the *dual* solution. The TPLP allocates resources from an area of supply to an area of demand in order to minimise costs. This is known as the primal solution to the TPLP. The companion dual solution is formulated in terms of an objective of opposite direction i.e. in this case - cost maximisation (see Hay, 1977; Killen, 1979, 1983). In the case of the TPLP, this objective can be interpreted as maximising the return to be realised from

allocating resources from supply to demand areas. Specifically, the objective function of the dual is to maximise the difference between the cost of a unit of resource at a supply area and the cost of receiving a unit of that resource at a demand area. The relative differences in these costs on a zone by zone basis are referred to as *shadow prices*. Under conditions of optimal flow, a destination shadow price differs from an origin shadow price by the difference in the transport cost associated with the two routes (see Killen, 1983). Thus, in terms of transport costs, the dual values for a set of supply or demand areas may be interpreted as the relative advantage of one location over another within the context of the optimal solution. In the case of origins (home in the case of home to work trips), a higher shadow price implies locational advantage; in the case of destinations (work in the case of a home to work trip), a higher shadow price implies locational disadvantage.

Formulation and Interpretation

Assume V_j is the associated shadow price for a unit of resource at a destination j (d_j) and U_i is the shadow price for a unit of resource at an origin i (o_i). Then, the dual problem maximises S , the value added to the unit of resource by the transfer between origin and destination (i.e. the transport cost). Using the same notation as for the primal of the TPLP, the dual transportation problem is to find U_i and V_j such as to:

$$\text{Maximise } S = \sum_j^n d_j V_j - \sum_i^m o_i U_i \quad (1)$$

$$\text{s.t. } V_j - U_i \leq c_{ij} \text{ for all } i, j \quad (2)$$

where U_i = the marginal cost of increasing o_i ; V_j = the marginal cost of increasing d_j ; and both U_i and V_j are unrestricted in sign. Constraint (2) ensures that the shadow price difference between each origin and destination must be less than or equal to the transportation costs. To elaborate, if we assume that $(V_i - U_i)$ and c_{ij} are the shadow price differential and the per unit transport costs respectively, it is only profitable to bring a route into use if the shadow price differential exceeds the per unit cost of transport. Mathematically, therefore, it is only profitable to bring a route into use if:

$$(V_j - U_i) - c_{ij} \geq 0 \quad (3)$$

If it is a requirement that condition (3) holds for a route to be brought into use, then constraint (2) ensures that at optimality, $V_j - U_i = c_{ij}$ for each route being used.

For a set of m origins with a set of m associated shadow prices U_i, \dots, U_m , one may interpret origins with the highest associated shadow prices as those with the most advantageous location within the optimum flow pattern. This is logical since an advantageous location is one which sends its flows along relatively low cost route(s) to a destination where it commands a relatively high price. The marginal location advantage of one origin location over another is given by the shadow price difference. In the literature, the marginal shadow price differential between each origin is a measure of locational premium and is generally referred to as *location rent* (Stevens, 1961; Maxfield, 1969; Pitfield and Benabi, 1982). Following Stevens (1961) one can determine the spatial rent that one location can demand over another based on its ability either as origin or destination to send out or receive flow over relatively low costs routes.

The interpretation of dual values is different for demand areas. For a set of n destinations with n associated shadow prices V_i, \dots, V_n , one may interpret destinations with the highest associated shadow price as those with the most disadvantageous location relative to the overall flow pattern. This is logical since a disadvantageous location is one which receives its flows via relatively high cost route(s). In the literature, the marginal locational disadvantage of one destination over another is given by the shadow price difference.

Figure 1. Schematic of shadow prices for hypothetical city regions

How then might one expect the pattern of location advantage to look for typical urban structures? To illustrate, consider a city region where the bulk of employment (trip destinations) is located in the central area and where the volume of residences (trip origins) declines gradually from the central area outwards (Figure 1(a)). In this case, the pattern of location advantage will decline from the central area outwards. Assuming that the distribution of residences remains similar to that of Figure 1(a), the opposite will be true of a city region where the bulk of employment is located at the periphery (Figure 1(b)). In this situation, location advantage will decline from the periphery towards the central area. Consider also a city region where considerable employment is located in the city centre and at surrounding sub centres and where the

volume of residences declines gradually from the central area and from sub centres outwards (Figure 1(c)). In this instance, the pattern of location advantage will decline from the central area to the periphery and from sub centres outwards.

Figures 1(a-c) effectively assume that ease of movement is equal in all directions. Figure 1(d) shows the expected impact on the solution in Figure 1(a) of a high quality transport corridor: the zones served by this corridor gain spatial advantage. The high quality radial corridors have the effect of reducing inter-zonal separation for trips that use them thereby conferring advantage on the zones that they serve. Thus, from a planning perspective, one of the advantages of mapping dual solutions is that it provides a method of quantifying the impact of such corridors.

Related research

Mapping shadow prices can be extremely useful in providing a basis from which future planning decisions can be made. One example within a planning context is that of Maxfield (1972) who used the TPLP to determine the assignment of students to schools that minimised total travel distance. The dual solutions were then analysed in order to determine the relative advantage/disadvantage of one school location over another. The results suggested that students near core areas tended to have the greatest efficiency with respect to location but that many of the schools in these areas were overcrowded, precisely because of their locational advantage. Maxfield concluded that the dual analysis would allow a determination of a school transportation tax based on residential location, or a tax increase for parents who moved to suburban areas thereby burdening outlying school districts with transporting their children to school.

Other examples of the use of the dual of the TPLP within a planning context are those of Maxfield (1969) who determined the minimum cost method of satisfying the overseas demand for United States hard red spring wheat in 1964. In similar research, O'Sullivan (1972) determined the optimal patterns of deliveries (from a distance minimising perspective) for a selection of commodity groups within the UK while Lord (1972) conducted similar research for the United States broiler industry. Horton and Wittick (1969) used the transportation model to analyse work trips in Waco, Texas. They suggested that maps of the dual

values could be used to identify areas which are disadvantageously located in terms of residences and/or workplaces. In fact, Horton and Wittick (1969) showed that it is possible to increase the total number of workers in an urban area and yet decrease the aggregate distance travelled provided that supply and demand additions are made at the optimal locations. This is something that must surely interest transport and land-use planners given the obvious wide-ranging implications for environmental sustainability. Theoretically at least, it implies that careful additions to the residential and employment stock can be made in such a way as to minimise increases in distance travelled or even reduce it.

In each of the foregoing studies, emphasis is placed on the shadow prices of origins and destinations and hence of the overall geography of locational advantage/disadvantage. Looking at the literature as a whole, it is worth remarking that despite its potential importance within a planning context, the employment of dual solutions to inform land-use and planning decisions has been very limited. To some extent, this is explained by the fact that many of the relevant studies were conducted in the 1960s and 1970s when the availability of suitable data and the computational capacity of computers were relatively limited.

Study area, data and objectives

Dublin

Prior to the late 1990s, Dublin was best characterised as having an essentially monocentric urban structure where the majority of employment was located in the city centre with scattered employment at the periphery (Murphy, 2004; MacLaran et al, 2010). After the Celtic Tiger economic boom beginning in the 1990s, the morphology of Dublin's built environment witnessed dramatic changes. This period of prosperity was characterised by industrial restructuring and in particular, a move towards more service-based industry away from heavy manufacturing (Williams and Shiels, 2002). A substantial proportion of this new service-based employment has located in the suburbs while many older manufacturing industries have also relocated to the there. This has led to a major restructuring in the development of office and industrial space away from the central towards 'edge city' locations at the periphery (see MacLaran etl al, 2010). Simultaneously, rapid population increases fed a rapid acceleration in housing development and much of that development has

occurred in a highly dispersed manner at the periphery of the region and beyond (Williams and Shiels, 2002; MacLaran, 2010). The failure to provide adequate public transport infrastructure to facilitate emerging employment centres at the periphery has led to increased demand for car-based commuting with its associated negative environmental consequences for the region. A monocentric city, with the majority of employment and service functions located in the core and with the majority of residences located in outlying areas, generates a primarily radial pattern of trip making with significant flows along a limited number of major radial routes. By way of contrast, a city where residential and non-residential functions are more interspersed (i.e. polycentric cities), as is now the case in Dublin, generates a more diffuse pattern of trip making. Specifically, these land use changes have created geographies of travel that have become increasingly complex and dispersed characterised by greater numbers of inter-suburban, cross-city and reverse commutes with a greater reliance on the private car to serve those trips. Between 1991 and 2002, the proportion of individuals driving to work increased in all counties of the Greater Dublin Area (GDA) with the greatest increases, in relative terms, occurring in the outlying counties (Murphy, 2009). Over the same period, the proportion of public transport trips decreased despite significant public transport investment. Thus, the revised pattern of journey to work flows appears to have placed a premium on car-based travel.

Data

The area of the current study comprises most of the Greater Dublin Area (GDA)¹. The data for the study was derived from a Dublin Transportation Office (DTO) traffic simulation model for the 2001 peak (8-9AM) and off-peak periods (2-3PM) and a comparable DTO model for the same periods in 1991. The total number of trips recorded for 2001 was 327 001 while the corresponding figure for 1991 was 234 834. The model has two main components: the Highway Model (primarily car based) and the PT Model (public transport based). In the PT model both bus and rail trips are modelled and interchanges between PT modes is permitted. The model is based upon a 463 zonal sub-division of the Dublin Region and is derived primarily

¹ The Greater Dublin Area (GDA) consists of the four administrative districts of county Dublin as well as the surrounding counties of Meath, Kildare and Wicklow.

from the Irish District Electoral Division (DED) system. The model components have been independently validated to within five percent of actual travel patterns using goodness-of-fit statistics; WSP concluded that the basic structure of the overall model was appropriate for modelling Dublin's traffic and that 'the current model components are in line with expectations' (WSP, 2003, 88).

The 1991 and 2001 data sets are based on the same zonal units. The fact that temporal data is available for analysis is useful because it means that changing relative locational advantage and disadvantage can be deciphered within the context of the rapid land-use and transportation re-organisation that took place in Dublin over the period (see Vega and Reynolds-Feighan, 2008, 2009; Murphy, 2009). Two types of journey-to-work flow matrices were available for the peak and off-peak period: private transport trips and public transport trips. Thus, the data was such that, unusually for such studies, comparisons could be made by time of day as well as between 1991 and 2001.

Road network distances were available for each mode of transport thus this was taken as a proxy for the cost of physical separation between zonal units. Other costs can be used including Euclidean distance and travel time which have been used in previous studies (see Horner, 2002; Merriman et al 1995); and also some generalised cost measure is possible although data availability is a major issue in this case. Intra-zonal travel distances were estimated by assuming that each zonal unit is approximately circular in shape (see Frost et al, 1998). In a similar manner to other studies, this study excluded those trips originating and destined for locations outside the study boundary (see Frost et al, 1998; Horner, 2002; Murphy, 2009).

The current analysis had three objectives. The first was to determine and map patterns of locational advantage for each trip origin (U_i values) and destination (V_j values) within the study area. This necessitated solving the dual of the TPLP. The second was to compare the degree to which marginal locational advantage differs between public and private modes of transport. The third was to determine where further residential and employment additions should be located if optimum commuting efficiency is to be maintained.

Results

Figures 2 and 3 show the shadow prices (dual variables) for trip origins and destinations for the morning commute for public and private transport in the study area for 1991 and 2001. In order to maintain consistency of interpretation where green and red are areas of locational advantage and disadvantage respectively, the legend colouring is inverted for origin and destination shadow prices. Each legend represents sextiles. Bearing in mind the foregoing interpretations placed on the shadow prices for origins and destinations, a number of interesting points emerge from the results.

Figure 2. Peak origin and destination shadow prices for public transport, 1991 and 2001

Figure 3. Peak origin and destination shadow prices for private transport, 1991 and 2001

First, and looking at the origin and destination shadow prices for both modes, it is clear that in locational terms, the central area² possesses a considerable degree of spatial advantage over more peripheral locations. The essential spatial picture demonstrated is that of a monocentric city as per Figure 1(a) despite the fact that considerable employment decentralisation took place in the GDA over the period. The fact that workers originating in the central area can travel to their destinations along low cost routes (i.e. within relatively close spatial proximity) confers spatial advantage upon them relative to workers originating at the outskirts who must travel via high cost routes to reach their destinations. In spatial terms, the higher dual variable values for central city zones as origins are representative of the locational rent premium attributable to those zones due to their greater accessibility to employment opportunities. Similarly, because city centre zones receive workers over relatively low cost routes, their shadow prices as destinations are high. As can be seen, the converse is true of peripheral locations where the shadow prices are low for origins and high for destinations.

Second, it is clear that in spatial terms the central area has become even more attractive in locational terms between 1991 and 2001 for users of both public and private transport. Visually, it can be seen that the area of relative advantage at the centre has moved outwards specifically for private transport. This is due to the movement of many jobs to inner suburban areas which confers relative advantage upon those locations.

² The central area is taken to be the area within the outer 'C'-ring motorway.

Overall, the greater juxtaposition of jobs-housing opportunities in these areas allows for more efficient spatial interaction in terms of journey distance travelled.

Third, looking specifically at public transport, it can be seen that the pattern of location advantage has become more consolidated in the central area between 1991 and 2001. In relative terms, it seems that some locations at the periphery have lost their ability to send and receive workers via low cost routes since 1991. This suggests that the public transport system has become even more focussed on the central area over the period despite the aforementioned decentralisation of many employment functions and the development of residences at the periphery. In fact, recent public infrastructure developments such as the Luas light rail system have a strong city centre focus. A similar trend is visible when one considers the spatial information for locational advantage by private transport (Figure 3). Taken together, the results for public and private transport demonstrate that the primary pattern of location advantage has not altered significantly between 1991 and 2001. On the contrary, it has become even more focussed on the central area in a manner similar to that depicted in Figure 1(a).

Finally, because dual solutions measure the marginal cost of increasing demand and supply at each origin and destination in the study area, they suggest information about where additions to the housing and employment stock may be provided given existing demand and supply imbalances within the system. In the case of the current study, origins with low shadow prices indicate areas that are disadvantageously located in terms of, on the one hand, residential places available (low U_i values) and, on the other, job opportunities (high V_j values). Looking at the results emerging from Figures 2 and 3, the data suggests that further supply (residences) or demand (employment) additions, should be targeted strategically at outlying areas which are disadvantageously located in terms of residential places and job opportunities available relative to the overall optimum flow pattern.

Figure 4. Off-peak origin and destination shadow prices for public transport, 1991 and 2001

Figure 5. Off-peak origin and destination shadow prices for private transport, 1991 and 2001

Turning to the off-peak results (Figure 4 and 5), they include work and non-work trips but are heavily weighted towards the latter. Thus, the differences between the two sets of maps reflect the differing spatial

patterns of supply and demand during the two travel periods. The results emerging are noteworthy for a number of reasons. First, there has been a dramatic shift in the spatial distribution of location advantage in Dublin for off-peak trip-making between 1991 and 2001 for private transport in particular. Effectively, the pattern of advantage has transferred from the central area to the periphery. The decentralisation of many of off-peak opportunities such as retailing, recreation, leisure and entertainment (see Murphy, 2004; MacLaran et al, 2010) means that they can now be accessed via much lower cost routes at the periphery rather than in the central area. Thus, the pattern of spatial advantage and the associated distribution of origin and destination activities has moved more towards Figure 1 (b). This dramatic change reflects the fact that at the level of the individual, off-peak destinations can often be changed very easily whereas work locations are relatively fixed. The off-peak results for private transport show that travel patterns and hence relative locational advantage/disadvantage can alter very rapidly over relatively short periods of time. The fact that peripheral locations are advantageously located for private transport users in the off-peak may have implications for residential location decision-making. If, as some have surmised, the journey-to-work is becoming less important in residential location decisions, it may be that the locational advantage of outer locations for off-peak activities is a significant pull factor for those making residential decisions.

By way of contrast to private transport, the pattern of relative locational advantage for off-peak trips by public transport has remained relatively stable over the period. As with the peak public transport maps for 1991, there is evidence that the railway lines to the west and along the southeast coast confer a significant advantage upon public transport users for both trip origins and destinations thus conforming to the hypothetical pattern outlined in Figure 1 (d).

Overall, the results for car and public transport indicate that the decentralisation of off-peak functions has occurred in a manner which confers spatial advantage upon peripheral private transport users but not upon their public transport counterparts. From this perspective, one could argue that considerable thought needs to be given to the way in which public transport is provided in the off-peak period. Given that the pattern of spatial interactions appears to be radically different for this period, the results suggest that the provision of public transport services should not simply be a 'watered-down' version of what is provided in

the peak period. Rather, it may be the case that the entire public transport route network needs to be altered considerably during the off-peak period if it is to cater adequately for the actual patterns of travel demand.³

Conclusion

The foregoing results demonstrate that in terms of the morning commute period, Dublin continues to have a relatively dominant monocentric-like city (akin to Figure 1(a)). The results show that the central area is the most advantageously located both as an origin and destination. However, in relation to the off-peak period, the spatial organisation of land-uses is more similar to that described in Figure 1(b); the peripheral area is the most advantageous location for trip-making both as origin and destination. Interestingly, considerable employment decentralisation over the period does not appear to have affected the overall pattern of location advantage. What this appears to suggest is that further targeted decentralisation of employment may be the appropriate course of action for future development. Not only would this lead to a greater juxtaposition of residences and employment but it would also aid the viability of a more comprehensive public transport route network which is less city centre focussed. From a policy standpoint, any further decentralisation of employment opportunities would need to be accompanied by appropriate public transport network resources in order to curb modal shift to private transport. The foregoing results demonstrate the desirability of locating decentralised jobs adjacent to high quality public transport corridors. In the particular case of Dublin, this would surely mean locating employment opportunities adjacent to suburban railway stations.

From the perspective of land-use and transport planning considerations, the results show that much greater cognisance needs to be given to the off-peak period. The results presented here suggest that travel patterns for the two periods are radically different: for the peak period, the focus of trip-making is towards the city centre whereas the opposite is the case for the off-peak period. Looking specifically at public transport provision, it appears that the route network is overly focussed on the central area and this is

³ Obviously, this is not entirely feasible for the rail network but certainly is for the bus network.

particularly so for the off-peak period. Realigning the route network to better serve the different journey characteristics of the off-peak period would likely increase patronage levels considerably.

In a situation where travel patterns become more complex, it generally becomes more difficult to serve those trips adequately by public transport and patronage levels tend to suffer as a result. By way of evidence, the results show that private transport users have the ability to react more rapidly to changes in the distribution of origin and destination activities than their public transport counterparts and this tends to confer significant advantage to users of that mode. This implies in its turn that there should be ongoing reviews of the public transport route network to ensure that it is reacting satisfactorily to changes in the spatial distribution of major trip-generating land-uses. From a policy-making perspective, it is interesting to note that the different journey characteristics between the peak and off-peak period were not considered in a recent design review of the Dublin Bus public transport network (Deloitte & Touche, 2009).

Finally, it must be borne in mind that the interpretation of dual variables occurs within the context of a somewhat unrealistic pattern of spatial interaction (i.e. the perfect efficiency assumptions of the TPLP). Nevertheless, the approach provides a useful method for determining not only comparative location advantage but, more importantly, where future resources (residences and workplaces) might be directed in order to maintain optimum spatial interaction so that the possibility of reducing observed commuting costs is at least provided for. Moreover, the importance of understanding changes in the spatial advantages of different locations within individual study areas, whether this is due to changes in the dynamics of locational supply and demand over time or changes in locational accessibility (in terms of commuting cost efficiency) due to network improvements, is highly revealing from the viewpoint of travel efficiency and land-use and transport planning considerations.

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Figure 1. Schematic of shadow prices for hypothetical city regions

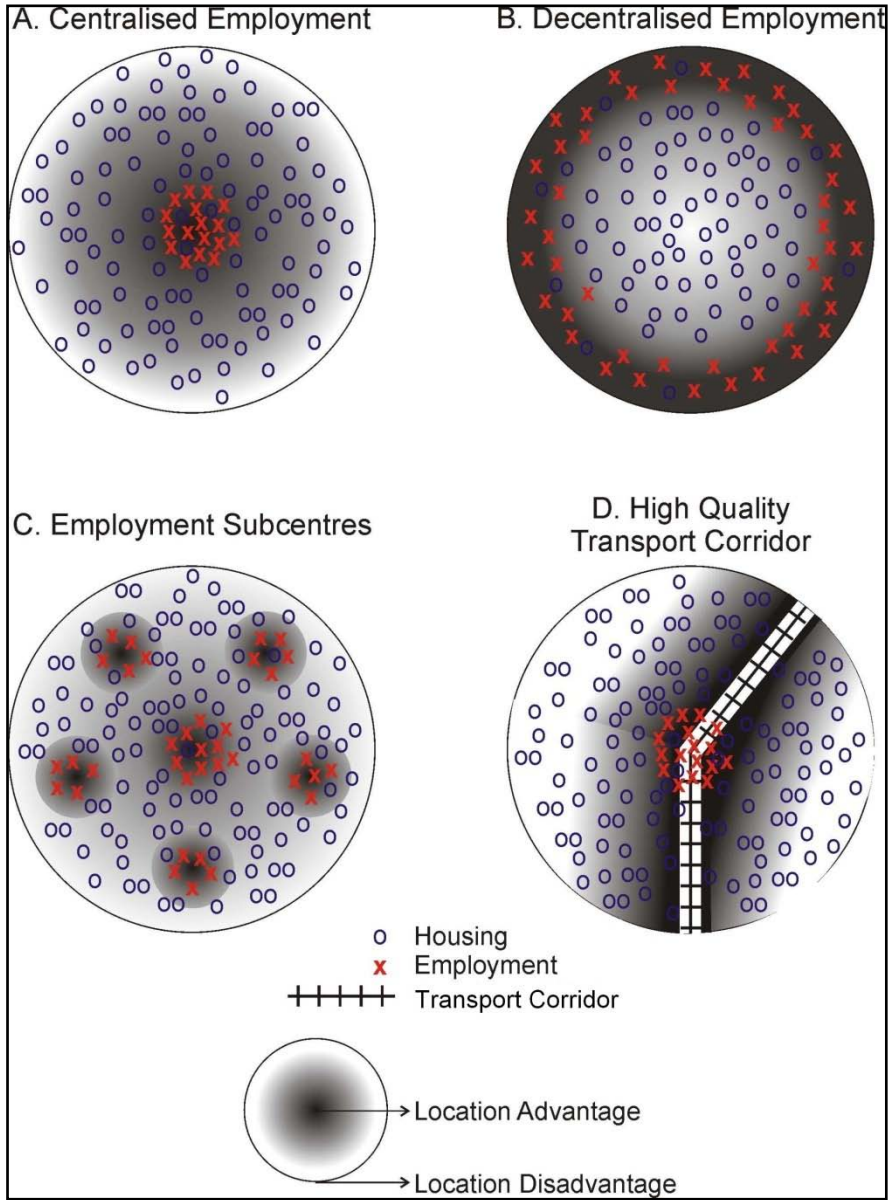
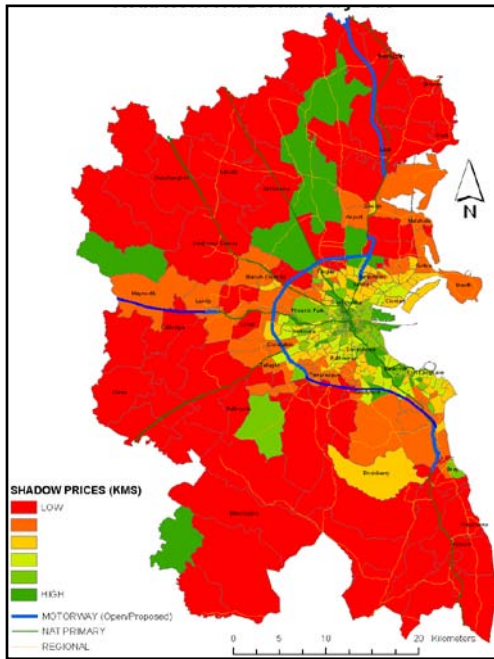
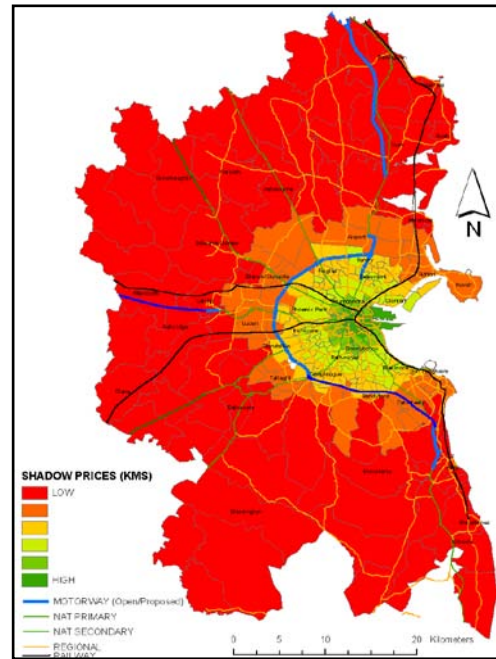


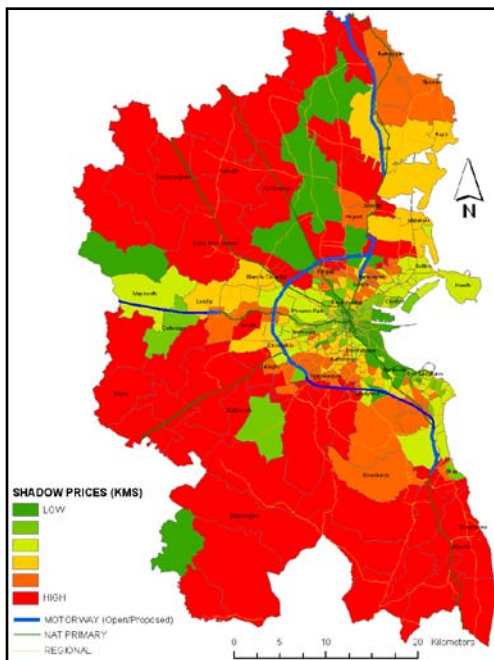
Figure 2. Peak origin and destination shadow prices for public transport, 1991 and 2001



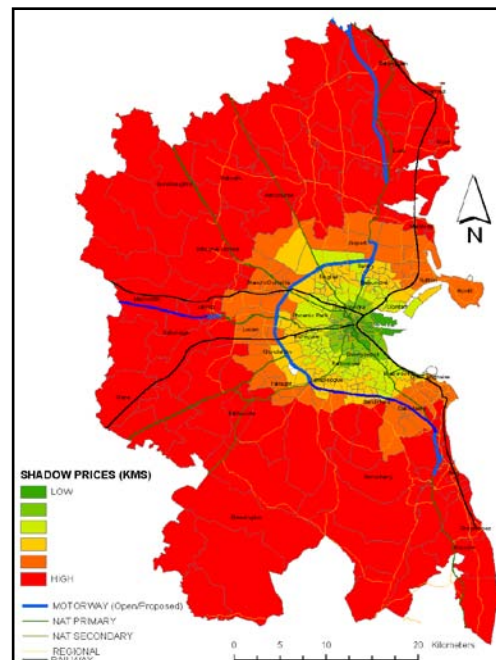
(a) Origin shadow prices for public transport, 1991



(b) Origin shadow prices for public transport, 2001

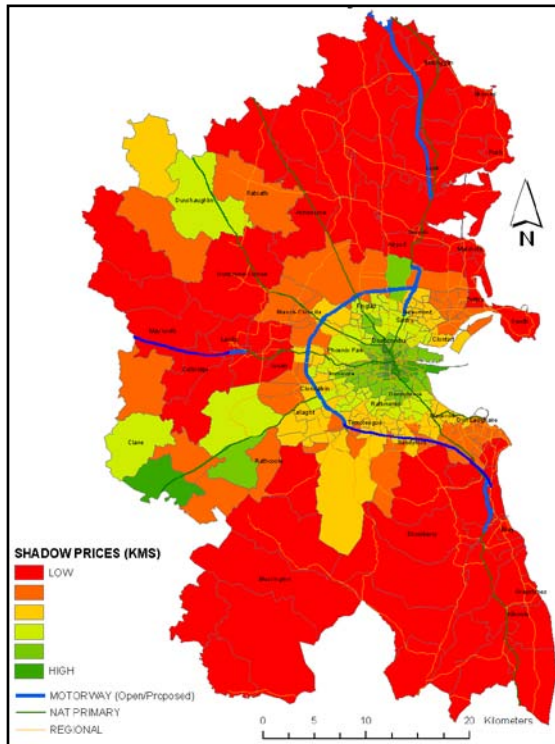


(c) Destination shadow prices for public transport, 1991

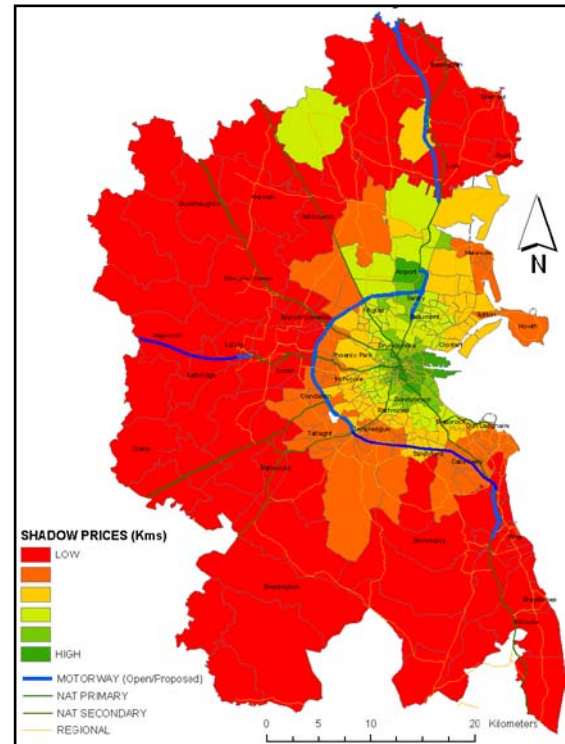


(d) Destination shadow prices for public transport, 2001

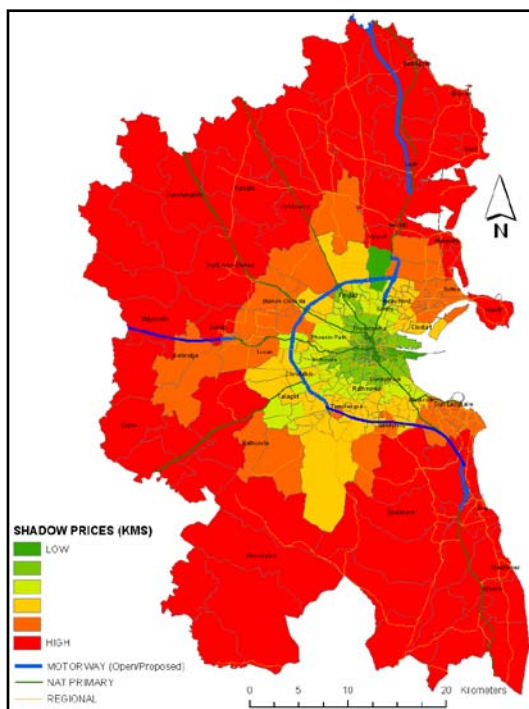
Figure 3. Peak origin and destination shadow prices for private transport, 1991 and 2001



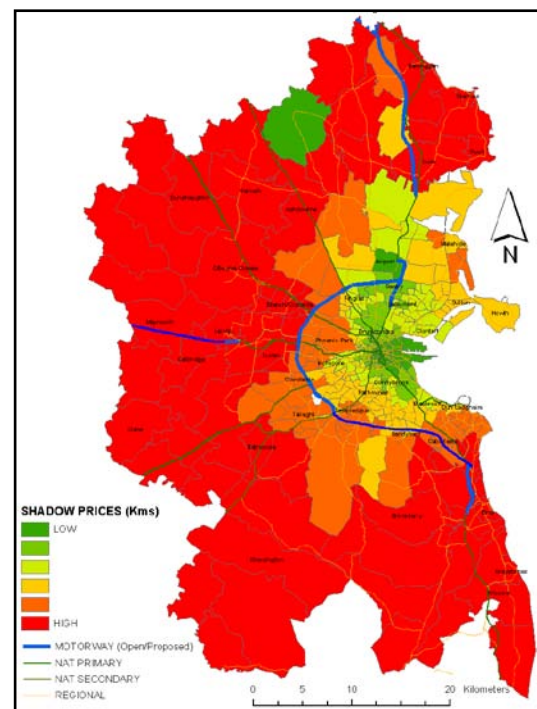
(a) Origin shadow prices for private transport, 1991



(b) Origin shadow prices for private transport, 2001

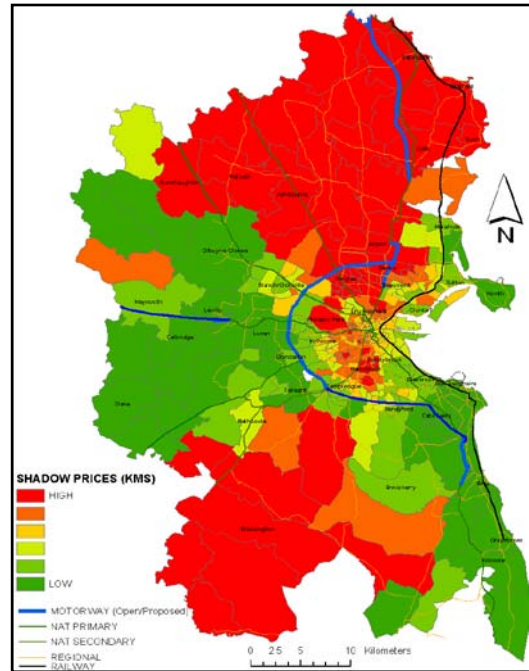
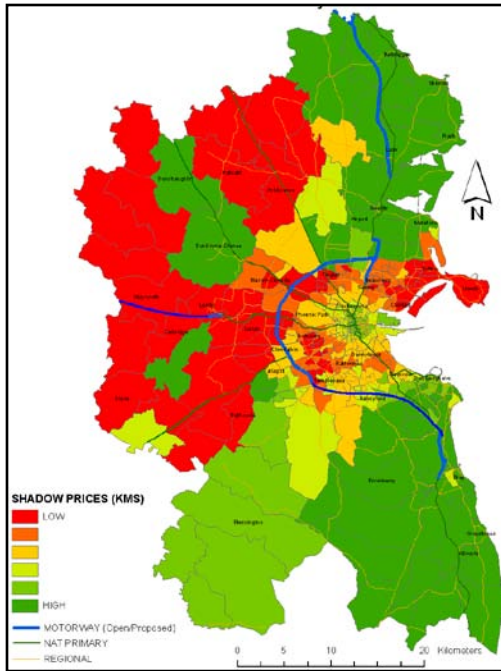


(c) Destination shadow prices for private transport,



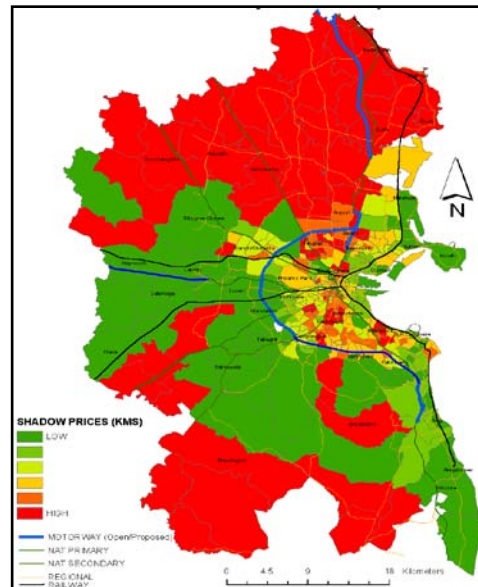
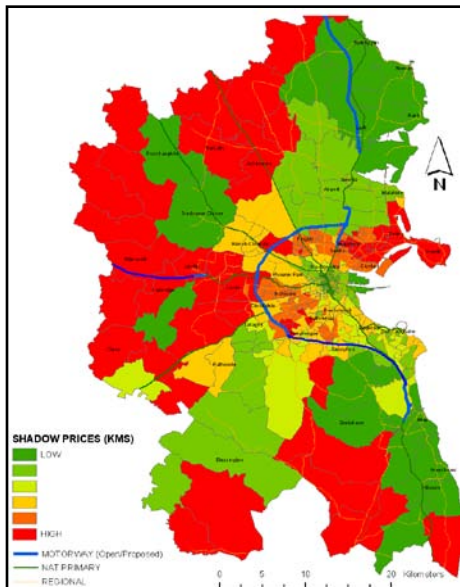
(d) Destination shadow prices for private transport,

Figure 4. Off-peak origin and destination shadow prices for public transport, 1991 and 2001



(a) Origin shadow prices for public transport, 1991

(b) Origin shadow prices for public transport, 2001



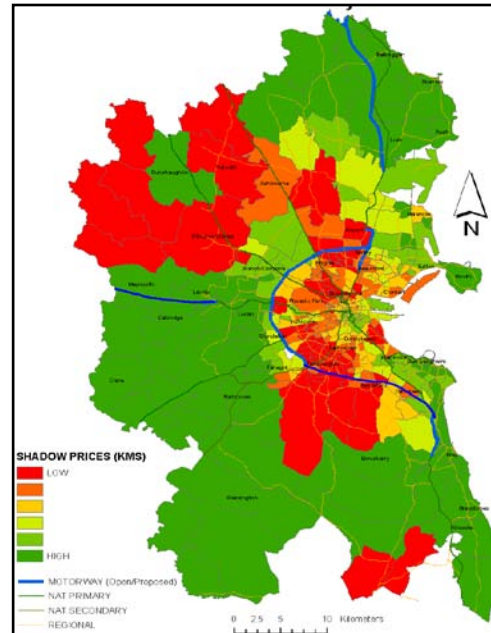
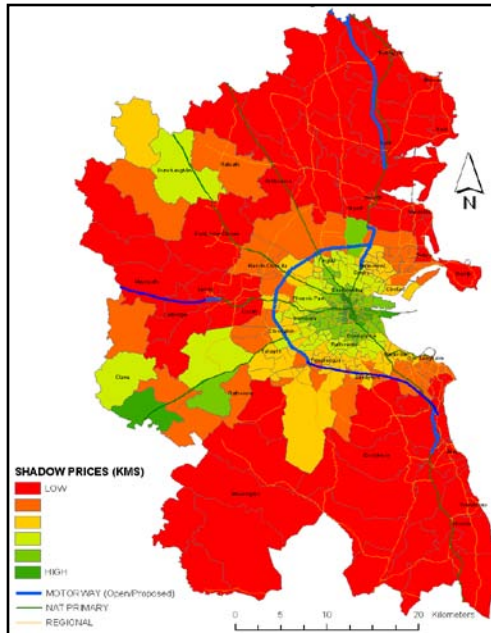
(c) Destination shadow prices for public transport,

(d) Destination shadow prices for public transport,

1991

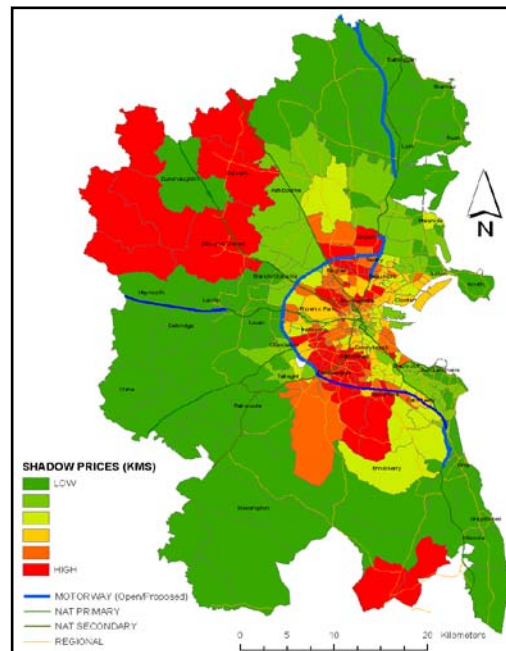
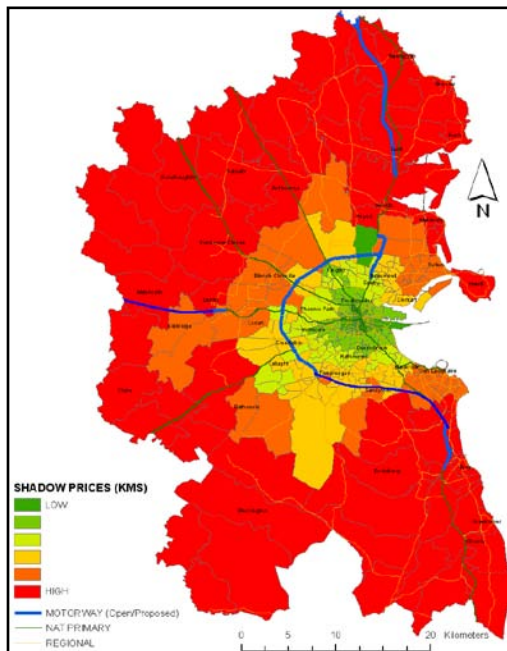
2001

Figure 5. Off-peak origin and destination shadow prices for private transport, 1991 and 2001



(a) Origin shadow prices for private transport, 1991

(b) Origin shadow prices for private transport, 2001



(c) Destination shadow prices for private transport,

(d) Destination shadow prices for private transport,

1991

2001

