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Long-Run International Diversification

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Preliminary - Comments Welcome

Abstract

Prevailing wisdom in finance suggests long-run investors have a competitive advantage, since they can ride out short-run fluctuations and mispricing, and pursue illiquid investments. This paper investigates if this advantage holds in a portfolio context, examining benefits of international diversification across short- and long-run horizons. Employing a multi-horizon non-parametric filter, increased long-run correlations between international equity markets are detailed, even for synchronized markets. A model replicating the temporal aggregation properties of inter-market correlation is developed, indicating that short-run correlations are downward biased by frictions. Finally, the impact on portfolio allocation is investigated, demonstrating decreased risk reduction benefits in the long-run.

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1. Introduction

The preferred allocation of wealth may diverge according to investment horizon, with conventional wisdom advocating superiority of stocks over the long-run (Spiegel, 2014). Investing for the long-run allows investors to take advantage of relatively decreasing long-horizon variance and predictability in returns (Campbell and Viceira, 2005; Barberis, 2000). Moreover, a focus on the long-run allows investors to ride out short-term fluctuations and mispricing, and pursue illiquid investments. However, recent research has suggested that mean reversion, a contributor to enhanced long-run investment opportunities, may be mitigated by parameter uncertainty (Pástor and Stambaugh, 2012). This paper investigates the advantages for long-run investors in a portfolio context, examining the risk-reduction benefits of international diversification across short- and long-run horizons. We find that long run benefits are somewhat negated compared to conventional wisdom.

Quantifying comovement between asset returns is one of the most fundamental aspects of modern finance. In international finance, the existence of low cross-correlations between global equity markets forms the basis of risk reduction through international diversification. The risk of holding a portfolio comprised of a range of international equity markets has long been shown to be lower than the risk of component assets (Levy and Sarnat, 1970; Grubel, 1968). Whilst evidence for risk reduction through international diversification is strong, the extent of long-run risk reduction may be obfuscated by the common use of weekly or monthly data.

The extant literature has tended to represent international investment opportunities with measures of broad market performance such as equity indices. However,

international equity indices may have characteristics, such as non-synchronous trading hours and serial correlation, which may perturb accurate measurement of diversification benefits. The primary finding in this paper is that the benefits of international diversification are not equally dispersed across heterogeneous horizons. In particular, long-run comovements between international equity markets are shown to be significantly larger than those measured at short horizons. This finding is only partially diminished for synchronized markets, suggesting non-synchronous trading is not the sole driver of increased long-run correlation. Moreover, starting with a monthly sampling interval, increasing correlation is found to persist to horizons of up to five years. While short-run findings are in keeping with the traditional literature on international diversification, the long-run results follow recent literature which question the benefits of international diversification (Christoffersen *et al.*, 2012; You and Daigler, 2010).

The finding of increased long-run correlation between international equity markets has important implications. In order to shed light on possible sources, we model the long-run intertemporal correlation between markets using only short-horizon data, whilst accounting for characteristics of international indices such as serial correlation and cross-serial correlation due to non-synchronous trading hours and other frictions. The implication from the model is that short-run correlations are, in fact, downward biased. Our finding of bias in the measurement of correlation complements previous research which demonstrates short-run bias in estimation for other common financial characteristics such as volatility and systematic risk, partially a consequence of non-synchronous trading (Lo and MacKinlay, 1990; Cohen *et al.*, 1983). The implication for investors, even those with short-run horizons, is that perceived risk reduction benefits may be overstated using short

horizon data.

Finally, our remaining contribution relates to methodology. In the presence of non-synchronous trading hours, common in international markets, correlation estimation may be biased. We introduce a novel non-parametric multi-horizon estimation methodology which helps to identify the synchronized correlation between two non-synchronized time series. The methodology is based upon an optimal weighting of consecutive returns but, in contrast to recent work such as Ortu *et al.* (2013), it consists of identifying long-run correlation rather than behaviour in a narrow spectrum. Using a simulation study we show that the methodology provides correlation estimates with smaller error than those found with traditional subsampling when presented with non-synchronous data.

The remainder of this paper is organized as follows. In the next section we review some literature relevant to asset dependence and international diversification. Section 3 describes the methodology, while Section 4 employs a simulation study to demonstrate the benefits of our empirical methodology in estimating correlation. The data investigated is discussed in Section 5. Our main results are described in Section 6, while Section 7 provides a summary and conclusion.

2. Related Literature

International diversification has long been documented as a natural risk reduction extension of the classic Markowitz portfolio selection theory (Levy and Sarnat, 1970; Grubel, 1968; Markowitz, 1952). A variety of studies have demonstrated low levels of cross-correlation between international equity markets and interpreted this as an opportunity to improve portfolio risk-return trade-off (Berger *et al.*, 2011; Goetzmann *et al.*, 2005; Levy and Sarnat, 1970; Grubel, 1968). However,

recent research has indicated that the low levels of correlation found between international equity markets may be deceptive, particularly when considered from a downside risk perspective (Christoffersen *et al.*, 2012; You and Daigler, 2010). We contribute to this debate on the benefits of international diversification, examining whether the benefits of international diversification persist in the long-run.

Various methods to augment the benefits of international portfolio allocation have been proposed; Guidolin and Timmermann (2008) suggest an alteration in diversification benefits as market regimes and preferences on skewness and kurtosis are taken into account. Eun *et al.* (2009) demonstrate additional international diversification benefits from small-cap stocks. Systemic risk, captured through simultaneous market jumps, is shown to reduce the benefits of international diversification (Das and Uppal, 2005). Moreover, time varying market regimes are found to have significant impact on the risk management of international equity portfolios (Okimoto, 2008; Ang and Bekaert, 2002). While significant diversification benefits from international investment are demonstrated in these papers, the long-run performance of international diversification has not been considered in detail, with previous research predominantly focussed on weekly or monthly return intervals.

Considerable evidence exists surrounding the influence of trading frictions on the effective measurement of fundamental financial characteristics. For example, quantification of non-diversifiable systematic risk, or beta, of an asset may be biased by delays in the trading process often attributed to liquidity (Kamara *et al.*, 2013; Perron *et al.*, 2013; Gençay *et al.*, 2005). Various authors have derived methods linking the sensitivity of measured betas to underlying frictions using, for example, leading and lagging serial and cross-serial correlations between an

asset and the market (Perron *et al.*, 2013; de Jong and Nijman, 1997; Cohen *et al.*, 1983). Moreover, a vast array of approaches have been developed to estimate important financial characteristics such as correlation and systematic risk (Hollstein and Prokopczuk, 2015; Cohen *et al.*, 1983). Considering international equity indices, significant serial correlation has been comprehensively observed, often with positive serial correlation at short intervals and negative serial correlation at longer return intervals (Ahn *et al.*, 2002; Poterba and Summers, 1988). Moreover, international equity markets often have non-synchronous trading hours, which induces cross-serial correlation between markets due to common information being incorporated in prices at different times (Schotman and Zalewska, 2006; Martens and Poon, 2001). In this paper, we model the long-run intertemporal correlation between international equity markets. To this end, we take short-run return correlation and incorporate a correction accounting for serial and cross-serial correlation between markets.

The impact of investment horizon on asset risk and return has been examined in some detail, with early research focussing on data with different return intervals¹ to disentangle how financial characteristics alter with horizon. Considering the relationship between financial assets, Epps (1979) documents increased correlation between financial assets as the horizon at which price changes are measured increases.² Recent contributions have considered the importance of time horizon in

¹Differing return intervals are created by subsampling price data or by summing over high frequency logarithmic returns and calculating long horizon or low-frequency returns. For example, daily price data may be sub-sampled every Friday to create weekly data. One of the concerns with this approach is the lack of motivation concerning sub-sample timing.

²For this reason, the phenomena of decreasing correlation between assets at high frequency is sometimes known as the *Epps effect*. While Epps (1979) considered a small number of domestic stocks at short horizons, in this study we consider a range of international equity markets and find characteristic increases in correlation in the long-run.

determining market predictability (Pástor and Stambaugh, 2012; Boudoukh *et al.*, 2007; Barberis, 2000) and horizon-based asset pricing (Kamara *et al.*, 2013; Bandi and Perron, 2008).

In order to overcome potential bias due to non-synchronous trading hours between international equity markets, many studies examining the benefits of diversification employ either weekly or monthly return intervals.³ However, beyond monthly horizons, little work has been done in determining the long-run benefits of international diversification. Traditional data sampling to long return intervals may result in a reduction in statistical power due to sample reduction (Ortu *et al.*, 2013). To mitigate this, we apply a non-parametric multi-horizon filter and show, via a simulation study, that this results in more accurate estimation of long-run correlation than subsampled return intervals in the presence of non-synchronous trading.

Our filtering approach consists of decomposing each time series of international equity returns into short- and long-run components using wavelet analysis. Wavelet analysis has been comprehensively utilized across a range of problems in both economics⁴ and finance to understand horizon dependent characteristics. In particular, wavelet based techniques have been broadly applied to understand the time-frequency properties of financial time series (Ortu *et al.*, 2013; Rua and Nunes, 2009; In and Kim, 2006; Gençay *et al.*, 2005). Relative to these studies, the approach developed here consists of decomposing the time series into spec-

³For example, Christoffersen *et al.* (2012) and Bekaert *et al.* (2009) employ weekly returns, while Eun *et al.* (2009) and Longin and Solnik (2001) consider data having monthly returns.

⁴Some specific wavelet contributions to the economics literature include tests for serial correlation in panel models (Hong and Kao, 2004), long memory estimation in time series (Faÿ *et al.*, 2009) and multi-scale serial correlation tests (Gençay and Signori, 2015).

tral components, but our focus is on the residual long-run contributions rather than the previously examined short-run frequency bands. We next describe the methodology adopted in this paper.

3. Methodology

3.1. Wavelet Decomposition

The partitioning of economic and financial time series into short- and long-term contributions has garnered considerable interest in the literature (Perron *et al.*, 2013; Hansen and Scheinkman, 2009; Hodrick and Prescott, 1997). Low-frequency (long-run) features of a time series may overcome short-term noise or shocks and help reveal underlying economic relationships between variables. When considering pairs of financial time series, measurement of their short-run dependency may be biased by frictions in the trading process. In particular, international equity markets have non-synchronous trading hours and may be exposed to specific frictions such as price transmission delays. Aggregation over longer horizons may help to mitigate these biases, a problem we address here using a non-parametric multi-horizon filter, based upon wavelet analysis.

In this section, we outline the wavelet approach to time series partitioning. Specifically, we describe the discrete wavelet transformation (DWT), a mathematical tool that projects a time series onto a set of orthogonal basis functions (wavelets) resulting in a set of wavelet coefficients or filtered time series associated with distinct frequencies (time horizons). A wavelet is defined as a small wave which can grow and decay in a limited time period, capturing localized features of a time series. The DWT provides a time-frequency (time horizon) representation of

a signal, detailing the frequency content of a signal as a function of time. We provide a concise description of wavelet decomposition using the Haar wavelet. While previous work has tended to focus on the short-run time series contributions associated with particular frequency bands, this provides little information regarding the long-run behaviour of financial time series (Ortu *et al.*, 2013; In and Kim, 2006; Gençay *et al.*, 2005). In contrast, our approach has a number of advantages; First, short-run frequency bands disaggregate information in a fashion that may not be of use to investors. In contrast, long-run wavelet correlation corresponds to the long-run comovement between assets without the bias associated with short-run frictions and is relevant for investment purposes. Second, using long-run wavelet correlation allows us to relate the behaviour at the very shortest horizons to the differential comovement found at the longest horizons, providing an intuitive basis for horizon based effects. Finally, using a simulation study we demonstrate the benefits of long-run wavelet correlation in estimating the synchronized correlation between non-synchronous markets.

3.1.1. Discrete Wavelet Transformation

In this section, we introduce the discrete wavelet transform, taking the most elementary Haar wavelet filter as an example. The Haar wavelet filter coefficient vector of length $\tau_1 = 2^1$, corresponding to scale or horizon one, is given by $h = (h_0, h_1) = (1/\sqrt{2}, -1/\sqrt{2})$ and has the following properties:

$$\sum_l h_l = 0, \quad \sum_l h_l^2 = 1, \quad \sum_l h_l h_{l+2n} = 0 \quad \forall \text{ integers } n \neq 0. \quad (1)$$

These properties ensure (i) the wavelet filter sums to zero and identifies changes in the data, (ii) the wavelet filter has unit energy, resulting in variance preservation

between the data and the decomposition, and (iii) orthonormality of the set of functions derived from h , facilitating multiresolution analysis of a finite energy signal.

The wavelet filter is complemented by the Haar wavelet scaling filter $g = (g_0, g_1) = (1/\sqrt{2}, 1/\sqrt{2})$, viewed as a local averaging operator and has properties:

$$\sum_l g_l = \sqrt{2}, \quad \sum_l g_l^2 = 1, \quad \sum_l g_l g_{l+2n} = 0 \quad \forall \text{ integers } n \neq 0. \quad (2)$$

Similar to the wavelet filter, the scaling filter has unit energy and is orthogonal to even shifts. The first property ensures that the scaling filter averages consecutive blocks of data, as opposed to differencing them.

Applying the Haar DWT filter of length $\tau_1 = 2$ to a return series, $\{r_t\}$, produces the following wavelet, $w_{1,t}$, and scaling, $v_{1,t}$ coefficients:

$$\begin{aligned} \sqrt{2}w_{1,t} &= h_0 r_{2t-1} + h_1 r_{2t}, & t = 1, 2, \dots, T/2, \\ \sqrt{2}v_{1,t} &= g_0 r_{2t-1} + g_1 r_{2t}, & t = 1, 2, \dots, T/2 \end{aligned} \quad (3)$$

Gathering together the coefficients associated with different points in time, the vector of wavelet coefficients \mathbf{w}_1 corresponds to a set of weighted differences between consecutive returns, representing the high frequency content of the time series. In contrast, the vector of scaling coefficients \mathbf{v}_1 corresponds to local averages of length two of the original returns data. While coefficients, \mathbf{w}_1 , associated with the wavelet filter represent a band of high frequency oscillations, the wavelet scaling coefficients capture low-frequency content. Collecting both sets of coefficients into a matrix $\mathbf{w} = (\mathbf{w}_1, \mathbf{v}_1)$ results in a filtration of the returns data into two orthogonal vectors.

In order to derive longer horizon wavelet and scaling coefficients, we first need to calculate higher order filters. To derive the scaling filter of order 2, we define a filter $g' = (g_0, 0, g_1) = (1/\sqrt{2}, 0, 1/\sqrt{2})$ which corresponding to a Haar scaling filter with a zero between the two coefficients. Then, the order 2 Haar wavelet scaling filter is defined as:

$$g_{2,i} = \{g * g'_i\} = \sum_{j=0}^{\tau_j-1} g_j g'_{i-j}, \quad i = 0, \dots, 3. \quad (4)$$

This allows determination of the scale 2 wavelet filter, $g_2 = (1/2, 1/2, 1/2, 1/2)$ with length $\tau_2 = 2^2 = 4$.⁵ The Haar scaling filter of order 2 is a simple average of four consecutive returns. Scaling coefficients are then calculated as follows:

$$2v_{2,t} = g_0 R_{4t-3} + g_1 R_{4t-2} + g_2 R_{4t-1} + g_3 R_{4t}, \quad t = 1, 2, 3, \dots, T/4. \quad (5)$$

Similarly, the scale 3 scaling coefficients may be generated by increasing the number of zeros inserted between the scaling filter coefficients. The scaling filter coefficients are found using $g_3 = \{\{g * g'_i\} * g''_i\}$, where $g'' = (g_0, 0, 0, g_1) = (1/\sqrt{2}, 0, 0, 1/\sqrt{2})$, giving

$$g_3 = \left(\frac{1}{\sqrt{8}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{8}}, \frac{1}{\sqrt{8}} \right) \quad (6)$$

A similar process may continue up to the scale $J = \log 2^T$ and for wavelet coefficients. Further detail may be found in Gençay *et al.* (2001).

⁵Similarly, a Haar wavelet filter, h_2 , may be defined that first averages two pairs of returns and then proceeds to difference them. Further detail is given in Gençay *et al.* (2001).

Decomposition of the original time series into wavelet and scaling coefficients allows us to express the coefficients in terms of frequency bands. After the first decomposition, returns $\{r_t\}$ are decomposed into wavelet coefficients \mathbf{w}_1 associated with high frequency content, $1/4 < f \leq 1/2$ and wavelet scaling coefficients \mathbf{v}_1 which span the lower half of frequencies, $0 \leq f \leq 1/4$. Repeating the procedure for the scale two coefficients, the wavelet scaling coefficients are associated with frequencies $0 \leq f \leq 1/8$. Continuing in a similar fashion, the remaining scaling coefficients span the residual frequencies $0 \leq f \leq 1/2^{(j+1)}$ (and associated horizons greater than $2^{(j+1)}$).

The Haar DWT scaling coefficients are closely related to time series aggregation, the process by which a set of τ_j -day returns, $\{R_t(\tau_j)\}$, can be created by summing over non-overlapping individual logarithmic returns, r_t ,

$$R_t(\tau_j) = \sum_{m=0}^{\tau_j-1} r_{\tau_j t - m}, \quad t = 1, 2, 3, \dots, \frac{T}{\tau_j}. \quad (7)$$

Later, we demonstrate equivalence between cross-correlation calculated for aggregated time series and Haar DWT scaling coefficients. Moreover, we show how long horizon wavelet scaling correlations can be expressed as a function of correlations calculated using original, unfiltered, data plus a correction for serial and cross-serial correlation.

A variation of the DWT is the maximum overlap discrete wavelet transformation (MODWT). Analogous to the DWT, the MODWT produces wavelet coefficients $\tilde{\mathbf{w}}_j$ and scaling coefficients $\tilde{\mathbf{v}}_j$ associated with a particular frequency band or horizon. In contrast to the DWT, the MODWT is not limited to dyadic time series, instead retaining coefficients associated with all times. As the MODWT

does not eliminate coefficients, this removes the alignment effects of the DWT and leads to a more efficient time series decomposition at multiple time horizons. For these reasons, the MODWT is adopted for the empirical analysis in this paper. Further details on the MODWT may be found in Gençay *et al.* (2001) and Percival and Walden (2000), amongst others.

The Haar wavelet decomposition described has a simple interpretation in terms of differences and averages of consecutive returns. When decomposing a time series of returns, we are separating layers of information associated with different frequencies that decrease for longer horizons. A variety of more sophisticated wavelet types also possess this interpretation as differences and averages of a time series.⁶ In contrast to the Haar wavelet, longer wavelet filters such as the Daubechies wavelets, the least asymmetric wavelets, and the Coiflet have a more refined weighting of the elements of the time series. We later demonstrate that accurate estimates of long-run correlation may be produced by employing these sophisticated wavelet filters.

3.2. Wavelet Long-Run Correlation

An important characteristic of wavelet analysis is the ability to partition the variance and covariance of a time series, informing regarding the short- and long-run contributions of each. Percival and Mofjeld (1997) proved the variance preserving properties of the MODWT. That is, the variance of the original time series is perfectly captured by the variance of the coefficients from the MODWT. In particular, total variance from a time series may be decomposed using the MODWT

⁶A detailed exposition of the array of sophisticated wavelets available to researchers is beyond the scope of this manuscript, with the Haar wavelet described as a base case. Interested readers are referred to Gençay *et al.* (2001) and Percival and Walden (2000) for further detail.

wavelet and scaling coefficients as follows:

$$\| \mathbf{R} \| = \sum_{j=1}^J \| \tilde{\mathbf{w}}_j \|^2 + \| \tilde{\mathbf{v}}_J \|^2 \quad (8)$$

where $\| \tilde{\mathbf{w}}_j \|^2 = \sum_{t=1}^{T/2^j} \tilde{w}_{t,j}^2$ is the variance associated with horizon $\tau_j = 2^j$ and $\| \tilde{\mathbf{v}}_J \|^2 = \sum_{t=1}^{T/2^J} \tilde{v}_{t,J}^2$ corresponds to the variance associated with horizon greater than $\tau_j = 2^J$. In this paper, our focus differs from the extant literature (Ortu *et al.*, 2013; In and Kim, 2006; Gençay *et al.*, 2005). Instead of considering the short-run contributions to variance $\| \tilde{\mathbf{w}}_j \|^2$, we concentrate on the long-run contribution $\| \tilde{\mathbf{v}}_J \|^2$.

Unbiased estimates of the wavelet long-run scaling variance, $\sigma_{m,J}^2$, for asset m at horizons greater than τ_J are given by

$$\sigma_{m,J}^2 = Var(\tilde{v}_{m,J}), \quad (9)$$

where $\tilde{v}_{m,J}$ are the wavelet scaling coefficients for asset m at long-run horizons greater than $\tau_j = 2^J$. Similarly, unbiased estimates of the wavelet long-run scaling covariance, $\sigma_{mn,J}^2$, between two distinct assets m and n at horizons greater than τ_j are defined by

$$\sigma_{mn,J}^2 = Cov(\tilde{v}_{m,J}, \tilde{v}_{n,J}), \quad (10)$$

where $\tau_j = 2^j$ and $\tilde{v}_{m,J}$ and $\tilde{v}_{n,J}$ wavelet scaling coefficients for assets m and n at horizons greater than $\tau_j = 2^J$. While the wavelet covariance decomposes the covariation on a scale-by-scale basis, the empirical results in this study will

focus predominantly on the wavelet scaling correlation, corresponding to long-run horizons (greater than $\tau_J = 2^J$) defined as

$$\rho_{mn,J} = \frac{\sigma_{mn,J}^2}{\sigma_{m,J}^2 \sigma_{n,J}^2}. \quad (11)$$

This captures the level of long-run correlation between two time series, characterizing the long-run dependence between two financial assets, vital in the context of long-term asset allocation.⁷ Previously, literature has measured wavelet correlation using wavelet coefficients, revealing the dependence structure between specific frequency bands associated with a time series (Ortu *et al.*, 2013; In and Kim, 2006; Gençay *et al.*, 2005). One of the difficulties with this approach lies in the practical implementation of frequency band analysis for investors. In contrast, our focus on wavelet scaling correlation reveals the long-run correlation between time series after short-run effects, often associated with frictions, have been removed. We later demonstrate, using a simulation approach, that wavelet scaling correlation provides more accurate estimation of synchronized dependence than simple aggregated (weekly or monthly) returns.

3.3. Model of Long-Run Correlation

An assortment of previous studies have demonstrated increased long-run correlation between financial assets. Moreover, a variety of studies have employed wavelets to examine the horizon dependent properties of cross-market correlations between assets, but with little intuition regarding the underlying drivers (Ortu *et al.*, 2013; Rua and Nunes, 2009; In and Kim, 2006). In this paper we con-

⁷In calculation of correlation, all wavelet coefficients associated with the boundary are removed. This results in an unbiased estimate of long-run correlation.

sider the long-run Haar wavelet correlation, which allows us to model long-run correlation as a function of just the original unfiltered returns plus a correction to account for serial and cross-serial correlation between markets. This provides insight regarding the origin of horizon dependent correlation. In order to develop this model, we first show that the elementary Haar DWT scaling correlation, $\rho_{mn,J}$, can be related to that calculated using aggregated logarithmic return time series at the same horizon.

Proposition 1. *The Haar DWT long-run (scaling) cross-correlation between changes in two stationary, finite time series m and n at horizon $\tau_J = 2^J$ is equivalent to cross-correlation between original time series aggregated at horizon $\tau_J = 2^J$.*

Proof: See Appendix

This result demonstrates that the elementary Haar DWT provides identical estimation of wavelet scaling correlation to that found using aggregation. While this would suggest little benefit in calculating correlation using the Haar DWT, more sophisticated wavelets exist which aggregate time series information in a more optimal fashion. We later apply these sophisticated wavelets in a simulation to show that they result in accurate estimation of correlation in the presence of non-synchronous trading hours. This notwithstanding, Proposition 1 allows us to derive a model capturing the long-run Haar wavelet correlation as a function of the original short horizon (1-day) data combined with information on serial and cross-serial correlation.

Proposition 2. *The Haar DWT long-run (scaling) cross-correlation between changes in two stationary, finite time series of logarithmic returns r_n and r_m at horizon*

$\tau_J = 2^J$ can be expressed as a function of the original time series as follows:

$$\begin{aligned} \rho(v_{m,J}, v_{n,J}) &= \rho(R_m(\tau_J), R_n(\tau_J)) \\ &= \rho(1) \times \left[\frac{\tau_J + \sum_{s=1}^{\tau_J-1} \left(\frac{\rho^s(r_m^1, r_n^1) + \rho^{-s}(r_m^1, r_n^1)}{\rho(r_m^1, r_n^1)} \right)}{\left(\tau_J + 2 \sum_{s=1}^{\tau_J-1} \rho^s(r_m^1) \right) \left(\tau_J + 2 \sum_{s=1}^{\tau_J-1} \rho^s(r_n^1) \right)} \right]. \end{aligned} \quad (12)$$

$v_{m,J}$ and $v_{n,J}$ are wavelet scaling coefficients associated with time series m and n at horizon τ_J respectively, and $R_m(\tau_J)$ and $R_n(\tau_J)$ are aggregated returns at horizon τ_J . $\rho(1)$ is the cross-correlation between the original untransformed returns r_m^1 and r_n^1 before aggregation. ρ^s and ρ^{-s} correspond to leading and lagging inter-temporal correlations of order s between the original series.

This proposition relates the wavelet long-run correlation to the short-run correlation plus a correction for serial and cross-serial correlation. The proposition suggests that changes in correlation at different horizons are a consequence of serial and cross-serial correlation, frequently observed in financial time series. Moreover, sources for serial and cross-serial correlations in equity indices include trading frictions, nonsynchronous trading, trading volume, time-varying risk premia and analyst coverage (Chordia *et al.*, 2011; Lo and MacKinlay, 1990). Cross-serial correlations are also related to the speed of adjustment hypothesis, whereby some assets adjust more slowly than others to economy-wide information (Chordia *et al.*, 2011). Later, we use Proposition 2 to demonstrate how increased long-run correlations between international equity markets may be modelled using only short-run data.

4. Wavelet Long-Run Correlation - A Simulation Study

To motivate our application of wavelet analysis to measuring the benefits of international diversification, we first employ a simulation study to highlight some advantages of measuring correlation using wavelet scaling coefficients versus sub-sampling. Biased estimates of cross-market correlation are induced through the use of non-overlapping trading hours in the simulation, replicating international equity markets. Wavelet long-run correlation is then shown to produce estimates of the true synchronized market correlation with smaller bias and error than sub-sampled returns. While previous papers have adopted the wavelet transform to examine correlation within specific frequency bands, one of the contributions of this paper is the novel use of long-run wavelet correlation to provide improved estimation of synchronous codependence.

4.1. Simulation Set-Up

The simulation is based on the relationship between returns for two markets, each having systematic exposure to a single common factor (the ‘world market’), resulting in correlation between the markets.⁸ Biased estimation of the true synchronous correlation between markets is induced by defining three non-overlapping trading periods per day. Returns for each market, $M1$ and $M2$, are defined by:

$$\begin{aligned} R_t^{M1} &= \alpha_1 + \beta_1 R_{t,1}^W + \beta_1 R_{t,2}^W + \beta_1 R_{t,3}^W + \varepsilon^{M1} \\ R_t^{M2} &= \alpha_2 + \beta_2 R_{t-1,3}^W + \beta_2 R_{t,1}^W + \beta_2 R_{t,2}^W + \varepsilon^{M2} \end{aligned} \tag{13}$$

⁸A similar approach was applied by Lo and MacKinlay (1990) to examine whether non-synchronous trading leads to serial correlation in portfolios.

where $R_{t,i}^W$ corresponds to returns of the zero-mean, independent and identically distributed World market factor on day t during trading period i , where $i \in 1, 2, 3$. $\varepsilon^{M1,M2}$ is zero-mean idiosyncratic noise, uncorrelated both cross-sectionally and temporally. The constants, α_1 and α_2 , are set to zero in the simulation, without prejudicing the results. Market $M1$ has three trading periods, each of which are perfectly synchronized with the world market. In contrast, market $M2$ is closed during trading period 3 and price information from this period is incorporated during the first trading session of the following trading day. This is captured in equation 13 through the $\beta_2 R_{t-1,3}^W$ term. This setup results in daily returns of $M2$ incorporating price information from both days $t - 1$ and t .⁹

This model is analogous to the trading structure found between US and European markets. On a given day, trading begins on European markets, followed by a period during which US and European markets are open, concluded by a period when only the US is open. Information common to both markets is incorporated into US markets during the final trading period, but European markets only incorporate this information into prices when trading recommences the following morning. Non-synchronous trading has been previously shown to result in biased estimation of cross correlation between markets as a consequence of induced serial cross-correlation (Schotman and Zalewska, 2006; Martens and Poon, 2001). Increasing the horizon over which returns are aggregated has the effect of reducing the proportion of non-synchronous trading hours between two markets, resulting in lower estimation bias for correlation.

⁹A more advanced model could incorporate additional features such as further non-synchronous trading or information delays. However, the set-up detailed is sufficient to examine performance benefits of wavelet scaling correlation.

Using simulation, we contrast the ability of wavelets and subsampling to measure the true synchronized correlation between markets. To this end, world market returns (R^W) are simulated using a Gaussian Brownian Motion both with and without jumps. Markets $M1$ and $M2$ are both assumed to have a beta coefficient or systematic risk exposure of 0.8 with the world market.¹⁰ In this study, we compare the effectiveness of wavelet scaling correlation to subsampled or aggregated correlation using both bias and mean square error. In each case, we are comparing the measured correlation to the actual synchronized correlation at a particular horizon. MODWT scaling coefficients are used to measure the long-horizon correlation and all boundary coefficients are removed, resulting in unbiased estimation.

4.2. Simulation Results

To illustrate the improvement in estimation using wavelet long-run correlation versus subsampled correlation, we simulate non-synchronous markets as described in equation 13. Differing specifications are examined, including simulation of long and short time series both with and without jumps. The accuracy of the wavelet long-run scaling correlation is compared to that measured using subsampled data both in terms of bias and error.¹¹

Results are detailed in table 1, for time series of length 8,192.¹² Across all measures, the level of bias is shown to decrease at long horizons, which corresponds to increased trading synchronicity. Sophisticated wavelets (LA8, C6 and D8) are

¹⁰A variety of values for the systematic risk coefficient were tested resulting in no qualitative impact on the results found. Wavelets were found to provide better estimation of synchronized correlation in all cases.

¹¹Bias is defined as $\mathbb{E}[\hat{\rho} - \rho]$, while error is defined using the mean square error $\mathbb{E}[(\hat{\rho} - \rho)^2]$.

¹²The time series were chosen to be dyadic in order to ensure optimal subsampling. A choice of non-dyadic time series would further improve the quality of the wavelet correlation estimation relative to subsampling.

found to have considerably lower bias for all time series lengths, both with and without jumps. In particular, without jumps the level of bias for the D8 wavelet ranges from 9.4% to 80.4% of that from subsampling, with similar findings with jumps. In terms of mean square error (MSE), the Daubachies wavelet of length 8 has the smallest error for horizons of up to 16 days without jumps in the simulation. When jumps are included in the time series, the Haar wavelet has lowest MSE at long horizons, as this wavelet filter accurately captures discontinuities in data. These findings suggest that wavelets provide more accurate estimates of contemporaneous correlation when market returns are recorded non-synchronously.

[Table 1 about here.]

The ability of the wavelet long-run correlation to estimate synchronized correlation for short time series is examined in Table 2. With and without jumps, wavelet long-run correlation is shown to have smaller bias and error than subsampled correlation. Without jumps, bias is shown to be as low as 18.4% of subsampled bias for the D8 wavelet, while with jumps the improvement in bias is found to be up to 24.5% of subsampled bias. Moreover, in terms of MSE, wavelets are found to have better performance than sub-sampled correlation across all horizons examined. Given the prevalence of trading lags and associated correlation biases between international equity markets, these results support our application of wavelets in this paper.

[Table 2 about here.]

Our simulation demonstrates that improved estimation of synchronous correlation from non-synchronous data is possible using wavelets. In our analysis, the

Daubachies wavelet of length 8 is shown to dominate other wavelets in terms of bias and MSE, with comparable performance found for the least asymmetric wavelet of length 8. For this reason, the Daubachies wavelet is used throughout the analysis. However, specific choice of wavelet filter is found to have little qualitative impact on results.

5. Data

Data for the study was obtained from DataStream, a division of Thompson Reuters. The data consists of daily prices for a range of international equity indices from January 1, 1980 through May 26, 2011, a total of 8,193 days. For each country, the index chosen represents a broad coverage of equities, with data available over the entire sample period. In total, equity indices from 22 countries were selected for the study, chosen to reflect a diverse range of developed and emerging markets with geographical diversity. The countries examined in the study are listed in Table 3 along with details of the equity index considered.

[Table 3 about here.]

To remove the impact of exchange rate fluctuations from the study, each local index is converted to a common currency.¹³ In this study we choose to measure the benefits of international diversification to a U.S. investor and thus select the U.S. dollar as the common currency. Throughout the study, the base data considered are daily logarithmic returns. Various studies considering international finance

¹³This is standard in international finance studies; Pukthuanthong and Roll (2009) suggest that “such conversions represent a ubiquitous practice in empirical studies of international financial markets”.

have considered daily (Berger *et al.*, 2011; Pukthuanthong and Roll, 2009), weekly (Christoffersen *et al.*, 2012) and monthly (Rua and Nunes, 2009) base data. In this study we will use wavelet analysis to estimate the long-run relationships between equity indices.¹⁴

6. Empirical Results

6.1. Summary Statistics

Descriptive summary statistics for the set of international equity markets studied may be found in Table 3. There are considerable cross-sectional differences in the characteristics of the raw returns. The MSCI Denmark Index had highest mean return over the extended period examined, albeit one of the lowest median returns. The Malaysian Kuala Lumpur Composite Index (KLCI) was found to have lowest mean return. The average standard deviation of daily returns is 1.44%, with Asian markets of Hong Kong, Malaysia and South Korea found to have standard deviations much greater than average. The lowest daily returns of -37.01% , -30.31% and -26.48% were experienced by Singapore, Australia and Malaysia. The majority of indices demonstrate negative skewness and positive excess kurtosis, with the Jacque Bera statistic rejecting the hypothesis of normally distributed returns for all. First order serial correlation is significant for fifteen indices at a 1% level. The majority of indices display positive first order serial correlation, with only indices from the USA and South Korea having significant negative serial correlation. The Ljung-Box test rejects the hypothesis that the first 20 autocorrelations in absolute returns are zero in all cases.

¹⁴Robustness checks are also performed using monthly base data, with consistent results found, Section 6.6.

6.2. Correlation Analysis

The examination of long-run benefits of international diversification begins with the measurement of unconditional long-run cross-correlation between representative international equity markets. First, as we are interested in measuring the benefits of international diversification to a U.S. investor, we measure the average unconditional correlation between the U.S. (S&P 500) and each world market. Since bivariate correlations between the U.S and other world markets only contribute partially to overall diversification benefits, we also investigate the average correlation between each market excluding the U.S.. Considering correlations in this way allows us the opportunity to distinguish between the various contributors to international diversification risk reduction.

The impact of time-horizon on the unconditional correlation between markets is examined in a selection of eight time-cohorts between 1980 and 2011 and for the entire period. Motivated by Section 4.2, the MODWT Daubechies wavelet of length 8 is employed to decompose the time series data for each equity index into a series of components corresponding to different time horizons.¹⁵ Then, unconditional long-run correlations between wavelet scaling coefficients are calculated using Equation 11.

Average horizon dependent long-run correlations between the U.S. (S&P 500) and each world market are detailed in Table 4. Considering first the average correlation between 1980 and 2011 at each distinct horizon, this is found to differ considerably across the various horizons studied. The 128 day correlation of 0.662 is 2.7 times greater than the correlation found at the original daily horizon and 1.2

¹⁵In the remainder of the analysis we refer to time horizons, which refers to the residual frequencies $0 \leq f \leq 1/2^{j+1}$ and corresponding horizons greater than or equal to 2^{j+1} .

times that measured at a 16 day horizon. The latter is similar to the monthly interval often used through the literature to calculate benefits of international diversification and demonstrates the considerable difference with long-run correlation. Using a Jennrich test, a significant difference is further shown between correlation measured at the shortest and longest horizons. Our initial results suggest that short horizon investors appear to reap further benefits from international diversification than their long horizon counterparts. However, correlation measured at daily horizons may be biased downwards by a lack of contemporaneous trading. Our later results suggest that adopting a weekly or monthly return interval may be insufficient to eradicate all friction related biases.

[Table 4 about here.]

The dynamic nature of asset correlation has been well documented. In order to examine the consistency of our results at differing points in time, we examine the correlation between international equity markets in different cohorts. Table 4 details the long-run average unconditional correlation within each cohort for various horizons. Results are found to be consistent, with increasing average correlation between the S&P 500 and each world market moving from short to long horizons. Jennrich tests confirm a significant difference between correlations measured at short horizons and those measured at a 128 day horizon.

Average inter-market correlations between each of the world indices excluding the U.S. are shown in Table 5. In agreement with our earlier results, average inter-market correlation is found to increase moving from short to longer horizons. A 32% increase in correlation is found when moving from a 4 day to a 128 day horizon. Considering the differing cohorts detailed, significant time variation in

unconditional correlation is further demonstrated in all cohorts.

[Table 5 about here.]

The increasing level of dependency detailed between world markets at long horizons suggests that differing diversification benefits accrue to investors with heterogeneous investment horizons. The lowest level of dependence between international equity markets is found at short horizons, suggesting increased opportunity for diversification. In contrast, long-run equity market interdependence is found to be larger, reducing the potential for diversification. While a short horizon investor may regularly adjust their portfolio weights, a long horizon investor may be a “buy-and-hold” investor, who adjusts their weights irregularly. One perspective on this suggests that, to increase the benefits of diversification, it may be necessary to adjust portfolio weights regularly, potentially increasing transaction costs. However, in Section 6.4 we model the increased long-run correlation as a function of short-run correlation and a correction for serial- and cross-serial correlation. Findings from this model suggest that short-run correlation are downward biased in a similar fashion to systematic risk when measured at short horizons (Gençay *et al.*, 2005; Cohen *et al.*, 1983).

6.3. Synchronized Returns

In order to isolate the impact of non-contemporaneous market trading and price transmission delays on inter-market correlations, we now consider a reduced set of contemporaneously measured international equity markets.¹⁶ As suggested

¹⁶The indices were selected due to the availability of a common index price measured daily at 16:00 GMT. Data was sourced from Datastream and stretches from July 2003 to May 2011, a total of 2,048 daily returns.

earlier, the use of non-synchronized data may induce cross serial-correlation between markets, in turn downward biasing estimation of cross-correlation (Martens and Poon, 2001). We now determine whether the increasing long-horizon correlations previously detailed are purely a consequence of non-synchronous markets or whether there may be additional frictions at work.

Average horizon dependent unconditional correlations for both synchronized and unsynchronized markets are shown in Table 6. Considering first the average correlation between the USA (S&P 500) and the range of other markets, we find significant increases in cross-correlation moving from short to long horizons, in keeping with our previous findings. For example, the average synchronized correlation using daily data is 0.756 while average correlation at a 128 day horizon is 0.89, a 17.7% increase. If non-contemporaneous market trading hours are the only driver of raised long horizon correlations, then we would expect this effect to be mitigated using synchronized data. However, we find that the differential between long and short horizons, while reduced, remains strongly and significantly evident. The findings for synchronized markets suggest that the increased correlation at long horizons detailed is not exclusively a consequence of non-synchronized trading.

[Table 6 about here.]

We next consider the average inter-market correlation between the range of markets excluding the USA. The remaining markets are all pan-European, suggesting that the timing gaps between market trading hours should be minimal. This is in keeping with our findings, with little evident divergence between synchronized and unsynchronized correlations at a particular horizon. However, for

both synchronized and unsynchronized markets we continue to find a persistent increase in correlations from short to long horizons. The long term correlation between markets using synchronized data is 0.863 compared to the daily return correlation of 0.739, a 16.8% increase, similar to that found for US synchronized correlations. This suggests that the benefits of international diversification are mitigated at long horizons. We further consider this shortly, when we consider the impact on portfolio optimization at differing horizon.

6.4. Why do international equity correlations increase at long horizons?

In previous sections, we have detailed increased long-run correlations between international equity markets, even for contemporaneously measured markets. Previous literature has suggested that non-contemporaneous trading may induce cross-serial correlation between markets, in turn downward biasing the measurement of correlation (Schotman and Zalewska, 2006; Martens and Poon, 2001). The existence of intertemporal dependence for non-synchronous markets suggests suggests the existence of additional frictions that alter correlation measurement. These findings are in keeping with previous studies investigating the temporal characteristics of markets with synchronized trading hours but potential lags in information transmission due to liquidity differentials (Schotman and Zalewska, 2006; Ahn *et al.*, 2002; Cohen *et al.*, 1983). In particular, for equity indices the latest index price may reflect shocks to the largest stocks with smaller stocks lagging due to information transmission delays (Hou and Moskowitz, 2004). Informed trading is transmitted from large liquid stocks to smaller more illiquid stocks with a lag, inducing cross-autocorrelations between stocks (Chordia *et al.*, 2011). Moreover, the high levels of observed serial correlation found for equity indices is partially a

consequence of microstructure frictions and information transmission delays from large to small stocks (Ahn *et al.*, 2002; Lo and MacKinlay, 1990).

In this section, we show how changes in correlations from short to long-run horizons may be a result of serial- and cross-serial correlation between markets. To this end, we model wavelet long-run correlations using only short horizon (daily) data and information on leading and lagging dependencies for and between markets. These intertemporal equity market dependencies are firstly a consequence of non-synchronous trading between markets, resulting in markets impounding common information at differing times. Further, inter-dependencies may be a result of price-transmission delays between markets, previously linked to observed index serial correlation (Ahn *et al.*, 2002). While our findings on increased correlation for synchronized markets suggest possible price transmission delays between international equity markets, perhaps a consequence of slow moving capital, we leave a detailed examination of the mechanism underlying this for further study.

Proposition 2 outlines how long-run Haar scaling correlation between two stationary, finite time series can be expressed as a function of the original time series plus a correction for serial and cross-serial correlation between the original series. In this section, we use this link to model the long horizon cross-correlation using original one day interval returns. To model the long-run τ_J -day wavelet scaling cross-correlation using original untransformed (daily) data, we first measure serial correlation and all lagging and leading cross-serial correlations up to interval τ_J between each market.

Table 7 shows τ_J horizon long-run correlations modelled using only one-day returns. In each case, an average over all pairs of international equity markets is shown. Long-run correlations are modelled for a range of different long-run

horizons. For example, the 8 day horizon corresponds to the long-run wavelet correlation at horizon $\tau_3 = 2^3$. When no serial or cross-serial corrections are accounted for, Equation 12 reduces to the cross-correlation between the original, daily, time series, which underestimates the long-horizon correlation by 18.2%. Incorporating information on serial and cross-serial correlation for up to 7 lags results in an increased modelled correlation of 0.469, consistent with the average measured correlation of 0.466.

[Table 7 about here.]

Considering our model of long-run correlation at longer horizons, similar results are found. Considering the scale $\tau_6 = 128$ day horizon, we demonstrate a consistent increase in the modelled correlation as further intertemporal lags are included in the model. For example, incorporating information on serial and cross-serial correlation for up to 3 lags results in an increase of 21.8% in average modelled cross-correlation relative to daily. While this captures almost 50% of the total increase required to replicate the measured correlation, at least 31 intertemporal lags, corresponding to approximately a two month horizon, are required to accurately model the average long-run measured correlation.

Previous research examining the intertemporal behaviour of financial characteristics such as systematic risk has established that short horizon measurements are biased (Gençay *et al.*, 2005; Hawawini, 1980). These findings have been largely related to frictions in the trading process manifesting in differential price adjustment delays and resulting in cross-serial correlation between securities. Moreover, the presence of serial correlation in equity indices has been extensively documented (Ahn *et al.*, 2002; Lo and MacKinlay, 1990). This is consistent with our model,

where measured short horizon correlations are biased as a consequence of serial and cross-serial correlations. Our empirical findings suggest that the correlation between international equity indices is, on average, downward biased at short horizons. The result of this, from the perspective of investors, is that perceived benefits of international diversification may be overestimated at short horizons. We further examine the implications for investors in the following Section.

6.5. Implications for International Diversification

The evidence presented thus far indicates that correlation between international financial markets is a function of the measurement horizon. This, in turn, suggests that diversification opportunities available to international investors differ between short- and long-run horizons. In this section, we investigate the potential benefits of international diversification in two ways. First, considering an equally weighted portfolio, we determine the level of risk reduction achieved at different horizons as the number of portfolio assets increases. Next, we examine whether the mean-variance efficient frontier alters as a function of time horizon. This helps to determine whether the benefits of international diversification accrue equally to short- and long-term investors.

In Figure 1, we investigate the level of risk reduction achieved by international investors as a function of the number of equally weighted international equity market investments held. The level of risk reduction, measured using variance, is calculated as a proportion of the average risk of a single randomly chosen asset. A simulation approach is used, with the level of risk reduction measured for portfolios made up of randomly chosen assets. Risk reduction is then averaged across 10,000 simulations. Results demonstrate that the level of risk reduction achievable is a

function of horizon. For example, for a diversified portfolio made up of 15 equally weighted international markets, an investor with a short horizon of 1 day removes 33.2% of the risk involved in holding a single randomly chosen asset, while an investor with a 128 day horizon only removes 22.2%. Robustness tests for these results are provided in Section 6.6, where results are shown to be independent of the wavelet methodology.

[Figure 1 about here.]

One potential problem with measuring only the level of risk reduction, is the potential for higher returns as compensation for the greater risk associated with long horizons. We address the risk-return trade-off now by determining the efficient frontier across the range of time horizons studied, Figure 2.¹⁷ The top figure displays the efficient frontier measured using all data from 1980 – 2011. For a given level of return, we see that the level of risk increases from short to long horizons. For example, for a 10% annualized expected return, the associated risk is 14.21% at the shortest horizon studied but 17.32% at the longest horizon, an increase of 22%. Investors with short time-horizons appear to achieve much lower levels of risk for a given level of return than long horizon investors, given the same investment set. This explicitly demonstrates the impact on portfolio allocation of the high long-run correlation between international equity markets, with greater associated levels of risk due to negated diversification benefits.

Figure 2 also shows a series of mean-variance efficient frontiers associated with the different time cohorts studied in Tables 4 and 5. Results across the cohorts

¹⁷A risk aversion level of 3 was used for the mean-variance allocation. Results were found to be consistent for differing aversion levels. The mean-variance analysis presented is ex-post.

are consistent with findings for all data, as short horizon investors experience substantially lower risk than long horizon investors across the majority of cohorts. The recent cohort, 2007 – 2011, is worth focussing on, as the difference between short and long term risk is found to be considerable for low levels of return. For example, for a 7% annualized expected return, risk is shown to increase from 16.45% to 22.51% moving from the shortest to longest horizon, an increase of 36.9%. This suggests that long horizon investors might have struggled to reduce risk during the global market crisis, contrary to market folklore.

[Figure 2 about here.]

6.6. *Robustness*

To ensure that our results are not simply an artefact of the estimation procedure, we test our results without wavelets, using sub-sampled return intervals. Table 8 displays the average correlation between the U.S. (S&P 500) and each international equity market for a range of sub-sampled return intervals.¹⁸ The trend in correlation is found to be consistent with previous wavelet results, showing increased correlation at long return intervals.

[Table 8 about here.]

Throughout this study, we have focussed on a daily sampling interval, allowing us to examine the impact of horizon on the benefits of diversification in different cohorts. In Table 9, we examine the robustness of our results using a monthly

¹⁸Similar results were obtained for the average correlation between world markets, excluding U.S.

sampling interval, to determine if our findings are a result of the base return interval chosen. Reduced data quantities mean that results are only available over the entire 1980 – 2011 period. In keeping with earlier findings, correlations are shown to increase moving from short to long horizon. Consistent with previous results, the average U.S correlation with world markets increases from 0.244 at the original daily horizon to 0.637 at a 4 month horizon, with a further increase to 0.721 at a 64 month horizon. This is substantially above the weekly or monthly correlations often used in studies of international diversification. Similarly, the inter-market correlation excluding the USA is shown to have a 64 month average of 0.72, also greater than daily or monthly measured correlations. This further supports our hypothesis that long-run investors appear to achieve much lower diversification benefits than those measured in the short-run, and that measurement of international diversification benefits using weekly or monthly data is insufficient.

[Table 9 about here.]

Optimal asset allocation decisions have been found to differ considerably during bull and bear market regimes (Okimoto, 2008; Guidolin and Timmermann, 2008). In order to ensure that our main results relating to increased unconditional correlation at long horizons are robust to varying market conditions, we explore the horizon dependence during NBER categorized economic expansion and contraction cohorts. In Table 10 we detail the average correlation between all world markets across short and long-horizons during both recessionary and expansionary periods. Results are shown to be consistent with earlier findings, displaying increased correlation in the long-run across all cohorts. This suggests that the results documented are not a consequence of asymmetric dependence structure

during bull and bear market regimes.

[Table 10 about here.]

Building on the risk reduction analysis in Section 6.5, we finally consider the benefits of international diversification using sub-sampled return intervals, Figure 3. The impact of adding additional assets to a portfolio made up of a randomly chosen selection of international equity markets is examined at different sub-sampled time horizons. The results suggest that our previous findings were not a consequence of the wavelet method adopted. As found previously, long horizon investors achieve significantly less diversification benefits than short horizon counterparts.

[Figure 3 about here.]

These robustness tests suggests that our findings of increased long-run correlation between international equity markets are not a consequence of methodology, original data periodicity or cohort.

7. Conclusions

Many investors, such as pension funds and private equity investors, have an ability to hold investments for the long-run. A common view is that such long-run investments are advantageous, in part due to an improved risk-return trade-off. However, little focus has been given to the advantages of long-run investment in a portfolio context. In this paper, we examine the portfolio risk-reduction benefits of international diversification across short- and long-run horizons. Our primary finding is that long-run portfolio risk reduction benefits from international diversification are smaller than at short horizons.

To overcome biased measurement of correlation for non-synchronous data, a novel non-parametric multi-horizon methodology is first developed. In contrast to earlier work, this paper uses wavelet scaling coefficients to provide correlation estimates with smaller errors than those found with traditional subsampling when presented with non-synchronous data. A simulation study demonstrates improvements in reducing correlation bias of up to 90.6% using the wavelet scaling methodology.

The main empirical contribution of the paper relates to the long-run diversification benefits from international diversification. Considering international equity indices, we find robust evidence of increased intermarket correlation at long-run horizons. Considering the average correlation between the U.S. and World equity markets from 1980 to 2011, correlation at a 128 day horizon is shown to be 1.23 times that at a monthly horizon, with the latter interval often considered in studies on international diversification. Examining the portfolio allocation implications, we demonstrate decreased risk reduction benefits in the long-run. Our finding of increased long-run correlation is only partially diminished for synchronized markets, suggesting that non-synchronous trading is not the sole driver of increased long-run correlation. Moreover, starting with a monthly sampling interval, increasing correlation is found to persist to horizons of up to five years.

To shed light on why long horizon international equity correlations are found to increase, a model accounting for frictions in the trading process is described. Incorporating delays in information transmission and non-synchronous trading between international markets using leading and lagging intertemporal correlations, we detail how elevated long horizon correlations may be generated using short horizon data. The implication from the model is that short-run correlations are,

in fact, downward biased. This bias is a consequence of characteristics of financial time series such as serial correlation, often attributed to frictions. Our interpretation of bias in the measurement of correlation relates to previous research that demonstrates bias in estimation for other common financial characteristics such as volatility and systematic risk as a consequence of non-synchronous trading (Lo and MacKinlay, 1990; Cohen *et al.*, 1983). The implication for investors, even those with short-run horizons, is that perceived risk reduction benefits may be overestimated using short horizon data. In summary, our results show that the benefits of international diversification are not equally dispersed across heterogeneous horizons. When measured at short horizons, the benefits of international diversification may appear to be large but this may be a function of characteristics specific to international indices such as serial correlation. For long-run investors international diversification is shown to reduce portfolio risk, but perhaps not to the extent previously thought.

Appendix

Proof of Proposition 1.

We demonstrate that determination of the Haar DWT scaling correlation at horizon $\tau_J = 2^J$ is equivalent to calculating correlation between aggregated data at the same dyadic horizon. At horizon $\tau_1 = 2^1$, the Haar scaling filter of length 2 is given by $g = (g_0, g_1) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$. Applying the Haar scaling filter to a set of returns $\{r_t\}$ and rearranging the scaling coefficient formula (Equation 2), we get

$$2v_{1,t} = r_{2t-1} + r_{2t}, \quad t = 1, 2, 3, \dots, T/2. \quad (14)$$

The orthogonality of the DWT and associated decimation gives $T/2$ scaling coefficients at horizon τ_1 , in a similar fashion to non-overlapping time series aggregation.

Next, we represent the aggregated time series at horizon $\tau_1 = 2^1$ as a summation over consecutive non-overlapping returns,

$$R_{m,2t}(\tau_1) = r_{2t-1} + r_{2t}, \quad t = 1, 2, 3, \dots, T/2. \quad (15)$$

Common terms in equations 14 and 15 are now exploited to determine the relationship between wavelet and aggregated correlations at horizon τ_1 . For time series r_m and r_n for assets m and n , aggregated covariance and variance at horizon $\tau_1 = 2^1$ may be written in terms of wavelet scaling coefficients $v_{m,1}$ and $v_{n,1}$,

$$Cov(R_m(\tau_1), R_n(\tau_1)) = 2^2 Cov(v_{m,1}, v_{n,1}) = 4\gamma_{mn,J}^2 \quad (16)$$

$$Var(R_m(\tau_1)) = 2Var(v_{m,1}) = 2\sigma_{m,j}^2 \quad (17)$$

$$Var(R_n(\tau_1)) = 2Var(v_{n,1}) = 2\sigma_{n,j}^2 \quad (18)$$

In turn, this relates the aggregated correlation between $R_m(\tau_1)$ and $R_n(\tau_1)$ to the wavelet scaling correlation

$$\rho(R_m(\tau_1), R_n(\tau_1)) = \frac{4Cov(v_{m,1}, v_{n,1})}{4Std(v_{m,1})Std(v_{n,1})} = \frac{4\gamma_{mn,J}^2}{4\sigma_{m,J}^2\sigma_{n,J}^2} = \rho(v_{m,1}, v_{n,1}) \quad (19)$$

Similarly, Haar DWT scaling coefficients, representing the average or long run trend at horizon $\tau_2 = 4$ can be rearranged as

$$2^{3/2}\tilde{v}_{2,t} = x_{4t-3} + x_{4t-2} + x_{4t-1} + x_{4t}, \quad t = 1, 2, 3, \dots, T/4. \quad (20)$$

Aggregated returns at horizon $\tau_2 = 2^2$ can similarly be written as

$$R_{m,4t}(\tau_2) = x_{4t-3} + x_{4t-2} + x_{4t-1} + x_{4t}, \quad t = 1, 2, 3, \dots, T/4 \quad (21)$$

As previously shown for horizon $\tau_1 = 2^1$, we write the aggregated variance and covariance as a function of wavelet scaling coefficients at horizon $\tau_2 = 2^2$. Aggregated correlation at horizon $\tau_2 = 2^2$ is then written in terms of wavelet scaling coefficients,

$$\rho(R_m(\tau_2), R_n(\tau_2)) = \frac{2^3Cov(v_{m,2}, v_{n,2})}{2^3Std(v_{m,2})Std(v_{n,2})} = \frac{2^3\gamma_{mn,2}^2}{2^3\sigma_{m,2}^2\sigma_{n,2}^2} = \rho(v_{m,2}, v_{n,2}) \quad (22)$$

Adopting a similar approach, we may demonstrate that the Haar wavelet scaling correlation is equivalent to the aggregated data correlation at any horizon $\tau_J = 2^J$,

$$\rho(R_m(\tau_J), R_n(\tau_J)) = \rho(v_{m,J}, v_{n,J}) \quad (23)$$

□

Proof of Proposition 2.

The estimation of cross- and serial-correlations from non-synchronous data has long been of interest (de Jong and Nijman, 1997; Lo and MacKinlay, 1990). Following Cohen *et al.* (1983), we outline how horizon τ_j cross-correlation may be written as a function of original (1 day) correlation with a correction for serial and cross-serial correlation. Applying proposition 1, long-run wavelet scaling correlation are then related to original (1 day) correlation with correction terms.

Assume that changes (returns) in two sets of stationary, finite variance time series of returns $\{R_m\}$ and $\{R_n\}$ are additive. In other words, by summing over non-overlapping individual returns $r_{m,t}^1$ and $r_{n,t}^1$, at the shortest interval (daily return intervals in this study), we achieve a τ_j -day return,

$$R_{\{m,n\},t}(\tau_1) = \sum_{i=0}^{\tau_j-1} r_{\{m,n\},\tau_j t-i}^1 \quad (24)$$

The aggregated τ_j day covariance between R_m and R_n is then given by

$$\begin{aligned} \sigma_{mn}(\tau_j) &= Cov \left[r_m^1(\tau_j), r_n^1(\tau_j) \right] \\ &= Cov \left[\sum_{i=0}^{\tau_j-1} r_{m,\tau_j t-i}^1, \sum_{k=0}^{\tau_j-1} r_{n,\tau_j t-k}^1 \right] \\ &= \sum_{i=0}^{\tau_j-1} \sum_{k=0}^{\tau_j-1} Cov \left[r_{m,\tau_j t-i}^1, r_{n,\tau_j t-k}^1 \right] \end{aligned} \quad (25)$$

The diagonal elements of the time-covariance matrix $\sigma_{mn}(\tau_j)$ are each equal to the contemporaneous covariance between r_m^1 and r_n^1 at the base horizon (1 day in our analysis). The off-diagonal covariances are leading and lagging cross-covariances. Assuming stationarity, all covariances where the lead and lag, $s = i -$

k , are the same are equal and there are $(\tau_j - s)$ of these (Similarly, for $-s = i - k$).

Considering the various component elements of $\sigma_{mn}(\tau_j)$, we can write

$$\begin{aligned}
i = k, \quad \text{Cov} \left[r_{m,\tau_j-i}^1, r_{n,\tau_j-k}^1 \right] &= \text{Cov} \left[r_m^1, r_n^1 \right] \\
i > k, \quad \text{Cov} \left[r_{m,\tau_j-i}^1, r_{n,\tau_j-k}^1 \right] &= \text{Cov} \left[r_{m,\tau_j-i}^1, r_{n,\tau_j-(i-s)}^1 \right] \\
&= \rho_{mn}^{+s} (r_m^1, r_n^1) \sigma_m \left(r_{m,\tau_j-i}^1 \right) \sigma_n \left(r_{n,\tau_j-(i-s)}^1 \right) \\
&= \rho_{mn}^{+s} (r_m^1, r_n^1) \sigma_m (r_m^1) \sigma_n (r_n^1) \\
k > i, \quad \text{Cov} \left[r_{m,\tau_j-i}^1, r_{n,\tau_j-k}^1 \right] &= \rho_{mn}^{-s} (r_m^1, r_n^1) \sigma_m (r_m^1) \sigma_n (r_n^1)
\end{aligned}$$

For brevity, we drop the common $r_{\{m,n\}}^1$ notation and then substitute each term into Equation 25, giving

$$\sigma_{mn}(\tau_j) = \tau_j \sigma_{mn}(1) + \sum_{s=1}^{\tau_j-1} (\tau_j - s) \rho_{mn}^{+s} \sigma_n + \sum_{s=1}^{\tau_j-1} (\tau_j - s) \rho_{mn}^{-s} \sigma_m \quad (26)$$

Since $\sigma_m \sigma_n = \frac{\sigma_{mn}}{\rho_{mn}}$, the covariance between r_m and r_n at horizon τ_j is

$$\sigma_{mn}(\tau_j) = \sigma_{mn}(1) \left[\tau_j + \sum_{s=1}^{\tau_j-1} \frac{\rho_{mn}^{+s} + \rho_{mn}^{-s}}{\rho_{mn}} (\tau_j - s) \right] \quad (27)$$

Similarly, the variance of an aggregated time series at horizon τ_j can be written as

$$\sigma_m(\tau_j) = \sigma_m(1) \left[\tau_j + 2 \sum_{s=1}^{\tau_j-1} \rho_{mn}^{+s} (\tau_j - s) \right] \quad (28)$$

Combining equations 28, 27 and 23, the aggregated correlation between returns r_m and r_n at horizon $\tau_j = 2^j$ can be expressed as a function of the original (one

day) time series cross-correlation plus a correction as follows:

$$\rho_{mn}(\tau_j) = \rho_{mn}(1) \times \left[\frac{\tau_j + \sum_{s=1}^{\tau_j-1} \left(\frac{\rho_{mn}^s + \rho_{mn}^{-s}}{\rho_{mn}} \right)}{\left(\tau_j + 2 \sum_{s=1}^{\tau_j-1} \rho_m^s \right) \left(\tau_j + 2 \sum_{s=1}^{\tau_j-1} \rho_n^s \right)} \right]$$

Proposition 1 demonstrated that wavelet long-run correlation between two time series is equivalent to aggregated correlation at the same horizon. Applying proposition 1, we write the wavelet scaling correlation in terms of correlation between original (one day, $r_{\{m,n\}}^1$) time series plus a correction for leading and lagging serial and cross-serial correlation,

$$\rho(v_{m,\tau_j}, v_{n,J}) = \rho(R_m(\tau_j), R_n(\tau_j)) \quad (29)$$

$$= \rho_{mn}(1) \times \left[\frac{\tau_j + \sum_{s=1}^{\tau_j-1} \left(\frac{\rho^s(r_m^1, r_n^1) + \rho^{-s}(r_m^1, r_n^1)}{\rho(r_m^1, r_n^1)} \right)}{\left(\tau_j + 2 \sum_{s=1}^{\tau_j-1} \rho^s(r_m^1) \right) \left(\tau_j + 2 \sum_{s=1}^{\tau_j-1} \rho^s(r_n^1) \right)} \right] \quad (30)$$

□

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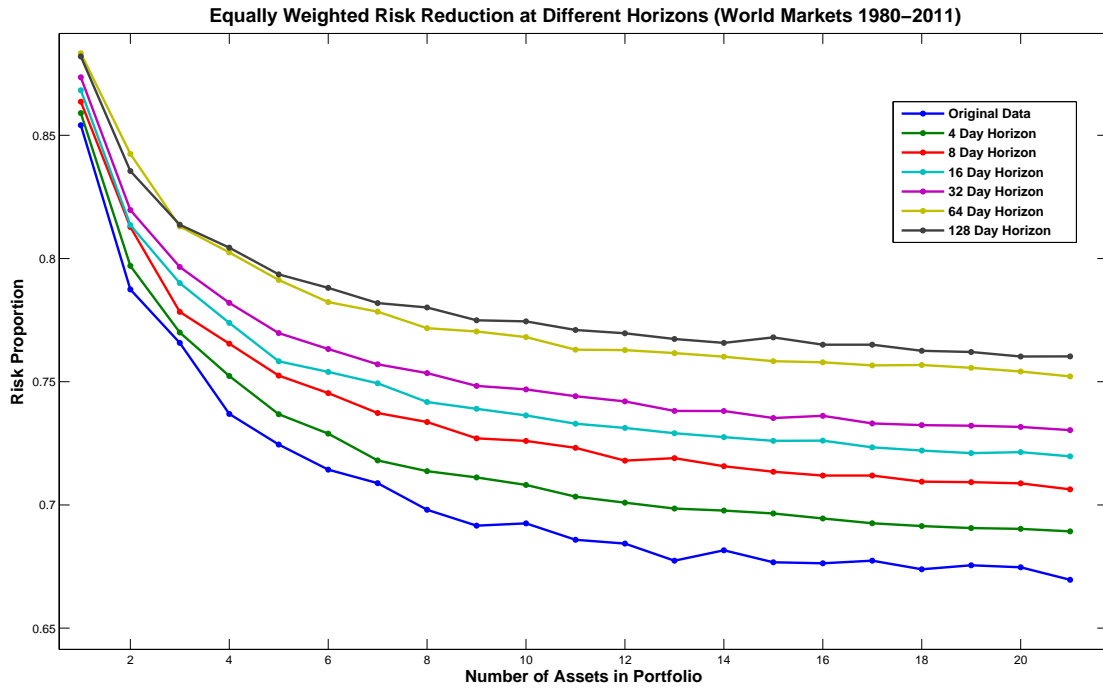


Figure 1: **Risk Reduction at a Range of Time Horizons for Differing Portfolio Sizes (1980 – 2011).**

The average risk of a portfolio made up of a number of randomly chosen assets is measured as a proportion of the average variance of a single randomly chosen asset. The Daubechies wavelet scaling coefficients are used to decompose returns data into the range of horizons detailed. Portfolio risk is determined using all available data for each asset from 1980 – 2011. A simulation approach is applied, with risk averaged over 10,000 randomly generated portfolios for each portfolio size.

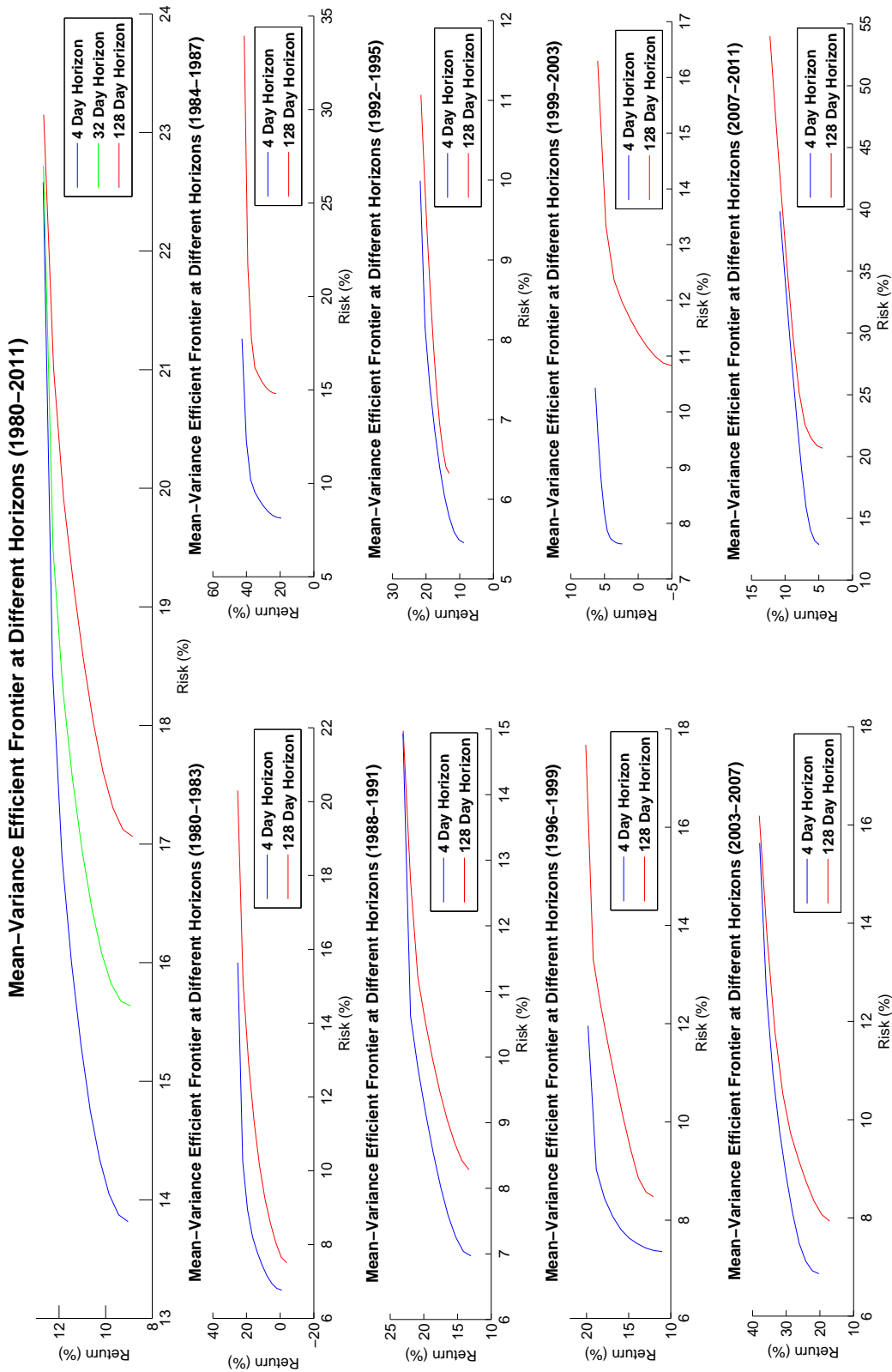


Figure 2: Mean-Variance Efficient Frontiers at different Horizons (1980-2011).

Sample based mean-variance efficient frontiers are calculated across a range of horizons and cohorts. The top figure shows the mean-variance efficient frontier using data from 1980-2011 at horizons of 4, 32 and 128 days. The following plots demonstrate the long horizon (128 day) and short horizon (4 day) mean-variance efficient frontiers in the following cohorts 1980-1983, 1984-1987, 1988-1991, 1992-1995, 1995-1999, 1999-2003, 2003-2007, 2007-2011 with covariance and mean return estimated within each cohort. The Daubechies wavelet scaling coefficients are used to decompose returns data into the range of horizons detailed, and long-run correlations are then calculated at each horizon. Mean variance allocations are made assuming a risk aversion coefficient of 3.

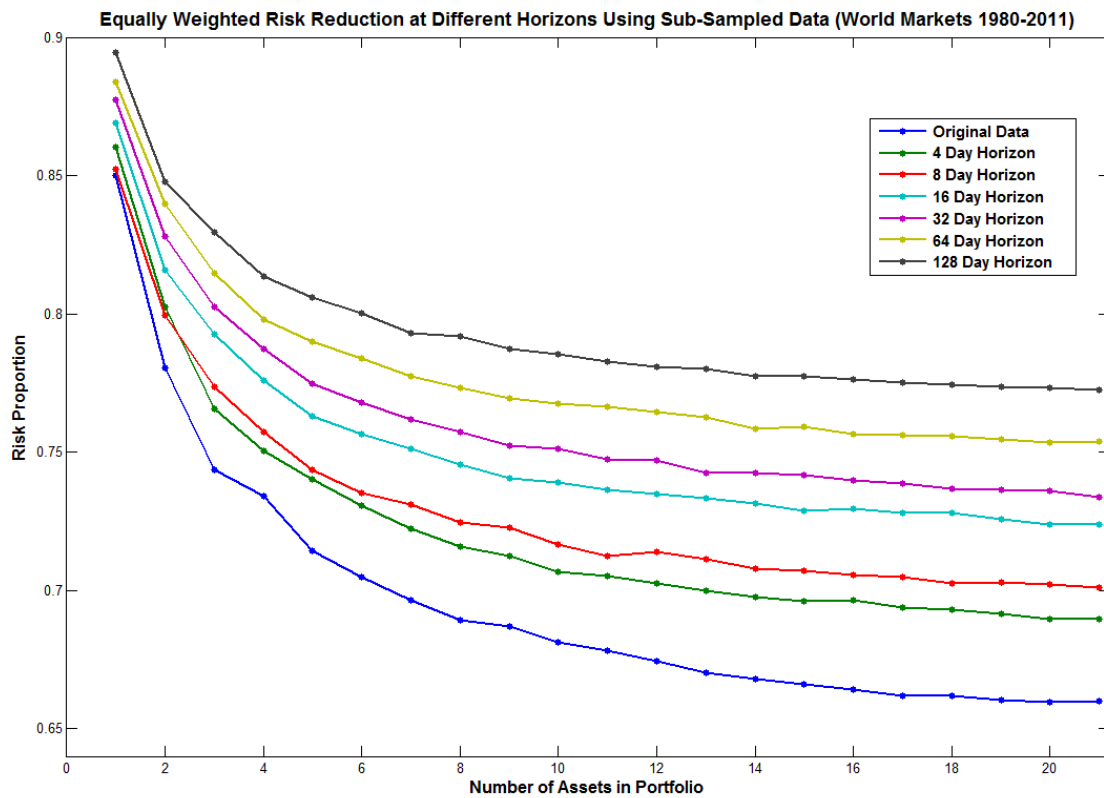


Figure 3: Risk Reduction at a Range of Sub-Sampled Time Horizons for Differing Portfolio Sizes (1980 – 2011).

The average risk of a portfolio made up of a number of randomly chosen assets is measured as a proportion of the average risk of a single randomly chosen asset. Subsampled returns are created by sampling the original data. Portfolio risk is determined using all available data for each asset from 1980 – 2011. A simulation approach is applied, with risk averaged over 10,000 randomly generated portfolios for each portfolio size.

(a) No Jumps, $T = 8, 192$

Measure	Bias					Error						
	4	8	16	32	64	128	4	8	16	32	64	128
Sub-sampled	-0.15196	-0.07603	-0.03812	-0.01923	-0.00989	-0.00549	0.15211	0.07634	0.03885	0.02128	0.01539	0.01707
Haar	-0.15197	-0.07604	-0.03809	-0.01915	-0.00979	-0.00528	0.15206	0.07619	0.03847	0.02033	0.01343	0.01376
	(1.000)	(1.000)	(0.999)	(0.996)	(0.991)	(0.961)	(1.000)	(0.998)	(0.990)	(0.956)	(0.873)	(0.806)
LA8	-0.12214	-0.03599	-0.00966	-0.00261	-0.00096	-0.00082	0.12225	0.03627	0.01101	0.00764	0.01021	0.01463
	(0.804)	(0.473)	(0.253)	(0.136)	(0.097)	(0.149)	(0.804)	(0.475)	(0.283)	(0.359)	(0.663)	(0.857)
C6	-0.13227	-0.04673	-0.01507	-0.00469	-0.00164	-0.00100	0.13237	0.04694	0.01595	0.00840	0.00989	0.01395
	(0.807)	(0.615)	(0.395)	(0.244)	(0.166)	(0.181)	(0.870)	(0.615)	(0.410)	(0.395)	(0.643)	(0.817)
D8	-0.12212	-0.03596	-0.00963	-0.00258	-0.00092	-0.00078	0.12222	0.03624	0.01098	0.00763	0.01020	0.01461
	(0.804)	(0.473)	(0.253)	(0.134)	(0.094)	(0.143)	(0.804)	(0.475)	(0.283)	(0.359)	(0.663)	(0.855)

(b) With Jumps, $T = 8, 192$

Measure	Bias					Error						
	4	8	16	32	64	128	4	8	16	32	64	128
Sub-sampled	-0.06294	-0.03159	-0.01603	-0.00839	-0.00496	-0.00391	-0.06442	0.03680	0.03109	0.03880	0.05395	0.07670
Haar	-0.06294	-0.03153	-0.01596	-0.00824	-0.00466	-0.00316	0.06401	0.03504	0.02677	0.03172	0.04388	0.06209
	(1.000)	(0.998)	(0.996)	(0.983)	(0.940)	(0.807)	(0.994)	(0.952)	(0.861)	(0.818)	(0.813)	(0.809)
LA8	-0.05061	-0.01495	-0.00424	-0.00143	-0.00111	-0.00151	0.05209	0.02272	0.02490	0.03526	0.05051	0.07265
	(0.804)	(0.473)	(0.265)	(0.170)	(0.224)	(0.386)	(0.809)	(0.617)	(0.801)	(0.909)	(0.936)	(0.947)
C6	-0.05480	-0.01940	-0.00646	-0.00229	-0.00134	-0.00151	0.05611	0.02545	0.02442	0.03385	0.04835	0.06923
	(0.871)	(0.614)	(0.403)	(0.273)	(0.270)	(0.387)	(0.871)	(0.692)	(0.785)	(0.873)	(0.896)	(0.903)
D8	-0.05060	-0.01494	-0.00423	-0.00141	-0.00110	-0.00145	0.05208	0.02271	0.02489	0.03524	0.05047	0.07257
	(0.804)	(0.473)	(0.264)	(0.169)	(0.221)	(0.371)	(0.808)	(0.617)	(0.800)	(0.908)	(0.936)	(0.946)

Table 1: **Simulation Study Contrasting Wavelet Long-Run Scaling Correlation with Subsampled Correlation for Long Time Series.**

Notes: The accuracy of wavelet scaling correlation estimation is contrasted to subsampled correlation for various specifications. Accuracy as a proportion of subsampled correlation is shown in brackets below each measurement. Bias is defined as $\mathbb{E}[\hat{\rho} - \rho]$, while error is defined using the mean square error $\mathbb{E}[(\hat{\rho} - \rho)^2]$. The scaling coefficients associated with maximum overlap discrete wavelet transform are used in the calculation of wavelet correlation. The model generating returns for two individual markets (equation 13) was simulated 50,000 times for long and short time-series, both with and without jumps. The world market is specified to have volatility of 20%, each market has a beta of 0.8 and jumps are specified to have frequency 0.01 and volatility of 200%. Sub-sampled data corresponds to original data sampled at differing horizons, Haar is the Haar wavelet, LA8 is the least asymmetric wavelet of length 8, C6 is the Coiflet wavelet of length 6 and D8 is the Daubechies wavelet of length 8. Correlation is estimated at 4, 8, 16, 32, 64 and 128 day horizons. The most accurate measurement is highlighted in bold at each horizon.

(a) No Jumps, $T = 1,024$

Measure	Bias					Error						
	4	8	16	32	64	128	4	8	16	32	64	128
Sub-sampled	-0.15245	-0.07661	-0.03907	-0.02074	-0.01262	-0.01110	0.15366	0.07899	0.04463	0.03381	0.03832	0.05557
Haar	-0.15244	-0.07661	-0.03886	-0.02035	-0.01177	-0.00886	0.15316	0.07781	0.04180	0.02837	0.02965	0.04041
	(1.000)	(1.000)	(0.995)	(0.981)	(0.933)	(0.798)	(0.997)	(0.985)	(0.937)	(0.839)	(0.774)	(0.727)
LAS	-0.12255	-0.03655	-0.01045	-0.00409	-0.00391	-0.00922	0.12340	0.03873	0.01870	0.02238	0.03456	0.06525
	(0.804)	(0.477)	(0.268)	(0.197)	(0.310)	(0.831)	(0.803)	(0.490)	(0.419)	(0.662)	(0.902)	(1.174)
C6	-0.13264	-0.04723	-0.01580	-0.00599	-0.00410	-0.00690	0.13343	0.04895	0.02189	0.02174	0.03163	0.05350
	(0.870)	(0.617)	(0.404)	(0.289)	(0.325)	(0.622)	(0.868)	(0.620)	(0.490)	(0.643)	(0.825)	(0.963)
D8	-0.12238	-0.03632	-0.01020	-0.00381	-0.00356	-0.00846	0.12323	0.03850	0.01851	0.02220	0.03419	0.06394
	(0.803)	(0.474)	(0.261)	(0.184)	(0.282)	(0.762)	(0.802)	(0.487)	(0.415)	(0.657)	(0.892)	(1.151)

(b) With Jumps, $T = 1,024$

Measure	Bias					Error						
	4	8	16	32	64	128	4	8	16	32	64	128
Sub-sampled	-0.06455	-0.03267	-0.01747	-0.01093	-0.01016	-0.01378	0.07518	0.06247	0.07706	0.10745	0.15455	0.22560
Haar	-0.06452	-0.03252	-0.01692	-0.00984	-0.00727	-0.00818	-0.07235	0.05379	0.06292	0.08696	0.12358	0.17691
	(1.000)	(0.995)	(0.969)	(0.900)	(0.716)	(0.594)	(0.962)	(0.861)	(0.817)	(0.809)	(0.800)	(0.784)
LAS	-0.05192	-0.01547	-0.00464	-0.00263	-0.00372	-0.00896	0.06262	0.05128	0.07142	0.10515	0.16017	0.26594
	(0.804)	(0.473)	(0.266)	(0.240)	(0.366)	(0.650)	(0.833)	(0.821)	(0.927)	(0.979)	(1.036)	(1.179)
C6	-0.05616	-0.02005	-0.00702	-0.00351	-0.00398	-0.00808	0.06567	0.05089	0.06806	0.09902	0.14670	0.22884
	(0.870)	(0.614)	(0.401)	(0.321)	(0.392)	(0.586)	(0.874)	(0.815)	(0.883)	(0.922)	(0.949)	(1.014)
D8	-0.05186	-0.01539	-0.00462	-0.00267	-0.00397	-0.00964	0.06255	0.05115	0.07124	0.10483	0.15923	0.26274
	(0.803)	(0.471)	(0.265)	(0.245)	(0.391)	(0.699)	(0.832)	(0.819)	(0.924)	(0.976)	(1.030)	(1.165)

Table 2: **Simulation Study Constraining Wavelet Long-Run Scaling Correlation with Subsampled Correlation for Short Time Series.**

Notes: The accuracy of wavelet scaling correlation estimation is contrasted to subsampled correlation for various specifications. Accuracy as a proportion of subsampled correlation is shown in brackets below each measurement. Bias is defined as $\mathbb{E}[\hat{\rho} - \rho]$, while error is defined using the mean square error $\mathbb{E}[(\hat{\rho} - \rho)^2]$. The scaling coefficients associated with maximum overlap discrete wavelet transform are used in the calculation of wavelet correlation. The model generating returns for two individual markets (equation 13) was simulated 50,000 times for long and short time-series, both with and without jumps. The world market is specified to have volatility of 20%, each market has a beta of 0.8 and jumps are specified to have frequency 0.01 and volatility of 200%. Sub-sampled data corresponds to original data sampled at differing horizons, Haar is the Haar wavelet, LAS is the least asymmetric wavelet of length 8, C6 is the Coiflet wavelet of length 6 and D8 is the Daubechies wavelet of length 8. Correlation is estimated at 4, 8, 16, 32, 64 and 128 day horizons. The most accurate measurement is highlighted in bold at each horizon.

Country	Index	Standard Deviation				Skewness	Kurtosis	1st Order	
		Minimum	Maximum	Deviation	Auto Correlation			LB(20)	JB
Australia	DS Index	-0.3031	0.084	0.0142	-1.87	33.90	0.052***	69.55***	397,033***
Austria	DS Index	-0.1235	0.1028	0.0124	-0.24	9.21	0.137***	225.45***	29,024***
Belgium	DS Index	-0.1181	0.0973	0.0117	-0.23	7.16	0.075***	105.04***	17,557***
Canada	S&P TSX Composite Index	-0.1998	0.1375	0.0121	-0.92	30.74	0.069***	126.04***	323,665***
Denmark	MSCI Denmark	-0.1399	0.1379	0.0139	0.60	22.27	0.001	33.55**	169,852***
France	DS Index	-0.1068	0.1067	0.0131	-0.24	6.25	0.054***	78.49***	13,413***
Germany	Dax 30	-0.1306	0.1237	0.0146	-0.20	6.22	-0.005	54.7***	13,276***
Hong Kong	Hang Seng	-0.4177	0.1727	0.0182	-2.06	44.68	-0.01	53.24***	687,271***
Ireland	DS Index	-0.1682	0.0934	0.0136	-0.73	10.25	0.063***	92.91***	36,623***
Italy	DS Index	-0.1088	0.1129	0.015	-0.19	5.36	0.087***	153.47***	9,856***
Japan	Topix	-0.1449	0.1144	0.0137	-0.07	3.06	0.037	55.59***	12,540***
Netherlands	DS Index	-0.1146	0.1022	0.0124	-0.30	8.26	0.01	69.30***	23,394***
Norway	DS Index	-0.2251	0.139	0.0166	-0.61	10.91	0.050***	84.01***	41,168***
Malaysia	KLCI Composite	-0.3701	0.2341	0.0162	-1.41	58.55	0.101***	187.13***	1,172,943***
Singapore	DS Index	-0.2648	0.1419	0.0139	-0.94	25.50	0.129***	197.67***	223,145***
South Africa	DS Index	-0.1624	0.167	0.0169	-0.53	8.15	0.078***	108.94***	23,074***
South Korea	Korea SE Composite	-0.2048	0.2744	0.0229	-0.10	12.32	-0.036***	98.60***	51,811***
Spain	Madrid SE General	-0.1104	0.1522	0.0135	-0.10	8.89	0.079***	107.08***	26,989***
Sweden	OMS Stockholm	-0.1714	0.1405	0.0163	-0.13	6.94	0.047***	88.18***	16,460***
Switzerland	DS Index	-0.1109	0.0906	0.0109	-0.33	6.78	0.035***	59.72***	15,848***
United Kingdom	FTSE 100	-0.176	0.1222	0.0127	-0.48	15.80	0.012	84.53***	85,492***
USA	S&P 500 Composite	-0.2283	0.1096	0.0113	-1.22	28.86	-0.027***	56.49***	286,275***
Average		-0.1864	0.1344	0.0144	-0.56	16.82	0.046		

Table 3: **Summary Statistics for Daily International Equity Index Returns (1980-2011)**

Notes: Each country and associated equity index is listed along with summary statistics denominated in USD. The period considered is from January 1980 to December 2011. Mean, median, minimum, maximum and standard deviation are in terms of daily returns. LB(20) is the Ljung-Box test for autocorrelation of order 20, JB is the Jacque-Bera test for normality based on excess skewness and kurtosis. 'DS Market' corresponds to the Datastream compiled representative value-weighted equity index for the country. ** indicates significance at a 5% level, while *** indicates significance at a 1% level.

Horizon	Average Correlation										All Cohorts
	1980-1983	1984-1987	1988-1991	1992-1995	1995-1999	1999-2003	2003-2007	2007-2011			
Original	0.098 [0.037, 0.158]	0.143 [0.084, 0.202]	0.164 [0.105, 0.222]	0.096 [0.036, 0.156]	0.191 [0.133, 0.248]	0.267 [0.211, 0.321]	0.251 [0.193, 0.306]	0.419 [0.369, 0.467]			0.244 [0.0.224, 0.264]
4 Days	0.198 [0.113, 0.279]	0.411 [0.337, 0.480]	0.425 [0.353, 0.493]	0.365 [0.289, 0.436]	0.461 [0.392, 0.524]	0.516 [0.453, 0.573]	0.675 [0.626, 0.718]	0.780 [0.745, 0.810]			0.508 [0.486, 0.530]
8 Days	0.223 [0.139, 0.304]	0.427 [0.354, 0.495]	0.451 [0.380, 0.517]	0.374 [0.298, 0.445]	0.508 [0.442, 0.568]	0.552 [0.492, 0.607]	0.727 [0.684, 0.765]	0.810 [0.779, 0.837]			0.527 [0.505, 0.549]
16 Days	0.260 [0.176, 0.340]	0.506 [0.439, 0.568]	0.459 [0.387, 0.525]	0.383 [0.306, 0.455]	0.497 [0.429, 0.559]	0.572 [0.513, 0.626]	0.748 [0.707, 0.784]	0.829 [0.800, 0.854]			0.550 [0.528, 0.571]
32 Days	0.202 [0.114, 0.287]	0.544 [0.478, 0.604]	0.498 [0.427, 0.563]	0.406 [0.330, 0.477]	0.566 [0.503, 0.622]	0.618 [0.561, 0.669]	0.748 [0.706, 0.785]	0.860 [0.836, 0.881]			0.571 [0.551, 0.591]
64 Days	0.271 [0.183, 0.355]	0.569 [0.502, 0.631]	0.570 [0.502, 0.632]	0.475 [0.400, 0.544]	0.609 [0.548, 0.664]	0.687 [0.637, 0.732]	0.825 [0.792, 0.853]	0.900 [0.881, 0.916]			0.623 [0.604, 0.642]
128 Days	0.249 [0.145, 0.348]	0.639 [0.570, 0.699]	0.482 [0.393, 0.562]	0.567 [0.499, 0.628]	0.611 [0.541, 0.674]	0.684 [0.624, 0.737]	0.853 [0.821, 0.881]	0.878 [0.851, 0.900]			0.662 [0.645, 0.679]
Jennrich Test											
P-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00			0.00
P-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00			0.00

Table 4: **Average Unconditional Wavelet Long-Run Correlation between US and World Markets (1980-2011).**

Notes: The average unconditional correlation between the US (S&P 500) and a range of 20 world equity markets is calculated at different time horizons using data from 1980-2011 and within 4 year cohorts. The Daubechies MODWT scaling filter of length 8 is used to decompose returns data into the range of horizons detailed, and long-run correlations are then calculated at each horizon. Correlations found using the original untransformed data are also shown. 95% confidence intervals for correlations are shown in brackets. The equality of correlation matrices constructed using short and long horizon data is tested using the Jennrich test.

Horizon	Average Correlation										All Cohorts
	1980-1983	1984-1987	1988-1991	1992-1995	1995-1999	1999-2003	2003-2007	2007-2011			
Original	0.229 [0.172, 0.285]	0.281 [0.225, 0.336]	0.376 [0.382, 0.418]	0.277 [0.221, 0.331]	0.346 [0.293, 0.397]	0.357 [0.305, 0.407]	0.525 [0.482, 0.566]	0.629 [0.594, 0.662]	0.400 [0.433, 0.466]		
4 Days	0.245 [0.164, 0.323]	0.299 [0.220, 0.375]	0.392 [0.318, 0.461]	0.267 [0.187, 0.344]	0.370 [0.296, 0.440]	0.412 [0.341, 0.479]	0.595 [0.538, 0.646]	0.700 [0.656, 0.740]	0.431 [0.406, 0.455]		
8 Days	0.269 [0.188, 0.346]	0.303 [0.223, 0.379]	0.391 [0.317, 0.461]	0.282 [0.201, 0.358]	0.414 [0.342, 0.481]	0.452 [0.383, 0.516]	0.647 [0.595, 0.694]	0.739 [0.699, 0.774]	0.453 [0.429, 0.476]		
16 Days	0.287 [0.206, 0.365]	0.376 [0.300, 0.448]	0.403 [0.328, 0.472]	0.291 [0.210, 0.368]	0.412 [0.339, 0.481]	0.506 [0.440, 0.566]	0.686 [0.638, 0.729]	0.764 [0.727, 0.797]	0.477 [0.454, 0.500]		
32 Days	0.241 [0.156, 0.323]	0.433 [0.359, 0.502]	0.441 [0.368, 0.510]	0.296 [0.214, 0.375]	0.441 [0.369, 0.508]	0.556 [0.493, 0.614]	0.707 [0.660, 0.749]	0.793 [0.759, 0.823]	0.495 [0.472, 0.518]		
64 Days	0.238 [0.149, 0.323]	0.477 [0.403, 0.546]	0.474 [0.399, 0.544]	0.343 [0.260, 0.422]	0.482 [0.410, 0.548]	0.613 [0.553, 0.667]	0.762 [0.719, 0.799]	0.843 [0.815, 0.867]	0.530 [0.508, 0.551]		
128 Days	0.333 [0.236, 0.424]	0.549 [0.471, 0.620]	0.447 [0.360, 0.526]	0.451 [0.369, 0.527]	0.477 [0.398, 0.550]	0.668 [0.604, 0.724]	0.794 [0.750, 0.831]	0.803 [0.763, 0.837]	0.570 [0.549, 0.590]		
Jenrich Test											
P-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		
P-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		

Table 5: **Average Unconditional Wavelet Long-Run Correlation between World Markets Excluding USA (1980-2011).**

Notes: The average correlation between World markets excluding the USA is calculated at different time horizons using data from 1980-2011 and within 4 year cohorts. The Daubechies MODWT scaling filter of length 8 is used to decompose returns data into the range of horizons detailed, and long-run correlations are then calculated at each horizon. Correlations found using the original untransformed data are also shown. 95% confidence intervals for correlations are shown in brackets. The equality of correlation matrices constructed using short and long horizon data is tested using the Jenrich test.

Horizon	Average Correlation USA		Average Correlation Excluding USA	
	Synchronized	Unsynchronized	Synchronized	Unsynchronized
Original	0.756 [0.737, 0.774]	0.506 [0.474, 0.538]	0.739 [0.719, 0.758]	0.747 [0.727, 0.765]
4 Days	0.787 [0.770, 0.803]	0.693 [0.670, 0.714]	0.764 [0.737, 0.788]	0.748 [0.720, 0.773]
8 Days	0.796 [0.780, 0.811]	0.763 [0.745, 0.781]	0.768 [0.742, 0.791]	0.750 [0.722, 0.775]
16 Days	0.808 [0.793, 0.822]	0.801 [0.785, 0.816]	0.776 [0.751, 0.799]	0.759 [0.733, 0.784]
32 Days	0.838 [0.824, 0.850]	0.834 [0.820, 0.847]	0.802 [0.779, 0.823]	0.791 [0.767, 0.813]
64 Days	0.869 [0.858, 0.880]	0.834 [0.820, 0.847]	0.835 [0.815, 0.854]	0.823 [0.801, 0.843]
128 Days	0.890 [0.880, 0.900]	0.891 [0.881, 0.901]	0.863 [0.844, 0.879]	0.850 [0.830, 0.868]
Jennrich Test				
P-value	Equality of 128 Day and Original Correlation Matrices 0.00		0.00	
P-value	Equality of 128 Day and 3 Day Correlation Matrices 0.00		0.00	

Table 6: **Unconditional Synchronized and Unsynchronized Wavelet Long-Run Correlations (2003-2011) for World Markets.**

Notes: Correlations calculated using daily synchronized data measured at 16:00 GMT at different horizons and using unsynchronized data measured at market close. Data is from 2003 to 2008. The Daubechies MODWT scaling filter of length 8 is used to decompose returns data into the range of horizons detailed, and correlations are then calculated at each horizon. Correlations found using the original untransformed daily data are also shown. The equality of correlation matrices constructed using short and long horizon data is tested using the Jennrich test. 95% confidence intervals are shown below each correlation. Equity market indices for each country were selected as follows: Austria - ATX Index, Denmark - KFX Index, France - CAC 40, Germany - DAX 30, Holland - AEX Index, Ireland - ISE Index, Switzerland - SWX Index, USA - S&P 500.

(i) Modelled τ Horizon Average Correlations

No of Intertemporal Lags	Horizon						
	Original	4	8	16	32	64	128
0	0.381	0.381	0.381	0.381	0.381	0.381	0.381
3		0.443	0.454	0.460	0.462	0.464	0.464
7			0.469	0.481	0.488	0.491	0.492
15				0.493	0.509	0.517	0.521
31					0.521	0.545	0.556
63						0.543	0.549
127							0.563

(ii) Measured Long-Run Average Correlations

Return Interval	Horizon						
	Original	4	8	16	32	64	128
Average Correlation	0.381	0.434	0.466	0.499	0.519	0.537	0.548

Table 7: **Modelled τ -Horizon Long-Run Correlations for Different Intertemporal Lags.**

Notes: Long horizon average τ -Day correlations between each pair of international equity indices is modelled using equation 12. The imputed correlation incorporates the contemporaneous correlation between each market in addition to a range of lagged and leading correlations, calculated using wavelet smooth. The number of intertemporal lags indicates the number of lagged and leading intertemporal variances and covariances used to impute the τ -Horizon correlation. Measured Correlations are found using wavelet scaling coefficients at different horizons. Correlations are modelled for each pair of international equity markets individually and then averaged cross-sectionally.

Horizon	Average Correlation										All Cohorts
	1980-1983	1984-1987	1988-1991	1992-1995	1995-1999	1999-2003	2003-2007	2007-2011			
1 Day	0.098 [0.037, 0.158]	0.143 [0.084, 0.201]	0.164 [0.105, 0.222]	0.096 [0.036, 0.156]	0.191 [0.133, 0.248]	0.267 [0.211, 0.321]	0.251 [0.193, 0.306]	0.419 [0.369, 0.466]			0.244 [0.224, 0.264]
4 Days	0.260 [0.144, 0.369]	0.286 [0.172, 0.392]	0.314 [0.202, 0.418]	0.198 [0.079, 0.311]	0.404 [0.299, 0.499]	0.470 [0.372, 0.557]	0.569 [0.482, 0.646]	0.581 [0.495, 0.656]			0.403 [0.367, 0.438]
8 Days	0.319 [0.159, 0.464]	0.363 [0.210, 0.500]	0.368 [0.212, 0.505]	0.214 [0.047, 0.370]	0.474 [0.333, 0.595]	0.562 [0.438, 0.667]	0.583 [0.459, 0.686]	0.733 [0.643, 0.804]			0.482 [0.435, 0.527]
16 Days	0.272 [0.037, 0.480]	0.470 [0.261, 0.637]	0.391 [0.171, 0.575]	0.214 [-0.026, 0.432]	0.542 [0.353, 0.690]	0.674 [0.522, 0.786]	0.538 [0.343, 0.690]	0.766 [0.641, 0.851]			0.525 [0.461, 0.584]
32 Days	0.369 [0.042, 0.626]	0.500 [0.202, 0.716]	0.496 [0.190, 0.716]	0.166 [-0.184, 0.480]	0.617 [0.361, 0.789]	0.690 [0.469, 0.832]	0.483 [0.174, 0.707]	0.838 [0.689, 0.919]			0.575 [0.488, 0.651]
64 Days	0.442 [-0.022, 0.753]	0.504 [0.037, 0.794]	0.540 [0.095, 0.810]	0.288 [-0.225, 0.679]	0.344 [-0.162, 0.709]	0.561 [0.121, 0.821]	0.668 [0.293, 0.869]	0.896 [0.716, 0.965]			0.590 [0.466, 0.691]
Jennrich Test	Equality of 64 Day and Original (1-Day) Correlation Matrices										
P-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 8: Average Unconditional Correlation between US and World Markets 1980-2011 Calculated using Sub-Sampled Data.

Notes: The average correlation between the S&P 500 and world markets is calculated at different time horizons. The original data is subsampled and associated correlations found at each horizon. 95% confidence intervals for correlations are shown in brackets. The equality of correlation matrices constructed using short and long horizon data is tested using the Jennrich test.

Horizon (Months)	Average U.S. Correlation	Average World Correlation
4	0.637 [0.545, 0.715]	0.548 [0.441 ,0.639]
8	0.675 [0.589, 0.747]	0.595 [0.495 ,0.680]
16	0.726 [0.649, 0.789]	0.632 [0.536 ,0.712]
32	0.706 [0.621, 0.776]	0.646 [0.546 ,0.729]
64	0.721 [0.614, 0.803]	0.720 [0.609 ,0.804]
Original Daily (daily)	0.244 [0.224, 0.264]	0.400 [0.382, 0.418]
Jennrich Test	Equality of 64 Month and Original Correlation Matrices	
P-value	0.00	0.00
	Equality of 64 Month and 4 Month Correlation Matrices	
P-value	0.00	0.00

Table 9: **Average Unconditional Wavelet Long-Run Correlation between International Markets (1980-2011) for Monthly Base Data.**

Notes: The average unconditional correlation between a range of world equity markets is calculated at different time horizons using data from 1980-2011. Monthly sub-sampled data is used as the base-data set upon which the wavelet transform is applied. The Daubechies MODWT scaling filter of length 8 is used to decompose returns data into the range of horizons detailed, and long-run correlations are then calculated at each horizon. Correlations found using the original untransformed daily data are also shown. The equality of correlation matrices constructed using short and long horizon data is tested using the Jennrich test. 95% confidence intervals are shown below each correlation.

Horizon	Average Correlation												Jennrich Test
	Q1 1980 - Q2 1980 Recession	Q2 1980 - Q2 1981 Expansion	Q3 1981 - Q4 1982 Recession	Q1 1983- Q2 1990 Expansion	Q3 1990 - Q2 1991 Recession	Q2 1991 - Q2 2001 Expansion	Q2 2001 - Q4 2001 Recession	Q1 2002 - Q4 2007 Expansion	Q1 2008 - Q2 2009 Recession	Q3 2009 - Q2 2011 Expansion			
Original Data	0.220 [0.069, 0.361]	0.217 [0.096, 0.333]	0.242 [0.146, 0.333]	0.265 [0.224, 0.304]	0.405 [0.286, 0.511]	0.313 [0.279, 0.347]	0.393 [0.282, 0.494]	0.464 [0.426, 0.500]	0.625 [0.567, 0.677]	0.611 [0.559, 0.659]			
4	0.206 [0.053, 0.351]	0.179 [0.055, 0.297]	0.242 [0.145, 0.334]	0.232 [0.190, 0.272]	0.459 [0.344, 0.561]	0.313 [0.279, 0.347]	0.468 [0.361, 0.563]	0.459 [0.420, 0.496]	0.636 [0.577, 0.690]	0.689 [0.642, 0.731]			
8	0.283 [0.135, 0.419]	0.222 [0.101, 0.338]	0.307 [0.214, 0.394]	0.287 [0.247, 0.326]	0.524 [0.417, 0.617]	0.384 [0.352, 0.416]	0.583 [0.492, 0.661]	0.581 [0.548, 0.612]	0.736 [0.689, 0.777]	0.776 [0.740, 0.808]			
16	0.479 [0.353, 0.589]	0.289 [0.173, 0.398]	0.326 [0.235, 0.412]	0.347 [0.308, 0.384]	0.529 [0.422, 0.622]	0.415 [0.383, 0.445]	0.693 [0.621, 0.755]	0.610 [0.579, 0.640]	0.799 [0.761, 0.831]	0.800 [0.766, 0.829]			
32	0.690 [0.605, 0.761]	0.291 [0.179, 0.396]	0.229 [0.135, 0.320]	0.402 [0.365, 0.437]	0.666 [0.581, 0.737]	0.429 [0.398, 0.459]	0.765 [0.709, 0.813]	0.651 [0.622, 0.678]	0.861 [0.834, 0.884]	0.802 [0.769, 0.831]			
64	0.804 [0.747, 0.850]	0.345 [0.249, 0.436]	0.322 [0.233, 0.407]	0.441 [0.406, 0.475]	0.781 [0.720, 0.831]	0.465 [0.435, 0.494]	0.781 [0.730, 0.825]	0.663 [0.635, 0.690]	0.926 [0.910, 0.939]	0.776 [0.738, 0.809]			
128	0.773 [0.691, 0.836]	0.362 [0.271, 0.448]	0.327 [0.238, 0.410]	0.490 [0.457, 0.522]	0.759 [0.700, 0.810]	0.474 [0.444, 0.502]	0.634 [0.552, 0.705]	0.713 [0.689, 0.737]	0.941 [0.928, 0.952]	0.745 [0.695, 0.789]			
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00			
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00			

Table 10: **Average Unconditional Long-Run Correlation between World Markets 1980-2011 using NBER Recession Data.**

Notes: Unconditional correlations between world equity markets are calculated at differing horizons for cohorts corresponding to NBER economic recession data. Approximate recession and expansion timings are given in each case. The Daubechies MODWT scaling filter of length 8 is used to decompose returns data into the range of horizons detailed, and long-run correlations are then calculated at each horizon. Correlations found using the original untransformed data are also shown. 95% confidence intervals for correlations are shown in brackets. The equality of correlation matrices constructed using short and long horizon data is tested using the Jennrich test.