Experimental Testing of a Cross-Entropy Algorithm to Detect Damage

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Abstract. Cross-entropy optimization has recently been applied to the damage detection in structures subject to static loading. The optimization procedure minimizes the error between the measured deflection data and theoretical deflection data obtained from artificially generated finite element models based on assumed statistical distributions of stiffness for each discretized element. Following a number of iterations, the finite element model with stiffness properties producing deflections closer to reality is established as the mathematical model closest to the true structure. However, while previous testing of the algorithm has been relatively successful, it has been limited to theoretical simulations. Therefore, this paper conducts lab experiments on a beam loaded statically to test the accuracy of the algorithm. Deflections are measured for beam scenarios under different loading levels. The accuracy of the results is discussed and recommendations are made to improve the performance of the algorithm when implemented in practice.

Introduction

The health of a beam structure can be characterized by its structural stiffness. The use of static data to monitor the health of structures requires the strain or deflection profile of a structure due to a static load. While damage only affects strain locally (i.e., the location of the measured strain needs to be close to the damaged location to appreciate changes due to damage), it does affect displacement to a larger extent (i.e., displacement will increase as a result of a loss of stiffness in the structure regardless the location of that stiffness loss and the measurement point although obviously some measurement locations will be more sensitive to damage than others). Deflection is a direct function of distribution of stiffness throughout the structure and boundary conditions. The measurement of deflection and the application of an optimization procedure to establish the stiffness distribution causing the measured deflection pattern is the method employed here to determine whether damage (i.e., a loss of stiffness) is present in the structure.

A number of authors have proposed methods to estimate the distribution of stiffness in a structure and thus locate damage based on a best fit to the response of a finite element (FE) model to the measured static displacements [1-7]. This paper focuses on the approach based on cross-entropy (CE) optimization introduced by [7] using a theoretical FE beam model (4 degrees of freedom per element). More recently, [8] and [9] extend the algorithm to structures made of plate elements (16 degrees of freedom per element). However, although results are promising, tests have been limited to theoretical simulations. The objective of this paper is to verify that this CE optimization damage detection algorithm is able to perform satisfactorily in a real life situation. For that purpose, a timber beam is statically loaded in the laboratory and deflection measurements are gathered for various scenarios. The latter are used as input to the algorithm to check its performance when predicting the stiffness distribution.

Cross-Entropy Optimization Damage Detection Algorithm

The static response of a beam to a given load is a function of the boundary conditions and the flexural stiffness, which can be described as the product of the beam's inertia (I) and its Young's modulus (E). As the deflection of a beam is a direct function of its stiffness (EI), it is theoretically possible to establish the stiffness of the beam by analyzing its deflection profile. However, determining this stiffness is a non-unique problem; i.e., EI does not need to be constant throughout the beam and a variety of stiffness combinations will give beams with similar static deflection properties. For this reason, CE optimization is employed as it allows the damage detection algorithm to consider a large number of possible stiffness profiles before refining them to achieve the value with the best performance. The theoretical response of a beam is represented using a discretized FE model based on Euler-Bernoulli beam theory. The elemental stiffness matrices $[K_i]$ of each discretized element i depends on the Young's modulus (E_i), the inertia (I_i) and the length (L_i) of each elementary beam. These elementary matrixes are combined to form the global stiffness matrix $[K_g]$ that is used to model the displacement response $\{y^t\}$ of the entire beam. The CE algorithm attempts to locate and quantify any possible damage in the true beam by creating a number of discretized FE trial beams (TBs) with varying stiffness profiles selected at random from a predefined distribution described by a mean μ_i^j and standard deviation σ_i^j at each iteration j. The algorithm then compares the structural response of each TB k ($y_r^{t,k}$) to that of the true beam (y_r^m) at each of the D measured locations. The TBs with the lowest deflection error (O_k), when compared to the measured beam, are retained and their mean stiffness and standard deviation are used as the new mean and standard deviation for the stiffness distribution of the next iteration as illustrated in Fig. 1.

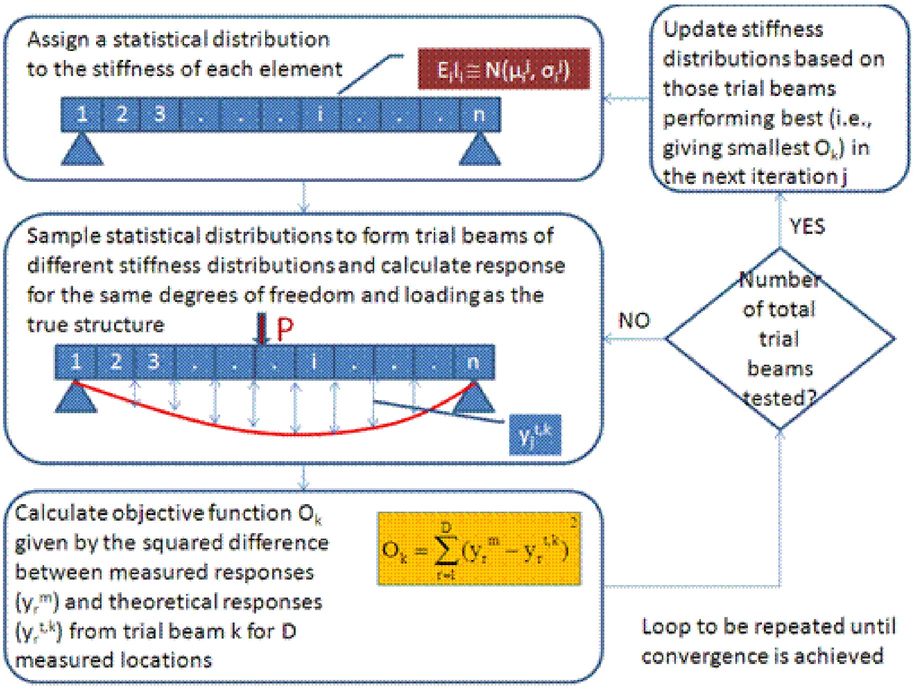


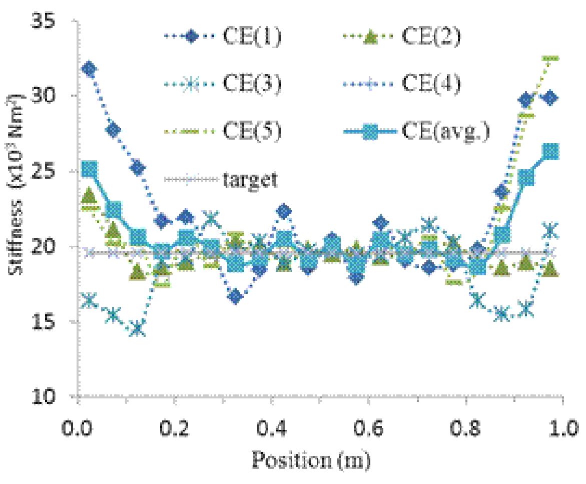
Fig. 1 – Cross-Entropy algorithm for estimation of stiffness distribution

Convergence is said here to have occurred when the rate of change of the error objective function falls by less than 0.000005% over 10 iterations of the algorithm. Then, injection (i.e., artificially inflating the standard deviation) is applied to prevent the algorithm from falling into a local minimum, allowing it to explore new combinations of stiffness values. This process continues until the system converges again. After each convergence, the solution is given by the mean of that iteration's TB elite set. A convenient method of analyzing the stiffness profile predicted by the algorithm to detect damage is by plotting a graph of stiffness against length. A potential damage location will be indicated by a local drop in the stiffness.

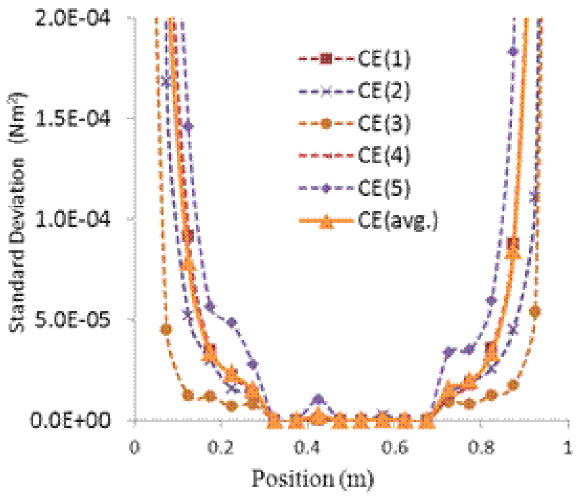
Theoretical Testing

Prior to the implementation of the test, theoretical tests are carried out to decide on the initial parameters for the CE algorithm (i.e., number of discretized elements of the FE model, number of TBs, initial values for mean and standard deviations of each elementary beam) and establish the expected level of accuracy. For that purpose, a FE discretized simply supported beam model with the same length (1 m between supports) and section ($I = 1.944 \times 10^{-6}$ m^4) as the beam in the lab is programmed in Matlab for preliminary testing using ideal theoretical results. This FE model will also be used in the experimental section. It is found twenty beam elements provide sufficient level of discretization (elements 5 cm long) as to capture the level of accuracy of the measurements (0.01 mm) without increasing computational time unnecessarily. Twenty elements also ensure that every second node in the model correspond to the location of a measured deflection value in the true beam (9 dial gages will be installed, one every 0.1 m). For preliminary testing, a theoretical healthy beam is assumed to have a constant EI value of 19573.80 Nm^2 which corresponds to an assumed modulus of elasticity of 10.071×10^9 N/m^2 (typical of timber which is expected to vary between 7×10^9 and 14 $\times 10^9$ N/m^2). Theoretical displacements are simulated at a number of beam locations for a static load of 5 kN to test the CE algorithm. For the first iteration of the CE algorithm, all discretized beams are assumed to obey to the same initial stiffness normal distribution. The mean value μ_i^1 of this initial stiffness distribution is chosen to be that of a healthy beam of the same size, shape and typical material properties altered by a random value for each individual element (i.e., sampled from a normal distribution with a mean 24% higher than the correct value), and the standard deviation σ_i^1 is set to be 20% of the initial stiffness value μ_i^1 to encompass all the possible expected values of stiffness along the beam, including damage. Additionally, injection is applied five times. The selected number of TBs to be tested in each iteration is 10000 to balance computational time and accuracy given that larger number of TBs did not improve results significantly.

The smaller the number of measurements, the larger the uncertainty of the predictions which will also depend on the location and severity of the damaged locations and their distances to the measurement points. Therefore, the impact of the number of measurements on the accuracy of the CE algorithm is tested first. For that purpose, 5, 7 and 9 displacements (all displacement locations are spaced by 0.1 m starting from mid-span. I.e., 5 inputs correspond to locations 0.3, 0.4, 0.5, 0.6 and 0.7 m from end support) are taken from the exact solution of a healthy beam of uniform stiffness, and used as input to the CE algorithm. The resulting distribution of stiffness predicted by different individual CE simulations is shown in Fig. 2 for the case of 5 available displacements.



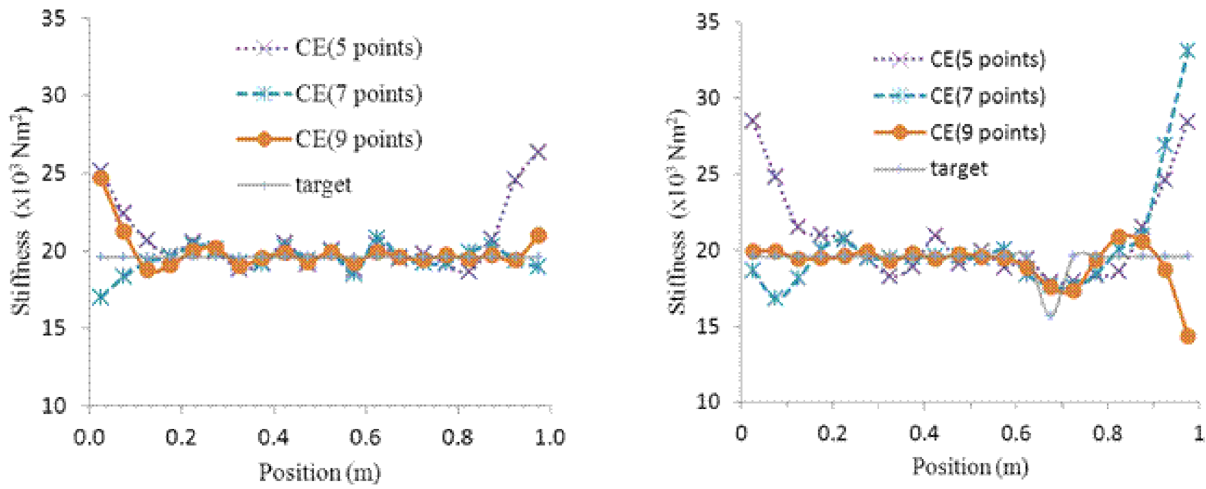
(a) mean



(b) standard deviation

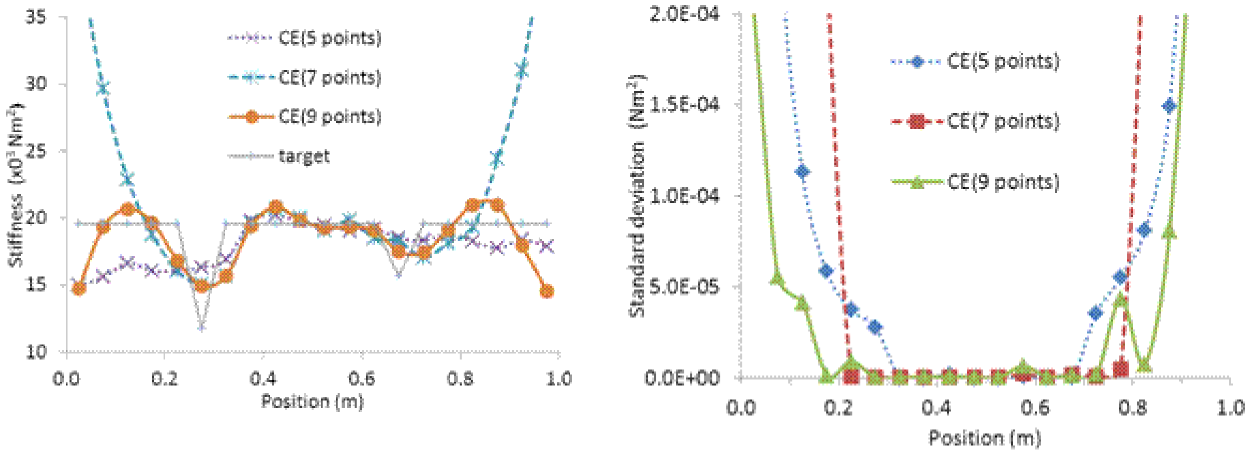
Fig. 2 – Individual CE predictions of stiffness distribution with 5 displacements

Although all individual predictions are close to the true value, there is a degree of uncertainty associated to the randomness of the process, and best results are achieved if using the average of a number of individual CE predictions. It can be noticed the inability of the algorithm to accurately capture the stiffness of elements near the supports. This is due to the relatively small contribution of these end-support elements to the overall deflections. However, the algorithm correctly identifies these predictions are not reliable by showing a large standard deviation compared to elements near mid-span where standard deviations are near zero and accuracy near 100% (this is the standard deviation of the distribution used to build the best TBs found). The potential correlation between standard deviation and accuracy of predictions is a significant virtue of the method. Fig. 3(a) compares the average of the prediction of mean and standard deviations of stiffness corresponding to five CE simulations for 5, 7 and 9 available measurements. Once the average of a number of CE simulations is calculated, the final result for this basic model of uniform stiffness is smooth and basically the same regardless the available points, except for inaccuracies near the supports that could possibly be overcome with the use of rotation measurements at these points. Fig. 3(b) carries out the same comparison but this time, the displacements used as input to the CE algorithm are the result of introducing a 20% loss of stiffness between 0.65 m and 0.70 m. It can be seen the algorithm is able to identify the damage, although it estimates it is of less severity and extended over a wider area than the true simulated damage. The latter is the consequence of modeling damage as an unrealistic sharp sudden loss of stiffness concentrated in element 14 as opposed to a gradual loss of stiffness. When comparing Figs. 3(a) and 3(b), it can be seen that the use of more points starts to show an improvement in the estimation of the stiffness profile (that should be uniform except the portion between 0.65 and 0.70 m).



(a) healthy beam (b) beam with one localized damage

Fig. 3 – Average CE predictions of stiffness distribution for different available measurement points



(a) mean (b) standard deviation

Fig. 4 – Average CE predictions of stiffness distribution for a beam with two damaged locations

Fig. 4 compares stiffness predictions using different number of measurements on a beam with two localized damages: a 20% loss of stiffness between 0.65 m and 0.70 m, and a 40% loss of stiffness between 0.25 m and 0.30 m. Comparing Figs. 3(a), 3(b) and 4(a), it can be seen how the impact of available number of measurements on accuracy becomes more evident as the stiffness profile gets less uniform. The CE based on 9 measurement points produces the most accurate stiffness profile and also the lowest overall standard deviation (Fig. 4(b)).

Experimental Set-up

Real-world measurements entail a number of inherent errors. A series of experiments are conducted in the laboratory to determine the accuracy of the algorithm when using the deflection profile of a beam subjected to various static loads. The timber beam is first tested in its original healthy state. The beam is then damaged and the tests repeated. The measured deflection profiles are used as the target deflection profile to be sought by the algorithm. The beam measures 1.2 m in length, 0.07 meters in depth and 0.068 meters in width. The frame used to support the test beams is constructed of steel box section (Fig. 5). The beam is simply supported in the unit with lateral clamps made from steel plates to hold it in place and prevent it from toppling during testing. The supports are composed of circular steel tubing, with a PVC sleeve (Fig. 6). The distance between the supports is 1 m. The beam is placed in the unit so that the overhang is 0.1 m at each support. The load cell applies a point load at the mid-span of the beam across the entire width via a semi-circular steel bar. The deflection of the beam is monitored using 9 dial gauges. A steel bracket is custom fabricated to support the dial gauges under the beam. These gauges are distributed evenly over the beam's length, which correspond to a spacing of 0.1 m between the dial gauges. Deflection data is acquired on the beam for varying load cases and damage scenarios. The latter include a healthy beam, a beam with one damaged location and a beam with two damaged locations as in the previous section.

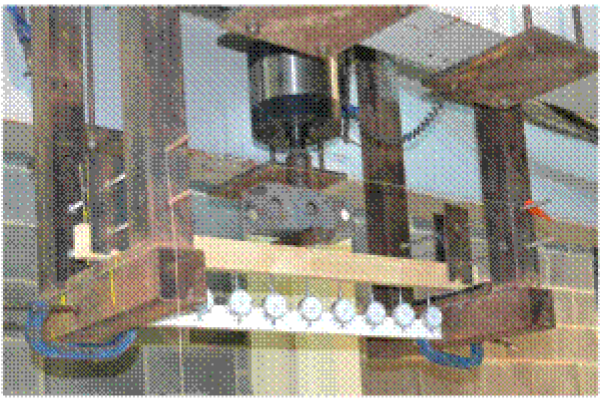


Fig. 5 – Experimental set-up

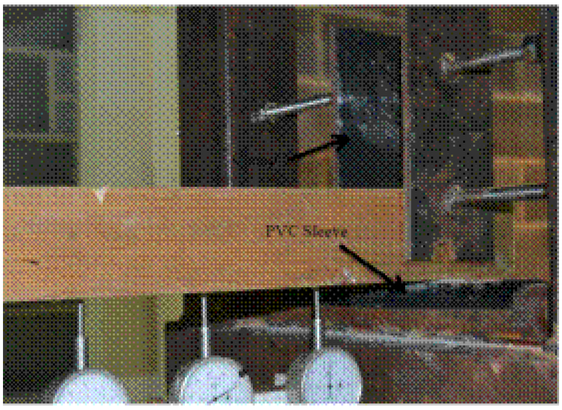
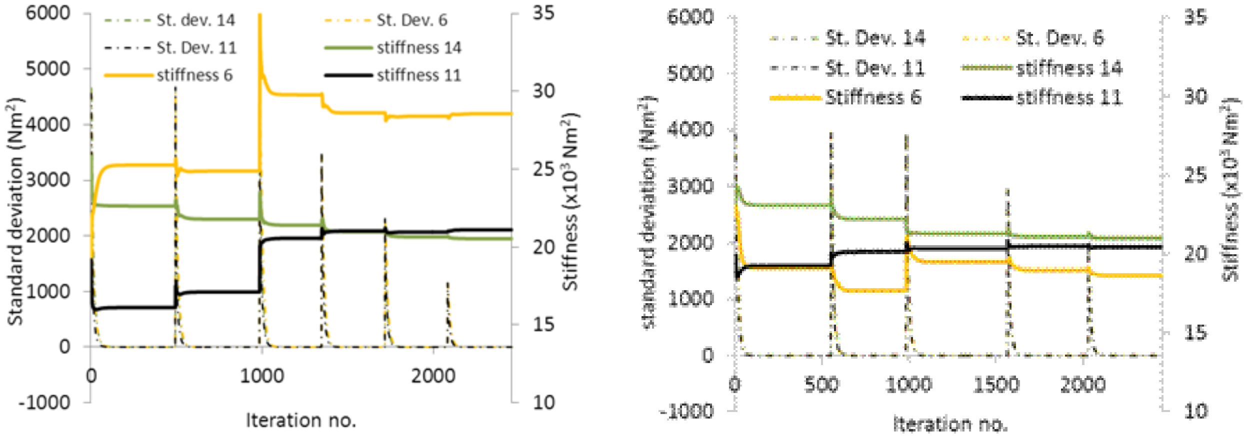


Fig. 6 – Details of support and lateral clamp

Experimental Testing

Measured deflection values show a non-symmetric shape due to a difference in the support levels and timber compressibility at the supports. The load on a timber beam tends to compress the timber fibers at points where the beam rests on supporting members. For these reasons, measured values are corrected based on the assumption of being the result of adding a rigid body movement to the beam deformation. Then, measurement points with greatest remainder error from the best-fit are identified and, given that two of these points (dial gauges located at mid-span and at 0.2 m to the right of mid-span) happened to be the same across all load states, it was decided to remove them from the analysis and repeat the best-fit with only seven points on the basis of potential faulty calibration or installation. Finally, experimental values are corrected by substracting the rigid body movement component, obtained from the seven-point best-fit. These values are then used as input for the CE algorithm.

Testing with Healthy Beam. Figs. 7(a) and (b) show the evolution of the stiffness distribution, its mean and standard deviation, through each iteration of the CE algorithm for two loading states (2 kN and 5 kN) in a CE simulation. Three representative elements of the FE model are shown: element 6 (between 0.25 and 0.30 m), element 11 (between 0.50 and 0.55 m) and element 14 (between 0.65 and 0.70 m). The iterations in which injection is applied to the stiffness distribution is clearly denoted by sudden peaks in the standard deviation.

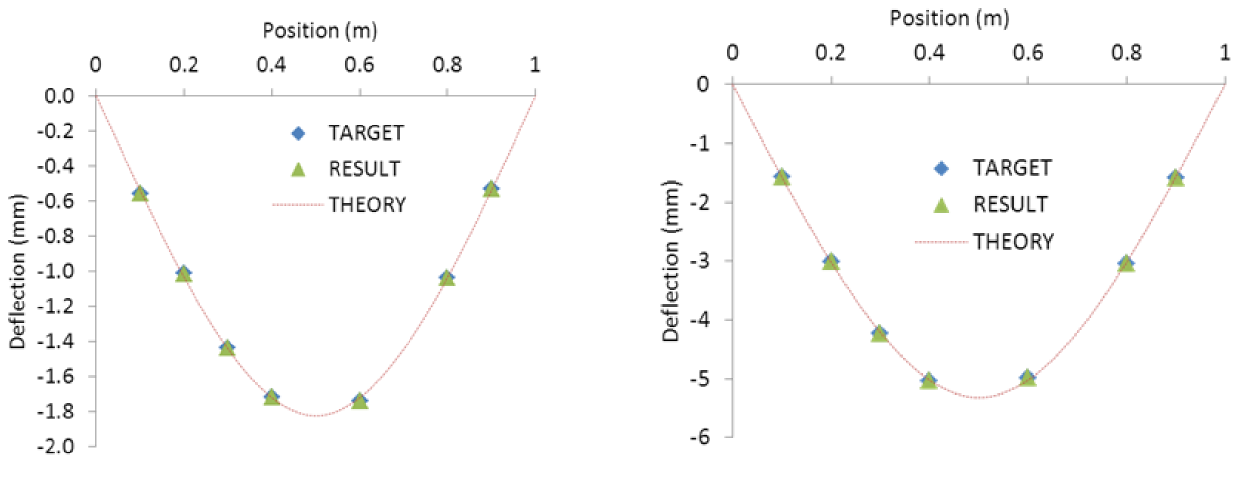


(a) 2 kN load (b) 5 kN load

Fig. 7 – Evolution of mean and standard deviation for three elements of a healthy beam

CE simulations have been repeated a number of times, using the same inputs and values for initial parameters, with only small variations in the final results (similarly to Fig. 2). For the 2 kN load, the average stiffness values found for elements 6, 11 and 14 in six CE simulations have been 28666.64, 23615.96 and 19718.72 Nm^2 respectively. For the 5kN load, average stiffness found for elements 6, 11 and 14 in five CE simulations have been 18050,85, 21517,82 and 19452.07 Nm^2 respectively. The algorithm provides stiffness values for elements 11 and 14 relatively close, whereas for element 6 is unrealistically higher in the 2 kN loading case and possibly affected by the proximity to the supports and a faulty/noisy reading. There are a number of points to notice here: (1) the predictions for element 6 are significantly different from the predictions for elements 11 and 14 in a beam of constant section; (2) the differences between the stiffness of three elements and the associated standard deivations in the 5 kN test are smaller than in the 2 kN test; (3) displacements measured for 2 kN are likely to be more affected by faulty/noisy readings than those larger measurements corresponding to 5 kN. Hence, predictions using the displacements due to 5 kN appear to be more reliable than those due to 2 kN, although a relative degree of uncertainty remains given that the objective function is not exactly zero and deviations from the expected theoretical response remain.

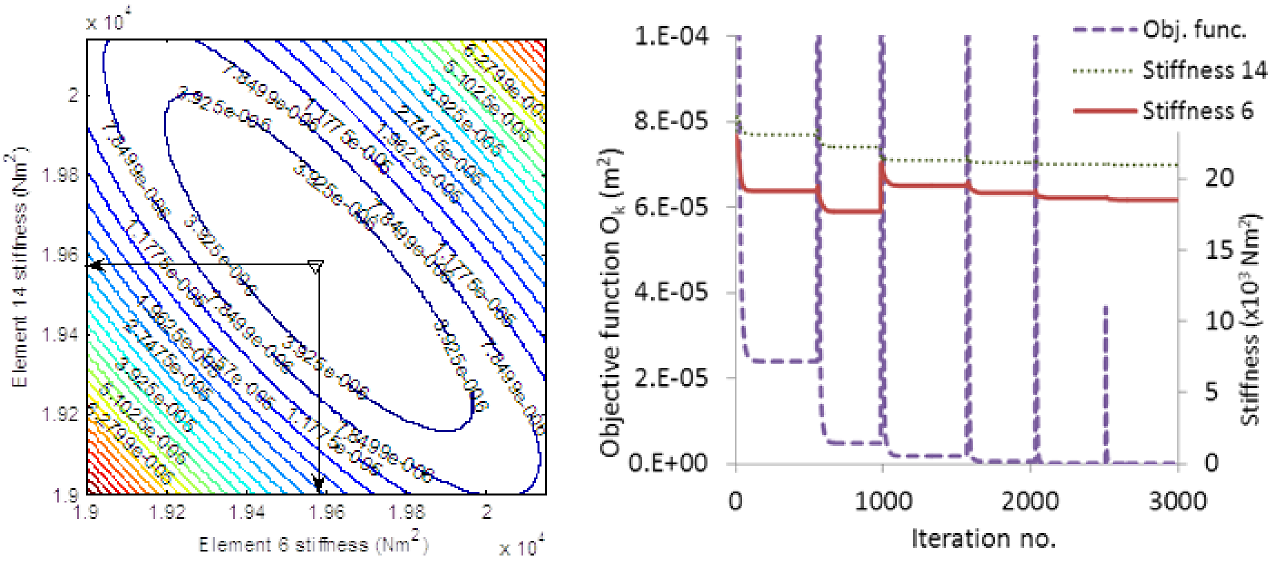
In Fig. 8, deflection values calculated with predicted stiffness results are compared to measured deflections (target) and to theoretical values obtained from a best-fit analysis to the exact equation of displacement for a beam of constant section. It is shown that the differences are very small, but sufficient to introduce inaccuracies (as in element 6 above) that would not appear using the 'ideal' measurements (as shown by Fig. 3).



(a) 2 kN load (b) 5 kN load

Fig. 8 – Measured and predicted deflections for healthy beam

Testing with Damaged Beam. Tests with damaged beams were unsuccessful. As demonstrated in the theoretical section, more complex stiffness require more accurate available measurements. If the number of measurements is small and/or corrupted (i.e., due to a faulty sensor or installation), and/or the mathematical FE model is unable to reproduce the true response (i.e., timber compresibility at supports, inaccurate boundary conditions, etc.), the algorithm will be unable to cope with the inaccuracies and will provide a non-realistic solution. In Fig. 9(a), the objective function, O_k, based on 7 theoretical ideal displacements (in the same locations as those selected in the experiment) due to 5 kN is illustrated for a discretized 20-element FE beam where the stiffness of all elements have been assumed to be known except for elements 6 and 14. As the stiffness of these unknown two elements is varied, O_k will vary and a minimum will be found at the exact solution (19573.80 Nm^2). However, small inaccuracies introduced by measurements prevent O_k from reaching a minimum for similar stiffness of elements 6 and 14 (as you would expect for a healthy beam of uniform section). This is illustrated by Fig. 9(b), which corresponds to the CE simulation of Fig. 7(b) using seven measurements, but here O_k is plotted together with stiffness of elements 6 and 14 versus iteration. It can be seen that the value of O_k corresponding to the solution (with a significant difference between the stiffness of both elements) is smaller than for previous combinations of closer values of stiffness as a result of deviations in the measurements.



(a) with 7 ideal theoretical displacements (b) with 7 measurements

Fig. 9 – Objective function versus stiffness of two elements

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Conclusions

This paper has used static deflection data and a cross-entropy optimization algorithm to locate and quantify damage. The stiffness distribution has been accurately predicted in theoretical simulations with healthy and damaged beams, except for elements near the supports. Best results have been obtained when calculating the average of the solutions provided by a number of CE simulations which reduces the randomness of the process and has the effect of smoothing the stiffness profile. The impact of the number of available measurements on the accuracy of the solution has been shown to become more significant for larger variations in the stiffness profile. Finally, the algorithm has been tested with measurements from a static load test on a timber beam. There has been a significant decrease in accuracy with respect to theoretical simulations. The algorithm has shown to be very sensitive to small deviations in measurements with respect to the response that a FE simply supported beam would expect. It is concluded that the successful implementation of the algorithm requires prior identification and removal of those outliers yielding a faulty or noisy measurement before its application.

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