

DYNAMIC AMPLIFICATION FACTOR OF CONTINUOUS VERSUS SIMPLY SUPPORTED BRIDGES DUE THE ACTION OF A MOVING LOAD

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ABSTRACT: This paper extends the research on dynamic amplification factors (DAFs) caused by traffic loading from simply supported to continuous (highway and railway) bridges. DAF is defined here as the ratio of maximum total load effect to maximum static load effect at a given section (mid-span). Another dynamic amplification factor FDAF can be defined as the ratio of the maximum total load effect throughout the entire bridge length to the maximum static load effect at a given section (mid-span). DAF/FDAF can be determined for both sagging and hogging bending moments in a continuous beam. Noticeable differences appear among DAF/FDAF of mid-span bending moment in a simply supported beam, DAF/FDAF of the mid-span bending moment in a continuous beam and the DAF/FDAF of the bending moment over the internal support in a continuous beam. Three span lengths are tested in the simply supported beam models as well as three continuous beams made of two equal spans. Each model is subjected to a moving constant point load that travels at different velocities. The location of the maximum total moment varies depending on the velocity. FDAF and DAF are plotted versus frequency ratio to allow for a generalisation of the results. The results show that FDAF is often greater than DAF in simply supported and continuous beams. Also, FDAF of sagging bending moment in continuous beam is about 12% greater than that the simply supported case. Moreover, FDAF of hogging bending moments is about 16% greater than those of sagging bending moments in the continuous beam. Consequently, all FDAF values in the continuous beam are larger than those of the simply supported case.

KEY WORDS: Dynamics; Vibration; Continuous bridges; Moving load.

1 INTRODUCTION

The dynamic behaviour of beam structures subjected to moving loads, such as bridges on highways and railways, has been investigated over a century [1]. Most of this research has been focused on simply supported structures even though continuous structures represent a significant proportion of the bridge stock. Therefore, this paper will focus on investigating the dynamic amplification of continuous structures due to a moving load and compare them to the simply supported case. From the structural point of view, the use of continuous deck reduces the bridge deck thickness. Also, the reduction of joints number in bridge structures represents substantial cost savings arising from the construction and maintenance costs of movement joints [2]. The influence of factors such as rail irregularity, ballast stiffness, suspension stiffness and suspension damping appears to be small for continuous bridges in comparison with simply supported bridges. These factors can affect drastically the riding comfort of the train cars traveling over bridges [3].

The amplification of the static response due to the vibration of the structure is characterized in many ways in literature. Cantieni [4] uses the term ‘dynamic increment’. Other terms are ‘impact factor’ or ‘dynamic load allowance’ [5], ‘dynamic increment factor’ [6], or ‘dynamic load allowance’ [7]. This paper uses the ‘Dynamic Amplification Factor’ (DAF) concept. However, there also exists different definitions for DAF. [7] defines DAF as the ‘increase in the design traffic

load resulting from the interaction of moving vehicles and the bridge structure and it is described in terms of the static equivalent of the dynamic and vibratory effects’. In [8], DAF is ‘a ratio between the maximum dynamic deflection in the moving mass problem and that in the corresponding moving force problem’, and in [9], is ‘the ratio of the maximum dynamic load effect induced by the vehicle to the maximum static load effect induced in the bridge from a vehicle’.

The static load effect at a given section is going to vary as the load crosses the structure, and it is possible to select a maximum static load effect from this time history. Similarly, the total (= static + dynamic) load effect will also vary with time and a maximum total load will be reached at some point during the load crossing. In this paper, DAF (Equation (1)) is going to mean the ratio of maximum total load effect to maximum static load effect for a given section location ‘A-A’ due to the crossing of a load. For the simply supported cases, the ‘A-A’ location will be mid-span (Figure 1). For the continuous cases, three different sections will be chosen as location ‘C-C’ and ‘B-B’ where these locations of these two sections are shown in Figure 2 as will be explained in section 3.2.

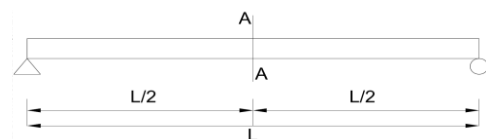


Figure 1: Location of the A-A section at the mid-length of simply supported beam

$$DAF = \frac{\text{Max. total load effect at sec. A} - A}{\text{Max. static load effect at sec. A} - A} \quad (1)$$

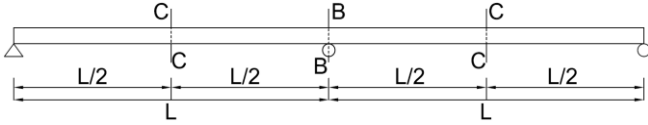


Figure 2: Location of the C-C sections at the mid-length of each of the two equal spans of continuous beam and the location of B-B section over the internal support.

Full Dynamic Amplification Factor (FDAF) is introduced first by Cantero et al. [10] and also used in this investigation for both simply supported and continuous bridges. In the latter, two FDAFs are suggested: one for sagging and another for hogging moments. FDAF gives an indication of the highest possible moment in any section of the bridge with respect to the maximum static moment at a selected section 'A-A' (Equation (2)). The numerator of Equation (2) is not necessarily the location 'A', but the section holding the highest moment during the load crossing. Therefore, FDAF must always be equal or greater than DAF.

$$FDAF = \frac{\text{Max. total load effect across the beam length}}{\text{Max. static load effect at sec. A} - A} \quad (2)$$

2 SIMULATION MODELS

1-D element Euler-Bernoulli beam elements with two degrees of freedom (2-DOF) at each node are assembled using finite element theory to build the bridge models. The length of each discretized element is 0.1 m. The bridge models are assumed to be homogenous with a modulus of elasticity of 35 GPa. A constant point load of 98.1 kN moving at velocities within a range from 1 km/h to 300 km/h is applied to the beam. This simple model of a single point load is used to represent a single vehicle/axle crossing simply supported and continuous bridges at constant velocity. This model has several significant simplifications. The mass of the vehicle is assumed to be small compared to the mass of the bridge. The vibration of the vehicle is ignored; therefore the interaction between the vehicle and the bridge is also ignored. This model is obviously different from the reality in many respects. It is employed here only to obtain an understanding of some of the principal factors which result in high amplification factors such the vehicle velocity. The Wilson- θ method [12] is used here to integrate the equations of motion and to obtain the displacements, velocities and accelerations along the beam traversed by the load. The time interval is 0.0005 and the damping ratio is 0.03. Further details on this method can be found in [12].

A one-dimensional model of the simply supported beam with a first natural frequency of 11.52 Hz is validated by comparison to the results of the closed form solution by [1] as it can be seen in Figure 3 for a load travelling at 20 m/s.

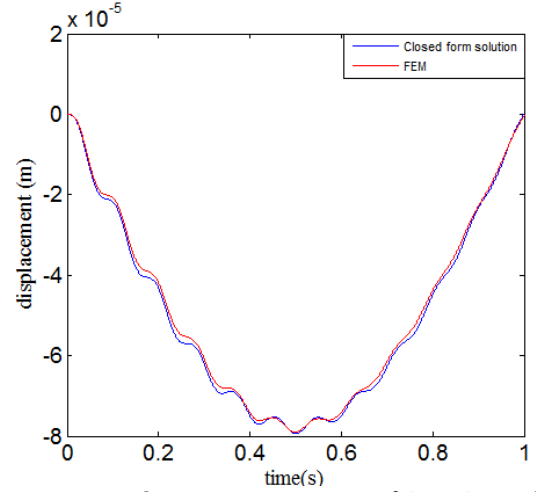


Figure 3: Displacement versus time of (20 m) simply supported beam subjected to a moving (constant) point load

A continuous beam model with first natural frequency of 11.52 Hz is also validated by comparison to approach by [13] as shown in Figure 4 for a load travelling at 20 m/s.

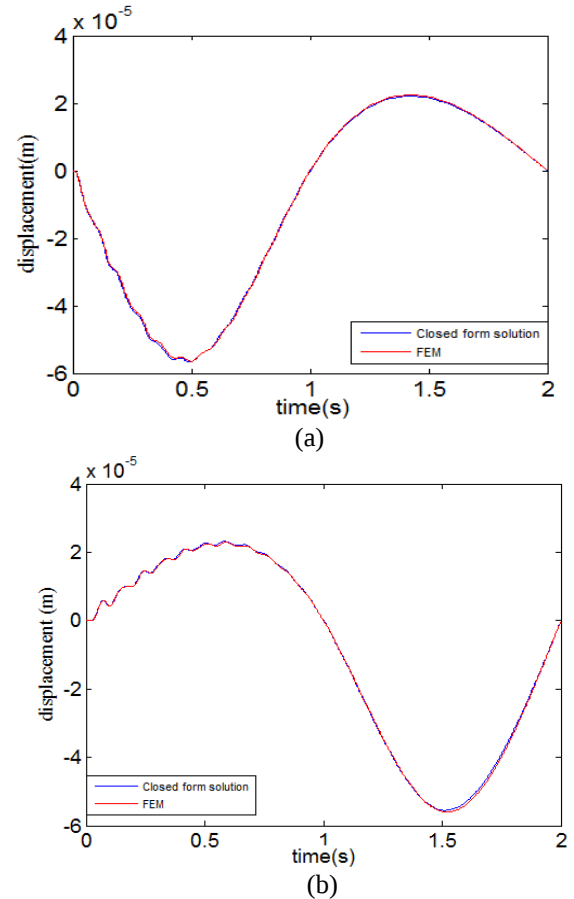


Figure 4: Displacement versus time of 40 m continuous beam with two equal spans subjected to a moving (constant) point load (a) Mid-length of first span (b) Mid-length of second span

Table 1 summarises the properties of the three simply supported beam models (10 m, 20 m and 30 m) as well as the three continuous beams made of two equal spans (10 m, 20 m

and 30 m each span) that will be analysed in the sections that follow. These properties are based on [11], where the bridge cross-sections are made of T-beam or Y-beams depending on the bridge span.

Table 1 General characteristics of the bridge models.

Type of the beam	Total length (m)	Mass per unit length (kg/m)	Second moment of inertia (m ⁴)	1st natural frequency (Hz)
S.S T-beam	10	18750	0.16783	8.79
S.S T-beam	20	37500	1.271233	4.27
S.S Y-beam	30	21080.5	2.2752	3.39
Con. T-beam	20	18750	0.16783	8.79
Con. T-beam	40	37500	1.271233	4.27
Con. Y-beam	60	21080.5	2.2752	3.39

3 DYNAMIC AMPLIFICATION FACTOR AND FULL DYNAMIC AMPLIFICATION FACTOR

3.1 Simply supported beams

Three beams with different span lengths are tested (10 m, 20 m and 30 m). As per Equation (1), DAF due to a constant moving point load is the ratio of maximum total load (static plus dynamic) effect to maximum static load effect at a given section (mid-span in this case). The static bending moment is calculated at the mid-span of the simply supported beam. The total bending moment (static plus dynamic) is also calculated at the mid-span [14]. As per Equation (2), FDAF is the ratio of the maximum total load effect across the entire bridge length to the maximum static load effect at a given section (mid-span in this case).

FDAF and DAF are plotted versus the non-dimensional Alpha parameter in Figure 5. Alpha is also known as speed parameter in [1] or frequency ratio in [9]. It is defined as the ratio of load circular frequency to the first circular frequency of the beam (Equation (3)).

$$\text{Alpha} = \frac{c}{2fL} \quad (3)$$

Where c is the velocity of the vehicle in m/s, f is the bridge natural frequency in Hz and L is the pan length in m.

As expected, DAF is less than FDAF [10]. The difference between DAF and FDAF generally increases as the location of the critical section (i.e., section holding the highest total moment across the beam length for the entire load crossing) moves away from mid-span due to changing of the load velocity. Maximum FDAFs and DAFs occur at a number of critical frequency ratios where the oscillatory nature of the dynamic component of the response reaches a maximum together with the maximum static.

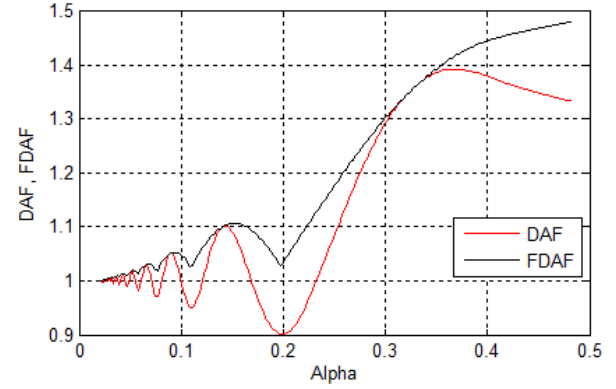
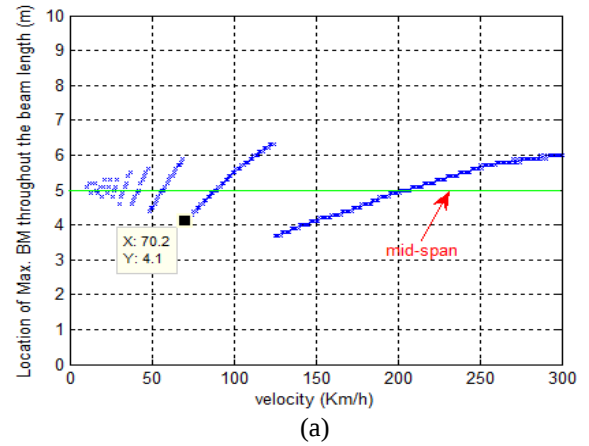


Figure 5: DAF and FDAF versus Alpha of 10 m simply supported beam

Figure 6 shows the variation of the location of maximum bending moment at the 10 m simply supported beam when subjected to a moving point load travelling at velocities from 1 km/h to 300 km/h (with a velocity increment 1 km/h). Figure 6(a) illustrates the location of the maximum total bending moment versus velocity of the moving point load acting on the beam. It is shown that, at the velocity of 70.2 km/h, the location of the maximum total bending moment is located at 4.1 m away from the left support of the beam. Figure 6(b) shows the total bending moment versus time (from the time the load enters the bridge) at the mid-span section and at a section 4.1 m away from the left support for a load velocity of 70.2 km/h. It can show the total bending moment at 4.1 m is greater than that at the mid-span of the beam.



Since different span lengths are used for simply supported bridges in this study; Figure 7 shows the FDAF versus Alpha. It can be seen that changes in the span lengths do not affect the FDAF value. Therefore, DAF and FDAF values due to a single point load can be generalized to any span length provided they are plotted versus the Alpha parameter. For a particular bridge length, Alpha parameter can be easily converted to speed or vice versa by simply applying Equation (3).

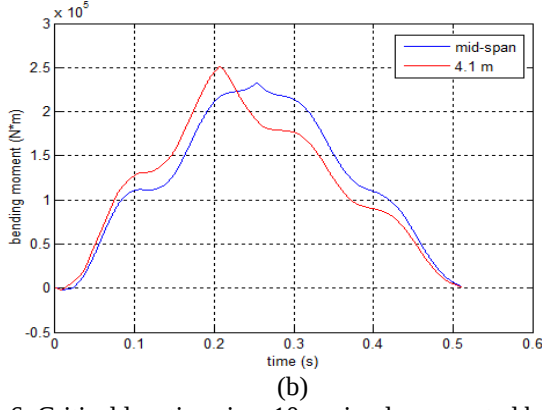


Figure 6: Critical locations in a 10 m simply supported beam: (a) location of max. total BM throughout the beam length, (b) Max. total BM in two locations

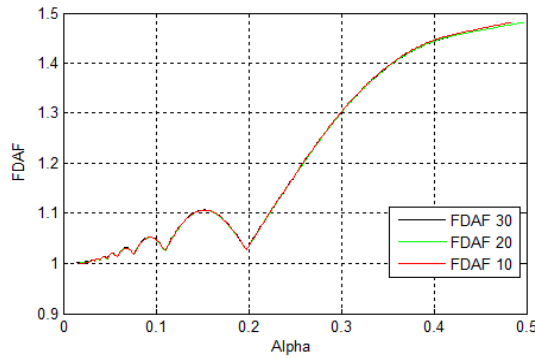


Figure 7: FDAF versus Alpha of 10 m, 20 m and 30 m simply supported beams.

3.2 Continuous beams of two equal spans

The main aim of this paper is to quantify the dynamic amplification factor of a continuous bridge compared to the simply supported. Here, continuous beams consist of two equal spans (10 m, 20 m and 30 m each span). Mid-length of the first and second spans are the two selected locations at which to refer the static sagging bending moments. Therefore, there are two DAFs of sagging moment: one for the first span (i.e., SDAF1 in Figure 8(a) which is DAF for a location 'C-C' in Equation (4) being mid-span of the 1st span) and another for the second span (i.e., SDAF2 in Figure 8(a) which is DAF for a location 'C-C' in Equation (4) being mid-span of the 2nd span).

$$SDAF = \frac{\text{Max. total load effect at Sec. C - C}}{\text{Max. static load effect at sec. C - C}} \quad (4)$$

A third DAF is introduced to characterize the hogging bending moment over the internal support (i.e., HDAF in Figure 8(b) which is DAF for a location 'B-B' in Equation (5) being the section at the internal support).

$$HDAF = \frac{\text{Max. total load effect at Sec. B - B}}{\text{Max. static load effect at sec. B - B}} \quad (5)$$

For clarity, FDAFs (Equation (4)) are redefined here for the three selected 'C-C' locations. FDAF for sagging bending

moment (i.e., FSDAF in Figure 8(a)) is the ratio of the maximum total sagging bending moment across the beam length to the maximum static sagging bending moment at the mid-length of the first span (Note: Maximum static moment at the mid-length of the first span and the second span are equal because it is a continuous beam of two equal spans). FDAF for hogging bending moment (i.e., FHDAF in Figure 8(b)) is the ratio of the maximum total hogging bending moment across the beam length to the maximum static hogging bending moment over the internal support (section B-B in Figure 2). It can be seen that FDAF is equal or larger than DAF. As before, the difference between DAF and FDAF increases as the location of the critical section varies across the beam length due to changes in load velocity. Maximum FDAFs and DAFs occur at a number of critical frequency ratios (i.e., critical speeds causing a larger dynamic amplification can be predetermined if the bridge frequency and length are known). Also FDAF and DAF of hogging bending moment is greater than that of sagging bending moment.

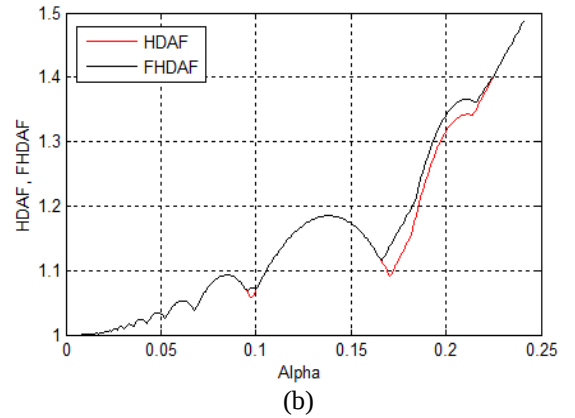
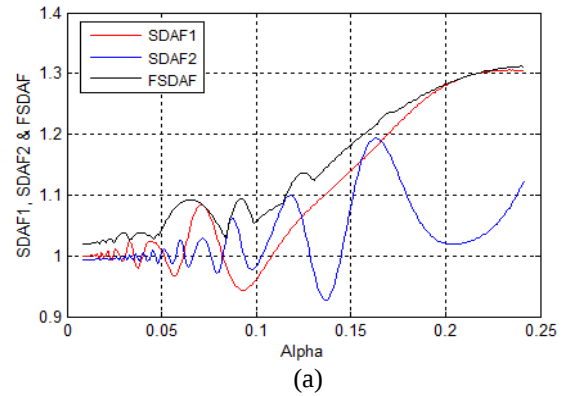


Figure 8: DAF and FDAF versus Alpha of 20 m continuous beam (a) DAF of 1st span and 2nd span, and FDAF of sagging bending moment versus Alpha (b) DAF and FDAF of hogging bending moment versus Alpha

Figure 9 shows the bending moment versus time at mid-length of the first span, mid-length of second span and over the internal support of a 20 m continuous beam with two equal spans subjected to a moving point load travelling at 1.08 km/h (0.3 m/s, i.e., practically the static component of any response to a moving point load at these locations). Figure 9(a) and (c) show the variation of the location of the maximum total hogging and sagging bending moment respectively with

velocity (with a velocity increment of 1 km/h) for a point load moving on a 20 m continuous beam.

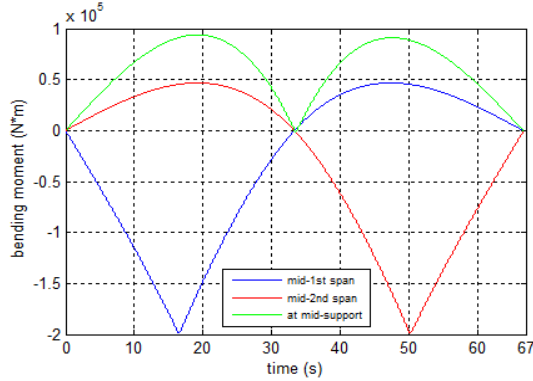
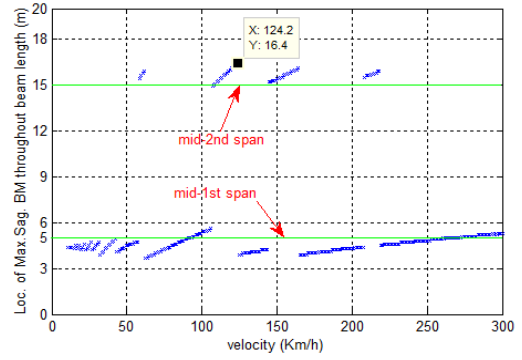
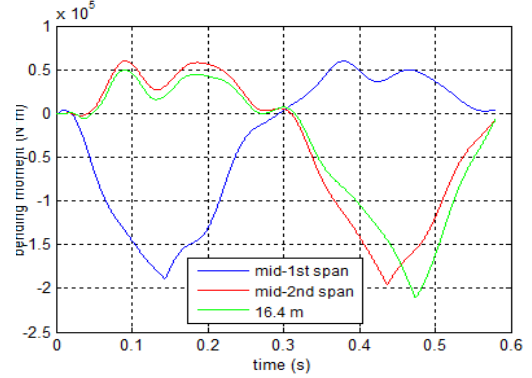


Figure 9: Bending moment versus time for a load velocity of 1.08 km/h on a 20 m continuous beam with two equal spans

In Figure 10(a), maximum total hogging moment commonly develops over the internal support, however, for a velocity of 232.2 km/h the maximum total hogging bending moment appears at 13.9 m of the beam length. Figure 10(b) illustrates the total hogging bending moment at the mid-support and 13.9 m of the beam length for this scenario at 232.2 km/h. Figure 10(c) illustrates the location of the maximum sagging bending moment for a range of load velocities (with a velocity increment of 1 km/h). Maximum sagging moment typically takes place in the 1st span, although there are a few velocities that are an exception. For example, at 124.2 km/h, the maximum total sagging bending moment is located at 16.4 m from the first support. Figure 10(d) illustrates this particular scenario where the maximum total sagging bending moment at 16.4 m exceeds both sagging moments at the mid-length of the two spans.

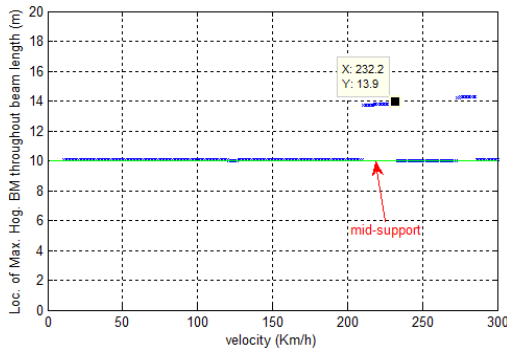


(c)

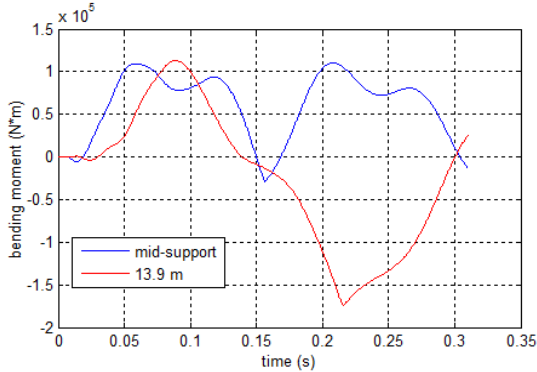


(d)

Figure 10: Critical locations in a 20 m continuous beam: (a) Location of max. total hog. BM throughout the beam length versus velocity of the moving load (b) Total BM versus time at two locations when velocity is 232.2 km/h (c) Location max. total sag. BM throughout the beam length versus velocity of the moving load (d) Total BM versus time at three locations when velocity is 124.2 km/h.

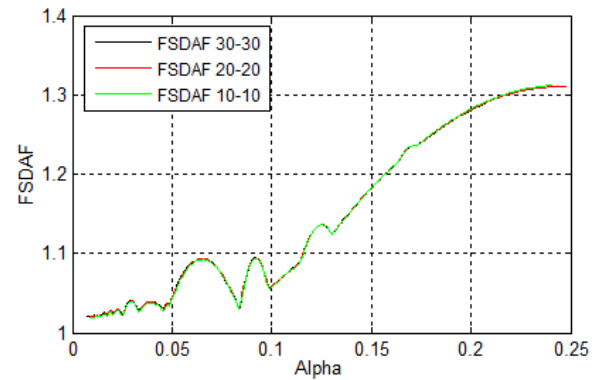


(a)



(b)

Figure 11(a) and (b) show the FDAF of sagging and hogging bending moment respectively versus Alpha for different lengths of the continuous beam. It can be seen that FDAF is not affected by changes in the span length for sagging or hogging bending moments. It can also be noticed that the FDAF of sagging bending moment in continuous beam is about 12 % greater than that the simply supported case. Moreover, the results showed that FDAF of hogging bending moments is about 16 % greater than those of sagging bending moments in continuous beam.



(a)

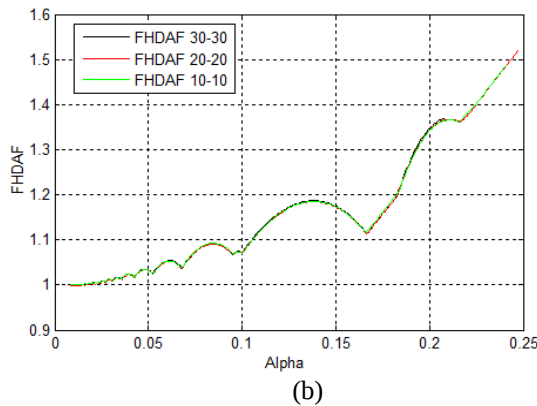


Figure 11: FDAF versus Alpha for 20 m, 40 m and 60 m continuous beams: (a) FDAF of sagging bending moment versus Alpha (b) FDAF of hogging bending moment versus Alpha

4 CONCLUSIONS

In this paper, the authors present a comparison on DAF and FDAF of continuous beams of two equal spans to those of simply supported beams. In continuous beams, two DAFs have been used to characterize sagging bending moment at the two mid-sections of the first and second spans. A third DAF has been employed to quantify the dynamic amplification for hogging bending moment over the internal support. FDAF for sagging bending moment and FDAF for hogging bending moment have also been calculated to assess the highest maximum moment and the critical sections at which they occur for each velocity of the moving load.

As expected, FDAF has been found to be greater than DAF in continuous and simply supported beams. The differences between DAF and FDAF have generally increased for those velocities where the critical section holding the maximum total moment has moved apart from the reference sections at mid-span and over the internal support. Maximum FDAFs/DAFs occur at a number of critical normalized frequency ratios (i.e., Alpha) that can be generalized to a beam model of any length when analysing the response to a single point load. DAF of sagging bending moment of continuous beam has been found to be greater than that of the simply supported beam. FDAF of sagging bending moment in continuous beam has been about 12% greater than that of the simply supported case. Finally, FDAF of the hogging bending moment has been about 16% greater than that of the sagging bending moment in a continuous beam. Therefore, this research has demonstrated that a location apparently with little dynamics as that over the internal support can lead to a more significant dynamic amplification of the hogging moment than other sagging locations.

5 ACKNOWLEDGEMENTS

The authors would like to express their gratitude to the Iraqi Ministry of Higher Education and Scientific Research (MoHESR) for the financial support towards this investigation.

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