Determination of Vertical Alignment of Track using Accelerometer Readings

Eugene J. OBrien[1], Cathal Bowe[2], Paraic Quirke[1,2]

[1]School of Civil, Structural and Environmental Engineering, University College Dublin, Dublin 4, Ireland.
[2]Iarnród Éireann Irish Rail, Technical Department, Engineering & New Works, Inchicore, Dublin 8, Ireland.

email: eugene.obrien@ucd.ie, cathal.bowe@irishrail.ie, paraic.quirke@irishrail.ie,

ABSTRACT: Railway track vertical alignment is an important indicator of serviceability condition. Through comparisons with past history, track alignment also informs maintenance planning. The vertical alignment of a railway track excites a dynamic response in a train which can potentially be used to determine that alignment. A method is proposed in this paper for the detection of the alignment through an analysis of vehicle accelerations resulting from the train/track dynamic interaction. The Cross Entropy optimisation technique is applied to determine the railway track profile heights that best fit the measured accelerations at and above a railway carriage bogie. Such an approach, using relatively low-cost accelerometers fixed to trains in regular service, would provide inexpensive daily 'drive-by' track monitoring to complement and compare data collected by the Track Recording Vehicle (TRV). The use of a TRV is the current preferred method used to determine railway track profiles using laser based methods. Numerical validation of the concept is achieved by using a 2-dimensional quarter-car dynamic model for the railway carriage and bogie to infer the track profiles in the longitudinal direction. The interaction model is implemented in MATLAB. The track is modelled as an infinitely stiff beam featuring various grades of rail irregularity which excite the vehicle inducing a dynamic response. Ten vertical elevations are found at a time which give a least squares fit of theoretical to measured accelerations. In each time step, half of these elevations are retained and a new optimisation is used to determine the next ten elevations along the length of the track. The optimised displacements are collated to determine the overall rail track profile over a finite length of railway track. This paper reports the results of the numerical simulations and the plans that are underway to further develop the model and test the concept in field trials.

KEY WORDS: Railway; Track; Vertical Alignment; Drive-by; Vehicle Rail Interaction; Cross Entropy.

1   INTRODUCTION

Increased demand on railway networks is reducing the time available to carry out inspections necessary to determine track condition. As a result, the collection of acceleration and other dynamic parameters from sensors mounted on in-service vehicles is becoming more desirable as a tool for monitoring the condition of railway track. Dynamic measurements are preferable to optical measurements which have a tendency to perform poorly in dirty railway environments. The drive-by nature of this continuous track monitoring (CTM) system has the potential to provide 'real time' feedback to railway infrastructure managers on the condition of their network. This makes possible the forecast of track defect development, verification of quality of repairs and the improvement of maintenance management [1]. A CTM system has been used by Deutsche Bahn since 2004. The system, using accelerometers fixed to axle boxes on their ICE 2 high-speed trains provides a periodic assessment of track geometry.

Traditionally, railway infrastructure managers assess the condition of their network using a Track Recording Vehicle (TRV): a specialised, instrumented train which periodically collects geometric data of the railway track including track gauge, longitudinal profile, alignment, superelevation irregularity (cross level or cant) and twist. European Standard EN13848 [2] is a series of standards defining the approach for evaluating railway track using TRVs for European Union member states. However, these vehicles are expensive to run and may disrupt regular services during their operation.

Other authors have investigated the use of vehicle acceleration measurements to determine the roughness of road profiles [3, 4]. Harris *et al.* [4] used the Cross Entropy combinatorial optimisation technique to characterise vehicle model parameters and road surface profiles using measured vehicle acceleration responses. Recently, the possibility of using inertial methods to estimate rail profiles using acceleration data measured on a vehicle has gained considerable interest. Real *et al.* [5] use a Fourier transform to solve the vehicle equations and find the transfer function that relates the input function and the output function in the frequency domain. The solution is then reverted into the time domain by applying an inverse Fourier transform to estimate the track profile. A mixed acceleration data filtering approach is used by Lee *et al.* [6] for stable displacement estimation and waveband classification of the irregularities in the measured acceleration. Investigations into the use of accelerometers and rate gyroscopes to estimate track geometry is presented by Weston *et al.* [7].

In this paper the Cross Entropy combinatorial optimisation method, as described by de Boer *et al.* [8], is adapted to determine rail profiles and the difficulties in making the transition from estimating road profiles to estimating rail profiles are discussed.

To the authors' knowledge, the Cross Entropy combinatorial optimisation method has not previously been used to determine rail profiles.

This paper is divided into the following sections. First a brief overview of existing profile estimation methods using vehicle accelerations is presented. The next section explains the numerical model used for the simulations. Following this, the Cross Entropy Method is described and the optimisation process used in this paper is discussed. This is followed by the results of the optimisation that validates the methodology for a range of profiles. Finally, an overview of the results and suggestions for further research are presented.

2   RAIL PROFILE AND VEHICLE MODEL

In order to assess the application of the combinatorial optimisation technique for estimating the rail profile for given accelerations, numerical simulation in MATLAB is used to generate the dynamic response of an instrumented vehicle. The generation of the rail profile and the procedure of calculating the vehicle dynamic responses are described in this section.

A series of rigid rail profiles are included in the simulations. It is the passage of the vehicle across the irregularities on these profiles that excite it and thereby invoke the dynamic response. A perfectly smooth profile featuring a bell-shaped normal distribution 'pothole' is first used to demonstrate the capabilities of the algorithm. Three random rail profiles of varying degrees of roughness are also generated and used in numerical tests.

Power Spectral Density (PSD) is used in several countries (including the US, Germany, China and France) to classify track quality according to its irregularity spectrum [9]. For this paper, three track profiles with random vertical irregularity are generated using theUS Federal Railroad Administration (FRA) PSD function $S(\Omega)$, **Eqn.1**[10]. The FRA function is chosen due to its common use in the literature [11, 12].

$$S(\Omega) = \frac{A_v \Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)} \tag{1}$$

where Ω is the spatial frequency, and coefficients A_v, Ω_r, Ω_c, are related to the grade of the track and are given in the **Table 1**.

Table 1. FRA American Railway Standard PSD Coefficients

Line Grade	Quality	A_v	Ω_r [rad/s]	Ω_c [rad/s]
Class 4	Very Poor	2.39×10^{-5}	2.06×10^{-2}	0.825
Class 5	Poor	9.35×10^{-6}	2.06×10^{-2}	0.825
Class 6	Moderate	1.5×10^{-6}	2.06×10^{-2}	0.825

The model of vehicle-rail interaction, which is used in this study, is shown in **Fig. 1**. The vehicle consists of two masses m_1 representing the quarter carriage mass and m_2 representing the suspension (bogie and wheelset) half mass. Each mass has a single degree of freedom (DOF), and are connected by an elastic spring k_1 and damping system c_1 representing the secondary suspension of the vehicle. The stiffness and damping of the bogie system are represented by k_2 and c_2 respectively and these parameters characterize the primary suspension of the vehicle. The sprung vehicle model is connected to the rail profile through its primary suspension system. The rail is modelled as an infinitely stiff beam of length L, with irregular vertical profile $r(x)$. This model is similar to vehicle descriptions used in other studies [13].

The vehicle properties are listed in **Table 2**. The vibrations of the main vehicle body mass are dependent on the primary and secondary suspension systems which are approximated by the adopted model. Vehicle properties have been taken from a recently published paper using a similar model [14]. The vehicle travels over the profiles at a constant velocity, v, of 108 km/hr (30 m/s). The authors acknowledge that the quarter car vehicle properties represent the upper half of the train vehicle, thereby only approximating the effect of the train wheels which exhibit high stiffness and negligible damping effects. The motive behind this decision is explained further in Section 4. This simplified vehicle is chosen to demonstrate the capabilities of the algorithm.

Table 2. Properties of the vehicle.

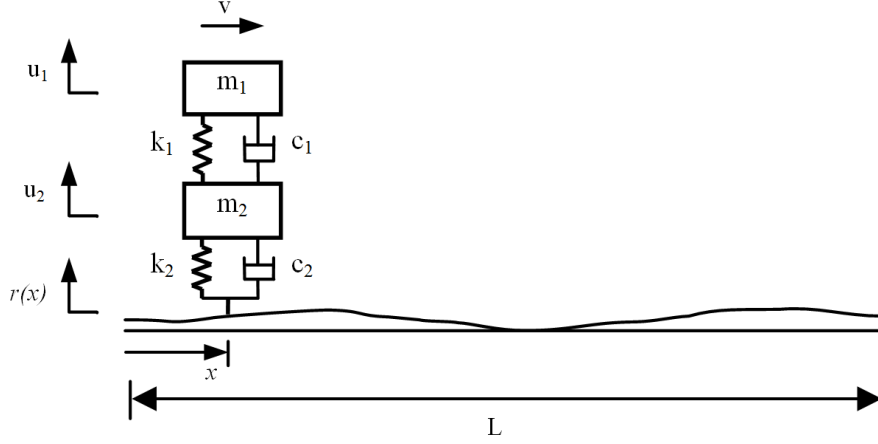| Property	Unit	Symbol	Value
Mass of main body	kg	m_1	7900
Mass of bogie and wheelset	kg	m_2	512.5
Damping of Secondary Suspension	Ns/m	c_1	15×10^3
Damping of Primary Suspension	Ns/m	c_2	5×10^3
Stiffness of Secondary Suspension	N/m	k_1	7.3×10^5
Stiffness of Primary Suspension	N/m	k_2	5×10^5
Vehicle velocity	m/s	v	30

Fig. 1. 2DOF Vehicle Rail Model.

A train/track interaction model is used to represent the vehicle travelling over the rail profile. The dynamic interaction between the vehicle and the rail is executed in MATLAB. The equations of motion of the vehicle model expressed in the time domain are as follows:

$$m_1\ddot{u}_1 + c_1(\dot{u}_1 - \dot{u}_2) + k_1(u_1 - u_2) = 0 \tag{2}$$

$$m_2\ddot{u}_2 - c_1\dot{u}_1 + (c_1 + c_2)\dot{u}_2 - k_1 u_1 + (k_1 + k_2)u_2 = c_2\dot{r} + k_2 r \tag{3}$$

where r is the rail profile and $\dot{r}$ is the first derivative of the rail profile with respect to time. These equations can be represented in matrix form as follows:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{Bmatrix} + \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1 + c_2 \end{bmatrix}\begin{Bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ c_2\dot{r} + k_2 r \end{Bmatrix} \tag{4}$$

i.e.,
$$\mathbf{M_v}\ddot{\mathbf{u}}_\mathbf{v} + \mathbf{C_v}\dot{\mathbf{u}}_\mathbf{v} + \mathbf{K_v}\mathbf{u_v} = \mathbf{f_v} \tag{5}$$

where $\mathbf{M_v}$, $\mathbf{C_v}$ and $\mathbf{K_v}$ are the mass, damping and stiffness matrices of the vehicle respectively. The vectors, $\ddot{\mathbf{u}}_\mathbf{v}$, $\dot{\mathbf{u}}_\mathbf{v}$ and $\mathbf{u_v}$ are the vehicle accelerations, velocities and displacements respectively. The displacement vector of the vehicle is, $\mathbf{u_v} = \{u_1 u_2\}^\mathrm{T}$. The vector $\mathbf{f_v}$ contains the time varying dynamic forces applied by the vehicle to the rail: $\mathbf{f_v} = \{0\ F\}^\mathrm{T}$ where F is the dynamic force,

$$F = c_2\dot{r} + k_2 r \tag{6}$$

The dynamic equations of motion of the system are solved using the Wilson-Theta integration method [15,16]. The value of the parameter, $\theta = 1.420815$, is used for unconditional stability in the integration scheme. This represents an optimal value as determined by [17].

3   CROSS-ENTROPY METHOD

The Cross-Entropy (CE) method is a combinatorial optimisation technique used in this paper to infer a series of rail profile elevations from measurements of vehicle response to them. Simply stated, the CE method is an iterative procedure which firstly generates a *population* of trial solutions according to a specified random mechanism and then updates the parameters of the random mechanism in order to produce an improved population of solutions in the next generation of estimates [4]. The second step involves minimising an objective function which is used to determine the fit of the data generated to the reference data being used.

For this study we wish to generate a population of rail profiles and determine which profile estimates generate vehicle responses most similar to the actual measured response, referred to from here as the reference acceleration signal. The reference acceleration signal is taken from the response of the vehicle described in Section 2, traversing a rail profile, as described in Section 4.

At the beginning of the algorithm, an initial normal distribution (defined by a mean and standard deviation) of values for each of the unknown rail profile elevations is assumed. Using the predefined mean and standard deviation, a population of rail profile estimates is pseudo-randomly generated. Following this, the vehicle track interaction is simulated for each profile in the

population, returning an acceleration signal for the DOF being analysed. An objective function is then minimised to determine the best matches to the reference acceleration signal. For this paper the objective function (**Eqn.7**) is defined as the sum of the squares of the differences between accelerations calculated for each trail profile, and the reference acceleration signal.

$$O(t) = \sum_{t=1}^{T} (\ddot{u}_{est,t} - \ddot{u}_{meas,t})^2 \tag{7}$$

where $O(t)$ is the objective function to be minimised, $\ddot{u}_{est,t}$ is the acceleration signal generated by the vehicle traversing the estimated profile, $\ddot{u}_{meas,t}$ is the reference acceleration signal, t is the timestep, and T is the total number of timesteps in the acceleration signal. The value of the objective function for each profile estimation in the population is ranked and, in this case, the best 10% of trial profiles are retained. The mean and standard deviation of the best profiles are calculated and used in a Monte Carlo simulation to generate a new population of profile estimates and the vehicle track interaction is simulated for the new population. This process is repeated until convergence of $\ddot{u}_{est,t}$ and $\ddot{u}_{meas,t}$ is reached. To increase computational efficiency, this iterative process is limited to 25 iterations during which it is observed that the desired convergence generally occurs.

Depending on the sampling interval and length of the profile, there may be a large number of unknowns in the problem. This means that a very large population size would be required and there is a risk that the algorithm may converge prematurely to a false solution. To overcome this problem the optimisation is split into a number of phases. Using this method, the phase location is represented by a 'window' of the profile. A number of unknowns, n, are determined within the window before moving to the next phase. At the end of each phase, the first $n/2$ best estimates of the profile heights are saved as the estimated profile, and the remaining $n/2$ profile heights are used as the first $n/2$ means for the next phase of n unknowns. To increase the efficiency of the algorithm, the remaining $n/2$ means values for the next phase are taken as the nth mean from the previous phase. Standard deviation is also reset to account for the relative uncertainty in profile heights further along the phase window being analysed. This is achieved by increasing the standard deviation in an array from 0.1 in increments of $1/n$ to 1. **Fig. 2** provides a schematic of the phasing procedure used to determine long profiles.
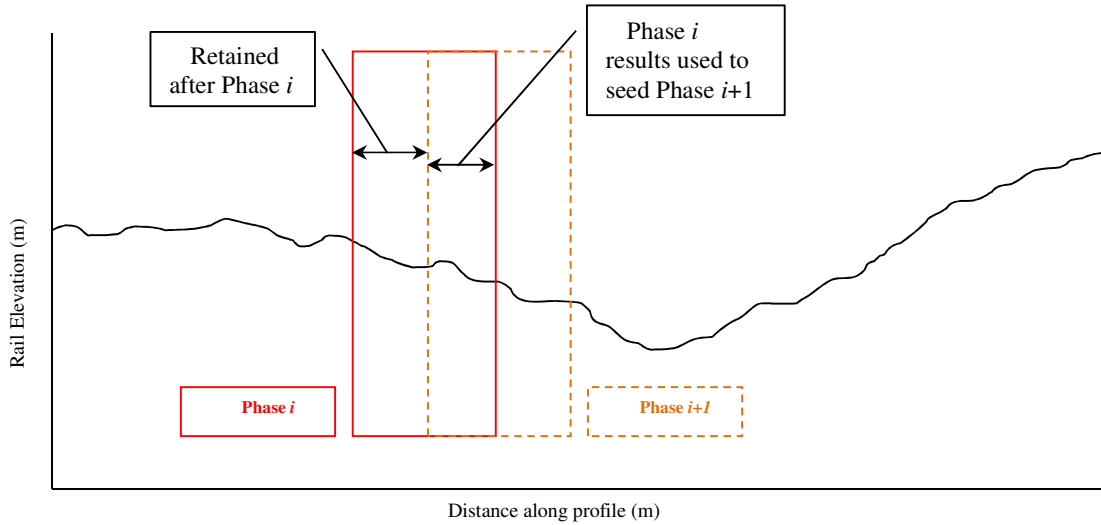

Fig. 2. 'Windowing' of Rail Profile in Phases

4   NUMERICAL ASSESSMENT

Test Profile

Numerical assessment of the Cross Entropy method is carried out in MATLAB. Firstly, a relatively smooth test rail profile, 20m in length, featuring a normal distribution 'bell' curve to represent an irregularity is used to demonstrate the capabilities of the algorithm. This 'pothole' irregularity, located at 5.0m, has a depth of 0.002m and a 'standard deviation' pothole width of 0.5. The quarter car vehicle is run across the profile and acceleration responses are generated. Following this, the Cross Entropy method, as described in Section 3, is executed in MATLAB with the parameters presented in **Table 3**. The acceleration signal from the bogie degree of freedom, m_2, is used as the reference acceleration.

Table 3. Cross Entropy Method Parameters

| Property	Value
Length of Profile Inferred per Phase	0.25m
Number of Profile Estimates per Phase	10
Initial Mean	0
Initial Standard Deviation	0.5
Size of Population of Estimates	150
Percentage of Population Transferred to Next Iteration	10%
Iterations per Phase	25

The computational time required to infer a profile 20m in length was about 2h using a 2.67 GHz processor and 6.0 GB RAM running on MATLAB. It is anticipated that improvements in algorithm efficiency and employment of more powerful parallel processors will improve on this computational time. These efficiencies are required to allow the inference of longer track profile lengths so that track geometry parameters within the wavelength ranges stipulated in EN 13848, [2] are determined and comparable track geometry results are generated from the algorithm.

The result of the rail profile estimation for the first test profile is shown in **Fig. 3**. An excellent estimate is generated with small errors: in the region of ~0.018mm.
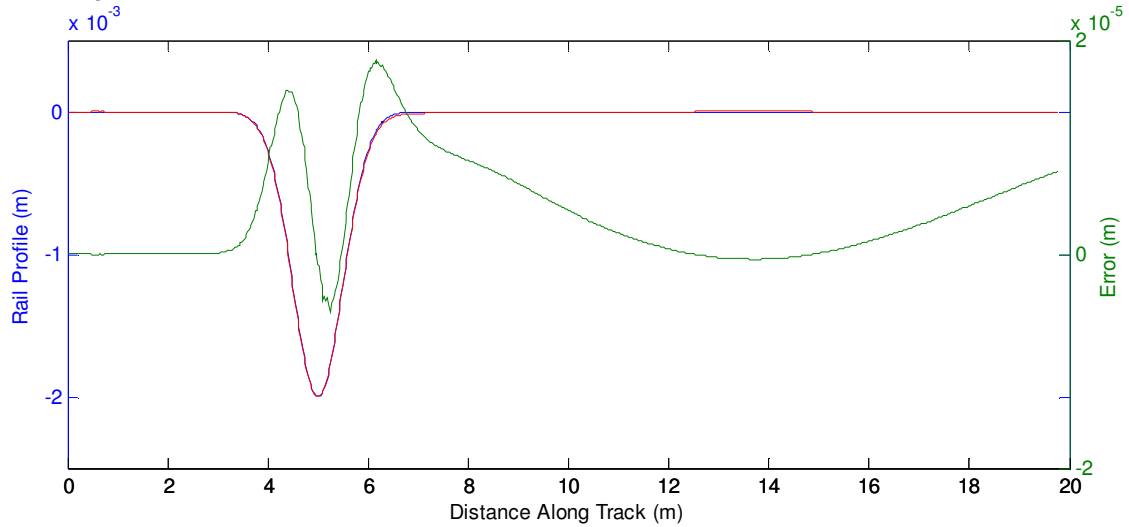
Fig. 3. Estimated (red) and actual (blue) rail profile verses distance, and errors in estimation (green) for Test Profile.

FRA Rail Profiles

The FRA PSD function presented in **Eqn. 1** is used to generate 3 rail profiles of varying roughness. The profiles range from Class 4 (roughest) to Class 6 (smoothest for non high speed rail applications). The PSDs for the profiles are presented in **Fig. 4**.
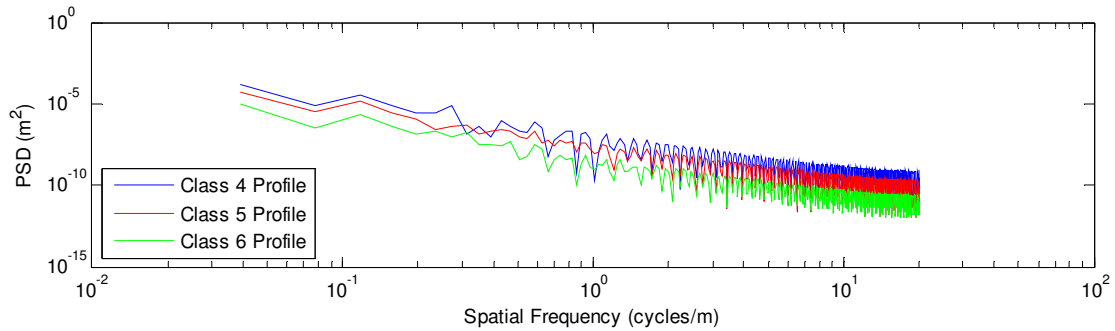
Fig. 4. Power Spectral Density for 3 FRA Rail Profiles used in analysis.

Again, the quarter car vehicle is run across the profiles and reference acceleration responses are generated. The increase in the number of irregularities and rate of elevation changes exhibited in the FRA profiles excite the vehicle in a more random fashion resulting in 'spikier' acceleration data. An alternative quarter-car vehicle model representing the train wheel and half-bogie sub-system was tested using the algorithm; however, the high stiffness of the train wheel (ignoring the influence of the non-linear wheel-rail contact phenomenon) resulted in very spiky acceleration signals. It was found that the objective function used in this

algorithm did not perform satisfactorily when processing the spiky data leading to the adoption of a simplified model ignoring axle box accelerations and using bogie sprung mass acceleration signals to infer the rail profile with more satisfactory results. Other authors [4] have employed the Cross Entropy method to determine road profiles using vehicle quarter cars exhibiting a relatively low tyre stiffness compared to the high wheel stiffness observed in rail vehicles, achieving good results.

The results of the rail profile estimation for the FRA profiles are shown in **Fig. 5**. Excellent estimates for all three profiles are generated by the algorithm, again with small errors.
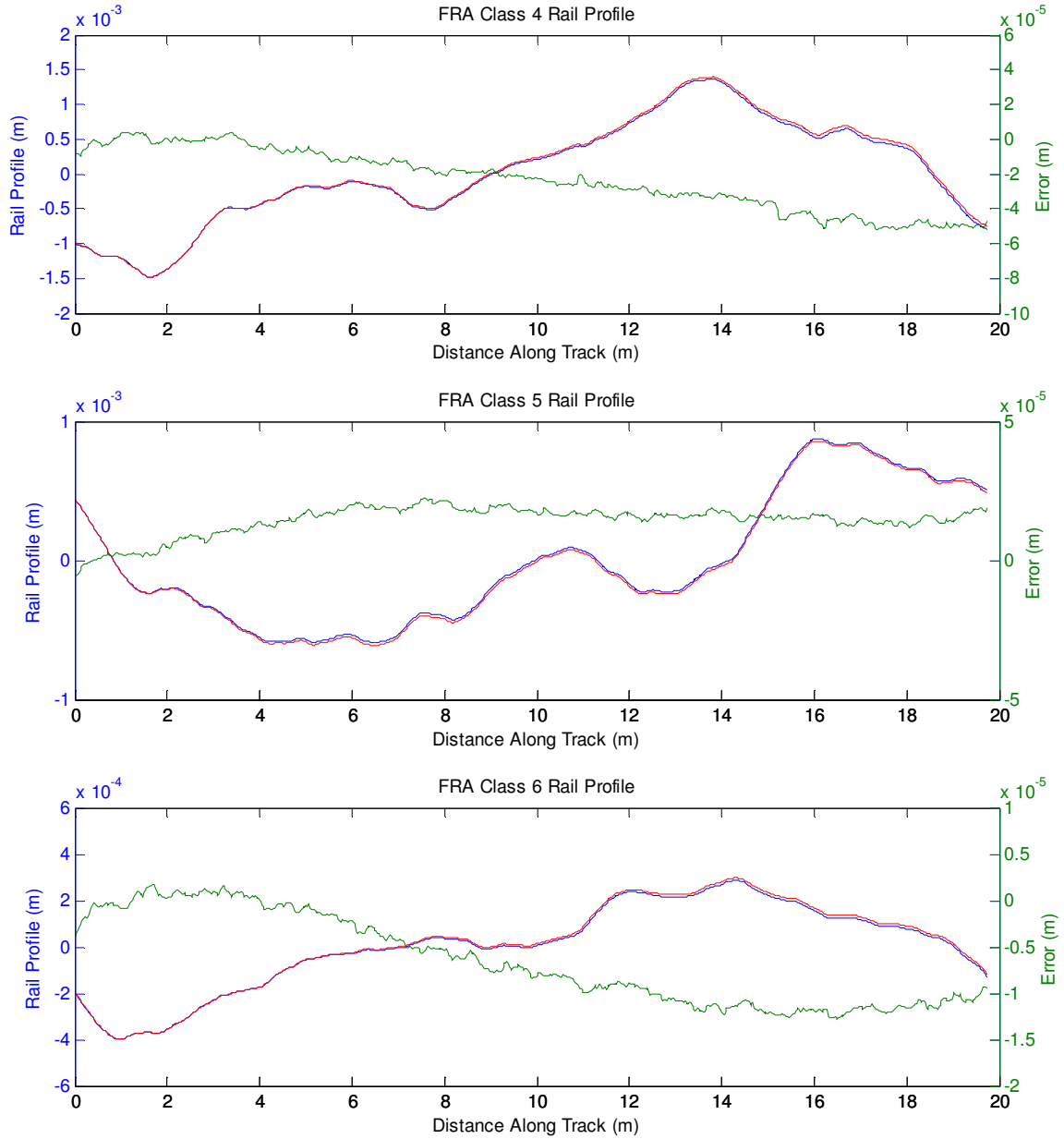

Fig. 5. Estimated (red) and actual (blue) rail profiles verses distance, and errors in estimation (green) for FRA Rail Profiles

It is observed that there is a gradual drift in the estimated rail profile which increases with distance from the origin. Harris *et al.* [4] attributed this drift to unavoidable errors in the estimation of target accelerations and any differences between the reference and observed accelerations will be apparent in the estimated profile as they are double-integrated with respect to time. It is not clear from the PSD comparison for the Class 4 profile (**Fig. 6**) whether this drift is due to low frequency or high frequency errors as the PSD profile for the actual profile and the estimate are well matched.
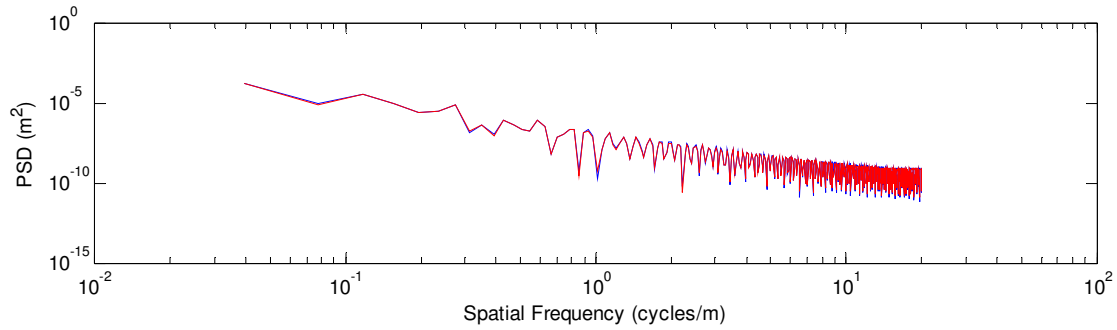
Fig. 6. Estimated (red) and actual (blue) rail profile power spectral densities for FRA Class 4 Rail Profile

5   SUMMARY AND DISCUSSION

A method for the estimation of the vertical alignment of various railway track profiles using the Cross Entropy combinatorial optimisation method has been presented. It is found that the estimated rail profiles produced by the method provide a very good fit to the actual profiles.

The analysis was carried out using a simplified railway vehicle quarter-car model using acceleration signals generated by the bogie to infer the rail profile with satisfactory results. It was found that spiky acceleration signals generated by the high stiffness properties of a train wheel in alternative vehicle arrangements were difficult to process in the algorithm. The authors acknowledge that the computational efficiency of the algorithm will need to be greatly improved prior to implementation.

Further work planned includes the transformation of the axle box acceleration signals into a form more suited to the optimisation process while retaining time-domain information. Methods such as the continuous wavelet transform (CWT) and Hilbert Transform are being investigated. The Cross Entropy method may then be used to determine the rail profile by changing the objective function to minimising the sum of squares of the differences between the intensity of the frequencies observed in the acceleration data at each time-step.

It is planned to increase the complexity of the vehicle model used in the algorithm, possibly to a more realistic configuration featuring four wheels and two bogie sets. The addition of a track model featuring rails (with profile), sleepers and ballast would allow for the deflection of track under the vehicle to be considered. The authors acknowledge that these improvements, and allowing for varying vehicle speed, must be considered prior to implementation.

The authors are in the process of acquiring accelerometers to attach to an Irish Rail train vehicle so that acceleration measurements can be obtained from a section of track. It is planned to use this data to estimate the vertical profile of the track and compare to values recorded on the TRV.

From the results shown in this paper, it can be concluded that the Cross Entropy method has the potential to be used to estimate and classify rail profiles using vehicle acceleration data. Accurate estimation of railway track vertical alignment using sensors mounted on in-service vehicles has the potential to provide a valuable tool to inform maintenance planning.

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