Developing Mathematics Teachers' Pedagogical Content Knowledge in Lesson Study: Case Study Findings

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Purpose: This study investigates the development of mathematics teachers' pedagogical content knowledge (PCK) over successive cycles of lesson study. Utilising the framework of Mathematical Knowledge for Teaching (MKT) (Ball, Thames and Phelps, 2008), this research classifies features of PCK as utilised by post-primary mathematics teachers in their planning and reflection conversations in lesson study. The development of these features of PCK is then traced over successive cycles.

Design/methodology/approach: Twelve teachers in two Irish post-primary schools participated in this research. Over the course of one academic year, these two groups of teachers completed a number of cycles of lesson study and qualitative data was generated through audio recordings of all lesson study meetings and through individual interviews with participants.

Findings: Analysis of teacher dialogue reveals distinct features of Knowledge of Content and Students (KCS) and Knowledge of Content and Teaching (KCT) incorporated by these teachers in their planning and reflection conversations, providing empirical evidence of MKT in lesson study. The occurrence of these features of KCS and KCT in lesson study conversations increased over successive cycles, demonstrating teacher learning.

Originality: This research contributes to the literature in expanding the theoretical underpinnings of teacher learning in lesson study. It also provides further empirical evidence of Mathematical Knowledge for Teaching (Ball et al., 2008) in teacher practice, specifically related to post-primary mathematics teachers.

Keywords: lesson study; mathematical knowledge for teaching; pedagogical content knowledge; professional development

Article Classification: research paper

Introduction

Since the international rise in popularity of lesson study, much research has documented teacher learning in this model of professional development (e.g. Lewis, Perry and Hurd, 2009). However, in recent years there have been calls to deepen the theoretical knowledge base about teacher learning in lesson study (Clivaz, 2015). In parallel, calls have also been made within the mathematics education research community to further investigate and detail the knowledge required to teach mathematics, particularly at post-primary level (Speer, King and Howell, 2015). This paper attempts to address these issues by investigating how post-primary mathematics teachers' pedagogical content knowledge (PCK) is incorporated in and developed through teacher dialogue on planning and reflection in lesson study.

Effective teaching requires distinct forms of subject-related knowledge and much research has identified the elements of both content and pedagogical knowledge required of mathematics teachers (e.g. Hil, Ball and Schilling, 2008). Currently in Ireland, there is an increased focus on the mathematical qualifications required to teach mathematics. This focus has come about, in part, due to the introduction of a new post-primary mathematics curriculum which emphasises changes in classroom practices from a traditionally didactic style of teaching (Lyons, Close, Boland, Lynch, Sheerin, 2003), to a more problem-solving approach based on constructivist approaches to teaching and learning (National Council for Curriculum and Assessment, 2012). Research has identified a serious issue in the high numbers (48%) of practising 'out-of-field' (non-subject specialist) teachers of mathematics (Ríordáin and Hannigan, 2011) and, in response, new subject-based, out-of-school, professional development courses have been designed and delivered nationwide for these out-of-field teachers. However, a recent report on the initial implementation of the revised curriculum has shown that there has been little change to classroom practices (Jeffes, Jones, Wilson, Lamont, Straw, Wheater, and Dawson, 2013) and

mathematics teachers have not felt supported in attempting to introduce new approaches to teaching and learning in their classrooms (National Council for Curriculum and Assessment, 2014). Professional development for out-of-field teachers has not been paralleled with forms of sustainable or school-based professional development for newly-qualified and experienced mathematics teachers. Furthermore, professional development continues to be seen as a voluntary 'add on' for post-primary teachers (Sugrue, 2006) since there is not yet official recognition or incentivisation for teacher engagement, thereby confining expertise and knowledge continue to isolated classrooms.

Encouraging reform of approaches to teaching and learning necessitates a focus on developing teachers' PCK (Yoshida and Jackson, 2011). In this research, in an attempt to investigate how teachers might be supported in developing their PCK, lesson study was introduced to two post-primary schools. Lesson study, built on the premise that teacher learning is encouraged and nurtured through collaboration (Elliott, 2012), has been incorporated in some pre-service primary teacher education programmes in Ireland (e.g. Corcoran, 2011) but has only recently been introduced to post-primary schools (see Brosnan, 2014). In this paper two lesson study communities, representing the majority of the mathematics teachers in both schools, participated in school-based lesson study (Takahashi, 2014) and planned, conducted, and reflected on specifically designed research lessons over successive cycles. Qualitative data, generated on both sites, was used to analyse the PCK incorporated and developed in planning and reflecting on successive research lessons. This research focuses on the evolution of teacher dialogue as an approach to documenting and demonstrating teacher learning (Murata, Bofferding, Pothen, Taylor, and Wischnia, 2012) and incorporates teachers' reflections on their own learning. It is situated within a sociocultural epistemology where teacher learning is analysed through their pedagogical conversations (Dudley, 2013) and extends the existing body of literature on mathematics teacher learning in lesson study.

Professional Development and Teacher Talk

Educational reform depends on teacher professional development which leads to improvements in students' learning experiences and achievement (Desimone 2009).

Research on the characteristics of professional development which lead to teacher learning suggests that such models should incorporate: a focus on content and how students learn that content (Garet, Porter, Desimone, Birman and Yoon, 2001), active learning which involves teacher collaboration on tasks (Desimone, Porter, Garet, Yoon and Birman, 2002); observation and reflection of classroom practices (Remillard and Bryans, 2004), and should occur over a sustained period of time (Garet et al., 2001) where there is opportunity to trial and reflect on classroom experiences (Jacobs, Lamb and Philipp, 2010). Further studies have concluded that sustainable and successful professional development occurs in teacher communities where dialogue is focused on pedagogy and practice (Little, Gearhart, Curry, and Kafka, 2003) and where teachers have opportunity to make their implicit knowledge explicit by detailing their approaches to teaching and learning.

Lesson study is one such model which incorporates the features of professional development suggested above. While general teacher talk within subject department meetings can often be discrete, condensed, and de-situated (Little et al., 2003), teacher talk in lesson study is explicitly directed towards the content and pedagogy of the research lesson (Lewis et al., 2009). The collective planning and reflecting ensure that persistent problems of curriculum and teaching receive the benefit of the group's experience (Fujii, 2016) and there is potential for teachers to individually and collectively develop the content and pedagogical content knowledge required to teach a particular topic (Tepylo and Moss, 2011). This practice therefore moves away from a culture of the isolated, bounded classroom, towards a collaborative approach to developing pedagogy.

Pedagogical Content Knowledge

Many studies have examined the roles which content and pedagogical content knowledge play in shaping teachers' practices (Speer et al., 2015). In this paper the focus is on PCK where teachers not only understand the content they are teaching, but also have expertise in knowing, investigating, and developing how to teach that content using, for example, metaphors, diagrams, and explanations that are attuned to students' learning and to the integrity of the subject matter (Ball et al., 2008, p392). Strong PCK supports good planning, observation, and discussion around the

teaching and learning of mathematics (Yoshida and Jackson, 2011) and should remain an important focus of teacher professional development (Timperley and Phillips, 2003).

In their proposed model of Mathematical Knowledge for Teaching (MKT), Ball and her colleagues (2008, p. 403) identified domains of PCK used in teaching elementary (or primary) mathematics as *knowledge of content and students* (KCS), *knowledge of content and teaching* (KCT), and *knowledge of content and curriculum* - of which the former two categories are the focus of this research. KCS was defined as a domain where knowledge of students and of mathematics is combined in, for example, anticipating student strategies (p. 401). KCT was defined as a domain of knowledge combining teaching and mathematics in, for example, sequencing content. Further details of the features of these domains are provided below.

Research Methodology and Analysis

The purpose of the research was twofold:

1) To identify the features of post-primary mathematics teachers' KCS and KCT incorporated in planning and reflecting on research lessons in lesson study
2) To analyse any development of KCS and KCT over successive cycles of lesson study.

This investigation was conducted on two sites over the course of one academic year (2012-2013) as a comparative case study between two lesson study communities (Bogdan and Biklen, 2007). Both schools, Doone and Crannog (all names used are pseudonyms), were both urban, post-primary schools with comparable numbers of students (approximately 600). Highlighted by Fullan (2003) as being necessary to reform, management in both schools were supportive of teachers taking part in the research and were flexible in scheduling meetings and research lesson times. There were, however, noticeable differences in the cultures of both schools. In Crannog there was widespread commitment to teacher learning with a designated work-space for teacher meetings and regularly scheduled subject department meetings. Teachers in this school, regularly shared classroom resources, and participated in the research in order to establish new norms of collaboration. In Doone there was no

culture of collaboration and teachers rarely met as a subject department. Within the subject department there was little or no sharing of resources and within the school there was no physical space for teachers to meet and collaborate.

None of the teachers were familiar with lesson study and, following an introductory presentation to both schools, five teachers in Doone and seven in Crannog volunteered to participate in this research[1]. These teachers varied in their qualifications and years of experience (see Table 1) and some identified themselves as out-of-field.

Table 1 Profile of Participating Teachers

Profile of Participating Teachers		
Name	**Field of University Qualification**	**Years of teaching experience**
Crannog		
Dave	Mathematics and Physical Education	5
Eileen	Business and Finance	3
Fiona	Mathematics	31
Judy	Mathematics	18
Martin	Mathematics and Geography	27
Stephen	Business (out-of-field)	9
Walter	Engineering	12
Doone		
Kate	Engineering (out-of-field)	3
Lisa	Business and Finance	7
Michael	Business (out-of-field)	4
Owen	Primary School Teaching (out-of-field)	1
Nora	Mathematics	35

Aligning with recommendations from Garet et al. (2001), the research was conducted over the course of one full academic year where teachers would have opportunity to participate in multiple, successive cycles of lesson study. Teachers had autonomy

[1] Teachers did not receive any bursary or recognition for their participation and volunteered their time and resources in participating in this research.Both schools received fixed funding from the National Council for Curriculum and Assessment to sponsor any additional substitute teaching requirements necessitated as part of the research.

over the scheduling and duration of their lesson study meetings and these lasted, on average, one hour with three cycles completed in Crannog and four in Doone.

The author, a former post-primary mathematics teacher, acted as a participant-observer in the research and was present for all of the meetings in both sites (Ko, 2012). Over the duration of the research, this participant-observer role evolved from that of a facilitator of lesson study in the initial cycles, to that of an observer in the final cycles.

Qualitative data was generated through audio recordings of all teacher meetings (both sites totalling 38 hours and 17 minutes) and each participating teacher was interviewed three times during the research. These semi-structured interviews investigated teachers' experiences of professional development and their reflections of their own learning as a consequence of participating in lesson study. Other data such as a researcher audio log, researcher field notes, teacher notes, and samples of student work were utilised in framing the analysis of the data and triangulating the findings. In an attempt to reduce any impact of the dual researcher role, analysis did not commence until all of the data had been generated.

Teachers' conversations were transcribed and analysed over four phases (outlined below) as a chronological evolution of KCS and KCT over successive cycles of lesson study. Teacher interviews were also analysed in order to investigate teachers' reflections on their own development of PCK due to their participation in lesson study.

Analysis of Teacher Learning: PCK

Phase 1

The transcript text of all teacher meetings was read to identify if the text qualified as a legitimate unit for analysis. A unit was defined as any episode of conversation between two or more teachers which:

a) was relevant to the lesson study cycle and

b) was relevant to constructing or reflecting on content of a research lesson

This parsing approach, utilised by Cajkler et al. (2013) in their analysis of initial teacher education in lesson study, aimed to encapsulate elements of teachers' conversations which incorporated features of KCS and KCT. As an example the following excerpt identifies an episode in the first post-lesson reflection discussion in Doone where Nora interpreted and shared her observation of a student. Nora had been absent for the previous planning meeting and had not had an opportunity to share her anticipations of students' strategies prior to the research lesson (taught by Lisa).

Nora	*Lisa, I have to tell you another very interesting observation. The little fella beside me, he was squaring them [the numbers] up to 5 and that was grand, but when he got to 6 he started to multiply*
Lisa	*By 2?*
Nora	*By 4.*
Kate	*By 4?*
Nora	*For perimeter! I was actually going to say that to you before class. I'd be surprised if somewhere along the line you're not going to get someone to get the perimeter from the area. It was funny though, when he started, it was very interesting - at a certain point he seemed to know, he seemed to realise he'd gone off the rails and he was getting them [the answers] wrong and he went back to them up top when he was halfway through filling in the squares and he started correcting them.*

This conversation excerpt qualified as a unit of analysis since it was both relevant to the lesson study cycle and to interpreting student thinking within the research lesson. It is interesting to note that Nora was the only teacher to have noticed, interpreted, and shared her reflection of students' strategies within this first post-lesson discussion – a practice which her colleagues began to follow in subsequent research lessons.

Phase 2

In analysing teacher learning it was important to first consider whether the same features of KCS and KCT determined by Ball et al. (2008) (interpreted in Table 2) were present in these teachers' planning and reflection conversations. This

framework was the basis of the second phase of analysis where chunks of data from phase 1 were associated, where relevant, to these features of KCS and KCT.

Table 2 Features of KCS & KCT, as interpreted from Ball et al. (2008)

KCS	KCT
Anticipate what students are likely to think and what they will find confusing (p. 401)	Sequence particular content for instruction (p401)
Predict what students will find interesting and motivating (p. 401)	Know different instructionally viable models (p. 402)
Hear and interpret students' emerging and incomplete thinking as expressed in the ways that pupils use language (p. 401)	Evaluate instructional advantages and disadvantages of representations or models used to teach a specific idea (p. 401)
Know common student conceptions and misconceptions (p. 401)	Select examples with pedagogically strategic intent (p401)

While Ball et al. (2008) reference that teachers "*must be able to talk explicitly about how mathematical language is used*" (p. 400), this form of knowledge is not explicitly incorporated within their descriptions of KCS and KCT and is therefore not included in the framework at this phase.

While there was strong alignment with a number of features from this original framework, a number of new codes were identified within the data. One obvious difference between the data generated in this research and the phase 2 framework in Table 2 were teachers' choices of mathematical tasks in planning. Ball et al. (2008) noted that teachers in their study used their PCK to choose mathematical examples and activities which students would find interesting and motivating.

However, in this research teachers chose or developed mathematical activities with regard to the *content* they were teaching and then adapted these tasks to have meaning and context for their students (Arbaugh et al., 2006). Due to this emphasis on mathematical content, this feature of PCK was therefore shifted from that of KCS to one of KCT. Similarly, although sequencing content for instruction remained an important element of teachers' conversations, it was evident that identifying and incorporating the prior knowledge of the specific class groups and students was a key element of teachers' planning. This was therefore included as a new feature of KCS. In this study, knowledge of common student conceptions and misconceptions was often incorporated as an anticipation of student mathematical thinking and these two features were therefore amalgamated as one element of KCS. In considering teachers' ability to *"hear and interpret students' emerging and incomplete thinking as expressed in the ways that pupils use language*" (Ball et al., 2008, p.401), it was relevant that teachers could not only 'hear' student thinking, but that they could notice student thinking from observing students' work during research lessons. This feature of KCS is therefore referred to as "noticing and interpreting" student thinking, as referenced in Jacobs et al. (2010). This refined framework was utilised in phase 3 of the analysis of teachers' conversations.

Phase 3

Phase 3 of analysis demonstrated that as teachers' participation in lesson study continued, they not only sequenced content for instruction within a research lesson, but also began to sequence content for instruction across a series of lessons (Suh, 2015). This additional aspect of sequencing was therefore incorporated as a modified feature of KCT. In addition, teachers explicitly identified the language utilised in teaching mathematical content as an important part of their planning (McMurry, 2010) and this was incorporated in a final framework of analysis. Furthermore, as well as selecting examples with pedagogical intent, teachers had begun to develop their own mathematical examples and activities (Fujii, 2016). This was an important development of teacher knowledge since these teachers had traditionally been dependent on textbooks as the sole guide for their classroom practice. This led to a final framework of features of KCS and KCT, separated into lesson study cycle phases of planning and reflection, which was the basis of the fourth and final phase of analysis (see Table 3).

Phase 4

The qualitative data was again coded and the frequency of these features was analysed as a chronological evolution of teacher talk over successive lesson study cycles.

Table 3 Features of KCS and KCT as present in the lesson study data

<table>
<tr><th>Phase of Lesson Study Cycle</th><th>KCS</th><th>KCT</th></tr>
<tr><td rowspan="3">Planning</td><td>Anticipate what students are likely to think, what they will find confusing, and identify common student conceptions and misconceptions.</td><td>Sequence content for instruction in research lesson and as a series of lessons.</td></tr>
<tr><td rowspan="2">Identify and incorporate students' prior knowledge.</td><td>Identify language that may assist or confuse students in teaching a specific idea.</td></tr>
<tr><td>Select or develop examples and activities with pedagogically strategic intent.</td></tr>
<tr><td>Reflection</td><td>Notice and interpret students' emergent and incomplete thinking as expressed in research lessons.</td><td>Evaluate instructional advantages and disadvantages of representations or models used to teach a specific idea.</td></tr>
</table>

Findings of the qualitative and quantitative analysis of these features of KCS and KCT are outlined below.

Results and Findings

As an initial finding of this research, the features of KCT and KCS outlined in Table 3 provide empirical evidence of post-primary mathematics teachers' PCK incorporated in the planning and reflection phases of lesson study. This is an important finding related to Ball et al.'s (2008) MKT which identifies and details features of KCS and KCT as related to planning and reflecting on research lessons.

Focusing on teacher learning in lesson study, an analysis of the features of KCS and KCT across successive lesson study cycles was undertaken. Initially, as might be expected, not all teacher talk during meetings was focused on the lesson study cycle. Teachers who were unused to regularly meeting with their colleagues discussed other discrete, de-situated topics (described by Little et al. (2003)) and therefore not all conversations involved planning or reflecting on a research lesson. However, as teachers continued to participate in successive cycles more of their conversations focused on the teaching and learning of mathematical content within the research lesson. This was particularly relevant in Doone where teachers had not been familiar working together as a subject department and where the incorporation of features of PCK became more and more prominent as their participation continued. Undertaking a quantitative analysis of features of KCS and KCT, the data reveals an increase in the proportion of teacher conversations relating to KCS and KCT over successive cycles of lesson study (Table 4).

Table 4 Percentage of transcript coded as KCS and KCT in teachers' lesson study conversations

	Doone (%)	Crannog (%)
Cycle 1	3.65	18.08
Cycle 2	13.07	26.18
Cycle 3	20.21	31.38
Cycle 4	29.66	---

The data presented in Table 4 provides evidence of teacher learning where each lesson study community began to more frequently incorporate their KCS and KCT -

focusing on their pedagogical practices and unpacking the mathematical content being taught to their students (Murata et al., 2012).

Figures 1 and 2 demonstrate the increased frequency of features of KCS and KCT across the successive lesson study cycles in both schools.

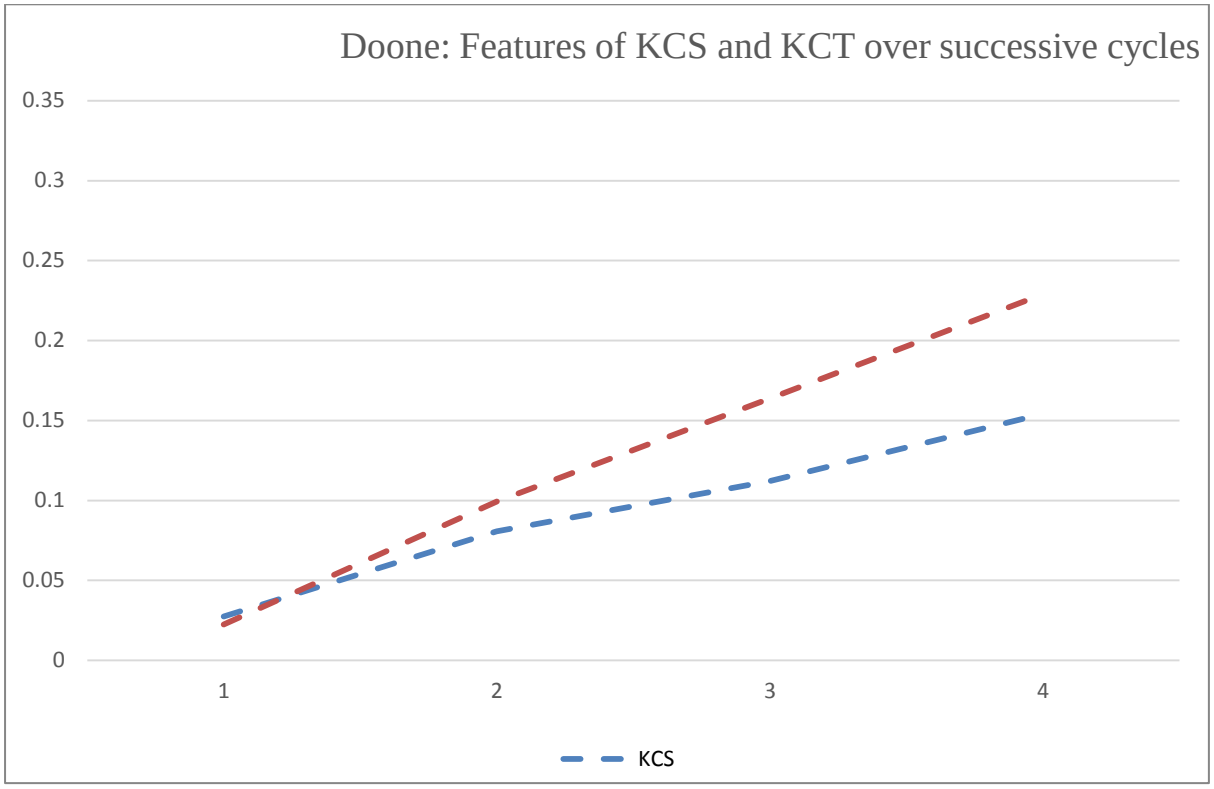

Figure 1 Features of KCS and KCT occurring as a percentage of teacher talk over successive cycles of lesson study in Doone

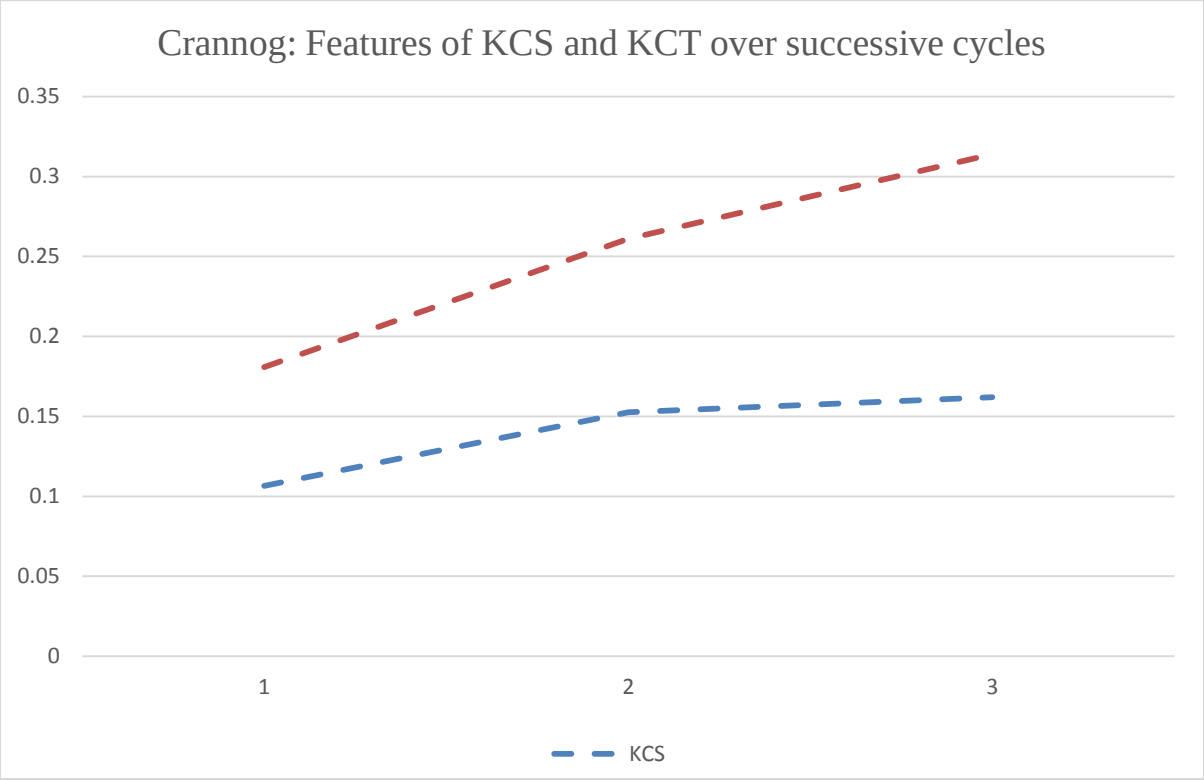

Figure 2 Features of KCS and KCT occurring as a percentage of teacher talk over successive cycles of lesson study in Crannog

This quantitative depiction of the qualitative data represents an overall increase of KCS and KCT incorporated as part of teachers' collective dialogue around planning and reflection of research lessons. A number of qualitative excerpts, outlined below, highlight how features of KCS and KCT developed as teachers' participation in lesson study continued.

Developing KCS and KCT: Qualitative Excerpts

In Crannog's first cycle teachers wanted to introduce the topic of quadratic patterns in a more innovative way. Teachers designed a lesson introducing quadratic patterns through inquiry and anticipated one strategy of how students might attempt this activity. However, having observed the research lesson, teachers reflected on the various (unanticipated) ways students had attempted the activity and began to reflect on how students had interpreted the mathematical task. Teachers also began to focus on the language students used during the activity (McMurry, 2010), something which had not been a feature of their planning. The following excerpt refers to student work collected and reviewed in the post-lesson discussion.

Dave *I think two or three of the groups got the thing [backwards], the levels 1, 2, 3, 4, your group did and yours as well.*

Eileen *Mine did, yeah.*

Walter *I don't think we spotted it as a potential.*

Dave *So did they spot that it was squared and then started firing down? Do you think they square rooted it or do you think they - ?*

Gemma *They did.*

Fiona *Mine spotted the fact that there was odd numbers; they were jumping up on odd numbers. They started talking about the odd numbers and the 1, 3, 5, 7 difference of odd numbers. They were discussing then trying to see if there was any way of doing this back with odd numbers…*

Walter *It was nice that they were coming up with sort of strategies or trying to.*

Fiona *But even the fact that they were using the language, the type of language. They were talking about odd numbers and they were talking about even numbers or things like that…that is what they need to do actually, is to speak it and to say it.*

Teachers gained insight into how the lesson could be improved by incorporating new terminology and being aware of other ways in which students might represent the pattern. This explicit noticing and reflecting on student thinking (Jacobs et al., 2010) led teachers to more carefully focus on developing student understanding in subsequent lesson study cycles.

In their second lesson study cycle, teachers began attempting mathematical activities during their planning as a way of better anticipating students' strategies. This led to teachers identifying and selecting more appropriate tasks with pedagogically strategic intent (Arbaugh et al., 2006), as demonstrated in the excerpt below:

Fiona *If I gave them something like that:* $2x^2 + 4x$ *and I will say "Ok now put that into a rectangle"*

Walter $2x^2 + 4x$ *[beginning to factorise the expression]*

Fiona *So they might put it like that, so you have* $2x(x+2)$*. Or they could put it like* $x(2x+4)$ *[writing out the two strategies for her colleagues to see]*

Walter *Which gives it to you then...*

Fiona *Now you see with these ones, if I gave them something like* $x(x+3)$*, I gave them that there now, there is only one way they can get it like that. With the* $2x(x+2)$ *there is going to be, they can do it that way or they can do it the other way. So they can see that there is two different ways. So I am going to do this with algebra tiles.*

This process of building and incorporating more features of KCS and KCT was also evident in Doone where there was little initial anticipation of students' strategies, but where this became a key element of their planning due to more focused noticing of students' work.

In Doone's first three cycles of lesson study teachers were reluctant to plan or discuss lessons prior to the research lesson. However, in their final cycle teachers realised that in order to meet their learning criteria of "*applying Pythagoras' theorem in a variety of real-life problems*" they would have to carefully outline what would occur in the two lessons leading to the research lesson:

Kate *If we're just going to observe a single class... Because to kind of introduce it - teach them how to use it and them teach them where to use it would be three classes…*

Michael *Oh yeah!*

Kate *You know, it wouldn't just be a single [lesson]…*

Lisa *It's the application of Pythagoras, we would then be demonstrating to that they understand what this guy [the equation] is all about…*

Nora *I think it's great that with all of [this], each lesson is planned and we know exactly where we're coming from.*

Kate *Yeah, this is the first time we've actually kind of planned a little scheme.*

Nora *For the whole thing. Take you and follow it through, because you know exactly where you stand or where they [the students] should stand.*

This was an important learning event for these teachers in structuring a sequence of student learning over a series of lessons (Suh, 2015). This learning event was also important since teachers realised that the content they taught did not have to be strictly guided or sequenced by the textbook (which introduced all applications of the theorem at once), but rather they could decide to return and develop the content at a later point in students' learning.

Michael *I really like the idea of doing it [the research lesson] with just the hypotenuse, I have to say I really like that. I really like the idea of coming back to it maybe at the end of second year or the start of third year, or at some stage there and bring in the other option. Because that is where they get lost, when you do it early on.*

Kate *And there's no point racing, rushing it.*

Lisa *That is...when you try and teach it too early.*

Teachers recognised that they could make informed decisions on developing students' understanding, independent of the traditional textbook approach.

As a result of their participation in lesson study, teachers developed in their ability to anticipate and interpret mathematical content through the eyes of a student (Fernandez, Canon and Chokshi, 2003). This is exemplified in the excerpt below where Lisa realised why students commonly misunderstood the labelling of sides of triangles when using Pythagoras' Theorem:

Lisa *The thing about learning for the students is that they can learn the theorem, but then it is confusion when the diagrams are labelled in any given way...We think it's saying* $a^2 + b^2 = c^2$ *but it's meaningless to them when you give them a thing and '* a*' is the hypotenuse and then you go,* $a^2 + b^2 = c^2$*...They don't actually understand.*

Owen *So that's rote learning.*

Kate *Yeah, concept rather than formula...*

Lisa *We know it. We know that this is the formula but we don't look at it from the kids [perspective]. And it's only that you talking about it today – if they label the hypotenuse '*a*' – I hadn't actually realised that that is what's causing the problem.*

This common student error was not previously apparent to Lisa and the conversation excerpt demonstrates the deepening of her PCK through these collaborative planning conversations. Similar conversations within the data provided evidence that the planning and reflection conversations were affording teachers with opportunities to develop their KCS and KCT.

In their individual interviews, all but one teacher (Nora) reflected that their participation in lesson study was beginning to positively impact their teaching outside of lesson study. In an interview at the end of the research Eileen noted:

> *Eileen* *I probably would ask myself a bit more how would they [the students] react to this or what questions will they have...pre-empt their questions or pre-empt their confusion. Yeah, I would think about that a little bit more.*

Nora, in her final interview, reflected that while she had always maintained her focus on student thinking, her participation in lesson study had increased her knowledge of designing specific mathematical tasks and her knowledge of the curriculum.

Discussion and Conclusion

Lesson study has long been identified as an effective model of teacher professional development. In this research, the development of post-primary mathematics teachers' PCK through participation in successive cycles of lesson study is investigated. Undertaken as case studies in two schools, this research demonstrates how teachers' conversations became more and more focused on the pedagogical elements of successive research lessons and demonstrates teacher learning over the duration of the research. Features of KCS and KCT, as identified by Ball et al. (2008) in their model of MKT, are modified as relevant to the data generated in this research and are detailed and analysed in teachers' planning and reflection conversations. The detailing of these features provides empirical evidence of MKT in post-primary teachers' lesson study conversations and verifies this model as relevant to post-primary mathematics teachers. Quantitative analysis of the features of KCS and KCT, as a chronological evolution of teacher dialogue over successive cycles of lesson study, shows an increased frequency of these features and is interpreted as

teacher learning (Cajkler et al., 2013). Over each successive cycle teachers began to more frequently anticipate, notice, and reflect on students' mathematical strategies (KCS), explicitly develop sequences of learning for students (KCT), and develop contextualised content related to students' prior knowledge (KCS and KCT). This research extends the theoretical underpinnings of MKT at post-primary level, as has been called for within the mathematics education research community (Speer et al., 2015). In addition, the alignment of Ball et al.'s (2008) MKT framework with this model of professional development serves to deepen the knowledge base about teacher learning in lesson study (Clivaz, 2015).

It is necessary to consider the limitations of this research and while these case-studies represented two urban post-primary schools, they may not be representative of all school-based lesson study groups, particularly in smaller or more rural schools. It is also important to note the high proportion of out-of-field teachers participating in the research, particularly in Doone, which may have impacted the content of teachers' conversations. The author was the sole researcher in analysing the data and, for this reason, extensive data was generated through multiple sources as a way of triangulating the findings of this study. Teachers' reflections of their learning in interview and final group discussions corroborated a reflected increase in their PCK as a result of participating in successive cycles of lesson study.

Professional development is not yet common practice for post-primary teachers in Ireland. This research demonstrates that school-based lesson study can be successfully introduced to Irish post-primary schools on a voluntary basis and as a model of supporting curriculum reform. In their final interviews and discussions teachers suggested that the success of such a collaborative, school-based model of teacher learning was dependent on teachers' 'buy in' and contrasted their voluntary participation with compulsory forms of teacher professional development, such as described by Brosnan (2014). These teachers did not see lesson study as an 'add on' (Sugrue, 2006) but rather as something which benefitted their practice, knowledge, and collaboration. It is worthy to note that due to the support of school leadership in Crannog, teachers continued to participate in lesson study in the following academic year.

While this paper demonstrates empirical evidence of developing teacher PCK in lesson study, further research is required to investigate the potential effects of the differing stages of teacher community and school culture on teacher learning. Teachers in Crannog had previously collaborated with their colleagues and had a greater sense of community within their subject department when compared to Doone. The established collaborative practices in Crannog may have positively impacted on teachers' capacity to focus their conversations on pedagogical issues and increased the incorporation of elements of KCS and KCT in their conversations. Further research will examine more closely the influence of the stages of teacher community on teacher learning in lesson study.

References

Arbaugh, F., Lannin, J., Jones, D. L., and Park-Rogers, M. (2006). Examining Instructional Practices in Core-plus Lessons: Implications for Professional Development. *Journal of Mathematics Teacher Education, 9*, 517-550.

Ball, D. L., Thames, M. H., and Phelps, G. (2008). Content Knowledge for Teaching: What Makes It Special? *Journal of Teacher Education, 59*(5), 389-407.

Bogdan, R. C., and Biklen, S. K. (2007). *Qualitative Research for Education: An Introduction to Theory and Methods* (5th ed.). USA: Pearson.

Brosnan, A. (2014). Introducing lesson study in promoting a new mathematics curriculum in Irish post-primary schools. *International Journal for Lesson and Learning Studies, 3*(3), 236-251.

Cajkler, W., Wood, P., Norton, J. & Peddar, D. 2013. Lesson Study: towards a collaborative approach to learning in Initial Teacher Education? *Cambridge Journal of Education*.

Clivaz, S. (2015). French Didactique des Mathématiques and Lesson Study: a profitable dialogue? *International Journal for Lesson and Learning Studies, 4*(3), 245-260.

Corcoran, D. (2011). Learning from Lesson Study: Power Distribution in a Community of Practice. In L. Hart, A. S. Alston, & A. Murata (Eds.), *Lesson Study Research and Practice in Mathematics Education* (pp. 251-268). New York: Springer.

Desimone, L., Porter, A., Garet,. M, Yoon, S., & Birman, B. (2002). Effects of Professional Development on Teachers' Instruction: Results from a Three-year Longitudinal Study. *Educational Evaluation and Policy Analysis, 42(2), 81-112.*

Desimone, L. (2009). Improving Impact Studies of Teachers' Professional Development: Toward Better Conceptualizations and Measures. *Educational Researcher, 38*(3), 181-199.

Dudley, P. (2013). Teacher learning in Lesson Study: What interaction-level discourse analysis revealed about how teachers utilised imagination, tacit knowledge of teaching and fresh evidence of pupils learning, to develop practice knowledge and so enhance their pupils' learning. *Teaching and Teacher Education, 34*(0), 107-121.

Elliott, J. (2012). Developing a science of teaching through lesson study. International Journal for Lesson and Learning Studies, 1, 108-125.

Fernandez, C., Cannon, J., & Chokshi, S. (2003). A US-Japan lesson study collaboration reveals critical lenses for examining practice. *Teaching and Teacher Education, 19*(2), 171-185.

Fujii, T. (2016). Designing and adapting tasks in lesson planning: a critical process of Lesson Study. *ZDM*, 1-13.

Fullan, M. (2003). *The Moral Imperative of School Leadership*. USA: Corwin Press, Inc.

Garet, M. S., Porter, A. C., Desimone, L., Birman, B. F., & Yook, K. S. (2001). What Makes Professional Develpment Effective? Results From a National Sample of Teachers. *American Educational Research Journal, 38*(4), 915-945.

Grossman, P., Wineburg, S., & Woolworth, S. (2001). Toward a theory of teacher community. *Teachers College Record, 103*(6), 942-1012.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking Pedagogical Content Knowledge: Conceptualizing and Measuring Teachers' Topic-Specific Knowledge of Students. *Journal for Research in Mathematics Education, 39*(4), 372-400.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional Noticing of Children's Mathematical Thinking. *Journal for Research in Mathematics Education, 41*(2), 169-202.

Jeffes, J., Jones, E., Wilson, M., Lamont, E., Straw, S., Wheater, R., & Dawson, A. (2013). *Final Report: Research into the impact of Project Maths on student achievement, learning and motivation* Retrieved from UK:

Ko, P. Y. (2012). Critical conditions for pre-service teachers' learning through inquiry: The Learning Study approach in Hong Kong. *International Journal for Lesson and Learning Studies, 1*(1), 49-64.

Lewis, C., Perry, R., and Hurd, J. (2009). Improving mathematics instruction through lesson study: a theoretical model and North American case. *Journal of Mathematics Teacher Education, 12*(4), 285-304.

Little, J. W., Gearhart, M., Curry, M., and Kafka, J. (2003). Looking at Student Work for Teacher Learning, Teacher Community, and School Reform. *Phi Delta Kappan*(November), 185-192.

Lyons, M., Lynch, K., Close, S., Sheerin, E., and Boland, P. (2003). *Inside Classrooms: The Teaching and Learning of Mathematics in Social Context*. Dublin: Insitute of Public Administration.

McMurry, S. (2010). *Mathematics as a Language*. Austria: Living Edition Publishers.

Murata, A., Bofferding, L., Pothen, B. E., Taylor, M. W., & Wischnia, S. (2012). Making Connections Among Student Learning, Content, and Teaching: Teacher Talk Paths in Elementary Mathematics Lesson Study. *Journal for Research in Mathematics Education, 43*(5), 616-650.

National Council for Curriculum and Assessment. (2012). *Junior Certificate Mathematics Syllabus: Foundation, Ordinary & Higher Level*. Dublin: NCCA.

National Council for Curriculum and Assessment. (2014). *Maths in Practice: Report and Recommendations*. Dublin: NCCA.

Remillard, J. T., & Bryans, M. B. (2004). Teachers' Orientations toward Mathematics Curriculum Materials: Implications for Teacher Learning. *Journal for Research in Mathematics Education, 35*(5), 352-388.

Ríordáin, M. N., & Hannigan, A. (2011). Who teaches mathematics at second level in Ireland? *Irish Educational Studies, 30*(3), 289-304.

Speer, N. M., King, K. D., & Howell, H. (2015). Definitions of mathematical knowledge for teaching: using these constructs in research on secondary and college mathematics teachers. *Journal of Mathematics Teacher Education, 18*, 105-122.

Sugrue, C. (2006). Irish teachers' experiences of professional learning: Implications for policy and practice. *Journal of In-Service Education, 28*(2), 311 - 338.

Suh, J., & Seshaiyer, P. (2015). Examining teachers' understanding of the mathematical learning progression through vertical articulation during Lesson Study. *Journal of Mathematics Teacher Education, 18*, 207-229.

Takahashi, A. (2014). The Role of the Knowledgeable Other in Lesson Study: Examining the Final Comments of Experienced Lesson Study Practitioners. *Mathematics Teacher Education and Development, 16*(1), 4-21.

Tepylo, D. H., & Moss, J. (2011). Examining Change in Teacher Mathematical Knowledge Through Lesson Study. In L. C. Hart, A. S. Alston, & A. Murata

(Eds.), *Lesson Study Research and Practice in Mathematics Education* (pp. 59-78). USA: Springer.

Timperley, H. S., & Phillips, G. (2003). Changing and sustaining teachers' expectations through professional development in literacy. *Teaching and Teacher Education, 19*, 627-641.

Yoshida, M., & Jackson, W. C. (2011). Response to Part V: Ideas for Developing Mathematical Pedagogical Content Knowledge Through Lesson Study. In L. Hart, A. S. Alston, & A. Murata (Eds.), *Lesson Study Research and Practice in Mathematics Education* (pp. 279-289). USA: Springer.