

Moving Force Identification as a Bridge Damage Indicator

Sevillano, E.¹, O'Brien, E.J.¹, Fitzgerald P.C.¹,

¹ *School of Civil Engineering, University College Dublin, Ireland*

Abstract

Information on the condition of bridges is primarily obtained through the use of visual inspection methods. These methods are unreliable due to an overdependence on human judgement and also inconsistencies due to human objectivity. A scientific approach is a more suitable alternative. Some authors use changes in the natural frequencies of the bridge to indicate possible damage but these methods aren't suitable for local damage detection. Mode shapes have also been used but are more difficult to infer from measurements.

This paper investigates the use of a Moving Force Identification (MFI) algorithm in conjunction with bridge deflection data. MFI back-calculates a vehicle applied axle forces to a bridge. It has been found that damage in a bridge changes the calculated axle forces substantially. These calculated axle force histories are used to infer damage from. The damage indicator used here is based on a linear regression analysis of the axle force histories. It is found that the absolute value of the slope of the linear regression fit increases with damage. Hence, by monitoring this parameter, information on possible bridge damage may be supplied on a vehicle by vehicle basis.

Keywords: Moving Force Identification, Deflection, Bridge, Damage, Camera-Based Monitoring

Corresponding author's email: paul.fitzgerald.3@ucdconnect.ie

Introduction

Visual inspections are the primary method today of obtaining bridge condition information. A large drawback of this approach is inconsistency due to human objectivity and a lack of science in the approach. This results in unreliable information on the true condition of a bridge. An adequate bridge monitoring scheme could prevent catastrophic occurrences such as the Interstate 34 Bridge collapse in Minneapolis in 2007 [1]. Early damage detection also makes it easier to undertake maintenance and repair works as they can be organised more easily. Traffic disruptions can also be managed more efficiently. Vibration sensor-based monitoring has become more popular in recent years as a result of the drawbacks of visual inspections. This has been facilitated by a reduction in instrumentation costs as well as an increase in computational technology.

Natural frequency based methods were one of the first vibration monitoring techniques used. This is primarily due to the ease at which a structure's natural frequencies can be measured. Damage in a structure causes the natural frequencies to change so it is logical that an

indication of damage may be inferred from this frequency change. A lot of the early literature on the use of frequency applied to damage detection focuses on simple structures such as bars. Adams et al. [2] use vibration data in the form of frequency modes to analyse the location and severity of a damaged region in a structure. The location of damage is predicted by superimposing several pairs of modes with the intersection of these modes corresponding to the possible locations of damage. The method is proved to be quite effective for one dimensional structures but has shown to be ineffective for some cases of higher damage severities.

Other authors have investigated the use of frequency changes as a damage indicator also. An extensive review is carried out by Salawu [3]. Banks et al. [4] and Weissenburger et al. [5] shed a bit of light on the drawbacks of frequency approaches. Weissenburger shows that if miniscule damage (represented as a loss of stiffness) occurs at certain points in a structure, it can result in large frequency changes. Furthermore, it is demonstrated that certain modes of vibration can remain unaffected by damage if damage exists at the nodes of vibration. Banks et al. conclude that the geometry of the damage explains why frequency methods work for some cases but are ineffective for other cases. Changes in mode shapes have also been used to provide information on damage location and severity. A high level of accuracy is required in these methods however, making it more difficult to detect damage from measurements [6].

Wu et al. are one of the first authors to examine the use of neural networks applied to damage detection [7]. Neural networks can be used to recognise patterns between the responses of damaged and undamaged structures and in many cases, have the advantage of negating the requirement of having an underlying mathematical model of the structure. Wu et al. apply a neural network to a simple three degree of freedom structure and have some limited success in predicting the damaged location in the structure. Lee et al. [8] use a neural network approach in conjunction with a baseline finite element model allowing for a degree of error in the baseline model with the inputs to the neural networks being the ratios of the mode shape components before and after the presence of damage. Modal sensitivity change must be less than the errors due to model inaccuracy in order to detect damage, which is a large drawback of the approach.

Camera technology has improved greatly in recent years along with affordability, paving the way for possible damage indicators to be inferred from camera data. A high-resolution camera could measure the deflection of a point or a number of points along a bridge as a vehicle traverses the bridge. This could be made possible by using one camera with an appropriate setup or else by using multiple cameras. Khuc and Catbas use a camera combined with computer vision techniques and achieve comparable accuracies to LVDTs (linear variable differential transformers) in measuring the deflection responses of a structure [9].

O'Brien et al. have previously investigated the use of moving force identification (MFI) for the purpose of damage detection based on deflection responses [10]. MFI is the process of calculating the axle force histories based on bridge measurements. The concept of statistical spatial repeatability [11] is used which is the fact that the daily mean applied axle forces for a population of vehicles of a certain category is repeatable. Hence, any change in this mean is an indicator of damage. This changes the output of the MFI algorithm and effectively, the axle forces are over predicted as the model of the bridge is unaware of the damage. A damage indicator is then created by using the root mean squared differences (RMSD's) between the mean of the calculated forces for a batch of vehicles crossing over a healthy bridge and then a damaged bridge. This paper also investigates the use of MFI as damage

indicator but a different damage indicator is investigated. Rather than using a damage indicator based on batches of vehicles, the method in this paper uses the calculated axle force histories from a single vehicle to infer damage from. The damage indicator is based on a linear regression analysis of the axle force histories. It is found that the absolute value of the slope of the linear regression fit increases with damage. Hence, by monitoring this parameter, information on possible bridge damage may be supplied on a vehicle by vehicle basis.

Moving Force Identification

MFI is a process which back-calculates the axle force histories from bridge measurements. This is achieved by calculating the axle forces such that the difference between the bridge measurements (deflections in this case) and the corresponding parameters from the dynamic equations is minimised. The original authors to propose this method were Law et. al [12] and the method was subsequently improved upon by Gonzalez et al. [13] by using a first order regularisation process. The fundamental equations of the MFI algorithm are now explained. The matrix differential equation for structural dynamics is represented by

$$[M_g]\{\ddot{u}\} + [C_g]\{\dot{u}\} + [K_g]\{u\} = F(t) \quad (1)$$

where $\{u\}$ is a vector of displacements, $F(t)$ is a forcing function and M_g , C_g and K_g are the mass, damping and stiffness matrices respectively. A more appropriate representation for the purpose of calculating the unknown forces is

$$[M_g]\{\ddot{u}\} + [C_g]\{\dot{u}\} + [K_g]\{u\} = [L]\{g(t)\} \quad (2)$$

where $g(t)$ is a vector of time dependent force magnitudes which are distributed to the relevant degrees of freedom by the location matrix, $[L]$, which takes into account the applied force locations on the bridge at a particular time. By defining

$$\{v\} = \{\dot{u}\} \quad (3)$$

and

$$\{a\} = \{\dot{v}\} \quad (4)$$

it may easily be shown that

$$\{a\} = -[M_g]^{-1}[C_g]\{v\} - [M_g]^{-1}[K_g]\{u\} + [M_g]^{-1}[L]\{g(t)\} \quad (5)$$

Equations (3) to (5) may now be combined to give

$$\begin{Bmatrix} \dot{u} \\ \dot{v} \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -[M_g]^{-1}[K_g] & -[M_g]^{-1}[C_g] \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} + \begin{bmatrix} 0 \\ [M_g]^{-1}[L] \end{bmatrix} \{g(t)\} \quad (6)$$

By defining a state vector, $\{X\}$, containing displacements and velocities, $\{u\}$ and $\{v\}$, equation (6) may be represented by

$$\{\dot{X}\} = [A]\{X\} + f(t) \quad (7)$$

where

$$[A] = \begin{bmatrix} 0 & I \\ -[M_g]^{-1}[K_g] & -[M_g]^{-1}[C_g] \end{bmatrix} \quad (8)$$

and

$$f(t) = \begin{bmatrix} 0 \\ [M_g]^{-1}[L] \end{bmatrix} \{g(t)\} \quad (9)$$

By using e^{At} as an integration factor on equation (7), followed by discretising and applying Padé approximations [14,15] to the result, equation (7) becomes

$$\{X\}_{j+1} = [M]\{X\}_j + [P]\{g\}_j \quad (10)$$

where

$$[P] = [M - I][A]^{-1} \begin{bmatrix} 0 \\ [M_g]^{-1}[L] \end{bmatrix} \quad (11)$$

and

$$M = e^{[A]h} \quad (12)$$

with h being the time step between two consecutive intervals.

The optimisation problem is to find the forcing function, $\{g\}$ that minimises the error, E , defined by

$$E(X_j, g_j) = \sum_{j=1}^N ((QX_j - d_j), W(QX_j - d_j)) + (g_j, Bg_j) \quad (13)$$

where N is the number of time-steps, d_j is a vector of measurements for the j^{th} interval, Q is a vector extracting the relevant state variables from the state vector, W is the identity matrix, and B is a regularisation parameter introduced due to the ill-conditioned nature of the problem. The notation (x,y) denotes the vector product in equation (13). The optimal smoothing parameter, B , is obtained from a method known as the L-Curve method [16] which seeks a trade-off between an acceptable least squares solution and ill-conditioning. The optimal regularisation parameter is located at the corner of the L-Curve, which corresponds to the point of maximum curvature [17].

A first order regularisation method is used in this paper which regularises the derivative of the forces as opposed to the forces themselves which would be the case in zeroth order regularisation. The minimisation process is solved by using dynamic programming [18] and Bellman's principle of optimality [19].

Damage Detection using Moving Force Identification

To demonstrate how the MFI algorithm is used for damage detection purposes, a simulated example of a vehicle crossing a bridge is conducted. A bridge is represented by a 20 m simply supported beam. It is modelled using the finite element method and it is broken up into twenty, one metre elements. Each node has two degrees of freedom. The beam has a cross sectional area, density, modulus of elasticity and second moment of area of 10m^2 , 2446 kg/m^3 , $35 \times 10^9\text{ N/m}^2$ and 1.15 m^4 respectively. An ISO class 'A' road profile (classified

| Vehicle Parameter | Axle 1 | Axle 2 |
|---|--------------------|-------------------|
| Unsprung mass (kg) | 1000 | 1000 |
| Suspension stiffness (N m) | 4×10^5 | 1×10^6 |
| Suspension Damping (N s m^{-1}) | 10×10^3 | 20×10^3 |
| Tyre Stiffness (N m) | 1.75×10^6 | 3.5×10^6 |

Table 1: Vehicle Parameters

as 'very good') is generated on the bridge. The vehicle is modelled as a two-axle 'half-car'. The 'half-car' allows for vehicle hop, pitch and bounce motions and has four degrees of freedom. The vehicle has a gross vehicle weight and velocity of 20,000 kg and 40 km/h respectively. The remaining vehicle parameters are listed in Table 1 below.

The vehicle above is simulated crossing the bridge. The MFI process uses the deflection measurements at every interior node (19 nodes) to calculate the forces. Noise is added to the deflection responses, d_{calc} , in order to create realistic measurements values, d_j , for each time step. Noise is introduced using the formula

$$d_j = d_{calc} + E_p d_{\max} N_{noise} \quad (14)$$

where N_{noise} is a normally distributed vector with a mean of zero and a standard deviation of one, d_{max} is the maximum deflection at the centre of the bridge as the vehicle passes and E_p is the noise level which was chosen to be five per cent in this case.

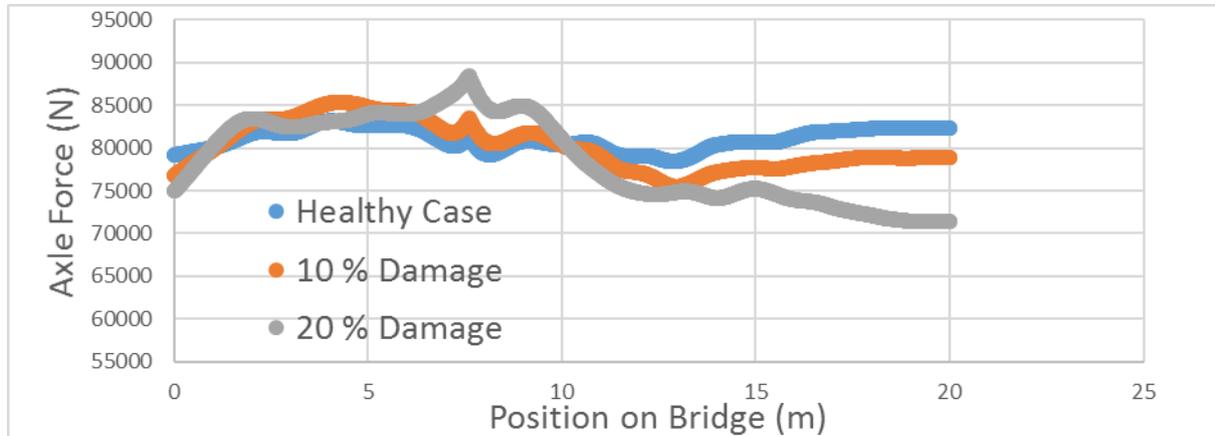


Figure 1: Axle 1 calculated force histories

Damage is now introduced as a percentage loss of stiffness in the region between the 6 m and 7 m mark along the bridge. Figure 1 shows the calculated Axle 1 force histories for the healthy case and for two levels of damage. It is observed that as damage increases, the calculated axle force histories deviate further from the healthy case. This led O'Brien et al. [10] to apply the concept of statistical spatial repeatability [11] to form a damage indicator based on the RMSD's between the mean calculated axle force histories of batches of similar vehicles crossing a healthy and then a damaged case. A contour plot of damage indicator value versus damage location and severity is then obtained for all possible cases of damage. The method is poor for predicting the location of damage however. This paper investigates an alternative form of damage indicator that uses a linear regression analysis on the calculated axle force histories.

Linear Regression Analysis of Axle Force Histories

This section investigates a damage indicator inferred from the calculated axle force histories of a single vehicle. By applying a linear regression analysis to the calculated axle force histories, it is seen that the absolute value of the slope of the regression line increases with damage. Figure 2 shows the regression analysis for Axle 1 using the same vehicle as examined previously.

The damage is also in the same region. The change in the magnitude of the slope of the regression line is quite large and is clearly sensitive to damage. This parameter also increases with damage. The same can be said for the calculated axle force histories for Axle 2 whose changes are shown in Table 2. The changes are represented as a 'factor of change', where 'factor of change' is defined as the absolute value of the ratio of the healthy slope to the damaged slope.

| Damage Case | Axle 1 Forces: Factor of change | Axle 2 Forces: Factor of change |
|-------------|---------------------------------|---------------------------------|
| 10 % Damage | 69.3 | 13.2 |
| 20 % Damage | 151.5 | 26.55 |
| 30 % Damage | 310 | 55.2 |

Table 2: Slope of Regression Line – Factor of Change

Table 2 shows that the factor of change is different for each Axle. Axle 2, which is the heavier axle, changes by less of a factor. An interesting observation from Table 2 is that the increase in factor of change is almost linear with damage severity for each level of damage. Figure 3 plots the data from Table 2 and this linear result is seen a bit more clearly.

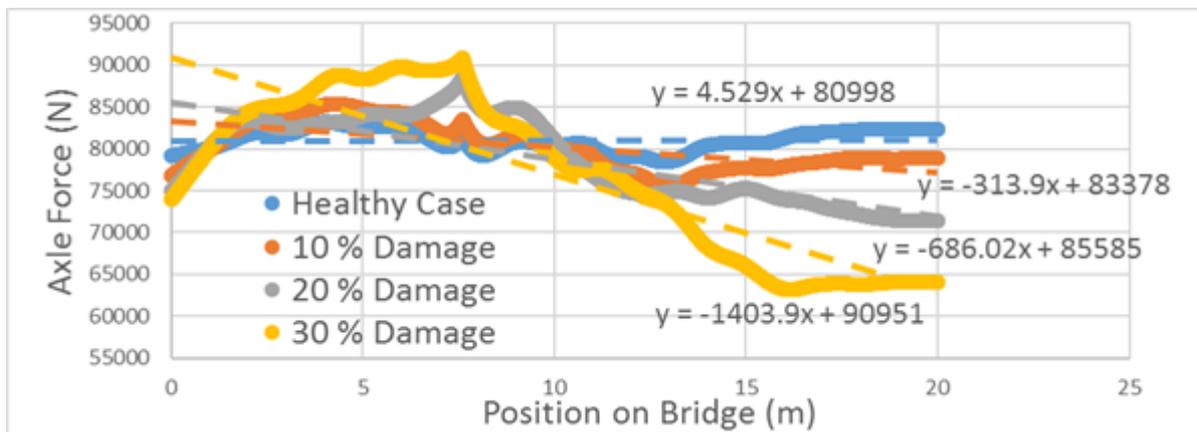


Figure 2: Linear Regression Analysis of Axle 1 force histories

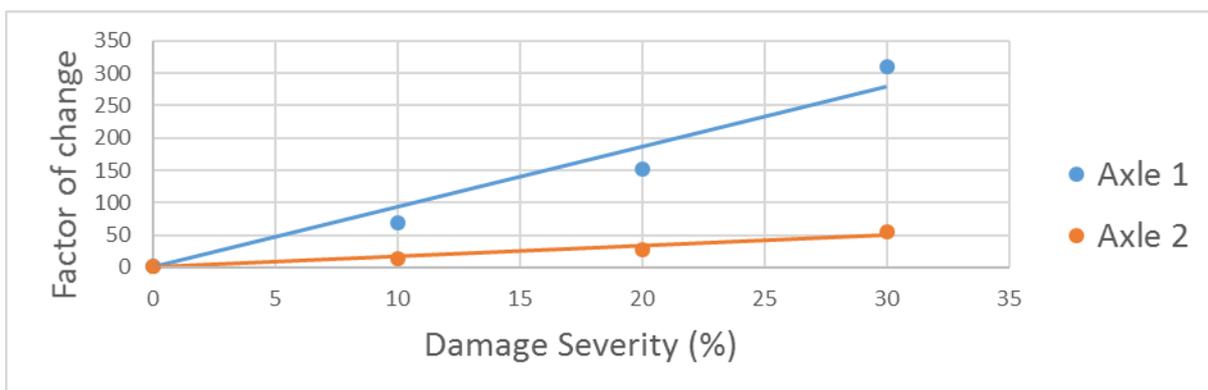


Figure 3: Factor of change vs Damage Severity

Conclusion

The analysis in this paper has shown that a linear regression analysis on the calculated axle forces has scope for potential damage detection. The slope of the regression line changes

by a significant amount when there is an area of damage in the bridge. This parameter increases with an increase in damage. Interestingly, the increase in the factor of change in the magnitude of the slopes between a damaged and healthy case is showing a linear trend, as is shown in Figure 3. This is a useful finding as damage severities for different factor of changes could be obtained from this relationship.

The findings in this paper are useful but still somewhat limited as only one vehicle and one area of damage was investigated in the analysis. A more detailed study using many vehicles and different areas of damage would need to be carried out in order to verify whether the parameter in question is suitable for damage detection purposes.

Acknowledgements

The authors wish to acknowledge the financial support received from Science Foundation Ireland under the US-Ireland Research Partnership Scheme towards this investigation.

References

- [1] M. Azarbayejani, A. El-Osery, M. Taha, Entropy-based optimal sensor networks for structural health monitoring of a cable-stayed bridge, *Smart Structures and Systems* 5(4) (2009) 369-379.
- [2] R. Adams, P. Cawley, C. Pye, B. Stone, A vibration technique for non-destructively assessing the integrity of structures, *Journal of Mechanical Engineering Science* 20(2) (1978) 93-100.
- [3] O.S. Salawu, Detection of structural damage through changes in frequency: a review, *Engineering Structures* 19(9) (1997) 718-723.
- [4] H.T. Banks, D. Inman, D. Leo, Y. Wang, An experimentally validated damage detection theory in smart structures, *Journal of Sound and Vibration* 191(5) (1996) 859-880.
- [5] J. Weissenburger, Effect of local modifications on the vibration characteristics of linear systems, *Journal of Applied Mechanics* 35(2) (1968) 327-332.
- [6] J.-T. Kim, Y.-S. Ryu, H.-M. Cho, N. Stubbs, Damage identification in beam-type structures: frequency-based method vs mode-shape-based method, *Engineering Structures* 25(1) (2003) 57-67.
- [7] X. Wu, J. Ghaboussi, J.H. Garrett, Use of neural networks in detection of structural damage, *Computers & structures* 42(4) (1992) 649-659.
- [8] J.J. Lee, J.W. Lee, J.H. Yi, C.B. Yun, H.Y. Jung, Neural networks-based damage detection for bridges considering errors in baseline finite element models, *Journal of Sound and Vibration* 280(3-5) (2005) 555-578.
- [9] T. Khuc, F.N. Catbas, Completely contactless structural health monitoring of real-life structures using cameras and computer vision, *Structural Control and Health Monitoring* (2016).
- [10] E. O'Brien, C. Carey, J. Keenahan, Bridge damage detection using ambient traffic and moving force identification, *Structural Control and Health Monitoring* 22(12) (2015) 1396-1407.

- [11] T. O'Connor, E. O'Brien, B. Jacob, An experimental investigation of spatial repeatability, *International Journal of Heavy Vehicle Systems* 7(1) (2000) 64-81.
- [12] S.S. Law, Y.L. Fang, Moving force identification: Optimal state estimation approach, *Journal of Sound and Vibration* 239(2) (2001) 233-254.
- [13] A. González, C. Rowley, E.J. O'Brien, A general solution to the identification of moving vehicle forces on a bridge, *International Journal for Numerical Methods in Engineering* 75(3) (2008) 335-354.
- [14] R. Bellman, *Introduction to Matrix Analysis: Second Edition*, Society for Industrial and Applied Mathematics, 1997.
- [15] D.M. Trujillo, The direct numerical integration of linear matrix differential equations using pade approximations, *International Journal for Numerical Methods in Engineering* 9(2) (1975) 259-270.
- [16] P. Hansen, Analysis of Discrete Ill-Posed Problems by Means of the L-Curve, *SIAM Review* 34(4) (1992) 561-580.
- [17] P.C. Hansen, *The L-curve and its use in the numerical treatment of inverse problems*, IMM, Department of Mathematical Modelling, Technical University of Denmark, 1999.
- [18] D.M. Trujillo, Application of dynamic programming to the general inverse problem, *International Journal for Numerical Methods in Engineering* 12(4) (1978) 613-624.
- [19] R. Bellman, *Introduction to the Mathematical Theory of Control Processes: Nonlinear Processes*, Elsevier Science, 1971.