

Validating Two Novel Equivalent Impedance Estimators

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Abstract—Certain approaches to appraising voltage stability use an equivalent impedance to characterise the wider power system. This letter proposes two new ways of inferring an appropriate equivalent impedance from a power system’s admittance matrix. Continuation power flow simulations are used to validate the quality of the new estimators, and to benchmark them against some extant approaches.

I. INTRODUCTION

CIRCUIT theory shows that the maximum power deliverable to a load will occur when its impedance matches the complex conjugate of the feeding Thévenin impedance, and this concept underpins various approaches to appraising a bus’ voltage stability [1]–[3]. Such indices, whose relative merits are beyond the modest scope of the present letter, typically infer an equivalent impedance using sequential samples of local voltage and current [4]. A recent review [5] noted one shortcoming of such approaches: “*these indices are very sensitive to the small change of the data*” and went on to suggest that future work in voltage stability should propose a measure that “*considers the Thévenin network impedance and is insensitive to the small change of the two consecutive measurement data*.”

Accordingly, the present letter proposes and validates two new ways to directly infer an equivalent impedance from a system’s admittance matrix. While an equivalent impedance alone cannot capture all aspects of voltage stability (due to e.g. machine reactive power limits) their more accurate estimation can offer insights on how network structure affects bus loadability.

II. METHODOLOGY

Alongside the two novel approaches, two established techniques are also used to populate the vector of network equivalent impedance estimators as seen by each load, z_L .

A. Proposed new estimators

1) *Load submatrix impedance*: The \mathbf{Y}_{bus} is reordered, per [1], such that the m generator buses and n load buses are grouped together:

$$\begin{bmatrix} i_G \\ i_L \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{GG} & \mathbf{Y}_{GL} \\ \mathbf{Y}_{LG} & \mathbf{Y}_{LL} \end{bmatrix} \begin{bmatrix} v_G \\ v_L \end{bmatrix} \quad (1)$$

Manipulation of (1) gives:

$$v_L = \mathbf{Z}_{LL} i_L + \mathbf{F}_{LG} v_G \quad (2)$$

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Where $\mathbf{Z}_{LL} = \mathbf{Y}_{LL}^{-1}$ and $\mathbf{F}_{LG} = -\mathbf{Z}_{LL} \mathbf{Y}_{LG}$. Recent work [6] has shown that the rows of \mathbf{F}_{LG} sum close to one with negligible imaginary components: it thus shows the different participation each generator has in establishing the no-load voltage at a particular bus. Therefore, the diagonal elements of \mathbf{Z}_{LL} have a clear interpretation as system effective impedances at each bus, as they explicitly describes the voltage drop caused by local current consumption (see [7] for more on this paradigm.) Therefore, an impedance estimate is given by:

$$z_L^{\text{Sub}} = \text{diag}(\mathbf{Z}_{LL}) \quad (3)$$

2) *Klein resistance distance*: The \mathbf{Z}_{bus} matrix (elements z_{ij}) is the inverse of the \mathbf{Y}_{bus} matrix. According to Klein [8], the Thévenin impedance between buses i and j is calculated using these elements of the \mathbf{Z}_{bus} matrix:

$$z_{ij}^k = z_{ii} + z_{jj} - z_{ij} - z_{ji} \quad (4)$$

Various works have used the intuition that the electrically-nearest generator to a load represents information relevant to creating a Thévenin equivalent [9], [10]. While those works used approximations, the full \mathbf{Z}^k matrix of internode impedances allows the extraction of the explicit Thévenin distance between each load and its nearest generator:

$$z_L^{\text{Near}} = \min_{j \in L} z_{ij}^k, \quad i = 1, \dots, m \quad (5)$$

B. Benchmark estimators

1) *Shortest path impedance*: Work in [9] used network traversal techniques to find the shortest topological path between each load and its nearest generator. The sum of branch impedances along this geodesic path was used in [9] as an estimate of the system equivalent impedance: z_L^{Topo} .

2) *Driving point impedance*: The main diagonal of the \mathbf{Z}_{bus} matrix contains *driving point impedances* which describe the short circuit power available at a bus. Some authors have likened these with a Thévenin equivalent of the system, at least under faulted conditions [11].

$$z_L^{\text{Driving}} = \text{diag}(\mathbf{Z}_{\text{bus}}) \quad (6)$$

C. Estimator quality assessment

As a validation exercise in the domain of one potential application, each estimator is used to predict the maximum loadability of each load bus within a test system. At a unity power factor, the forecasted maximum active power point is given as function of $z_L (= r_L + jx_L)$ by [12]:

$$p^+ = \frac{v_L^2}{2(\sqrt{r_L^2 + x_L^2} + r_L)} \quad (7)$$

To validate the quality of the p^+ forecast, and the z_L equivalent it derives from, the empirical steady state loading

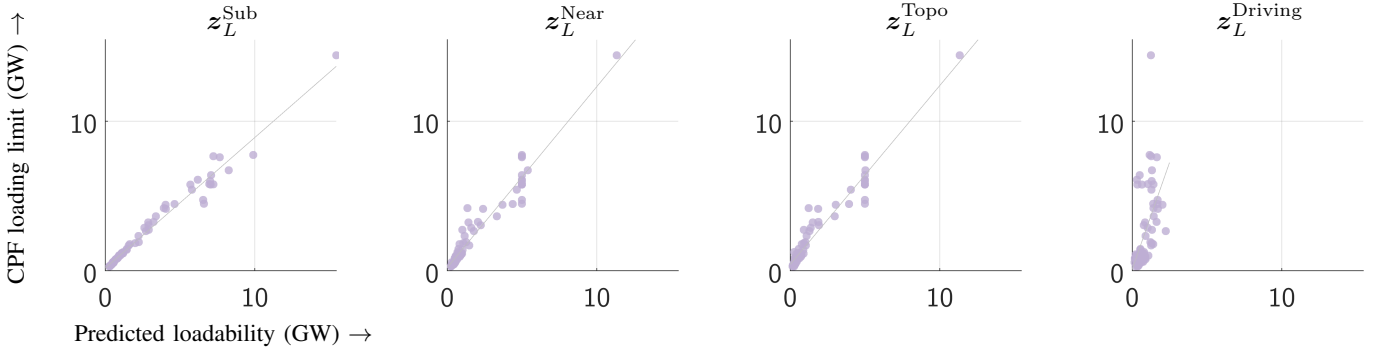


Fig. 1. Scatterplots showing the predictive efficacy of each estimator versus maximum active power loadability for each bus within *nesta_case118_ieee*

TABLE I.
QUALITY OF LOADABILITY FORECASTS p^+ USING EACH z_L ESTIMATOR

System Name	z_L^{Sub}	z_L^{Near}	z_L^{Topo}	z_L^{Driving}
<i>nesta_case30_ieee</i>	16	26	46	89
<i>nesta_case39_epri</i>	27	35	38	74
<i>nesta_case57_ieee</i>	12	14	40	80
<i>nesta_case73_ieee_rts</i>	14	28	37	52
<i>nesta_case89_pegase</i>	20	52	53	100
<i>nesta_case118_ieee</i>	7.7	25	39	47
<i>nesta_case162_ieee_dtc</i>	46	24	41	187
<i>nesta_case189_edin</i>	40	43	41	69
<i>nesta_case300_ieee</i>	24	28	38	49

limits at each load bus are calculated. This is achieved using simple continuation power flow techniques [13], where each load is individually increased. The increasing load is served from its local generators, as identified from the non-zero entries in the relevant row of the F_{LG} matrix. This analysis is undertaken solely to determine which estimator of z_L most accurately encapsulates the network through which the load sources its increasing power requirement: as such, machine active and reactive power limits are ignored, and v_L is uniformly set = 1.

III. RESULTS & CONCLUSIONS

Empiric loadabilities were calculated for *every* load bus in each of nine test systems [14]. This rich dataset [15] is then compared against the corresponding loadabilities p^+ that were predicted according to each of the four z_L estimators: the quality of these predictions is shown in Table I, which shows their *Mean Average Percentage Error*. For instance, considering every load bus in the *nesta_case30_ieee* system, the p^+ loadability prediction that was based on z_L^{Sub} typically deviated from the empiric value by 16%.

The z_L^{Sub} estimator consistently delivers the best loadability predictions. The z_L^{Near} estimator also performs well; it outperforms z_L^{Topo} , which doesn't properly account for the parallel nature of impedances within a meshed transmission system. Finally, Table I shows that z_L^{Driving} is wholly unsuited to predicting loadability limits.

A more granular view of the data is given in Fig. 1, which plots predicted versus empiric loadabilities at each load bus in the *nesta_case118_ieee* system. The clear linear trend for the z_L^{Sub} estimator is apparent, with most datapoints clustered tightly around the regression line. In conclusion, the Z_{LL} matrix is not sensitive to small changes in operating

conditions, and it contains information useful for bus loadability analysis.

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