Impact of short-duration acceleration records on the ability of signal processing techniques to derive accurate bridge frequencies

Kun Feng¹, Arturo González¹, Miguel Casero¹ ¹School of Civil Engineering, University College of Dublin, Ireland e-mail : <u>kun.feng@ucdconnect.ie</u>, <u>arturo.gonzalez@ucd.ie</u>, <u>miguel.caseroflorez@ucd.ie</u>

ABSTRACT: This paper envisions a scenario in which unmanned aerial vehicles gather data from low-cost and flexible wireless sensor networks, i.e., accelerometers. However, flight duration, coupled with limited sensor battery time, is a substantial technical limitation. In order to assess the impact of these constraints on bridge monitoring, this paper analyses the extraction of bridge dynamic features from short-duration acceleration records. The short acceleration record is simulated using the theoretical response of a simply supported beam subjected to a moving load. Estimated frequencies are obtained in free vibration and compared with the natural frequencies calculated from formula. Given that short records limit the resolution in the frequency domain, the error in the prediction of frequencies will typically decrease as the duration of the signal increases. Signal processing techniques for extracting dynamic features include the Fast Fourier Transform, Frequency Domain Decomposition, Continuous Wavelet Transform and Hilbert-Huang Transform. This paper carries out an assessment of the accuracy of these signal processing techniques in extracting frequencies as a function of the duration of the measurements. Edge effects and loss of resolution are shown to remain key issues to be addressed when the duration of the signal is too short.

KEY WORDS: Short-duration records; Acceleration; Frequency; Structural Health Monitoring; Bridge dynamics.

1 INTRODUCTION

With the technology advancement in Unmanned Aerial Vehicles (UAVs), the latter are becoming increasingly popular for Structural Health Monitoring (SHM) of bridges, mostly to carry out visual inspections via cameras [1]. In comparison, traditional data collection methods based on wired sensor networks require extensive lengths of cables to transmit recorded data to a centralized data repository. While using UAVs for SHM has several advantages over conventional data collection methods, such as efficient installation, low cost and high scalability [2] ;the limitations of short flight duration and limited sensor battery time cannot be ignored. To overcome this problem, further research on how to extract bridge features from short-duration acceleration records is needed.

Acceleration measurements can be processed to derive natural frequencies that depend on the distribution of mass and stiffness as well as on the boundary conditions of the bridge. External factors such as temperature variations and applied load (i.e., forced vibration), also have an impact on the bridge frequencies. Hence, this data can be used to monitor changes in the condition of a structure. If the stiffness of the structure decreased as a result of the presence of a crack, the bridge will be more flexible and the frequency of vibration of the structure will be diminish. Therefore, the frequency of vibration needs to be estimated very accurately to be able to detect a small change that could denote damage [3]. There are numerous signal processing techniques for extracting dynamic features from acceleration records, such as the Fast Fourier Transform (FFT), Frequency Domain Decomposition (FDD), Continuous Wavelet Transform (CWT) and Hilbert-Huang Transform (HHT), amongst others [4].

The response of a simply supported beam subjected to a moving constant force is simulated in this paper. Compared to

displacement and velocity, acceleration signal is easier to measure. Here, the acceleration response is calculated through a compact formula derived from the equation proposed by Kumar et al. [5]. Then, the frequencies of the first three modes are calculated exactly and compared to the frequencies from short-duration records obtained by different signal processing techniques. As expected, the error in the predicted frequencies is shown to decrease as the duration of the signal increases, given that short records imply a limitation of the resolution in frequency domain. Loss of resolution and edge effects remain key issues to be addressed when the duration of the signal is too short. The magnitude of the inaccuracies in frequencies allow assessing the ability of each signal processing techniques in detecting damage with limited observation times.

Section 2 describes the simply supported beam model used in the simulations. A simple and compact formula to determine the free vibration response of the beam is given. In Section 3, various techniques are applied to short-duration records and discussed. Then, Section 4, simulated results are discussed regarding short-duration acceleration records. Finally, Section 5 provides conclusions.

2 MODELING OF A SIMPLY SUPPORTED BEAM SUBJECTED TO A MOVING LOAD

2.1 Beam model

The response of a simply supported beam of total span length, L, subjected to a moving constant load, P, at a constant velocity, v, is simulated for the section at 3.75 m from the left support (Figure 1). This response will be used as a basis for the calculations in the sections that follow. And Table 1 gives the properties of the beam.



Figure 1. Bridge model.

Table 1. Properties of the bridge.

Properties	Symbol	Unit	Value
Length	L	m	15
Mass per unit	m	kg/m	28125
Modulus of elasticity	E	MPa	35000
Second moment of area	Ι	m ⁴	0.5273
Moving load	Р	Ν	98100
Moving velocity	v	m/s	15

Equation (1) is used to calculate the natural frequencies of structure from the bridge properties.

$$f_i = \frac{\pi \times i^2}{2 \times L^2} \times \sqrt{\frac{EI}{m}} \tag{1}$$

where f_i is the natural frequency of the mode of vibration (i = 1,2,3,...,n). Table 2 shows the frequencies of the first three modes of vibration based on Equation (1).

Table 2. Bridge natural frequency.

Mode No.	1 st	2^{nd}	3 rd
Natural Frequency (Hz)	5.6555	22.6220	50.8995

Equation (2) is used to calculate the error in predicting a frequency.

$$error_{i} (\%) = \left| \frac{f_{i}^{e} - f_{i}}{f_{i}} \right| \times 100$$
⁽²⁾

where f_i^e is the estimated frequency derived from various signal processing techniques, and f_i is the natural frequency (real value) corresponding to the *i*th mode of vibration.

2.2 Free vibration response

The frequency of free vibration needs to be estimated very accurately to be able to detect a small change that could denote damage. Analysis of free vibration is very effective and reliable to derive natural frequency, mode shape and damping ratio during the free vibration period, given that there are no deviations caused by forced vibration due to the moving load. Kumar et al propose a simple and compact formula to determine the free vibration response of a simply supported beam [5]. The response in free vibration due to the first three mode frequencies (i = 1,2,3) of a simply supported beam for t > L/v is as follows:

$$w(x,t) = \sum_{n=1}^{3} X_n e^{-\xi_n \omega_n t} \sin \frac{n\pi x}{L} \sin(\omega_{dn} t - \phi_n)$$
(3)

where ξ_n is the damping ratio, ω_n is the natural frequency for the *n*th mode, and ω_{dn} is the damped natural frequency. X_n is the amplitude (or magnitude) given by:

$$X_{n} = \frac{F_{0}K_{n}\sqrt{1+e^{-\frac{2\xi_{n}n\pi}{K_{n}}}-2e^{-\frac{\xi_{n}n\pi}{K_{n}}}\cos n\pi \cos\left(\frac{n\pi}{K_{n}}\sqrt{1-\xi_{n}^{2}}\right)}}{\omega_{n}^{2}\sqrt{\left(1-K_{n}^{2}\right)^{2}+\left(2\xi_{n}K_{n}\right)^{2}}\sqrt{1-\xi_{n}^{2}}}$$
(4)

where F_0 is 2P/mL, K_n is the non-dimensional speed that $K_n = \Omega_n/\omega_n$, and Ω_n is the excitation frequency that $\Omega_n = n\pi v/L$. In addition, ϕ_n is the phase angle of free vibration response of the simply supported beam for the n^{th} mode given by:

$$\phi_n = \tan^{-1} \{ \frac{-e^{-\frac{\xi_n n\pi}{K_n}} \sin(\frac{n\pi}{K_n} \sqrt{1-\xi_n^2})}{\cos n\pi - e^{-\frac{\xi_n n\pi}{K_n}} \cos\left(\frac{n\pi}{K_n} \sqrt{1-\xi_n^2}\right)} \}$$
(5)

In this paper, the damping ratio of the bridge, ξ_n , is considered to be zero. Thus, the simplified Equation (3) without damping ratio is as follows:

$$w(x,t) = \sum_{n=1}^{3} X_n \sin \frac{n\pi x}{L} \sin(\omega_n t - \phi_n) \quad (6)$$

By deriving Equation (6) twice with respect to time, the acceleration function, \ddot{w} , of the free vibration response can be extracted (Equation (7)).

$$\ddot{w}(x,t) = -\sum_{n=1}^{3} X_n \omega_n^2 \sin \frac{n\pi x}{L} \sin(\omega_n t - \phi_n)$$
(7)

Figure 2 shows a 0.5 s long response in free vibration at quarter-span (x = L/4) using Equation (6) for displacement and (7) for acceleration.



Figure 2. Displacement and acceleration response in free vibration.

3 TECHNIQUES APPLIED TO SHORT-DURATION RECORDS

3.1 Fast Fourier Transform (FFT)

FFT is an efficient algorithm for calculating discrete Fourier transform and it is also one of the most utilized and oldest methods for signal processing in SHM. FFT converts discrete samples of a continuous time series signal into a frequency domain representation. It has been used in many different types of structures to detect damage based on extracting the modal parameters.

Chae et al. apply the FFT technique to a 9.0 s long acceleration signal to successfully extract bridge frequencies,

being the estimated 1st frequency of the bridge 3.80 Hz [6]. In their research, a total of 45 sensors are installed in the bridge, whose locations and types are determined by two factors: bridge behaviour and wireless communication stability. The wireless communication stability and sensor data reliability can be tested from signals obtained through accelerometers. Wang et al. use a wireless sensor network and a permanent cablebased structural monitoring system to record the acceleration response of the Voigt Bridge concrete box girder [7]. Then, the FFT method is applied to an 8.0 s long signal to derive the 1st mode frequency (4.89 Hz) successfully. The measurements acquired using the wireless monitoring system are shown to be accurate for precise determination of the primary modal frequencies and operating deflection shapes of the bridge deck. Finally, Xiao et al. apply the FFT method to an 18.0 s signal to obtain the natural frequency (estimated corresponding frequencies, 1.50 Hz, 4.47 Hz and 4.90 Hz) as a characteristic index for damage identification [8]. The dvnamic characteristics of the structure are extracted by successfully converting the time domain data to frequency domain.

A major advantage of FFT is its implementational simplicity and versatility. Furthermore, the way the algorithm works is intuitive and any additional constraint can be incorporated in the iterations [9]. However, it is important to note that that FFT algorithm has significant limitations: 1) it is not capable of preserving the information on time domain, 2) it cannot be used for nonlinear and non-stationary signals, 3) it cannot depict the spectral changes over time that are fundamental in SHM.

Figure 3 shows the first three frequencies extracted by FFT for a 3.0 s long response of the model in Section 2.



Figure 3. First three frequencies extracted by FFT.

3.2 Frequency Domain Decomposition (FDD)

The FDD technique is a well-known non-parametric technique for operational modal analysis of structures. It is more generally used in the field of SHM to analyse and monitor the modal response of structures. The method is based on singular value decomposition of the cross-power spectral density matrix from simultaneous array recordings of ambient vibrations. Additionally, FDD is as an output-only algorithm, which is useful when the input data is unknown.

O'Brien and Malekjafarian apply short-time FDD to a 4.0 s long response to estimate the bridge mode shapes with a resolution sufficiently high for damage detection [10]. Thus, they propose a damage index based on mode shape squares to detect the presence and location of the damage. Zhang et al. include a brief but very illustrative compilation of the evolution of Frequency Domain techniques [11]. All of them are based on the evaluation of the spectral matrix of the structural response, the first and simplest of them being the Pick Peaking method (PP). However, given the limitations of PP methods as discussed in Gentile and Saisi, a more complex and advantageous technique labelled FDD, is proposed [12]. The difference lies on the calculation of the Singular Value Decomposition (SVD) of the spectral matrix in the case of FDD. Consequently, some limitations of the PP method are overcome, i.e. mode shapes are extracted more easily. Gentile and Saisi compare both methodologies using experimental data and they find that the results agree for both frequencies and mode shapes of the structure.

The advantage of FDD derives from being able to retrieve the resonance frequencies of structures the corresponding modal shapes without the need for an absolute reference. However, there is a significant drawback in that the classical implementation of the FDD technique requires some user interaction.

Figure 4 illustrates the first three frequencies extracted by FDD for the 3.0 s long response used in Section 3.1.



Figure 4. First three frequencies extracted by FDD.

3.3 Continuous Wavelet Transform (CWT)

In the past few years, CWT has been very popular as a powerful technique for identifying the damage in SHM by analysing dynamic structural response in the spatial domain. It is used to divide a continuous-time function into wavelets. Unlike the FFT method, the CWT possesses the ability to construct a timefrequency representation of a signal that offers very good time and frequency localization.

Hester and Gonzalez apply the CWT method to a 13.3 s long acceleration response from a simulated vehicle-bridge finite element interaction model to identify damage of the bridge structure [13]. They develop a novel wavelet-based approach using wavelet energy content at each bridge section, which proves to be more sensitive to damage than a wavelet coefficient line plot at a given scale as employed by others. However, the proposed method does not rely on identifying specific parameters of the structure, but rather on analysing the relative energy associated to the wavelet transform. It is found that it is possible to detect damage in a 40 m simply supported bridge beam model by analysing short-duration acceleration response to a moving load. Marchesiello et al. use the CWT technique to obtain natural frequencies and model shapes for a bridge-like structure traversed by loads successfully [14]. In their article, they consider the case of a slender pinned-pinned plate carrying a moving load, simulating the well-known example of a bridge crossed by vehicles or trains. The duration of the acceleration signal is 6.0 s and it is used to successfully

identify the structural frequencies. The estimated 1st mode frequency is 9.98 Hz, which is very close to the real value. Ülker-Kaustell and Karoumi use the CWT method to study the amplitude dependency of the natural frequency and the equivalent viscous modal damping ratio of the first vertical bending mode of a ballasted, single span, concrete–steel composite railway bridge [15]. In this case, the Morlet base is used instead and the viscous damping is characterized in addition to estimating the frequency of the system. For this purpose, the CWT is combined with the definition of viscous damping from a damped linear oscillator. The importance of edge effects is discussed in this paper, establishing a domain where they are negligible, i.e. between 5 s and 30 s in a 35 s long signal.

Figure 5 demonstrates the first three frequencies extracted by CWT for the same 3.0 s long response used before.



Figure 5. First three frequencies extracted by CWT.

3.4 Hilbert-Huang Transform (HHT)

The HHT technique is specially developed for analysing nonlinear and non-stationary data. The method consists of two parts: the EMD and the Hilbert spectral analysis. The key part of the method is the first step, the EMD, that decomposes a data set into a finite and often small number of the Intrinsic Mode Functions (IMFs).

Gonzalez and Aied apply the HHT method to a 5.0 s long acceleration response to obtain the frequencies of non-linear bearings [16]. In this article, a lead rubber bearing is idealized using the hysteretic Bouc-Wen model. The HHT is then employed to characterize the features of the non-linear system from IMFs of the bearing response to a time-varying force. Finally, the results using HHT (12 Hz) closely approximate the exact natural frequency (14 Hz) and stiffness of the system. Also, IMFs are shown to be a useful tool in detecting sudden damage to the bearings simulated by a reduction in the effective stiffness of the force-deformation loop. Additionally, Kunwar et al. propose a damage detection method based on applying HHT to a 6.0 s long acceleration signal from sensors on a bridge subjected to ambient vibration [17]. They test the proposed methodology on a scaled model in the laboratory, for which different levels of damage are simulated. The estimated frequencies for damage levels 1, 2 and 3 are 0.28 Hz, 0.18 Hz and 0.23 Hz, which implies reductions of 33%, 57% and 45%, compared with the natural frequency (0.42 Hz), respectively. The results show the interest of looking at the marginal Hilbert spectrum as an indicator of damage, as well as analysing instantaneous phases from the signal. Nonetheless, the authors note how the response of sensors away from the damage is barely altered in comparison with those closer to it.

Figure 6 displays the first three frequencies extracted by the HHT for a 0.5 s long response. An effective method to determine the estimated frequency from instantaneous frequency is proposed in this paper. Considering the edge effects at both ends, 11 points in the middle range are selected to estimate the frequency from the mean value at these points.



Figure 6. First three frequencies extracted by HHT.

4 RESULTS AND DISCUSSION

4.1 Comparison of results for different durations of the response

The duration of the response is varied from 0.1 s to 3.0 s with a 0.1 s increment, as shown in the horizontal axis of Figure 7. Figure 7 compares the estimated value of frequency (vertical axis) by various techniques with the real value for 1st mode of vibration. The difference between the estimated and real value can be clearly established when the response is shorter than 1.2 s. In particular, for a 0.1 s long response, the estimated frequency obtained by the CWT is nearly 2.5 times the real value of 3.91 Hz for a response shorter than 1.2 s. However, the difference between the estimated and real value of 3.91 Hz for a response shorter than 1.2 s. In the context of the beam under investigation, the results from Figure 7 indicate that both FFT and HHT are more effective than CWT and FDD when applied to the shorter duration responses.



Figure 7. Frequency comparison for the 1st mode.

Figure 8 compares the frequencies obtained for the 2^{nd} mode of vibration. For a short-duration response, the estimated value obtained from FFT is nearly the same as the real value (22.62 Hz). In the case of the HHT method, the difference is small and acceptable, but the estimated value fluctuates around the real value. However, the estimated frequency is a constant (23.44 Hz) when using FDD method. Therefore, the results from Figure 8 clearly support that FFT and CWT outperform the other two techniques. FDD is not efficient when applied to the shortest responses, and in the case of HHT, the difference is acceptable, but the estimated frequency is fluctuating up and down around the real value.



Figure 8. Frequency comparison for the 2nd mode.

Figure 9 illustrates the frequency comparison between the estimated frequency with the real value for the 3^{rd} mode of vibration. Regarding the 3^{rd} mode, the differences obtained from FFT, FDD and HHT are much smaller and the estimated frequency is nearly the same as the real value. This is expected, since higher frequency components can be fully defined within shorter periods of time. However, the estimated frequency derived from HHT still fluctuates around the real value. The value obtained by CWT is a constant (51.96 Hz) much larger than the real value. The results from Figure 9 show that FFT, FDD and HHT outperform CWT when calculating the frequency of the 3^{rd} mode of vibration.



Figure 9. Frequency comparison for the 3rd mode.

4.2 Error analysis regarding to various techniques

The error in the prediction of the frequency associated to the 1st mode of vibration is shown in Figure 10. For a 0.1 s long response, the error of the estimated frequency extracted from CWT is 146.2% and from FFT, HHT and FDD is around 36.1%, which are unacceptable margin of error for damage detection purposes. Additionally, the error associated to the HHT is around 3.03% and 1.33% when the duration of the response reaches 0.5 s and 1.2 s by HHT respectively. However, the error of using FDD method is 30.9% when applied to a response shorter than 1.2 s. Figure 10 suggests that both FFT and HHT are more efficient than CWT and FDD.



Figure 10. Error analysis for the 1st frequency.

Figure 11 shows the equivalent analysis in errors when predicting the frequency of the 2nd mode of vibration. The error by FFT is 0.45% for 0.1 s and decreasing until 0.04% for 1.0 s. Regarding the HHT method, the mean error is around 1.0%, but it is fluctuating up and down around the real value. However, the error by FDD is 3.62% for any duration shorter than 2.4 s. There is a negligible error of 0.03% regardless the duration of the response when using the CWT method, which is very close to the real value. Therefore, the results from Figure 11 clearly support that CWT and FFT methods are more reliable to extract the second frequency of bridge structure than the others. Regarding the HHT, the error is subject to many variations. Compared with others, the FDD again performs poorly for limited durations of the response.



Figure 11. Error analysis for the 2nd frequency.

Figure 12 shows the results of errors in frequency for the 3rd mode of vibration versus the duration of the available response. For a 3.0 s long response, the error is around 0.1% in the use of FFT, FDD and HHT. The error by CWT is 2.1% regardless the duration of the response. Summarizing, all techniques perform better when estimating the third frequency from a short-duration response, except the CWT method.



Figure 12. Error analysis for the 3rd frequency.

5 CONCLUSIONS

The acceleration response in free vibration has been obtained for a simply supported beam model traversed by a moving load. By comparing the exact frequency with the estimated natural frequencies, it has been possible to assess the accuracy of various techniques to extract the bridge frequencies for the first three modes of vibration from short-duration records. The results illustrate how the error in the predicted frequencies decrease as the duration of the available response increases. Overall, it can be drawn that the ability of FFT and HHT to derive accurate bridge frequencies is the most adequate of all four processing techniques for the shortest durations of the response. FDD leads to significantly larger errors for the three modes of vibration. Regarding the CWT, errors are negligible for the 2nd frequency, but increase for the other two. This paper has dealt with the challenge of deriving bridge frequency in cases with limited observation times. Further work will place focus upon the evaluation of mode shapes and damping from short data bursts.

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