

The use of wavelets on the response of a beam to a calibrated vehicle for damage detection

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Abstract

The monitoring of the dynamic properties of a structure as a tool to detect structural damage has been prevalent for some time. The fundamental theory behind this branch of non-destructive testing is that, if a crack was present in a structure it results in a localised loss in stiffness, which leads to a change in the modal properties of the structure. However, while it is reasonably easy to diagnose that a structure has suffered damage, the more challenging problem is to identify its location and severity. In recent years the wavelet transform has been used to locate discontinuities in measurements that could be associated to structural damage. This paper highlights the potential for using the wavelet technique to analyse the response of a beam to the passage of a moving load.

Résumé

Le suivi des propriétés dynamiques d'une structure comme outil pour détecter les dégâts structurels prévaut depuis longtemps. La théorie fondamentale derrière cette branche d'essai non destructif est que, si une fissure était présente dans une structure elle aboutirait à une perte localisée dans la rigidité, ce qui mène à un changement des propriétés modales de la structure. Cependant, tandis qu'il est raisonnablement facile de diagnostiquer si une structure a subi des dégâts, la vraie gageure est d'identifier ses emplacement et gravité. Ces dernières années, la transformée en ondelette a été utilisée pour localiser des discontinuités dans les mesures qui pourraient être associées aux dégâts structurels. Ce papier met en évidence le potentiel de la transformée en ondelette pour analyser la réponse d'une poutre au passage d'une charge mouvante.

Keywords

Dynamics, Moving load, Damage detection

1 Introduction

The process of damage detection almost always involves some form of non-destructive testing. Farrar [1] gives a useful overview of the whole area of structural health monitoring (SHM). In the field of SHM highway bridges are a particularly interesting set of structures for two reasons: Firstly, the bridge stock of many developed countries contains a significant portion of bridges that are approaching or have exceeded the design life that was envisaged for them when they were first built; Secondly, the increasing demand in transport is leading to heavier traffic loads, so effectively as the bridges get older they are required to work harder. If the service life of these older structures can be successfully extended, the potential economic benefits are very substantial. A significant step towards achieving the above would be the development of a reliable damage detection technique that could be applied to bridges. Obviously if the level of damage could be accurately measured and monitored, it would alert road authorities to the fact that there was a problem and allow them to take corrective action

before the level of damage became critical. Dobeling [2] gives a summary review of vibration based damage identification methods, and he describes how the fundamental principle involved in all vibration based damage identification techniques is that the modal parameters (such as frequency and mode shapes) are determined by the physical properties of the structure (such as stiffness, mass and damping), therefore any changes in the physical properties of the structure will cause detectable changes in the modal properties. A relatively new approach and one that has some potentially useful features is the one suggested by Zu [3], where he showed that by performing a continuous wavelet transform (CWT) on the deflection-time signal of a cracked beam subject to a constant moving load, it was possible to determine the position of the crack. In Zu's work, some small cracks were detected, however, most of the study focused on larger cracks, typically with a crack height to beam depth ratio of 0.5. It is the aim of this paper to investigate if it is feasible to identify small cracks and the application of a number of wavelet functions for comparison purposes.

2 Model of the Structural Response of a Cracked Beam to a Moving Load

A mathematical model of the response of a cracked bridge to a constant moving load is employed to simulate the 'measured' signals that will subsequently be analysed using a CWT. The governing equation of motion is shown in Eq. 1.

$$[M_g] \left(\frac{d^2 y}{dt^2} \right) + [C_g] \left(\frac{dy}{dt} \right) + [K_g] y = F(t) \quad (1)$$

where y is the vertical displacement of the model nodes, $[M_g]$ is the consistent mass matrix, $[C_g]$ is the damping matrix, $[K_g]$ is the stiffness matrix and $F(t)$ is the vector of forcing functions. In order to perform a dynamic simulation of a load crossing the beam, it is necessary to convert the equilibrium equations of motion into a discrete time integration scheme. There are various numerical methods that do this but the one used in this model was the exponential matrix method as implemented by Rowley [4]. Damping was not considered in the simulations. It was assumed that the mass of the structure was unaffected by the crack so the mass matrix for the damaged structure was the same as for the undamaged structure. Full details on the model can be found in Hester [5].

When creating the stiffness matrix for a cracked structure the two established methods to model a crack are the rotational spring method [3, 6] and the stiffness reduction method [7-9]. The stiffness reduction technique was adopted in this study and the method of Sinha [9] was used to calculate the elemental stiffness matrix for the cracked beam elements. The results of the model fell within 5% of the results published by Mahmoud [6] for a simply supported beam of length $L = 50\text{m}$, depth $h = 1.0\text{m}$ and width 0.5m . The Young's modulus and density of the beam were $2.1 \times 10^{11}\text{Pa}$ and 7860kg/m^3 respectively. These beam characteristics are maintained for all dynamic simulations throughout the paper.

3 Damage Detection using Wavelets

A wavelet is a waveform of effectively limited duration that has an average value of zero. Reda Tada [10] gives a practical review of the uses of the wavelet transform (WT) in the field of SHM. Wavelets tend to be irregular and asymmetric. In practical terms the way a wavelet transform works is as follows, the wavelet is compared to a section of the original signal, and the wavelet coefficient (C) is calculated. C represents how closely correlated the wavelet is with this section of the signal, the higher C is, the more the similarity, the entire length of the signal is checked in this way, then the wavelet is scaled (i.e., stretched) and the above process

is repeated. The results of the WT are many wavelet coefficients, and a local discontinuity in the signal can be identified by a local peak in the value of the wavelet coefficients.

3.1 Application of the wavelet technique

A 'pure' signal is defined here as a signal that contains no noise. If the bridge had a cracked section, there will likely be a discontinuity in the deflection-time response of the structure as a load passes over the damaged section. The basic principal behind analyzing the deflection-time signal using wavelets is to identify this discontinuity and consequently locating the damage. The structure described in the previous section was modelled as having a crack at one third of the beam length from the left support. The speed and magnitude of the moving load were 1m/s and 10kN respectively. The sampling rate was 100Hz. For a wavelet to be effective at detecting damage there needs to be a broad range of scales where the wavelet coefficients in the vicinity of the damage show a peak value relative to other parts of the structure. Fig. 1 shows a contour plot of the magnitude of the wavelet coefficients using the db2 wavelet function for a ratio crack height (a) to overall beam depth (h) of $a/h = 0.5$. The lighter colour indicates a higher value of wavelet coefficient. A peak at $1/3L$ from the left end of the beam is clearly evident indicating that there is a discontinuity in the signal.

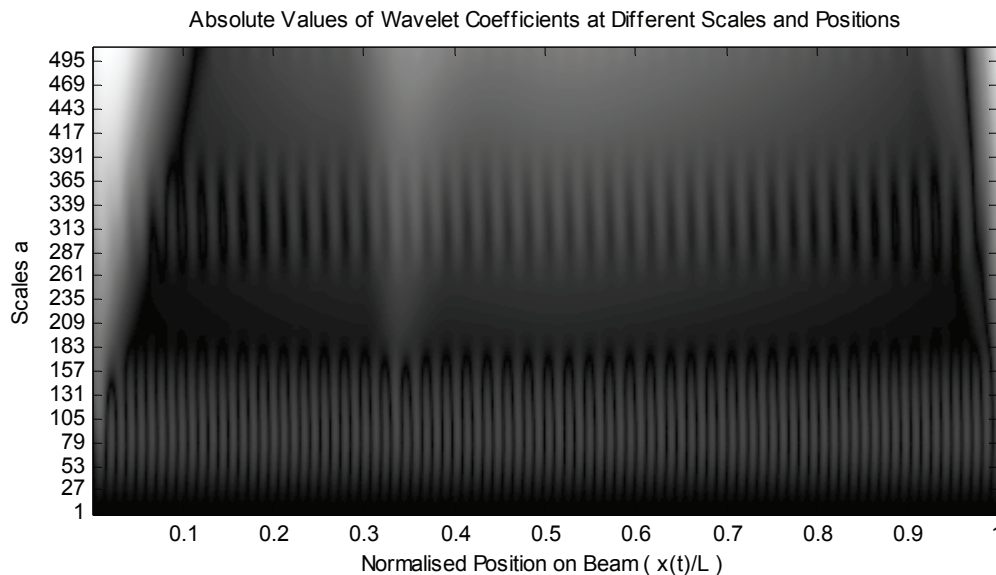


Figure 1. Plot of wavelet coefficients when signal analysed using db2

3.2 Results using a single measurement location

The ability of different wavelets functions to detect different levels of damage, and the effect of noise were investigated. The structure and the moving load were modelled using the parameters described in Section 3.1, except that the severity of the crack at the one third point, a/h , was varied from 0.1 to 0.5. When dealing with a pure signal wavelets were generally successful at detecting small levels of damage. However, real signals obtained from an instrumented structure will contain a certain amount of noise. In order to simulate corrupted measurements, 3% noise was added to the calculated response of the beam. Db2, Sym2, Corfil 1, Gauss2 and Mexican Hat wavelet functions were found unable to detect damage levels of $a/h = 0.1$. A small amount of noise present in the response can have a significant effect on the ability of the technique to detect damage. So, for $a/h = 0.2$ and 0.3 , the five families could detect damage if the signal was noise-free, but they failed to detect it from the corrupted signal. For $a/h = 0.4$, only Gauss2 and Mexican Hat clearly detected

damage using the noisy signal. Finally, when $a/h = 0.5$, all five functions were successful in detecting damage in the presence of 3% noise.

3.3 Improved performance using multiple measurements

It is recognised that for the technique to be of practical use it is necessary to reliably detect relatively small cracks. It has been found that if the wavelet coefficients of the measured signal are closer to the damaged section, peaks will be more distinctive than at a location far from the damaged section. For small amounts of damage, if the response signal was measured far from the damage location, the wavelet coefficients may show a peak at the damaged location, however, this peak is difficult to distinguish from many other local peaks. The fact that the wavelet transform for all signals, (regardless the measurement point) shows a local maxima at the damaged location is very useful, because it means that if the coefficient lines from different measurement locations are added together a discernable peak should be evident for the damaged location. For analysing the advantages of having multiple measurement locations, the displacements of the 50 m beam are simulated here at four additional beam locations: 10m, 20m, 30m and 40m from the left hand support (the damaged section is at 16.67m). 3% noise is added to all deflection signals. The Mexican Hat wavelet was selected based on the results of Section 3.2.

Figs. 2 shows the coefficient line plots for a scale of 170 when the midspan signal was analysed using the Mexican Hat wavelet for a crack height to beam depth ratio of $a/h = 0.2$. A coefficient line plot is simply a section through a contour plot similar to the one shown in Fig. 1. The scale of 170 was selected since it showed a reasonably clear peak compared to other scales where peaks may have a stronger influence from noise or other sources of interference. While there is a local minima at the 1/3 point of the structure it is not possible to identify it as damage due to the presence of many other undulations in the plot.

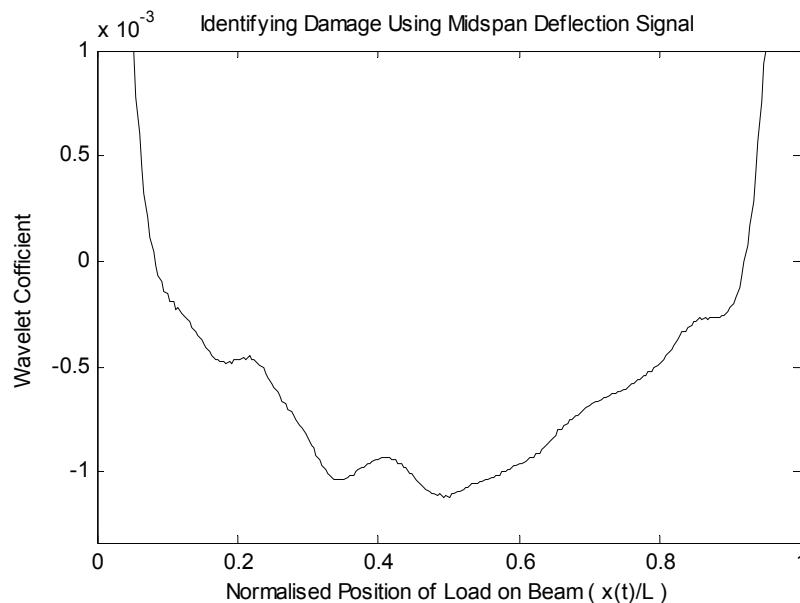


Figure 2. Identifying damage using midspan deflection signal

Fig. 3 is produced by performing a wavelet transform on the signals at 1/5th, 2/5th, 3/5th and 4/5th points of the beam, and then averaging the coefficients from the transforms at the 4 locations. The local minima is significantly more distinct than in Fig. 2 where only midspan deflection was employed (the same pattern occurs for all crack depths).

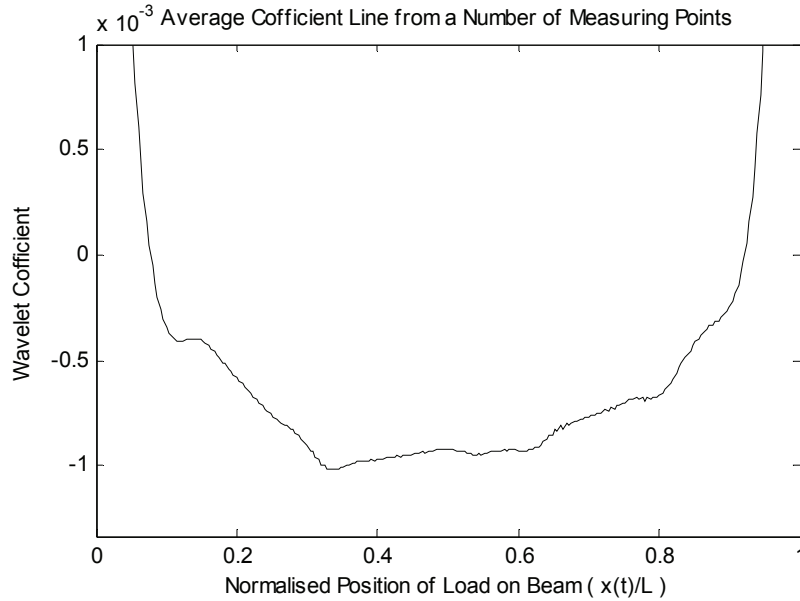


Figure 3. Identifying damage by averaging coefficients from a number of measuring points

Fig. 4 shows the individual coefficient line plots from the $1/5^{\text{th}}$, $2/5^{\text{th}}$, $3/5^{\text{th}}$ and $4/5^{\text{th}}$ points of the structure. Clearly, from all multiple recording locations, best results are found for the location closer to the damaged section, in this case, $2/5L$.

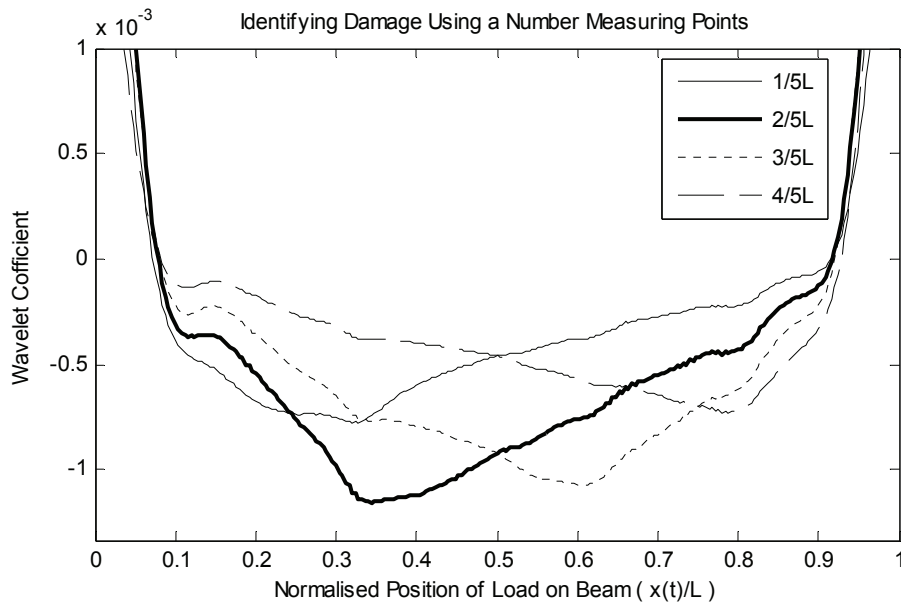


Figure 4. Identifying damage using a number of measuring points

It would appear that the most sensible approach to detecting damage is to identify a local minima in the average coefficient line plot (in the example proposed in this paper, the one third point) and then to look at the individual coefficient line for the measuring location closest to the suspected damage location, (in the same example, the $2/5^{\text{th}}$ point). The wavelet coefficients of the latter should reveal a distinct dip at the damaged location. However, if the coefficient line for the measuring point nearest the suspect location showed no clear dip at the point of suspected damage then, the suspected damaged section could be the result of noise,

dynamics or the passage of the moving load over the measurement section rather than a damaged section.

4 Conclusions

This paper has investigated the possibility of using a load moving at low speed over a beam structure as a form of non-destructive test to detect a loss of stiffness in the structure. It has become evident that to detect and accurately locate damage in a structure at a relatively early stage, it is necessary to use multiple measurement points. Results have shown that using an 'average' coefficient line from four measuring locations, it is possible to more accurately identify smaller cracks than using just one measuring location, mainly due to the fact that the probability of having a measurement location closer to the damaged location increases and this closer measurement location will be the one providing best results. While the analysis carried out in this paper has been limited by the simplifications of the structural model, it has been seen that wavelets can be employed to identify damaged sections by using the response of a structure to a moving load.

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