Sensitivity of SHM Sensors to Bridge Stiffness

Daniel Martinez[1], Eugene J. OBrien[1], Abdollah Malekjafarian[1], Yahya M. Mohammed[2], Nasim Uddin[2]
[1] School of Civil Engineering, University College Dublin, Dublin 4, Ireland
[2] Department of Civil, Construction and Environmental Engineering, University of Alabama, Birmingham, USA
emails: daniel.martinezotero@ucd.ie, eugene.obrien@ucd.ie, abdollah.malekjafarian@ucd.ie, yahya1@uab.edu, nuddin@uab.edu

ABSTRACT: Bridges play an important role in transport infrastructure and it is necessary to frequently monitor them. Current vibration-based bridge monitoring methods in which bridges are instrumented using several sensors are sometimes not sensitive enough. For this reason, an assessment of sensitivity of sensors to damage is necessary.

In this paper a sensitivity analysis to bridge flexural stiffness (*EI*) is performed. A discussion between the use of strain or deflections is provided. A relation between deflection and stiffness can be set by theorem of virtual work, expressing the problem as a matrix product. Sensitivity is obtained by deriving the deflection respect to the reciprocal of the stiffness at every analysed location of the bridge. It is found that a good match between the deflection and the bridge stiffness profile can be obtained using noise-free measurements. The accuracy of sensors is evaluated numerically in presence of damage and measurement noise. Field measurements in the United States are also described to identify the potential issues in real conditions.

KEY WORDS: Sensitivity; Accuracy; SHM; Bridge; Sensors; Displacement Transducers

1 INTRODUCTION

Sensor-based monitoring (or direct instrumentation) is gaining in importance compared to visual inspection strategies in bridge assessment. The main advantage of the former over the latter is that measured parameters are expected to be more accurate [1]. However, sensor measurements can be polluted by inaccuracies related to environmental noise [2]. These inaccuracies can be a great drawback in bridge damage detection.

The main objective of Structural Health Monitoring (SHM) is to identify damage in an engineering structure [3]. In this paper, bridges are considered. Ideally, the objective is to obtain the flexural stiffness (*EI*) throughout the bridge, but in most situations this is not possible. Damage detection can be categorised in four different classes [4]:

1. Damage identification on the bridge,
2. Damage location,
3. Damage assessment (location and quantification) and
4. Structure safety for a damage situation.

Inaccuracies can cause the misidentification of damage at some of these levels.

Bridge assessment can be performed using strain measured with a strain gauge. Strain is the deformation of a solid due to load and an elongation or a contraction results. Bridge assessment usually involve using strains or deflection [5]. Displacement transducers [6] are considered and deflection measurements are used in this paper. Deflection is related to flexural stiffness throughout the bridge by the theorem of virtual work. This formula can be expressed as a matrix product and all deflections can be calculated if all stiffnesses are known. Using this relationship, derivatives of the deflection respect to the reciprocal of the stiffness are used here to determine how sensitive deflection is to bridge damage.

A sensitivity analysis is performed in this paper. Sensitivities in a healthy bridge, a damaged bridge and under noisy conditions are considered. A static finite element model of two point loads traversing a simply supported bridge is adopted. A damage scenario including loss of stiffness is considered. Noise is added to the simulated deflection measurement and sensitivity is analysed. Three different loading locations are considered as well as an envelope of 26 different loading cases.

2 RELATION OF DEFLECTION TO STIFFNESS

The theorem of virtual work is a central concept in structural engineering. From this equation, the displacement at any point can be obtained [7]. This Unit Load Theorem formulation (Eq. 1) establishes the relation between deflections, bending moments and flexural stiffnesses:

$$u = \int_0^L \frac{M M_u}{EI} dx \tag{1}$$

where u is the bridge deflection at an instant of time, M is the bending moment of the bridge caused by the vehicle's loads, M_u is the bending moment caused by a unit load at the analysed location, EI is the flexural stiffness and L is the length of the bridge. This equation can be discretized and transformed into a vector product as shown in Eq. 2:

$$u = P \cdot J \tag{2}$$

where P represents the vector obtained by the element-by-element product of both bending moments in Eq. 1 and J is a vector of the reciprocals of the flexural stiffness components. In matrix form, Eq. 2 can be written as:

$$u = \{P\}_{1\times n} \times \{J\}_{n\times 1} \tag{3}$$

where P is a row vector with n elements and J is a column vector with the same number of elements. n is defined by the number of locations analysed on the bridge (elements of the finite element model). This means that if deflection is calculated at t different instants in time, Eq. 3 can be reformulated as a matrix product:

$$\{u\}_{t\times1} = [P]_{t\times n} \times \{J\}_{n\times1} \tag{4}$$

P is a matrix of bending moment products. It has to be adapted depending on the load distribution and the location where deflection is measured.

Measured deflections are affected by several sources of inaccuracy. White noise is considered in this example to calculate the sensitivity of deflection to stiffness. The introduced noise is formulated as a function of the maximum measured deflection [8]:

$$u_{noise} = u_{real} + N_{level} \times N_{noise} \times u_{max} \tag{5}$$

where u_{noise} is the noisy deflection signal, u_{real} is the theoretical real deflection, u_{max} is the maximum theoretical deflection, N_{level} is the noise level as a percentage and N_{noise} is a normal distribution with zero mean and a unit standard deviation.

Sensitivity can be defined as uncertainty in the output relative to the input [9]. Sensitivity can be calculated using the partial derivatives of the output with respect to the input [10]. The reciprocal of the stiffness at every location (input) contributes to the deflection calculation (output), so partial derivatives are needed for each of the measurement locations for sensitivity calculation [10]. This can be presented as in Equation 6.

$$S(i,j) = \frac{\partial u_i}{\partial J_j} \tag{6}$$

where $S(i,j)$ is the sensitivity of the deflection respect to the reciprocal of the flexural stiffness and J_j is the reciprocal of the flexural stiffness at element j.

3 NUMERICAL MODEL

A finite element beam model of a vehicle traversing a bridge is used in this paper. Two point loads separated by a distance Δx are considered to simulate the characteristics of a two axle vehicle. No vehicle-bridge dynamic interaction is considered in the model. The main features are represented in Figure 1.

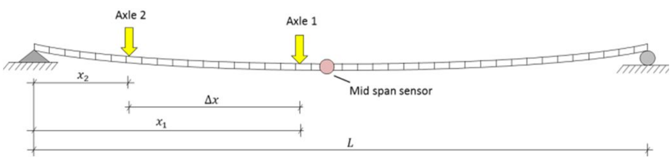


Figure 1. Beam model with two loads.

The loads and the properties of the Euler-Bernoulli beam elements used in this paper are defined in Table 1. A white noise level with $N_{level} = 5\%$ is considered. A single damage location is considered in this paper. A 40% loss of flexural stiffness is simulated. The exact damage location is shown in Figure 2.

Table 1. Point load values and geometrical and mechanical properties of the bridge

Properties	Notation	Value
Point load axle 1	P_1	80 kN
Point load axle 2	P_2	80 kN
Distance between loads	Δx	6 m
Number of elements	n	200
Length	L	20 m
Young's modulus	E	35×10^9 N/m^2
2nd moment of area	I	1.26 m^4

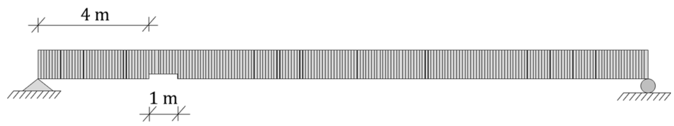


Figure 2. Single damage location for sensitivity analysis.

The sensitivity analysis is performed, considering three different load locations. Figure 3 shows these locations. Location (a) shows a situation in which only the first axle of the vehicle is on the bridge whilst in location (b) first axle is over the mid-span sensor. Location (c) considers a situation in which both the loads have passed mid-span.

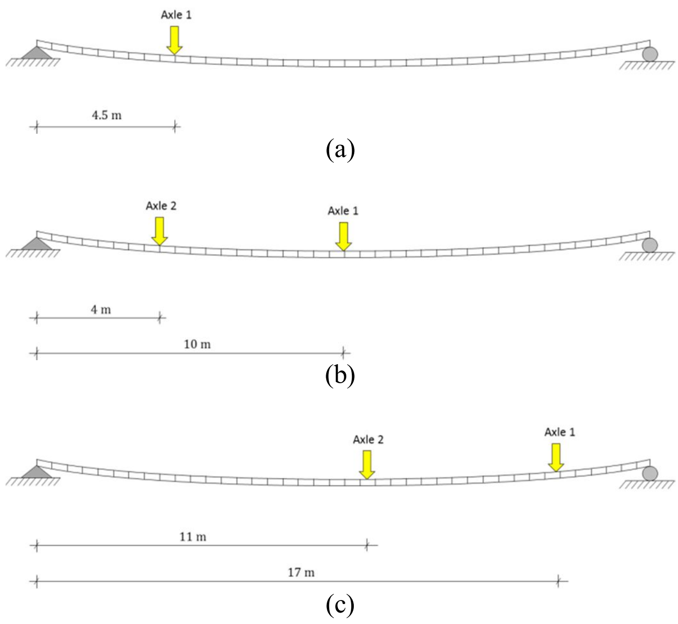
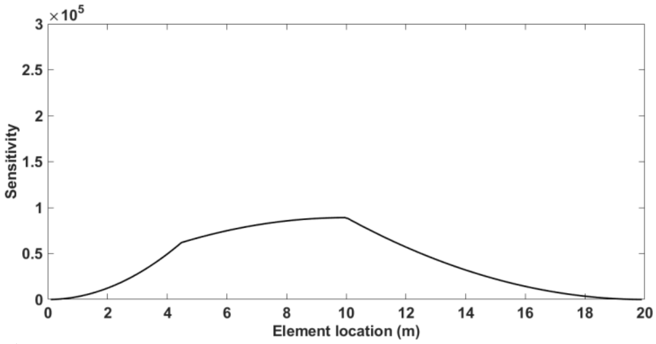


Figure 3. Load cases for sensitivity analysis when (a) 1st axle is at 4.5 m, (b) 1st axle is at mid-span and (c) 1st axle is at 17 m.

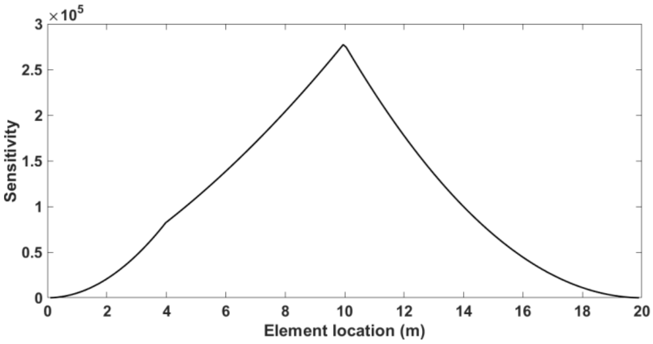
4 RESULTS AND DISCUSSION

Sensitivities are calculated in this section. Figure 4 illustrates the sensitivities of the deflection signal with respect to the reciprocals of the flexural stiffness (J) at each element location. A value is obtained for every derivative with respect to J_n (see Eq. 6). A continuous plot is created using these values. The sensitivity in Figure 4a has a discontinuity in slope at 4.5 m from the left support, i.e., at the axle location. The mid-span peak corresponds to the position of the sensor in the simply supported bridge. The greatest sensitivity to flexural stiffness is for the parts of the beam between these points. This trend also occurs in Figures 4b and 4c.

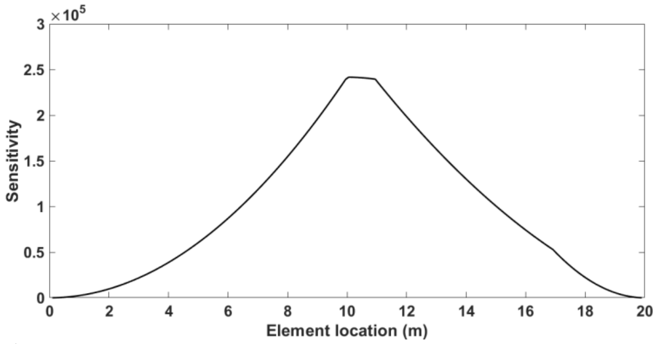
In all cases considered, the peak of the graph is at the sensor location. However, the total load on the bridge and the positions of these loads can influence the sensitivity. The sensitivity of Figure 4a is lower than the sensitivities in Figures 4b and 4c as only one load is located on the bridge and the moments are therefore less. In comparison, the magnitudes of the sensitivities in Figures 4b and 4c are similar. The differences between the former and the latter are caused by the differences in load positions.



a)



b)



c)

Figure 4. Sensitivities obtained from deflection measurements at load case a), b) and c) in Figure 3.

Even if the potential damage is not sensitive for a particular vehicle location, it may be sensitive for other locations. Consequently, an envelope of sensitivities is plotted in Figure 5. Equally spaced loading situations at every metre are considered, totalling 26 cases. The sensitivity envelope is roughly triangular, demonstrating that sensitivity is greatest at the sensor location at the centre of the bridge and reduces approximately linearly from there. It follows that the sensitivity to damage near the bridge supports is quite low.

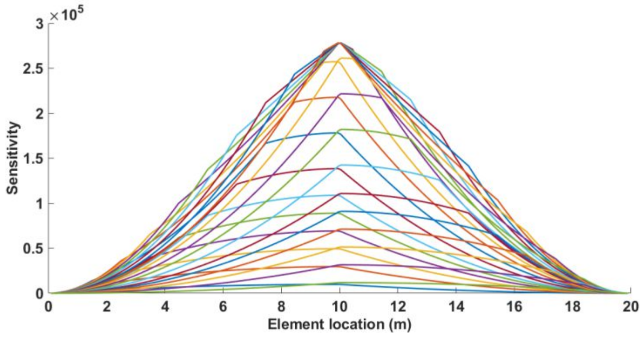


Figure 5. Envelope of the sensitivities

5 REAL MEASUREMENTS IN DRY CREEK BRIDGE

A test has been taken in Dry Creek Bridge in Alabama, Georgia. The bridge is composed by three simply supported spans and two lanes. Measurements were taken only at the first of the three spans. Five displacements transducers were installed along the cross section of the mid span at five different locations separated by a constant distance. A three axle experimental truck is used to traverse the bridge. One of the lanes is closed for the vehicle to cross the bridge whilst in the other second lane random traffic crosses the bridge. Simplified bridge and vehicle characteristics are displayed in Table 2.

Table 2. Vehicle and bridge mechanical properties

Properties	Notation	Value
Axle load 1	W_1	66.7 kN
Axle load 2	W_2	48 kN
Axle load 3	W_3	46 kN
Distance between Axle 1 and Axle 2	d_{1-2}	4.6 m
Distance between Axle 1 and Axle 3	d_{1-3}	6 m
Vehicle's speed	c	10.12 m/s
Sampling frequency	f_s	200 Hz
Length	L_b	21.34 m
Width	A	10.68 m

Using measured deflections, sensitivity of deflection to an assumed stiffness is analysed. Considering that the bridge has a constant flexural stiffness of 57.7×10^9 Nm^2, a deflection comparison between the real measurements and deflection theoretically obtained is performed. It is assumed that the measured deflection is the average of the three sensors closer to the lane that the truck is traversing. Figure 6 shows that theoretical results are far from the measured deflections as there are many sources of inaccuracy not considered in the model. It can be taken into consideration dynamics, road profile, or noise in displacement transducers.

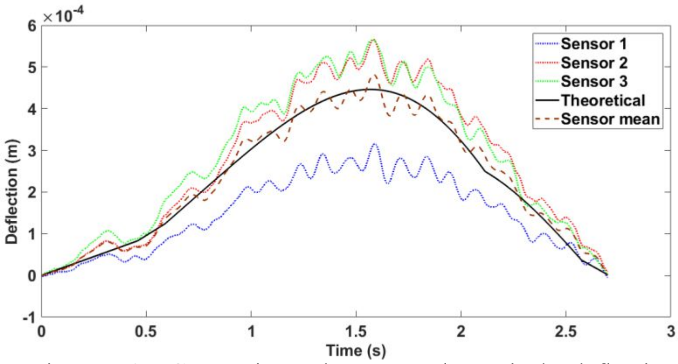


Figure 6. Comparison between theoretical deflection assuming a healthy bridge and the real measurements.

CONCLUSIONS

The paper analyses the sensitivity of deflection to bridge flexural stiffness. Sensitivity of deflection to stiffness is dependent on the load and the position of the load that traverses the bridge. Sensitivity is not affected by damage or noise. Unfortunately, measured deflections are very different from the simulated measurements. For this reason, a further analysis is necessary to adapt flexural stiffness to deflections in real conditions. Measurements taken from several vehicles crossing a bridge can improve the accuracy of the bridge deflection.

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