Behavioral Modeling for Digital Predistortion of RF Power Amplifiers: from Volterra Series to CPWL Functions

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***Abstract* — This paper gives an overview of behavioral modeling for digital predistortion of RF power amplifiers. It starts with discussing the specific system requirements and then explains what constraints must be satisfied when developing behavioral models for this application. Both theoretical aspects and practical implementation issues are discussed. Historical development and recent innovation are reviewed with a conclusion of future outlook in the end.**

***Index Terms* — behavioral model, canonical piecewise-linear function, digital predistortion, power amplifiers, Volterra series.**

I. Introduction

Digital predistortion (DPD) is a linearization approach that uses digital signal processing techniques to compensate for nonlinear distortion induced by radio frequency (RF) power amplifiers (PAs) in wireless transmitters. DPD allows PAs to be operated at highly nonlinear region for high power efficiency without losing linearity. It has been widely deployed in modern wireless systems, especially in high power cellular base stations. The principle of DPD is that a nonlinear function is built up within the digital domain that is the inverse of the distortion function exhibited by the PA. An accurate behavioral model must be developed first since only when the nonlinear characteristics are correctly modeled and thus reversed, the overall system response to a signal flowing serially through the cascade of DPD-PA can become linear.

In the past decades, many advanced behavioral models have been proposed [1]. From the signal processing point of view, if the PA is considered as a "black" box, modeling the PA can be simply treated as a general nonlinear system identification issue. One might think that there should be plenty of models available for use in DPD because nonlinear system identification is a very active and large field of research where many models and methodologies have been developed over the years. Looking back, one may be surprised that the majority of DPD models used today are either simplified or modified from the Volterra series. One may be wondering why the Volterra series is so popular while the others are not.

This paper will start with discussing the specific requirements of digital predistortion for RF PAs, and then explain what constraints must be satisfied when developing behavioral models for this application. We will discuss both theoretical aspects and practical implementation issues including system requirements and model characteristics. Historical development and recent innovation in behavioral modeling will be reviewed with a conclusion of future outlook in the end.

II. Modeling Requirements

Most predistorters today are implemented in digital baseband. The block diagram of the system is illustrated in Fig. 1, where the I/Q (in-phase/quadrature) baseband signals are first pre-processed by the DPD block to generate the predistorted signals that are then passed through digital-to-analog converters (DACs), modulated and up-converted to the RF frequency, and finally sent to the PA. To extract the coefficients of the DPD, a small fraction of the transmit signal is fed back and transferred to baseband via a down-converter and analog-to-digital converters (ADCs). The model parameter extraction unit compares the captured input and output data to extract and update the coefficients for DPD.

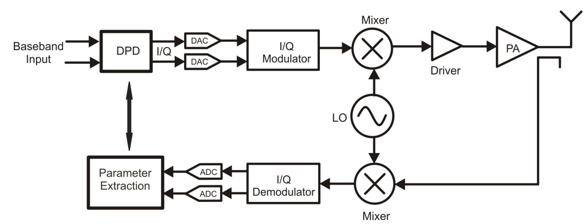

Fig. 1 The block diagram of DPD

DPD modeling can be considered as a nonlinear system identification problem, but there are certain conditions that we must consider when selecting a model for DPD:

1. In Discrete Time Domain

The model must be in the discrete time domain in order to facilitate implementation in digital circuits.

2. Deal with Complex-Valued Signals

It must be able to handle complex-valued signals because DPD is conducted in baseband.

3. Model both Static Nonlinearities and Memory Effects

As wireless communication evolves towards high data-rate and broadband services, RF PAs often exhibit frequency dependent behavior, also called memory effects, which leads that the output of the PA no longer only depends on its instantaneous input but also the previous inputs. To effectively linearize these systems, one must compensate for both static nonlinearities as well as memory effects. The behavioral model for DPD thus shall simultaneously take into accounts both static nonlinearities and memory effects.

4. Comply with First-Zone Constraint

In Fig. 1, we can see that the power amplifier is operated at RF frequency excited by real-valued signals while the DPD is run in baseband with complex numbers. The nonlinear behavior of the PA at RF must be modeled by using a baseband low-pass equivalent representation.

Although the signal spreads over many harmonic frequencies after amplification, only the first-zone spectral components are of interest, and those not lying near the carrier frequency can be filtered out and hence can be neglected. In [2], after rigorous derivations, it concluded that in the baseband representation, only an odd number of terms shall be included in the model, with the number of original terms greater than that of the conjugate terms by 1, e.g., $\tilde{x}^{p}(n)\tilde{x}^{*p-1}(n)$. This is usually called the "first-zone constraint".

This conclusion was based on that the PA nonlinearity is represented by a polynomial or Volterra function and the modulated signal is bandlimited. If we characterize the PA using an arbitrary function or derive the mapping, from the first-zone input to the first-zone output, from the curve fitting point of view, this constraint can be relaxed. In [3], Ding has shown that the model accuracy can be improved by including even terms. Even though, we have to emphasize that the so-called "even terms" in [3] is referred to the power of magnitude of the complex numbers, i.e.,$|\tilde{x}(n)|^{p}$. These terms do not come from any even-order harmonic terms in the passband. Why the model performance can be improved is mainly due to these terms contain a richer basis set that can fit the PA characteristics better than those using a limited number of the odd only terms in the model. As Ding pointed out in the paper, to obtain a better fit to the PA characteristics, one can choose between including even terms and keeping the maximum order low or allowing odd terms only and going for high orders. The first option is preferred, since low-order polynomials enjoy better numerical properties.

This issue was further discussed in [4], where the "relaxed first-zone constraint" is summarized into two conditions: (i) the equation must comply with odd-parity; (ii) the phase must preserve the unitary value of the scalar multiplying θ_n. Complying with the first-zone constraint is highly important in order to guarantee the model do not produce responses that can never be observed.

5. Linear-in-Parameters

While selecting a correct model equation is important, how the model coefficients can be extracted is also critical since it impacts not only the model accuracy but also the implementation complexity. In this regard, a model that is linear-in-parameters, meaning that the output of the model is linear in relation to its coefficients, is preferable so that linear system identification algorithms, such as least squares (LS), can be directly employed in model extraction.

In summary, the above conditions should be considered when selecting a model for DPD. As mentioned earlier, in the general nonlinear system identification field, there are many models available but the models that can simultaneously satisfy all the above requirements are very few. For instance, neural networks are very popular in nonlinear modeling but they are not linear-in-parameters and thus require using complicated nonlinear optimization algorithms to find the coefficients. Splines are very effective in curve fitting but they are not able to simultaneously model nonlinearities and memory effects.

III. Model Development

In the last decades, many advanced behavioral models have been developed and they are widely employed in the real systems.

A. Volterra Models

The Volterra series is a combination of linear convolution and nonlinear power series. Its complex baseband representation has the form,

$$\begin{aligned}\tilde{y}(n) &= \sum_{i=0}^{M} h_1(i)\tilde{x}(n-i)\\ &+\sum_{p=2}^{P}\sum_{i_1=0}^{M}\cdots\sum_{i_{2p-1}=0}^{M} h_{2p-1}(i_1,\cdots,i_{2p-1})\prod_{j=1}^{p}\tilde{x}(n-i_j)\prod_{k=p+1}^{2p-1}\tilde{x}^{*}(n-i_k)\end{aligned} \tag{1}$$

where $\tilde{x}(n)$ and $\tilde{y}(n)$ represents the baseband input and output signal, respectively, and h_{2p-1} is the complex Volterra kernel while $(\cdot)^{*}$represents the conjugate.

The Volterra series satisfies all the DPD conditions and it can be easily implemented in digital circuits. More importantly, it perfectly fits the nonlinear behavior of conventional RF PAs, such as class-AB, where the PA is linear in the small signal region and tends to become nonlinear when the amplitude of the signal increases, shown in Fig. 2(a). Many simplified versions have been developed, such as memory polynomial (MP) [5], generalized memory polynomial (GMP) [6], dynamic deviation reduction (DDR) Volterra model [7]. These models have been widely used in real systems.

Because its basis functions are polynomial-based, the Volterra models have some inherent limitations. For instance, as shown in [8], due to dynamic changes of the supply voltage, the envelope tracking (ET) PA exhibits very distinct characteristics in different power regions, shown in Fig. 2(b) where strong nonlinearity can be observed in the small signal region. It is difficult to use a single Volterra model to model this PA because the polynomial-based Volterra function cannot accurately fit it. To resolve this problem, the piecewise Volterra model proposed in [8] can be employed where the nonlinear function is divided into multiple pieces and different function can be selected to fit each piece separately. The piecewise model works reasonably well but it is still limited to the Volterra format and the system complexity can significantly increase if multiple segments are required.

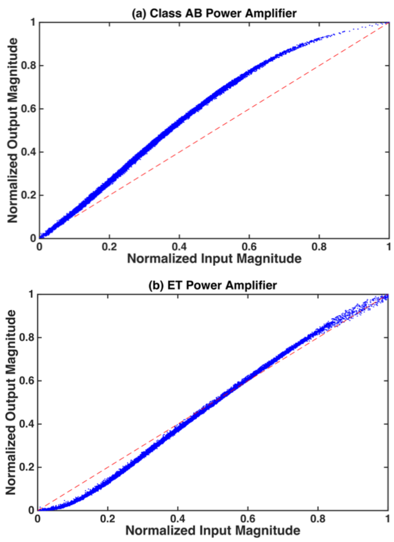

Fig. 2 AM-AM curves: (a) Class AB; (b) ET.

With stringent efficiency requirement, more advanced PA architectures, such as out-phasing, multi-way/multi-stage Doherty, have been developed, and more new architectures, e.g., coherent multiband and various switch-mode PAs, will also emerge. These PAs are to a greater or lesser extent using multiple transistors or building blocks in various combinations. The behavior of these PAs becomes very different from conventional single-ended versions. To model these PAs, the Volterra models face significant challenges.

B. CPWL Models

Originally proposed by Chua [9], the canonical piecewise-linear function (CPWL) has a very simple structure and it has been proved that it can be used to represent a wide range of continuous nonlinear functions with a high precision. In CPWL representation, the nonlinear function is approximated by a summation of a series of linear functions defined in multiple hyperplanes (partitions) using the "absolute" value operation. If we use CPWL to model a finite-memory nonlinear digital system, the function can be expressed as

$$y(n) = \sum_{i=0}^{M} a_i x(n-i) + b + \sum_{k=1}^{K} c_k \left| \sum_{i=0}^{M} a_{ki} x(n-i) - \beta_k \right| \qquad (2)$$

where $x(n)$ and $y(n)$ is the input and output, respectively. $|\cdot|$ denotes "absolute" (ABS) operation. K is the number of partition and β_k is the threshold that defines the boundary of the partition. M represents the memory length and a_i, b, c_k and a_{ki} are the coefficients.

Instead of using polynomials, the nonlinear operation in CPWL is simply achieved by using the "absolute" value operation, which only involves changing the sign of the input. Because the nonlinear functions are composed in piecewise manner, it does not have any restrictions on the shapes of the nonlinear curves. This model therefore is much more flexible and capable in modeling highly nonlinear and "unusual" power amplifiers compared to the Volterra models. The original CPWL function however cannot be directly employed in DPD because the existing function only satisfies two of the DPD modeling conditions: in the discrete time domain and simultaneously taking into account both static nonlinearity and memory effects. The rest of requirements are not satisfied.

In [10], a modification is made to conduct the absolute operation on each delayed sample instead of a full filter. The model thus becomes linear-in-parameters. To deal with complex signals, it proposes to conduct the ABS operation in four steps: (i) calculate the magnitude value of the signal; (ii) subtract away the threshold; (iii) apply an "absolute" operation; (iv) and finally restore the phase. The real-valued CPWL function in (2) is then converted into a complex format as,

$$\tilde{y}(n) = \sum_{i=0}^{M} a_i \tilde{x}(n-i) + \sum_{k=1}^{K} \sum_{i=0}^{M} c_{ki} \left| |\tilde{x}(n-i)| - \beta_k \right| e^{j\theta(n-i)} \qquad (3)$$

Because the nonlinear ABS operation is only applied on each individual sample, the model in (3) cannot take into account the interactions of the present and past samples which are often very important in the PA modeling. To improve the performance, various formats of cross-terms are introduced, e.g., the 1st-order basis can be multiplied with the magnitude of $\tilde{x}(n)$ to include more amplitude dependent information. The complete model can be found in [10]. Nevertheless, the basis functions of the new model are completely different from that used in the Volterra model. This model does not have limitations on the selections of nonlinearity orders. In other words, the new model can be used to characterize very high order nonlinearities at any power regions with a small number of terms, which provides great flexibilities in modeling unusual and emerging PAs.

IV. MODEL EXTRACTION

In existing DPD systems, model coefficients are usually extracted by comparing the input and output signals in the time domain. Because of spectral regrowth, the PA output signal occupies multiple times of the transmit bandwidth. Although there are many efforts made aiming to reduce the sampling rate of the ADC, a very high data rate is still required to capture the complete time domain information in the output in order to correctly model the nonlinearity of the PA. This situation becomes worse when the bandwidths are further increased in future systems.

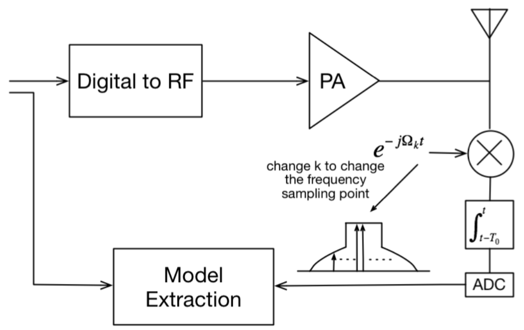

Fig. 3 Sequential demodulation.

Recently, Hammler proposed a sequential demodulation scheme in which the frequency domain samples can be captured directly from RF [11]. As shown in Fig. 3, the RF output signal is first multiplied with $exp(-j\Omega_k t)$, then integrated for a period of T_0 and finally sampled by an ADC. With this operation, the sample captured at the ADC output is equivalent to the frequency domain value calculated from Fourier transform. By sequentially changing k, frequency domain value at different frequencies can be captured. For the models with linear-in-parameters, the time domain coefficients can be extracted by using these frequency domain samples directly. In this approach, the ADC sampling rate is independent from the signal bandwidth. It provides great advantages for future wideband applications.

V. CONCLUSION AND FUTURE OUTLOOK

In this paper, we have given a quick overview of behavioral modeling for DPD. After many years development, DPD has become a mature and widely used technique, but this does not mean no further development is required. With increasing demands for higher data rate and higher power efficiency, wireless systems will face dramatic changes. Non-contiguous carrier aggregation and multiple input multiple output (MIMO) techniques will be deployed. The transmit power of the PA may instantaneously change with the incoming traffic. In small cell applications, cost and power consumption of digital circuits must be watched closely. Further research must be conducted and innovative solutions must be sought to deal with these new issues in future systems.

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