

# Method of Equivalent Currents for the Calculation of Magnetic Fields in Inductors and Magnets with Application to Electronics

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**Abstract**—Magnetic components are essential in many applications of electronics. Despite a very clear understanding of magnetic phenomena developed from first principles of Electromagnetics and Maxwell’s equations, modelling of the magnetic field, flux and force in a particular system can be a very challenging problem. Often, direct calculations are avoided, and a phenomenological model describing magnetic interactions is used instead. There are a number of methods which can be used for the modelling of the magnetic field due to magnetic materials and inductors and which can provide detailed and predictive information on such systems. Multi-physics scientific packages utilising finite-element methods are among the most common tools as they can solve a wide range of different problems and employ universal numerical algorithms. As a trade-off, they are very resource-intensive and have a low speed of execution. As an alternative, one can develop simulation techniques utilising magnetic dipoles or equivalent currents. These methods are less resource-intensive and very fast; however, they also have their limitations. This paper presents a method of equivalent currents developed for the fast calculation of the magnetic field and flux. We show the application of the method to inductors and permanent magnets that have a particular importance in power electronics and electromagnetic kinetic energy harvesting.

**Index Terms**—Magnetic interaction, numerical simulation, power inductors, magnets, energy harvesting

## I. INTRODUCTION

The modelling of the magnetic field and related characteristics due to inductors and magnetic materials is immensely important in a wide range of engineering applications. Despite a very clear understanding of magnetic phenomena developed from first principles of Electromagnetics, on many occasions the direct calculation of the magnetic field is a resource consuming and very challenging tasks. However, only the direct calculation of the magnetic field can provide a useful insight and predictive properties of magnetic components. It is the task worth undertaking, in particular, in light of recent applications.

One notable application includes power electronics where ferrite-core inductors are widely used in DC-DC converters. It is known that the inductance of a ferrite-core inductor changes very weakly at low currents (the region of weak saturation) but rolls off quickly as the amplitude of the current increases (the region of saturation). In recent years, there have been a number of papers proposing to use smaller inductors in the intermittent region of moderate saturation. The inductance displays a

dependence on current in this region, which makes the design of a converter more challenging, but such an inductor would consume less power and take up a smaller area [1]–[3]. The main aim of modelling in this case is to calculate the magnetic flux as a function of the external magnetic field and, as a result, the inductance of an inductor [4], [5].

Another application, which is particularly relevant to microelectronics, includes micro-scale kinetic energy harvesters employing electromagnetic induction as the coupling mechanism between the electrical and mechanical domains [6]. In such a system, a permanent magnet is attached to a mechanical resonator that oscillates in response to external vibrations. A coil is placed near the magnet, and a variable magnetic flux causes an induced voltage in the coil. The main aim of modelling is then to calculate the magnetic flux in three-dimensions, the induced voltage in the coil and the electromagnetic force acting on the magnet [6].

In all cases, one begins by calculating the magnetic flux density (also known as the magnetic induction or simply the magnetic field)  $\vec{B}(x, y, z)$ , which is a field in three dimensions. From the magnetic flux density, it is possible to determine primary quantities of interest such as the magnetic flux and inductance or the electromotive force (e.m.f) and electromagnetic force. There are two main approaches for calculating the magnetic induction  $\vec{B}(x, y, z)$ : (i) Solving Poisson’s equation in terms of the scalar magnet field potential and (ii) Integration of the magnetic field generated by individual magnetic dipoles in three dimensions to obtain the net field.

The first approach, combined with a finite-element method (FEM), is used in the COMSOL Multiphysics software [7]. It is widely accepted that this approach is among the best techniques since it allows one to obtain a moderately accurate solution for a system of any complexity. Despite this advantage, the method is poorly suited to electromechanical systems due to its huge numerical complexity. For example, the solution to a problem where a block magnet moves in the vicinity of a single wire loop — a typical problem in electromagnetic kinetic energy harvesting (emKEH), requires at least 30 minutes of calculation time on a modern PC, and the result has low accuracy and self-consistency [8]. The second approach requires integration in three dimensions and hence can be difficult to implement for some topologies. However, it gives significantly better accuracy and faster calculation time

when a system possess some symmetry.

The aim of this paper is to present a simulation technique allowing one to calculate the magnetic induction due to inductors and magnetic materials in three-dimensions from first principles of Electromagnetics, avoiding, however, FEM simulations or 3D integration. Employing such a technique, one can derive the secondary quantities of interests (inductance, magnetic flux, e.m.f. and force) over a range of configurations relevant for different applications in electronics. We begin by presenting the method. We then illustrate the application of the method to power inductors and energy harvesters.

## II. OUTLINE OF THE METHOD OF EQUIVALENT CURRENTS

For a magnetic system, our main aim is to calculate the magnet flux density  $\vec{B}(x, y, z)$  and the quantities of interests such as the magnetic flux  $\Phi$ , inductance  $L$ , electromotive force  $\mathcal{E}$  or magnetic force  $F_{EM}$ . In this paper, we propose a method to calculate the magnetic field that is based on equivalent currents. The main idea behind this method is to present a magnetic material (permanent magnet or a ferrite in the core of an inductor) as a stuck of line segments carrying a current density  $j$  where  $j$  denotes the amperage per unit height of the magnet (see Fig. 1).

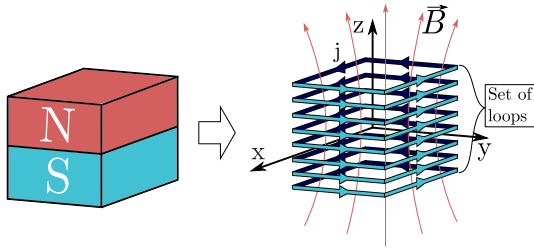


Fig. 1. A permanent magnet can be seen as a set of line segments, each carrying a current  $j$ , such that the net magnetic field produced by the coils is the same as the magnetic field due to the permanent magnet. In this study, the coils lie in the  $x$ - $y$  plane.

For instance, the expression for the magnetic field due to a segment of line carrying a current  $j\Delta h$  is well-known:

$$B = \frac{\mu_0 j \Delta h}{4\pi d} (\cos \alpha_1 - \cos \alpha_2), \quad (1)$$

where  $B$  is the magnitude of the magnetic flux density,  $\mu_0$  is the permeability of the material,  $\Delta h$  is the width of the line segment,  $d$  is the length of the segment, and  $\alpha_1$  and  $\alpha_2$  are the angles from the edges of the segment to the point in question. A counterpart expression exists for a coil of a circular shape. Using expression (1), the magnetic field generated by the stack of equivalent coils from Fig. 1 can be easily calculated [9].

As an example, one can take the magnetic flux density inside a very long permanent magnet of cylindrical shape. This quantity is a known table characteristics of the material [10] the magnet is made of (let us denote it  $\vec{B}_{rest}$ ) and is uniform. To relate this characteristic to the linear current density  $j$  of the equivalent stack of coils, one replaces the magnet with an equivalent coil configuration, runs a current  $j$  in it and calculates the resulting magnetic field  $\vec{B}(j)$ . The current  $j$  is adjusted so  $\vec{B}(j)$  becomes the same as reference  $\vec{B}_{rest}$ . This could be easily solved with a simplest numerical technique,

for example, the bi-sectional method, if one wants to speed up the calculations:

$$\vec{B}(j) - \vec{B}_{rest} = \vec{0}. \quad (2)$$

Once this relation is found, it allows one to use an equivalent current-carrying coil configuration instead of a continuous magnetic medium, as illustrated in Fig. 1. This is a significant simplification since the magnetic flux density  $\vec{B}(x, y, z)$  due to such a material can now be easily calculated in three-dimensions using the order-reduced expression (1). The method is adjusted to two different applications as discussed in the next Sections.

## III. APPLICATION TO FERRITE-CORE INDUCTORS FOR POWER ELECTRONICS

TABLE I  
GEOMETRY PARAMETERS OF THE STUDIED POWER INDUCTOR

Parameter	Value
Total height of the coil	6 mm
Diameter of the coil	12 mm
Height of the core	6 mm
Diameter of the core	11 mm
Number of loops	17

The first application we use in this study to illustrate the method is a ferrite-core inductor. We take the shielded power inductor MSS1260-103ML as one common example whose measured data is known [11]. Table I summarises the geometry parameters of the inductor. The inductor consists of an external coil and a ferrite core. It has to be pointed out that the data sheet for this inductor does not contain the precise information about the particular ferrite used in the core. For this reason, we use a reference ferrite material [10] with a simplified hysteresis characteristic as shown in Fig. 2(a). The hysteresis

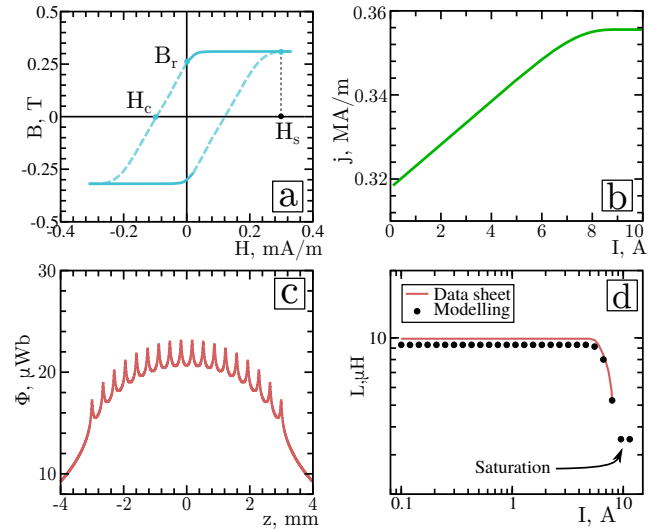


Fig. 2. Summary of the input data for the method and simulation results for the ferrite-core inductor under study: (a) Simplified hysteresis curve with the main reference points; (b) Relation between the amperage in the external coil  $I$  and the linear current density  $j$  for the equivalent coil model representing the behaviour of the ferrite core; (c) Total magnetic flux linkage when the inductor operates in saturation mode; (d) Inductance of the inductor versus current obtained through modelling and compared with the measurements from the data sheet.

TABLE II  
PARAMETERS OF THE NUMERICAL ALGORITHM IMPLEMENTING THE  
EQUIVALENT CURRENT METHOD FOR THE STUDIED POWER INDUCTOR

Parameter	Value
Number of polygon vertexes	1000
Number of flux calculating meshes	100
$\Delta I$ for the numerical derivative	$10^{-10}$ A
Number of virtual loops for the core	1000

characteristic is our primary (and only) input information when we model a magnetic material.

The inductance of this inductor can be calculated by employing the algorithm summarised in Fig. 3. First of all, we evaluate the magnetic field strength  $H$  generated by the external coil, which is, as expected, depends on its geometry and the current flowing in it. At this stage, we apply the equivalent current model and replace the ferrite core with an equivalent set of coils, emulating the behaviour of the core, as shown in Fig. 4. An additional model reduction can be made if, instead of a circular coil, we use a polygon-shaped loop since the flux density due to this loop can be easily calculated using expression (1). Hence, the whole inductor is modelled by two sets of coils, external and internal.

We then establish the relation between the current  $I$  flowing in the external inductor coil with the magnetic flux density  $B$  induced, as a result, in the ferrite core. To solve this problem, we again recall that for a stack of coils (with the stack height  $h$  being significantly greater than the radius of a coil  $R$ ), we can always calculate the longitudinal component of the magnetic field strength  $H$  if a current  $I$  runs through the stack. The magnetic field  $H$  is “inducing” or “external” with respect to the ferrite core. The induced magnetic field  $B$  in the ferrite is related to  $H$  through the material’s hysteresis curve, as shown in Fig. 2(a). Knowing the target induced magnetic flux density  $B$  in the ferrite, we determine what current density  $j$  must flow in the equivalent set of coils in order to observe exactly the same magnetic field. Performing these steps, we can directly relate the current  $I$  flowing in the external coil to the current density  $j$  that flows in the equivalent set of coils “mimicking” the ferrite core. The dependence  $j(I)$  is shown in Fig. 2(b).

For power inductors, the major quantity of interest is their inductance as a function of the current  $I$  flowing in the external coil. We introduce the magnetic flux  $\phi(I, z)$  through a single

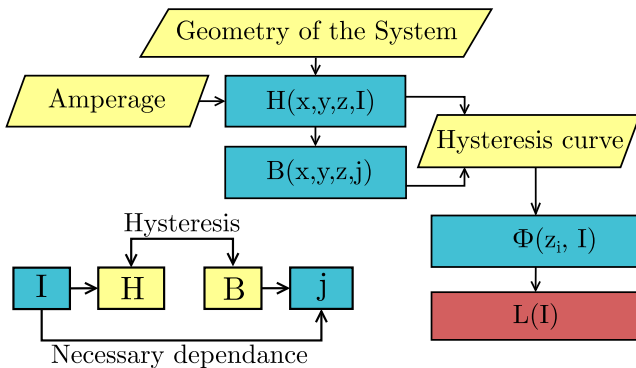


Fig. 3. Algorithm summarising the modelling step in the equivalent current framework. The primary quantity yielded by the algorithm is the magnetic field  $\vec{B}$ . Other quantities of interest can be obtained from  $\vec{B}$ .

loop, noting that the flux is a function of the current  $I$  inducing it and the position of the coil  $z$ . One can make use of the axial symmetry of the system, simplifying numerical integration to the following sum:

$$\phi(I, z) = \sum_{k=0}^N \pi(\Delta r)^2 \cdot (2k + 1) \cdot (B_z^{\text{coil}} + B_z^{\text{core}}), \quad (3)$$

Here,  $\Delta r$  is the elementary increment of the coil radius and  $B_z^{\text{coil}}$  and  $B_z^{\text{core}}$  are the  $z$ -components of the magnetic flux density due to the external coil and the ferrite core respectively, evaluated at the coordinates  $((k + 1/2)\Delta r, 0, z)$  in three-dimensions. The magnetic flux  $\phi(I, z)$  as a function of the position  $z$  at a given current  $I$  is shown in Fig. 2(c). The inductor, whose total height is 6 mm, is placed between  $z_1 = -3$  mm and  $z_2 = 3$  mm. As expected, the magnetic flux  $\phi$  decays with the distance  $z$  and displays local maxima at those values of  $z$  where we place (a finite number of) surfaces to calculate it numerically.

Finally, to determine the inductance  $L$  of the system, we use the definition:

$$L = \frac{d\Phi}{dI}, \quad (4)$$

where  $\Phi$  is the total flux linkage in the inductor. This gives us the graph of the inductance  $L$  versus the current  $I$  shown in Fig. 2(d). The result demonstrates a very good agreement with the data from the inductor’s data sheet. As expected, the inductance is constant when a small current (and hence a weak magnetic field) is applied. The model predicts the roll-off of the  $L - I$  curve at 8.0 A and the saturation region that follows after the roll-off.

#### IV. APPLICATION TO ELECTROMAGNETIC KINETIC ENERGY HARVESTERS

Electromagnetic kinetic energy harvesters (emKEH) are another interesting application that can make use of the proposed electromagnetic simulation technique. In emKEHs, the transduction mechanism performing the transfer of energy from the mechanical to the electrical domain is based on the Faraday induction principle. A typical schematic diagram of an emKEH is shown in Fig. 5(a). Such a harvester contains a proof mass  $m$  suspended on a nonlinear spring  $k$ . The mass oscillates when external acceleration  $a_{\text{ext}}(t)$  is applied.

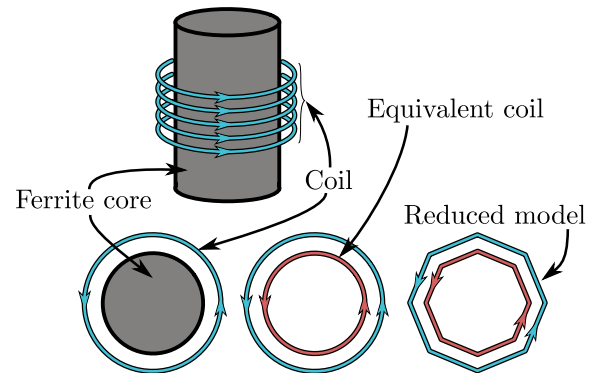


Fig. 4. Modelling of an inductor: its schematic diagram, representation of its core as a coil carrying an equivalent current and reduction of the equivalent coil to a polygon shape.

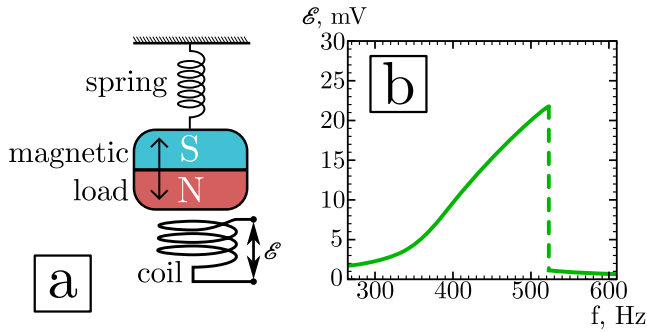


Fig. 5. (a) Lumped model of emKEH showing a proof mass with a magnet suspended on a spring in the vicinity of a coil; (b) The result of simulations showing the induced e.m.f.  $\mathcal{E}$  versus the frequency  $f$  of the actuating acceleration.

Usually, the mass has a permanent magnet attached to it. The motion of the mass and magnet induces a variable magnetic flux in a coil placed in their vicinity.

The modelling of such an electro-mechanical system is even more challenging since one has to solve coupled equations describing the dynamics of the mass in the mechanical domain and the state of the coil in the electrical domain. A lumped model of an emKEH can be written in the following form:

$$m\ddot{z} = -b \cdot \dot{z} - k(z) \cdot z + F_{EM}(z, \dot{z}) + ma_{ext}(t), \quad (5)$$

where  $b$  is the air damping coefficient,  $k(z)$  is the function describing the nonlinear spring and  $a_{ext}$  is the external acceleration. The latter is usually presented in the form  $A_{ext} \cos(\omega t)$  where  $A_{ext}$  is the amplitude and  $\omega$  is the frequency of the external acceleration. The equation contains the electromagnetic force  $F_{EM}(z, \dot{z})$  that couples the displacement of the mass  $z(t)$  and its velocity  $v_z(t) = \dot{z}(t)$  with the electrical quantities of the system and, hence, the converted power.

Again, the primary quantity that we obtain using the method of equivalent currents is the magnetic flux density  $\vec{B}(x, y, z, t)$  for the mass-magnet moving due to external vibrations. Knowing  $\vec{B}$ , we calculate the magnetic flux  $\Phi$  through the coil. We then use the Faraday law (6) to obtain the e.m.f. or induced voltage:

$$\mathcal{E} = \frac{d\Phi}{dt} = \frac{d\Phi}{dz} \frac{dz}{dt} = \frac{d\Phi}{dz} \cdot v_z. \quad (6)$$

Note how the electrical equation on e.m.f. contains the mechanical quantity  $v_z$ , highlighting again the coupling between the two domains. To calculate the electromagnetic force, we use first principles and write the force in the form:

$$d\vec{F}_{EM} = [\vec{B} \times I d\vec{l}] \quad \text{and} \quad \vec{F}_{EM} = \sum_{coil} \oint_{loop} d\vec{F}_{EM}. \quad (7)$$

The force is evaluated on the nodes of a discrete mesh and interpolated between the nodes. Taking into account that the force is linear with respect to velocity (however, it is nonlinear versus the displacement) the interpolation formula for  $F_{EM}$  can be easily found using polynomial approximations. The result of simulations is shown in Fig. 5(b) with the parameters of the devices taken from [12], [13]. One can compare the simulation result with the measurements of emKEHs provided in the cited

paper to see that indeed the model simulations demonstrate very good agreement with the measurements.

## V. CONCLUSIONS

This paper presents a method for the simulation of magnetic interactions and the calculation of the magnetic field as the primary quantity of interest for many practical applications in electronics. The method allows one to calculate the magnetic field in three-dimensions *in* magnetic materials and *due* to permanent magnets. Knowing the magnetic field, one can easily obtain other quantities of interest, such as the magnetic flux, electromotive force or electromagnetic force. The method, while still being a numerical technique, is compact and simple enough so that it does not require substantial computer resources. We demonstrate the application of the method to calculate the inductance of an inductor as a function of the current flowing in it and to calculate the e.m.f. generated by an electromagnetic kinetic energy harvester.

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