

The Distributional Characteristics of a Selection of Contracts Traded on the London International Financial Futures Exchange

ABSTRACT

This study examines the distributional properties of futures prices for contracts traded on LIFFE. A filtering process is employed to remove day of the week and holiday effects, a maturity effect, moving average effects and the influence of an asset's conditional variance from the raw returns series. Alternative distributional models from the stable paretian and ARCH families are examined for their applicability to futures data using a stability under additions. The results conclusively reject the hypothesis that futures returns are normally distributed with findings in favour of two related hypotheses - the mixtures of stable distribution and the ordinary stable distribution.

Keywords: Distribution of futures returns, stability under additions test, stable distribution, mixtures of distributions, GARCH-M

The Distributional Characteristics of a Selection of Contracts Traded on the London International Financial Futures Exchange

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1. INTRODUCTION

The London International Financial Futures Exchange, LIFFE, was launched on 30 September 1982. Despite initial scepticism about the viability of a financial futures exchange in London, LIFFE has grown at the impressive rate of 45 percent per annum. With a nominal daily turnover of close to £160 billion, the equivalent of trading the entire UK economy every week, LIFFE is now one of the largest exchanges in the world. When the futures exchange opened its doors for trading in 1982 only two futures contracts were traded. LIFFE now, however, facilitates the trading of 17 fixed interest and money market futures and options products in addition to four FTSE Index contracts and 70 equity options.

Empirical work detailing the behaviour of futures markets and the price movements of contracts traded in these markets has been wide ranging. Issues such as: the existence of a risk premia in

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futures contracts; the rival hypotheses of random walks and price trends; the day of the week and holiday effects; the impact upon price volatility of contract time to maturity; the distributional properties of futures prices have all been explored. However, the majority of empirical research has tended to focus upon the computation of appropriate hedge ratios and related issues such as duration effects and hedge ratio stability. Such concentration is to be expected given that the prime function of futures markets is to allow producers and investors to manage the risks from the underlying spot markets. In terms of the approach adopted to hedging, most research has been content to utilise the risk-minimisation approach. This approach implicitly assumes either that returns are normally distributed or that investors have quadratic utility functions. An increasing literature points to the assumption of quadratic utility being overly restrictive and also questions the assumption of normally distributed returns. Also, and of practical importance for investment managers, risk management techniques such as Value at Risk models are often estimated relying on similarly restrictive return characteristic assumptions.

In this paper we return to the issue of the distributional properties of futures prices with our focus that of a selection of contracts traded on LIFFE. In particular, we aim to make use of both the findings and indeed the techniques implicit in earlier work to filter price data of a selection of futures contracts. The filtered price data is then tested to determine whether the respective contracts can unanimously be characterised by a particular distributional form. More specifically, raw returns data is filtered to remove day of the week and holiday effects, a maturity effect, moving average effects as well as the influence of the asset's conditional variance from the raw returns series. The distributional properties of the filtered data, for each of the futures contracts under consideration, are then

scrutinised. In this context many alternative distributional models have been suggested for their applicability in correctly describing properties of futures price series. Those that have attracted strong attention include the stable paretian (Hudson et al., 1987; and So, 1987), the mixtures of normals (Hall et al., 1989), the mixtures of stables (Gribbin et al., 1992), ARCH (Fujihara and Park, 1990) and GARCH-M (McCurdy and Morgan, 1988). To examine the applicability of the above noted distributional forms the stability under additions procedure, as developed by McCulloch (1986), is employed. Among other things, this procedure will provide answers to questions such as whether it is possible, using filtered data, to rehabilitate normally distributed returns and consequently risk minimisation as an approach to hedging spot market exposure with futures contracts.

In terms of the format of the paper the following approach is taken. In the second section an overview of salient literature is provided. A two-stage approach is taken in this section. First, an overview of papers dealing with the distributional characteristics of financial time series is presented concentrating on futures price series. Second, literature focusing upon the presence of serially correlated variances in financial time series and the role of autoregressive conditional heteroskedastic, ARCH, models is described. The third section documents the methodological approach adopted. The characteristic function of the 'stable paretian' distribution is detailed, as is the stability under additions test. This section then continues by describing the procedure for filtering the raw futures price. Finally a rescaling of the data series is set out with the rationale for this process being that much of the serial dependence exhibited in the second moments of an asset's return distribution can be removed by standardising returns with an ARCH approach. The next section

focuses upon the data set employed which as indicated is a selection of futures price series traded on LIFFE - two stock index contracts, two bond contracts, and two short-term interest rate contracts. The key empirical findings are detailed in section six with the final section presenting some concluding comments.

2. LITERATURE REVIEW

Whilst an examination of distributional properties for futures prices has been overlooked relative to other speculative markets, past research has focused on two main areas. These are the stable paretian family; and more recently, given the documented characteristic of time varying variance, the ARCH related models. This paper breaks up the review of past studies according to these areas. The main distributional findings for futures are that the data is leptokurtotic and conditional heteroskedasticity is present. Consequently, futures returns are not normally distributed. Venkateswaran et al. (1993) indicate significant leptokurtosis for futures traded in the US. Also, in this market, there is a lack of independence caused by time varying variances (Fujihara and Park, 1990). To a lesser extent, skewness is documented, and it is systematic in these contracts (Junkus, 1991). Futures traded on LIFFE have not similarly been examined.

In the context of these findings, studies have concentrated on examining non-normal distributional hypotheses. Mandelbrot (1963) suggests that speculative prices have characteristics of a stable paretian distribution. In particular, this is supported by the existence of data that exhibit fat tails relative to normality. This characteristic corresponds to the presence of leptokurtosis. Within the stable paretian family, many alternative distributional models are suggested for their applicability in

correctly describing the leptokurtosis in futures series'. Those that attract strong attention include Mandelbrot's stable model (Hudson et al., 1987), the mixtures of stables (Gribbin et al., 1992) and the mixtures of normals (Hall et al., 1989). These hypotheses are tested using a stability under additions test, relying on an attribute of the stable paretian family (McCulloch, 1986).

Empirical analysis suggests that inconclusive findings regarding a particular type of distribution may be explained by a lack of uniformity of all the characteristics of data over time. The changing of a particular characteristic has led to suggestions that futures price series may involve mixtures of specific distributions (Gribbin et al., 1992). Also, favourable evidence is presented for spot markets (Anderson and Bollerslev, 1997). Two mixtures of distributional type are indicated; namely, the mixtures of stables and mixtures of normals.

The mixtures of stable paretian model involves either a changing of the scale or skewness parameters. In contrast, the mixtures of normals involves price series with the same mean and changing variance. Thus, both distributions documented for futures price series involve a combination of two or more distributions with the same mean but differing higher moments, and may be referred to as 'contaminated series' (So, 1987). Appropriate reasoning behind changing moments for futures prices are due to temporal factors, and are outlined in Hall et al. (1989). For instance, variance values would change, if the measure is proportional to the actual number of days in a week as opposed to the number of trading days. Non constant characteristics would also occur if one is examining price series with differing times to maturity (Milonas, 1986).

Although the models previously outlined indicate the existence of serially correlated variances (Hall et al., 1989), they do not specifically focus on the non stationarity in the second moments. Recently, models that incorporated this conditional dependence have been developed and applied to futures prices with varying success. Those models which have gained the widest support for US traded futures are the ARCH (Fujihara and Park, 1990) and related frameworks such as the GARCH-in-mean ~GARCH-M (McCurdy and Morgan, 1988). The stable and ARCH models have a close link as they assume non constant parameters, and more importantly, they both explain volatility clustering (Mills, 1996).

The set of ARCH related approaches have received voluminous empirical attention in finance, and comprehensive reviews of these studies can be found in Pagan (1996). The basis of these approaches has been that the source of the leptokurtosis is caused by heteroscedasticity in the variance of a distribution. Under these conditions, the traditional statistical tools are inefficient and care should be exercised on their application. There are two ways in which such a problem can be overcome. First, if the true theoretical framework is normally distributed, but with time invariant second moments, the issue becomes redundant in time periods sufficiently short enough so that the variance is unchanged. Secondly, heteroscedasticity can be removed resulting in a distribution consistent with normality.

In past studies, comparisons of how well the time varying models incorporate non stationarity in the second moments suggest that they have a similar ability to transform a series to a close approximation of normality. They are all unable to correctly adjust a data set to truly fit a normal distribution but they improve the situation considerably in contrast to findings for the original series. In particular, while the kurtosis findings for all these models show an improvement after adjusting for time varying variance, its still persists (Yang and Brorsen, 1993). Also, time varying models reduce skewness considerably, but do not completely remove it (Op. Cit.).

As many of these models are applicable to futures data, it is necessary to have a justified rationale for examining any particular ARCH specification. As we have seen, temporal and trading factors have an influence on time varying risk. In fact, the effect of these factors can be seen in both the mean and variance, resulting in a risk return relationship if both the first and second moment influences are incorporated together. This generally involves a risk premium where a risk adverse investor requires additional compensation for investing in a risky asset. Futures contracts do exhibit a risk premium (Taylor, 1986). Given this relationship, an ARCH-M model, or a generalised version detailed by McCurdy and Morgan (1988) can be applied. The premise of such a model is that risk premiums exist, and the conditional mean is an explicit function of the conditional variance.

3. METHODOLOGICAL ISSUES

A stability under additions test is applied to examine the distributional properties of futures traded on LIFFE. As well as examining compounded returns, two distinct sets of futures returns are tested using the stability under additions test. First, residuals from a filtering technique based on a GARCH-M specification are examined. This approach removes temporal and trading anomalies influencing the conditional mean returns. Second, the filtered returns are rescaled

using the estimate of the conditional variance. This standardising process indicates the distributional type of the data, as well as the ability of the filtering technique to remove ARCH effects.

(i) Stability Under Additions Test

The stability under additions test relies on the stable paretian model defined by its characteristic function:

$$\log \phi(t) = i\delta t - \gamma |t|^\alpha (1 + \beta(t/|t|) \tan(\pi\alpha/2)) \quad (1)$$

for $0 < \alpha \leq 2$

where t is any real number, δ is the location parameter, γ is the scale parameter, β is the skewness parameter, and α is the kurtosis parameter and is also the characteristic exponent.

The characteristic exponent is the most important parameter of the model and describes the shape of a distribution and in particular, the areas around the mean and the tails of the distribution. Specific types of stable distributions realise different values. For $\alpha = 1$, the model describes a cauchy distribution with a finite mean and infinite variance. The special case of the normal distribution is described when $\alpha = 2$. Surrounding these values, if $\alpha < 1$, the distribution has an infinite mean and variance, and if $1 < \alpha < 2$, the distribution has a finite mean, but the variance is infinite. In practice this infinite variance implies that erratic behaviour is shown by returns for large samples and this is indicated by larger standard deviations than for Gaussian processes. This conclusion falls neatly into the volatility clustering arguments that support time varying models such as GARCH where periods of high volatility can occur consecutively. In this case,

futures price series should have a characteristic exponent valued between 1 and 2.

In the context of examining futures prices in terms of the stable distributions, a second important parameter is the measure of skewness, β , which takes on values in the interval $-1 \leq \beta \leq 1$. When $\beta = 0$, the distribution is symmetrical and is consistent with normality. For non Cauchy distributions, a positive parameter coefficient indicates a distribution that is skewed to the right, whereas a negative coefficient refers to a distribution that is skewed to the left. The extent of the skewness coefficient value refers to the degree of skewness present. The location parameter, δ is not necessarily the mean of the distribution. This would only be the case when $\alpha \geq 1$, as in other situations the mean of a distribution is not defined. Finally, the scale parameter, γ has different associations with the variance depending on the specific type of stable distribution that one is detailing. For instance, with the special stable case of the normal distribution, γ is half the variance.

Distributional characteristics are tested for using the stability under additions test, developed by McCulloch (1986). The procedure operates by comparing the characteristic exponent across different sums of observations. An example might be monthly (combining 20 daily values), versus weekly (combining 5 daily values) versus daily data. In this case the sums of data would be 20, 5 and 1 respectively. The values of the characteristic component across these different sums of observation determines the futures prices distribution. If α remains constant, that is $\alpha_1 = \alpha_2 = \dots = \alpha_n$ the distribution describing the data is stable. Earlier versions of the testing methodology (for example, Fama and Roll, 1971) assume symmetry. Using this approach, a finding of a non constant α , with values increasing towards two as you move across the sums of observations indicates a mixtures of normals distribution, signified by a single mean and

changing variance. However, if skewness or scale parameters are allowed to vary, the result of α increasing towards 2 suggests mixtures of a stable distribution.

McCulloch's stability under additions test uses a fractal approach similar to Fama and Roll's (1971), but allows the skewness parameter to take on a range of values ($-1 \leq \beta \leq 1$). Five sample quantiles, and linear interpolations of tabulated index numbers are the basis of this technique. The sample quantiles are used to approximate for population values, X_p . Also, a skewness correction component is included, based on associating each ascending value of X_i with a linear extrapolation of two adjacent values of $X_{q(i)}$, where $q_i = (2i - 1)/2n$. This development ensures that spurious skewness is not present for finite samples, and leads to consistent estimates of different quantiles X_p . Indexes for both α and β coefficients are measured using the following:

$$V_\alpha = \frac{X_{.95} - X_{.05}}{X_{.75} - X_{.25}} \quad (2)$$

and

$$V_\beta = \frac{X_{.95} + X_{.05} - 2X_{.5}}{X_{.75} - X_{.25}}$$

(3)

where V_α and V_β are sample estimates of corresponding population values. These indexes are dependent on each other. In contrast, summary statistics assume that each coefficient is calculated with other parameters remaining constant. Interdependant values for β , and more importantly α based on (2) and (3) are tabulated by McCulloch (1986).

By approximating sample values, α estimates can be affected by sampling error, and it may be difficult to attribute increases in the calculated characteristic exponent to actual changes in this parameter. The extent of the sampling error could be compounded with the stability under additions test when you move to larger sums of observations, as the sample size decreases. To control for this problem, the number of different non overlapping sums of observations is limited to 20, where the classifications are 1, 2, 5, 10, 15 and 20 loosely corresponding to daily, bidaily, weekly, biweekly, triweekly and monthly continuously compounded returns.

(ii) Filtering Technique:

The purpose of the filtering technique is to remove documented anomalies from the futures data, thereby generating smooth returns series'. The approach used to filter futures returns is a GARCH-M model outlined in (4). By using (4) as the relationship between the sequences $\{Y_t\}$ and $\{Z_t\}$, the time varying model involves a joint estimation of the mean and the disturbance term plus the heteroskedastic process that estimates the second moments.

$$Y_t = b Z_t + c h_t + \varepsilon_t \quad (4)$$

As the GARCH-M model incorporates serial correlation of the second moment, it has theoretical and practical advantages over models that do not. For example, if the conditional variance is to change for any reason over a time series (for example the Gulf War effect on financial markets), the GARCH-M approach is set up to deal with such an event. However, Pagan (1996) notes that analysis of an GARCH-M type model is more complex than that of pure GARCH models due to the requirement of joint estimation.

The specific filtering process adopted here is similar in its objective to that of Pagan and Schwert (1990). Temporal anomalies, moving average effects and the influence of an asset's conditional variance are removed from the raw compounded returns series. The residuals from this process are the new series', known hereafter as, filtered returns. The recognition that returns are a function of a future's risk is accounted for by including a measure of its variance. As the degree of risk is time varying, one would expect that the risk averse investor would require analogous varying compensation. By including a time varying risk parameter in the filtering of an asset's returns, a GARCH-M model is being applied.¹ This model is estimated by maximum likelihood methods using the algorithm developed by Berndt et al. (1974). It involves a joint estimation of the mean equation incorporating the conditional variance, as well as the heteroscedastic process that estimates the second moments, and is given in (5).

$$Y_t = a_o + \sum_{s=1}^{10} A_s X_{t-s} + A_{11} D_M + A_{12} D_T + A_{13} D_W + A_{14} D_{Th} + A_{15} D_F + A_{16} D_H + A_{17} h_t + \varepsilon_t$$

(5)

Where Y_t is the filtered returns; X_{t-s} is the lagged raw returns series; the days of the week and holidays are given by dummy variables so that $D_M = 1$ if Monday, $D_T = 1$ if Tuesday, $D_W = 1$ if Wednesday, $D_{Th} = 1$ if Thursday, $D_F = 1$ if Friday and $D_H = 1$ if a holiday. Otherwise they are 0. The measure of the conditional variance is denoted by h_t . The residuals ε_t from this process are the filtered returns series.

Past studies have included a lag structure of order ten to remove autocorrelation in the mean (Taylor, 1986 and Pagan and Schwert, 1990), and the filtering equation includes a similar lagged structure to remove this dependence. In this case, the observed leptokurtosis will not be accounted for, but rather, it will help to obtain more precise results. Pagan and Schwert (1990) indicate that nonsynchronous data can induce dependence in stock returns. An obvious source of nonsynchronous data for futures series' is caused by the rolling over of different contracts across time in generating a single lengthy time series.

The risk measure is allowed to vary over time and is assumed to follow a GARCH (1,1) process given by equation (6). The changing variance model is adjusted for day of the week, holiday and maturity effects.

$$h_t = b_0 \alpha e_{t-1}^2 + \beta h_{t-1} + b_1 D_M + b_2 D_T + b_3 D_W + b_4 D_{Th} + b_5 D_F + b_6 D_H + b_7 M_t + b_8 M_t^2$$

$$E_t - V_{t-1} \sim N(O, h_t)$$

(6)

The maturity effect is captured by including the terms, M_t and M_t^2 ; where M_t is the number of days left in a specific contract divided by the number of days in the contract.

Junkus (1986) has documented seasonality and trading anomalies, for example, holiday and day of the week effects and these are filtered from our raw compounded returns series' by including respective dummy variables in both the return and risk equations. Specifically for futures data, Milonas (1986) finds that volatility and time to maturity are negatively related, and these maturity effects are modelled in the filtering process only through the volatility measure as there is no a priori reason for including maturity effects in the mean equation.

(iii) Rescaled Returns

A related set of returns to those generated by the filtering process are the rescaled returns, given in (7). The robustness of the filtering technique can be examined with these returns. On their own, they can also be used to determine distributional characteristics with the stability under additions test.

$$Z_t = \frac{Y_t - \bar{Y}}{\sqrt{h_t}} \quad (7)$$

where h_t is an estimate of the actual conditional variance.

This transformation uses the values for the filtered series with the sample mean and standard deviation. It is common to focus on a simplified variation of (7), where the filtered returns series is divided by the conditional standard deviation, as the results are similar. An examination of the properties of the kurtosis coefficient should indicate normal characteristics if the transformation process explains the excess kurtosis inherent in the original returns. It is also interesting to compare the kurtosis coefficient for raw and rescaled returns. The extent to which leptokurtosis is removed from the raw returns determines how well futures returns can be approximated by normality. In its favour is Venkateswaran et al. (1993) finding that for futures series, the most efficient rescaled returns involve a time varying approach.

Also, Taylor (1986) and Bollerslev (1988) suggest that much of the serial dependence exhibited in the second moments of an asset's return distribution can be removed if one examines the standardized returns after applying an ARCH approach. Bollerslev (1988) suggests a diagnostic to determine the goodness of fit of the ARCH related specification based on examining the autocorrelation function of the standardized filtered returns, ε_t/h_t . ARCH effects are recognised in the data if there are significant autocorrelation values for the squared returns series. Thus, a comparison of dependency for raw and rescaled returns indicates whether the influence of serial dependence in the second moments have been removed after filtering.

4. DATA CONSIDERATIONS

Six futures series traded on LIFFE are used for analysis, two each from the equity market, the bond market and the money market. Details of these contracts are given in Table 1.

INSERT TABLE 1 HERE

Table 1
Sample Period and Contract Months for LIFFE Futures

<i>Series</i>	<i>Months Used</i>	<i>Contract Dates</i>	<i>No. of Obs.</i>
<i>Stock Index Contracts</i>			
FTSE100	Mar, Jun, Sep, Dec	Jun 1984 Dec 1997	3,413
FTSE250	Mar, Jun, Sep, Dec	Jun 1994 Dec 1997	942
<i>Interest Rate Contracts</i>			
Sterling	Mar, Jun, Sep, Dec	Dec 1982 Dec 1997	3,790
EuroSwiss	Mar, Jun, Sep, Dec	Mar 1991 Dec 1997	1,705
<i>Bond Contracts</i>			
German Bund	Mar, Jun, Sep, Dec	Dec 1988 Dec 1997	2,272
UK Long Gilt	Mar, Jun, Sep, Dec	Dec 1982 Dec 1997	3,790

The futures series are developed using consecutive returns for each nearest contract, and upto the last trading day prior to the delivery month. The linkage method is consistent; and the rollover return incorporates the first day's price from the new contract, with the last day's price of the old contract. This approach minimises differences to maturity, maximises the number of observations used and has been commonly applied (Taylor, 1986). Studies have also used a similar system with differing rollover times (Venkateswaran et al., 1993). Finally, it should be noted that the return data, hereafter known as the raw returns, was computed as the first difference of the natural logarithms of all daily closing prices.

5. EMPIRICAL FINDINGS

(i) Summary Statistics

Summary statistics are shown in Table 2 where details of the first four moments of the data, plus the Jarque-Bera normality test are presented.

INSERT TABLE 2 HERE

Table 2
Summary Statistics of LIFFE Contracts^a

<i>Series</i>	<i>Mean %</i>	<i>Max %</i>	<i>Min %</i>	<i>Standard Deviation%</i>	<i>Skewness^a</i>	<i>Kurtosis</i>	<i>Jarque-BeraTest</i>
<i>Stock Index Contracts</i>							
FTSE100	4.55x10 ⁻² *	8.09	-16.72	1.07	-1.43*	22.66*	73,135.6*
FTSE250	2.04x10 ⁻² *	2.89	-5.33	5.50x10 ⁻¹	-1.17*	11.31*	5,007.0*
<i>Interest Rate Contracts</i>							
Sterling	6.11x10 ⁻⁴ *	2.92	-1.63	1.55x10 ⁻¹	2.56*	53.90*	474,266.0*
EuroSwiss	5.67x10 ⁻² *	8.43	-15.40	1.19	-1.65*	22.21*	35,038.7*
<i>Bond Contracts</i>							
German Bund	3.75x10 ⁻³ *	2.30	-3.74	3.62x10 ⁻¹	-0.76*	8.28*	6,685.4*
UK Long Gilt	4.56x10 ⁻³ *	3.94	-2.26	6.75x10 ⁻¹	-9.92*	336.54*	1785,280.0*

Notes:

(a) A normal distribution has a skewness and kurtosis coefficient of zero and three respectfully.

* represents significant at five percent level.

The mean values indicate positive returns for all futures contracts with the EuroSwiss interest rate contract offering the most favourable returns. A common feature of stock returns is the existence of a risk premium required to attract investments in risky projects. For stocks, a risk premium incorporates a measure of the risk free rate prevalent at the time. In essence, the premium is a measure of the expected return for an investment, given the prevalent risk free rate.

As there is no initial financial layout required for investment in futures, the risk premium is the expected return, not affected by the risk free rate. Taylor (1986) suggests using the *t*-statistic based on the mean value as a test for the existence of a risk premium for future series. The test of no premium against its alternative, a positive premium, compares the computed *t*-statistic with critical values. The results are robust, given the sample size; and the hypothesis that there is no risk premium is not accepted at traditional significance levels.

The risk in a futures contract is usually measured by its standard deviation. It describes the rate at which price changes occur for a series of data. Changing risk has been noted as an important

factor in the distributional findings for speculative price series (Anderson and Bollerslev, 1997). The risk findings are quite similar for the contracts analysed with the EuroSwiss futures having a slightly higher standard deviation than the others. The findings indicate that the risk measure is reasonably in line with other futures series (see for example, Yang and Brorsen, 1993). Combining the risk and return measures, the FTSE100 and EuroSwiss series' are characterised as having a higher return and risk relative to the other series.

The issue of normality is addressed by examining both the skewness and kurtosis coefficients. While there is no clear consensus regarding the sign of the skewness coefficient, generally the magnitude is small and close to zero for speculative prices. The results in this paper contradicts the hypothesis that the data is symmetric based on the test statistic ($t = \beta / (SE \beta)$).² Evidence of the most extreme values are shown by the maximum and minimum coefficients in Table 2. All futures series' exhibit negative skewness with the exception of the Sterling contract. This lack of asymmetry is incorporated in the stability under additions procedure when the kurtosis coefficient is examined interdependent on the skewness findings.

The general conclusion of leptokurtosis is accepted as significant kurtosis for all the future contracts.³ The test statistic ($t = \alpha / (SE \alpha)$) examines whether the kurtosis value is in line with a normal distribution or not. The findings suggest that the returns are leptokurtotic, with a consequent characteristic of too many observations being bunched too close as well as too far from the mean. The Long Gilt contract exhibits the most pronounced leptokurtosis. As all contracts exhibit significant kurtosis the assumption of normality is contradicted. The normality question is formally resolved by calculating a Jarque-Bera statistic. This statistic is a joint test of skewness and kurtosis which follows a chi-square distribution. The findings indicate that all

futures contracts are not normally distributed at conventional levels, supporting previous studies on the distributional properties of futures series.

(ii) Stability under Additions Findings

A key focus of this paper is testing distributional characteristics of futures traded on LIFFE. The test technique applied is the stability under additions test. Using this methodology, the futures data is tested under three headings. First, compounded returns are examined. This is followed by returns which are filtered for anomalies and non-synchronous data effects. Finally, the filtered returns are standardized using the measure of the conditional variance, obtained by a GARCH-M model.

Values of the characteristic exponent for raw compounded returns are shown in Table 3. Again, the null hypothesis of daily futures returns belonging to a normal distribution is rejected conclusively after using the stability under additions test, as all of the α coefficients have values significantly different from 2. This has obvious implications for methodologies that assume normality. A case in point is many of the models that underlie Value at Risk (VaR) measures. If these models assume a normal distribution, when the true distribution exhibits fat tails, this implies the true risk is understated. Otherwise, no uniform pattern emerges for all the series, with only individual futures contracts showing distinct formations. For instance, characteristic exponent findings for both bond contracts, the German Bund and UK Long Gilt, indicate that the underlying distribution characterising the data is a mixtures of stable distribution. A similar conclusion can be made for the EuroSwiss short term interest rate and FTSE100 stock index contracts. As the α coefficient values increase across the sums of observations, and are insignificantly different from 2 for monthly returns, this would suggest that these data sets can be

characterised by a distribution with constant mean and varying scale or skewness. In contrast, by comparing the individual sums of observations with the average value (last column) a different pattern emerges for the Sterling and FTSE250 contracts. These data sets show the attributes of a stable distribution with the characteristic exponent remaining constant across the sums of observations at conventional significance levels.

INSERT TABLE 3 HERE

Table 3
Estimates of the Characteristic Exponent for Raw Returns

<i>Series</i>	<i>n = 1</i>	<i>n = 2</i>	<i>n = 5</i>	<i>n = 10</i>	<i>n = 15</i>	<i>n = 20</i>	<i>Avg. n^a</i>
<i>Stock Index Contracts</i>							
FTSE100	1.85 (0.06)	1.80 (0.05)	1.64 (0.04)	1.94* (0.06)	1.91* (0.06)	2.00* (0.07)	1.86 (0.06)
FTSE250	1.44 (0.06)	1.63 (0.08)	1.55 (0.07)	1.85* (0.11)	1.53 (0.07)	1.64 (0.08)	1.61 (0.08)
<i>Interest Rate Contracts</i>							
Sterling	1.25 (0.03)	1.27 (0.03)	1.29 (0.03)	1.16 (0.03)	1.33 (0.03)	1.29 (0.03)	1.27 (0.03)
EuroSwiss	1.78 (0.07)	1.81 (0.08)	1.78 (0.07)	1.99* (0.10)	2.00* (0.10)	2.00* (0.10)	1.89* (0.09)
<i>Bond Contracts</i>							
German Bund	1.64 (0.05)	1.58 (0.05)	1.64 (0.05)	1.90* (0.07)	1.86 (0.07)	2.00* (0.08)	1.77 (0.06)
UK Long Gilt	1.66 (0.04)	1.65 (0.04)	1.89* (0.06)	1.94* (0.06)	1.87 (0.06)	2.00* (0.07)	1.84 (0.06)

Notes:

(a) Avg. n represents the average α value for the different sums ($n = 1, 2$ etc) examined.

* These values are insignificantly different from 2 at the five percent significance level.

In order to determine whether the stable paretian family, and in particular, the mixtures of stables is not only the most applicable classification, but also, and of more importance, whether futures series' actually follow this distribution, this issue is investigated further. The stability under additions methodology is also applied to the futures contracts after being filtered.⁴ Thus, trading

and seasonality effects are thereby removed from the compounded returns, and the characteristic exponent findings are shown in Table 4. Kurtosis values remain relatively unchanged from the raw to the filtered returns, with two exceptions. First, the FTSE250 series has a coefficient that is insignificantly different from 2 for monthly data and is now classified by the mixtures of stables distribution. Secondly, the UK Long Gilt bond contract displays relatively constant values corresponding to the stable distribution. These changes indicate a benefit of filtering futures returns as the distributional conclusions are more specific after applying a GARCH-M smoothing process, that is, the importance of the influence of data anomalies is made clear. Again the results here emphasise the importance of taking account of true non-normal distribution in for example, the computation of VaR measures.

INSERT TABLE 4 HERE

Table 4

Estimates of the Characteristic Exponent for Filtered Returns

<i>Series</i>	<i>n = 1</i>	<i>n = 2</i>	<i>n = 5</i>	<i>n = 10</i>	<i>n = 15</i>	<i>n = 20</i>	<i>Avg. n^a</i>
<i>Stock Index Contracts</i>							
FTSE100	1.82 (0.05)	1.83 (0.05)	1.71 (0.05)	1.92* (0.06)	1.94* (0.06)	2.00* (0.07)	1.87 (0.06)
FTSE250	1.52 (0.07)	1.80 (0.10)	1.68 (0.08)	1.85* (0.11)	1.63 (0.08)	1.95* (0.12)	1.74 (0.06)
<i>Interest Rate Contracts</i>							
Sterling	1.31 (0.03)	1.25 (0.03)	1.25 (0.03)	1.16 (0.03)	1.34 (0.03)	1.32 (0.03)	1.27 (0.03)
EuroSwiss	1.72 (0.07)	1.86* (0.08)	1.78 (0.07)	2.00* (0.10)	2.00* (0.10)	2.00* (0.10)	1.89* (0.09)
<i>Bond Contracts</i>							
German Bund	1.66 (0.05)	1.64 (0.05)	1.61 (0.05)	1.87* (0.07)	1.90* (0.07)	2.00* (0.08)	1.78 (0.06)
UK Long Gilt	1.67 (0.04)	1.68 (0.04)	1.81 (0.05)	1.95* (0.06)	1.74 (0.05)	1.69 (0.04)	1.76 (0.05)

Notes:

(a) Avg. n represents the average α value for the different sums ($n = 1, 2$ etc) examined.

* These values are insignificantly different from 2 at the five percent significance level.

After standardising the filtered returns using estimates of the conditional variance given by the GARCH-M model, the related characteristic exponents are calculated. The findings here determine how well the time varying GARCH-M model is able to remove the non-normal attributes of the futures contracts. Kurtosis values are given in Table 5. While there is a trend for the rescaled α coefficients to be higher than those of the raw returns, there still is a rejection of normality for daily rescaled returns. The conclusion is that three contracts have an α value that tends to increase towards 2 at the five percent significance level, indicating a mixtures of stables distribution; and three contracts have a relatively constant characteristic exponent, thereby representing support for the stable hypothesis. The examples of the stable distribution are the Sterling interest rate and the two bond contracts.

INSERT TABLE 5 HERE

Table 5
Estimates of the Characteristic Exponent for Rescaled Returns

<i>Series</i>	<i>n = 1</i>	<i>n = 2</i>	<i>n = 5</i>	<i>n = 10</i>	<i>n = 15</i>	<i>n = 20</i>	<i>Avg. n^a</i>
<i>Stock Index Contracts</i>							
FTSE100	1.81 (0.05)	1.81 (0.05)	1.68 (0.04)	1.83 (0.05)	2.00* (0.07)	1.92* (0.06)	1.84 (0.05)
FTSE250	1.51 (0.07)	1.89* (0.11)	2.00* (0.13)	2.00* (0.13)	2.00* (0.13)	2.00* (0.13)	1.90* (0.12)
<i>Interest Rate Contracts</i>							
Sterling	1.26 (0.03)	1.27 (0.03)	1.39 (0.03)	1.47 (0.03)	1.49 (0.03)	1.65 (0.03)	1.42 (0.03)
EuroSwiss	1.65 (0.06)	1.85* (0.08)	2.00* (0.10)	2.00* (0.10)	2.00* (0.10)	1.95* (0.09)	1.91* (0.09)
<i>Bond Contracts</i>							
German Bund	1.54 (0.04)	1.58 (0.05)	1.68 (0.05)	2.00* (0.08)	1.69 (0.05)	1.81 (0.07)	1.72 (0.06)
UK Long Gilt	1.57 (0.04)	1.63 (0.04)	1.76 (0.05)	1.92* (0.06)	1.78 (0.05)	1.84 (0.05)	1.75 (0.05)

Notes:

(a) Avg. n represents the average α value for the different sums ($n = 1, 2$ etc) examined.

* These values are insignificantly different from 2 at the five percent significance level.

Also, the rescaled findings can be used to examine the impact of the filtering process, and its measure of conditional variance. By comparing characteristic exponent findings for the three sets of returns, many of the changes that are caused by filtering remain in place after rescaling. Thus, these new findings for the specific contracts generally concur with the classifications indicated for the filtered returns series'. In fact, some findings are even more pronounced, for example, the characteristic exponent of the FTSE250 contract reaches 2 across the sums more quickly, thereby

following a mixtures of stable distribution.

(iii) Autocorrelation Findings

Characteristic exponent findings indicate a lack of normality even after rescaling futures returns. Reasoning behind these conclusions can be given by detailing levels of serial correlation exhibited in the futures returns. In particular, futures data are non-normal due to dependency in the first moments (Hall et al., 1989), and if a randomising procedure is followed, the resulting kurtosis coefficient equals 2. Another reason suggested for the leptokurtosis in futures data is the high levels of serial correlation in the variance of returns (Yang and Brorsen, 1993). Volatility clustering results from this dependency and this can be shown by a fat tailed distribution.

To determine whether LIFFE's futures exhibit any of these dependency characteristics, the levels of autocorrelation between returns is assessed. First moment dependence is examined by focusing on the level of autocorrelation in futures returns, whereas we can also determine whether serial correlation in the second moments occurs by examining the squared returns. Daily dependence is calculated for the futures series on an individual and aggregate basis. Individually, sample correlations for 30 lags are given using the following:

$$r_{\Gamma,x} = \frac{\sum_{t=1}^{n-\Gamma} (X_t - \bar{X})(X_{t+\Gamma} - \bar{X})}{\sum_{t=1}^n (X_t - \bar{X})^2} \quad \Gamma > 0$$

(8)

At the aggregate level (30 lags), the Ljung-Box Portmanteau Q -statistic is used to formally test

the hypothesis that all calculated autocorrelations are zero and the series is white noise, and is given as follows :

$$Q = T(T+2) \sum_{k=1}^n r_k^2 / (T-k) \quad (9)$$

The autocorrelation findings are given in Table 6. Individual autocorrelation coefficient results support the previous work on futures prices where serial correlation of the first moments is concluded, although at a very low level. The findings indicate very low levels of positive and negative dependence within the original futures series with no individual value exceeding 0.08. Only the German Bund contract has a negative first lag, whereas the first lags are positive for the other series'.

Not with standing the small absolute values of the autocorrelation coefficients, 27 of the lags are significantly different from zero at conventional levels. Also, all futures series have significant autocorrelations as shown by the Ljung-Box coefficients. Thus futures returns contradict the hypothesis that they follow a white noise process. The rescaling process removes much, but not all of the first order dependence as the Ljung-Box coefficients are much smaller for the rescaled series.⁵ Dependence is removed in the two stock indexes and the UK Long Gilt contract.

Table 6
Autocorrelation Values for LIFFE contracts^{a, b}

	$\{Y_t\}$	$\{Y_t^2\}$	$\{R_t\}$	$\{R_t^2\}$
<i>Stock Index Contracts</i>				
FTSE100	35.80*	1,730.00*	20.40	34.00*
FTSE250	51.60*	113.00*	25.80	38.80*
<i>Interest Rate Contracts</i>				
Sterling	62.20*	1,680.00*	39.60*	481.00*
EuroSwiss	31.50*	1,090.00*	28.10*	56.10*
<i>Bond Contracts</i>				
German Bund	63.70*	991.00*	41.00*	297.00*
UK Long Gilt	32.20*	266.00*	23.30	360.00*
Nos. $r_{F,x}$ Significant ^c	27	150	8	59

Notes:

(a) Y and R represent raw compounded and rescaled returns respectively.

(b) Standard errors may be underestimated in these cases due to non-normality of returns but the magnitude of the findings suggest they are robust.

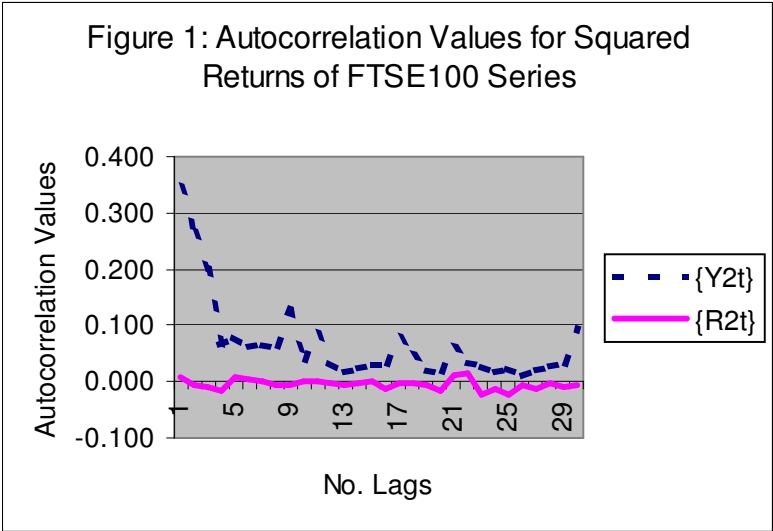
(c) Significant values are summed together for all futures contracts.

* represents the Ljung-Box Portmanteau Q -statistic being significant at the five percent level.

INSERT TABLE 6 HERE

Findings for the dependence in the second moments are also shown in table 6 after examining autocorrelation levels for squared returns. The findings conclusively support strong ARCH effects as all the futures contracts exhibit significant serial correlation, and nearly all of the individual lags indicate strong dependence (that is, 150 out of 180 lags are significant). Dependence in squared returns actually dominate the ordinary values indicating stronger serial correlation in the variance in comparison to the mean. However, much of this second moment serial correlation is removed by rescaling. This is verified by the reduction in the number of significant autocorrelation values after removing the influence of anomalies through filtering. In general, the filtering process removes quite a large proportion of the serial dependence of the futures series. This removal of second moment dependence can be more clearly seen in figure 1 which shows the levels of autocorrelation for the FTSE100 index before (Y_{2t} - squared raw

return series) and after (R^2_t - squared rescaled return series) rescaling. In Figure 1, the high serial correlation disappears after rescaling the futures returns.



INSERT FIGURE 1 HERE

6. CONCLUSION

This study examines the distributional characteristics of futures contracts traded on LIFFE. Contracts from the equity, bond and short term interest rate markets are analysed. The paper uses a stability under additions test to determine kurtosis values, allowing the moments of a distributions to be interdependent on each other. The key to the procedure is calculating the characteristic exponent of a stable model, which can take on distinct sets of values, thereby determining the underlying distribution of the data analysed. The futures series' are tested in three forms after applying a filtering process to remove data anomalies, as well as giving estimates of the conditional variance. These are filtered, rescaled (obtained from standardising the filtered returns) and raw compounded returns. An analysis of the kurtosis results is given determining the influence of dependency in both the mean and variance of a distribution.

The results conclusively reject the hypothesis that futures returns are normally distributed. Significant skewness and leptokurtosis is found for all contracts chosen. We find in favour of two related hypotheses for different contracts. Generally, a mixtures of stable distribution is supported for the data; with exceptions concurring with an ordinary stable distribution. These hypotheses assume that time varying parameters such as skewness and scale exist. ARCH effects do influence the kurtosis findings, whereas dependency in the first moments is comparatively

negligible. Also, the filtering technique, incorporating a GARCH-M specification is able to remove significant serial dependence in the variance for three of the contracts analysed.

Given the growth of futures trading, and their importance as a financial instrument, it is important that their practical applications work effectively. As indicated earlier, an area that has attracted attention recently is the use of Value at Risk (VaR) measures as a risk management tool. The popularity of this risk measurement method has been in no small way encouraged by the Riskmetrics® service provided by J.P. Morgan. VaR measures provide investors with single measures of their downside risk exposure on various investment stances. Also, a main area in the academic literature that has attracted attention is hedging risk effectiveness. Much of this literature has examined different hedging models on a comparative basis. Both areas tend to make assumptions about key methodological issues. For example, many VaR models and traditional risk minimisation econometric models have as its main assumption the idea that futures returns are normally distributed. However, this paper finds a lack of normality in the futures contracts selected, and as such, suggests that both academics and practitioners alike be aware of possible distortions in past work. As we have suggested VaR models assuming normal kurtosis findings underestimate risk measures for leptokurtotic data. For VaR models, development of measures that reasonably approximate futures' distributional characteristics would be beneficial. Also, it may not be possible to remove all the non-normal characteristics of futures returns, and hedging techniques that incorporate leptokurtosis and allow moment parameters to vary, should be the basis of future empirical work in this area.

NOTES

1 Different order specifications of the conditional heteroscedastic model are examined (for example, the GARCH-M (2,1) and GARCH-M(1,2) amongst others. It is found that the GARCH-M (1, 1), best characterises the behaviour of futures data.

2 Positive skewness generates a positive coefficient, and asymmetry a value of zero.

3 A normal distribution has a kurtosis value of 3, leptokurtosis values greater than 3 and platykurtosis less than 3.

4 UK Long Gilt returns are filtered by an ARCH-M (1) process as different versions of GARCH-M models are misspecified.

5 Filtered autocorrelation values are very similar to the rescaled returns and thereby not included in the analysis.

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