Simultaneous Uplink/Downlink Transmission Using Full–Duplex Single–RF MIMO

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***Abstract*—In this Letter, we introduce a full–duplex protocol for simultaneous transmission between the uplink and the downlink of cellular networks. The protocol takes advantage of the inactive antenna(s) in Multiple–Input–Multiple–Output (MIMO) systems with a single active Radio Frequency (RF) front–end. More precisely, for the downlink transmissions, we make use of Spatial Modulation (SM), and for the uplink, we make use of the Coordinate–Interleaved Orthogonal Design (CIOD) based Space–Time Block Code (STBC). We provide accurate mathematical expressions for evaluating the error–performances and the achievable diversity order at the Base Station (BS) and at the Mobile Terminal (MT) in the presence of self–interference. Our results demonstrate clearly the potential of SM and CIOD for full–duplex operation.**

***Index Terms*—Full–Duplex, Single–RF MIMO, Performance Analysis.**

I. Introduction

SINGLE-radio-frequency (RF) based Multiple–Input–Multiple–Output (MIMO) implementations are receiving considerable attention among the research community due to their ability to harvest the benefits of MIMO systems using a single active RF chain [1]. Two such single–RF MIMO schemes are Spatial Modulation (SM) [2] and Coordinate–Interleaved Orthogonal Design (CIOD)[1] based Space–Time Block Code (STBC) [3]. In SM and in CIOD, only a single transmit antenna is required to be active in any given time–slot. This leads to high energy savings at the transmitter side.

In SM, one part of the data to be transmitted in each channel use is encoded onto the conventional signal-constellation diagram, and the other part is encoded onto the transmit-antenna being activated. This encoding process allows SM to achieve a higher spectral efficiency compared to conventional single–antenna systems. In general, SM requires a large number of transmit antenna elements (with only one of them active) to be effective from a spectral–efficiency point–of–view. Hence, SM is particularly useful for the downlink of cellular systems. For the uplink, however, single–RF CIOD constructed using two transmit antenna elements has more potential. This is mainly because CIOD provides a full–rate and a second–order diversity when the transmit signal set is constructed to have a non–zero Coordinate Product Distance (CPD).

In spite of common belief, recent results have highlighted the feasibility of full–duplex operation in wireless networks [4]. This progress can be mainly attributed to the recent advances in self–interference cancellation techniques. Motivated by these results, more recently, some attempts to combine MIMO with full–duplex communication systems have been made [5], [6]. In this paper, we propose a full–duplex wireless protocol for simultaneous transmission between a Base Station (BS) and a single Mobile Terminal (MT). More specifically, due to the previously mentioned considerations, SM is used for downlink transmission, and CIOD is used for uplink transmission. Furthermore, we exploit the inactive antenna(s) in SM and CIOD systems for reception, while they transmit from one of the scheduled antennas. This way, we make the best use of the limited antenna resources by adaptively using them for the transmission and the reception of data. We provide accurate mathematical expressions for evaluating the error–performances at the BS and at the MT in the presence of self–interference. Our results clearly demonstrate the potential of SM and CIOD for full–duplex operation.

Notation: $\mathcal{P}\{\cdot\}$ denotes probability. $(\cdot)^*$ and $|\cdot|$ denote complex conjugate and absolute value operators, respectively. $\mathbb{E}_X\{\cdot\}$ denotes the expectation computed with respect to the Random Variable (RV) X. $j=\sqrt{-1}$ denotes the imaginary unit. $Q(x)=\left(1/\sqrt{2\pi}\right)\int_x^{+\infty}\exp\left(-t^2/2\right)dt$ denotes the Q-function. $\Gamma(x)=\int\limits_0^{\infty}t^{x-1}\exp(-t)dt$ is the Gamma function. $\mathrm{card}\{\cdot\}$ denotes the cardinality of a set. x_I and x_Q denote the real and imaginary part of a complex number x.

II. System Model

A. System Setup

A single–cell cellular network deployment is considered, where the BS communicates with a single MT in the full–duplex mode, i.e., at the same time on the same frequency. The BS and the MT are equipped with N_{BS} and N_{MT} antennas, respectively. Due to practical constraints, N_{MT} is usually small compared to N_{BS}. Due to the reasons outlined in Section I, the data from the BS to the MT is transmitted by using SM, and the data from the MT to the BS is transmitted by using CIOD. At the MT, a fixed pair of antennas is used for CIOD transmission; any extra antennas are simply used to enhance the reception quality.

In the conventional SM and CIOD systems, only one of the antennas is active in any given time–slot. During this time–slot, the remaining antenna(s) that are idle for transmission, are switched off in order to reduce the circuit power consumption. In the transmission protocol proposed in this paper, the idle antenna(s) ($\overline{N}_{\mathrm{BS}}=N_{\mathrm{BS}}-1$ antennas at the BS and $\overline{N}_{\mathrm{MT}}=N_{\mathrm{MT}}-1$ antennas at the MT) are not switched off.

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[1]Throughout this paper, we consider the CIOD scheme with only two antennas at the transmitter.

Instead, they are switched on in the receive mode. During the next time–slot, the antennas that are in the transmit and in the receive mode may change. Thus, the same antennas are adaptively used for the transmission and the reception of the data. However, this simultaneous transmission and reception results in self–interference, which is an inherent drawback of full–duplex wireless systems. Several techniques for self–interference cancellation are available in the literature. In this work, however, we are only interested in the quality of the employed self–interference cancellation technique, and not on any specific category of cancellation technique. As we will see later, the quality of the self–interference cancellation is explicitly taken into account in our analysis.

B. Channel Model

a) The useful channels are modelled as frequency–flat Rayleigh fading channels. Let b' and m' be the indices of the active antennas at the BS and the MT, respectively, in the current time–slot which we denote as TS1. Similarly, b'' and m'' be the indices of the active antennas at the BS and the MT, respectively, in the next time–slot which we denote as TS2. We denote by $h_{b'm}$ the channel impulse response of the wireless link from the b'–th antenna of the BS to the m–th antenna of the MT. Similarly, $h_{m'b}$ is the channel impulse response of the wireless link from the m'–th antenna of the MT to the b–th antenna of the BS. These channels are assumed to be independent complex Gaussian RVs with zero mean and variance σ_u^2 per dimension.

b) The self–interference channel from the b'–th antenna to the b–th antenna of the BS, with $b' \neq b$, is denoted as $h_{b'b}$. Similarly, $h_{m'm}$, with $m' \neq m$, is the self–interference channel from the m'–th antenna to the m–th antenna of the MT. These channels are also modelled as Rayleigh fading, assuming that the self–interference cancellation scheme is capable of completely removing the line–of–sight component [5]. Furthermore, the different self–interference channels at the node X (for X $=$ BS and X $=$ MT) has zero mean, and the variance (per dimension) is $\sigma_{\mathrm{X}}^2 = (1/2)\left(E_{\mathrm{X}}/N_0\right)^{-\lambda_{\mathrm{X}}}$, with E_{X} being the node X's average transmit energy per symbol, and λ_{X} being a small positive constant, which captures the quality of the self–interference cancellation technique [5], [7]. For instance, $\lambda_{\mathrm{X}} = 0$ refers to a poor self–interference cancellation process, and $\lambda_{\mathrm{X}} = 1$ refers to a high–quality cancellation process. As will become apparent from Section IV, the value of λ_{X} plays a crucial role in determining the performance of the proposed full–duplex wireless system.

C. Maximum–Likelihood (ML) Detection

Let $x_{1,\mathrm{BS}} \in \mathcal{A}_{\mathrm{BS}}$ and $x_{2,\mathrm{BS}} \in \mathcal{A}_{\mathrm{BS}}$ be the Phase Shift Keying (PSK) modulated symbols transmitted by the BS in TS1 and TS2, respectively. Similarly, let $x_{1,\mathrm{MT}} \in \mathcal{A}_{\mathrm{MT}}$ and $x_{2,\mathrm{MT}} \in \mathcal{A}_{\mathrm{MT}}$ be the rotated square Quadrature Amplitude Modulated (QAM) symbols of the MT, before coordinate interleaving, during TS1 and TS2. PSK is employed from the downlink transmission as it can outperform QAM when applied to SM systems [2]. Rotated square QAM is employed for the uplink transmission following similar considerations as in [3]. Also, let $\mathrm{card}\left\{\mathcal{A}_{\mathrm{BS}}\right\} = M_{\mathrm{BS}}$ and $\mathrm{card}\left\{\mathcal{A}_{\mathrm{MT}}\right\} = M_{\mathrm{MT}}$.

1) At the MT: Without loss of generality, we consider TS1, and we denote for simplicity $x_{1,\mathrm{BS}} = x_{\mathrm{BS}}$. The BS forwards its data to the MT by using SM. Accordingly the signal received at the m–th antenna of the MT, for $m = 1, 2, \ldots, N_{\mathrm{MT}}$ with $m \neq m'$, can be formulated as follows:

$$y_m = \underbrace{\sqrt{E_{\mathrm{BS}}} h_{b'm} x_{\mathrm{BS}}}_{\text{desired signal}} + \underbrace{\sqrt{E_{\mathrm{MT}}} h_{m'm}\left(x_{1I,\mathrm{MT}} + j x_{2Q,\mathrm{MT}}\right)}_{\text{self}-\text{interference}} + \underbrace{n_m}_{\text{AWGN}} \tag{1}$$

The conventional ML detector for SM is used in order to demodulate the received signal in (1). The ML detector can be formulated as follows:

$$\begin{cases} \left[\widehat{x}_{\mathrm{BS}}^{(\mathrm{MT})}, \widehat{b'}^{(\mathrm{MT})}\right] = \underset{\substack{\widetilde{x}_{\mathrm{BS}} \in \mathcal{A}_{\mathrm{BS}} \\ \widetilde{b'} \in 1,2,..N_{\mathrm{BS}}}}{\arg\min} \left\{\Lambda^{(\mathrm{MT})}\left(\widetilde{x}_{\mathrm{BS}}, \widetilde{b'}\right)\right\} \\ \Lambda^{(\mathrm{MT})}\left(\widetilde{x}_{\mathrm{BS}}, \widetilde{b'}\right) = \sum\limits_{m \in \Omega_{\mathrm{MT}}^{(\mathrm{Rx})}} \left| y_m - \sqrt{E_{\mathrm{BS}}} h_{\widetilde{b'}m} \widetilde{x}_{\mathrm{BS}} \right|^2 \end{cases} \tag{2}$$

where: i) $\widehat{x}_{\mathrm{BS}}^{(\mathrm{MT})}$ and $\widehat{b'}_{\mathrm{MT}}$ are the estimated symbol and the estimated active antenna index at the MT of the data transmitted from the BS; and ii) $\Omega_{\mathrm{MT}}^{(\mathrm{Rx})} = \{m = 1, 2, ..., N_{\mathrm{MT}}, m \neq m'\}$ is the set of all antennas that are in the receive mode at the MT.

2) At the BS: The MT transmits its data to the BS by using CIOD. Accordingly, the signal received at the b–th antenna of the BS in two consecutive time–slots can be formulated as follows:

$$\begin{cases} y_b^{(1)} = \underbrace{\sqrt{E_{\mathrm{MT}}} h_{m'b}\left(x_{1I,\mathrm{MT}} + j x_{2Q,\mathrm{MT}}\right)}_{\text{desired signal at TS1}} + \underbrace{\sqrt{E_{\mathrm{BS}}} h_{b'b} x_{1,\mathrm{BS}}}_{\text{self}-\text{interference at TS1}} \\ \qquad + \underbrace{n_b^{(1)}}_{\text{AWGN at TS1}} \quad ; \quad \text{for } b \in \Omega_{\mathrm{BS}}^{(\mathrm{Rx},1)} \\ y_b^{(2)} = \underbrace{\sqrt{E_{\mathrm{MT}}} h_{m''b}\left(x_{2I,\mathrm{MT}} + j x_{1Q,\mathrm{MT}}\right)}_{\text{desired signal at TS2}} + \underbrace{\sqrt{E_{\mathrm{BS}}} h_{b''b} x_{2,\mathrm{BS}}}_{\text{self}-\text{interference at TS2}} \\ \qquad + \underbrace{n_b^{(2)}}_{\text{AWGN at TS2}} \quad ; \quad \text{for } b \in \Omega_{\mathrm{BS}}^{(\mathrm{Rx},2)} \end{cases} \tag{3}$$

where $\Omega_{\mathrm{BS}}^{(\mathrm{Rx},1)} = \{1, 2, .., N_{\mathrm{BS}}; b \neq b'\}$ and $\Omega_{\mathrm{BS}}^{(\mathrm{Rx},2)} = \{1, 2, .., N_{\mathrm{BS}}; b \neq b''\}$ are the set of antennas at the BS that are in the receive mode during TS1 and TS2, respectively.

The ML detector at the BS can be formulated as follows:

$$\begin{cases} \left[\widehat{x}_{1,\mathrm{MT}}^{(\mathrm{BS})}, \widehat{x}_{2,\mathrm{MT}}^{(\mathrm{BS})}\right] = \underset{\substack{\widetilde{x}_{1,\mathrm{MT}} \in \mathcal{A}_{\mathrm{MT}} \\ \widetilde{x}_{2,\mathrm{MT}} \in \mathcal{A}_{\mathrm{MT}}}}{\arg\min} \left\{\Lambda^{(\mathrm{BS})}\left(\widetilde{x}_{1,\mathrm{MT}}, \widetilde{x}_{2,\mathrm{MT}}\right)\right\} \\ \Lambda^{(\mathrm{BS})}\left(\widetilde{x}_{1,\mathrm{MT}}, \widetilde{x}_{2,\mathrm{MT}}\right) \\ \quad = \sum\limits_{b \in \Omega_{\mathrm{BS}}^{(\mathrm{Rx},1)}} \left| y_b^{(1)} - \sqrt{E_{\mathrm{MT}}} h_{m'b}\left(\widetilde{x}_{1I,\mathrm{MT}} + j\widetilde{x}_{2Q,\mathrm{MT}}\right)\right|^2 \\ \quad + \sum\limits_{b \in \Omega_{\mathrm{BS}}^{(\mathrm{Rx},2)}} \left| y_b^{(2)} - \sqrt{E_{\mathrm{MT}}} h_{m''b}\left(\widetilde{x}_{2I,\mathrm{MT}} + j\widetilde{x}_{1Q,\mathrm{MT}}\right)\right|^2 \end{cases} \tag{4}$$

where: i) $\widehat{x}_{1,\mathrm{MT}}^{(\mathrm{BS})}$ and $\widehat{x}_{2,\mathrm{MT}}^{(\mathrm{BS})}$ are the estimates at the BS of the MT symbols $x_{1,\mathrm{MT}}$ and $x_{2,\mathrm{MT}}$, respectively.

III. Performance Analysis

In this section, we first compute the Average Pairwise Error Probability (APEP), and then the Average Symbol Error Probability (ASEP) is obtained from it with the aid of the

Nearest Neighbour (NN) approximation, for both the downlink and the uplink transmissions. In addition, we also characterise their achievable diversity order.

A. Downlink Transmission

1) APEP: Let $\mathbf{x}_{\rm BS} = [x_{\rm BS}, b']$ and $\widetilde{\mathbf{x}}_{\rm BS} = \left[\widetilde{x}_{\rm BS}, \widetilde{b'}\right]$ be the actual transmitted data from the BS and their hypothesis, respectively. Then, the PEP of demodulating $\widetilde{\mathbf{x}}_{\rm BS}$ in lieu of $\mathbf{x}_{\rm BS}$, by assuming that they are the only two SM symbols possibly being transmitted, can be formulated as follows:

$$
\begin{aligned}
&{\rm PEP}\left(\mathbf{x}_{\rm BS} \to \widetilde{\mathbf{x}}_{\rm BS} \left| \mathbf{n}^{\rm (MT)}, \mathbf{h}_{si}^{\rm (MT)}, \mathbf{h}_{u}^{\rm (MT)} \right.\right) \\
&\quad = \mathcal{P}\left\{ \Lambda^{\rm (MT)}\left(\widetilde{\mathbf{x}}_{\rm BS}\right) < \Lambda^{\rm (MT)}\left(\mathbf{x}_{\rm BS}\right) \left| \mathbf{n}^{\rm (MT)}, \mathbf{h}_{si}^{\rm (MT)}, \mathbf{h}_{u}^{\rm (MT)} \right.\right\}
\end{aligned} \tag{5}
$$

where: i) $\mathbf{n}^{\rm (MT)}$ is a short–hand used to denote all the AWGN contributions at the MT; and ii) $\mathbf{h}_{si}^{\rm (MT)}$ and $\mathbf{h}_{u}^{\rm (MT)}$ are the short–hands for all the self–interference channels and the useful channels at the MT, respectively.

In order to compute the APEP, we first condition the PEP in (5) upon $\mathbf{h}_{si}^{\rm (MT)}$ and $\mathbf{h}_{u}^{\rm (MT)}$ and remove the conditioning over $\mathbf{n}^{\rm (MT)}$ and $\mathbf{h}_{si}^{\rm (MT)}$. Subsequently, we remove the conditioning over $\mathbf{h}_{si}^{\rm (MT)}$. Accordingly, from (5), we have:

$$
\begin{aligned}
&{\rm APEP}\left(\mathbf{x}_{\rm BS} \to \widetilde{\mathbf{x}}_{\rm BS}\right) \\
&= \mathbb{E}_{\mathbf{h}_{u}^{\rm (MT)}}\left\{ \mathbb{E}_{\mathbf{n}^{\rm (MT)}, \mathbf{h}_{si}^{\rm (MT)}}\left\{ {\rm PEP}\left(\mathbf{x}_{\rm BS} \to \widetilde{\mathbf{x}}_{\rm BS} \left| \mathbf{n}^{\rm (MT)}, \mathbf{h}_{si}^{\rm (MT)}, \mathbf{h}_{u}^{\rm (MT)} \right.\right)\right\}\right\} \\
&= \mathbb{E}_{\mathbf{h}_{u}^{\rm (MT)}}\left\{ Q\left(\sqrt{ \sum_{m \in \Omega_{\rm MT}^{\rm (Rx)}} \frac{\left|E_{\rm BS}\right|^2 \left| h_{b'm} x_{\rm BS} - h_{\widetilde{b'}m} \widetilde{x}^{\rm (BS)} \right|^2}{2\left(N_0 + 2E_{\rm MT}\sigma_{\rm MT}^2\right)} } \right)\right\}
\end{aligned} \tag{6}
$$

By applying Craig's formula, and with the aid of some algebraic manipulations, (6) can be computed in closed–form as follows:

$$
\begin{aligned}
{\rm APEP}\left(\mathbf{x}_{\rm BS} \to \widetilde{\mathbf{x}}_{\rm BS}\right) &= \frac{1}{\pi} \int_0^{\pi/2} \left(\frac{\sin^2(\theta)}{\sin^2(\theta) + (\beta/2)\overline{\gamma}_{\rm MT}} \right)^{\overline{N}_{\rm MT}} d\theta \\
&\overset{(a)}{=} \alpha^{\overline{N}_{\rm MT}} \sum_{k=0}^{\overline{N}_{\rm MT}} \binom{\overline{N}_{\rm MT} - 1 + k}{k} (1-\alpha)^k
\end{aligned} \tag{7}
$$

where: i) $\overline{\gamma}_{\rm MT} = \left(\frac{2E_{\rm BS}\sigma_u^2}{N_0 + 2E_{\rm MT}\sigma_{\rm MT}^2} \right)$; ii) β is defined as follows:

$$
\beta = \begin{cases} (1/2)\left(\left|x_{\rm BS} - \widetilde{x}_{\rm BS}\right|^2\right) & \text{if} \quad b' = \widetilde{b'} \\ 1 & \text{if} \quad b' \neq \widetilde{b'} \end{cases} \tag{8}
$$

and iii) (a) follows from [8, 5A.58], with $\alpha = (1/2)\left(1 - \sqrt{\frac{(\beta/2)\overline{\gamma}_{\rm MT}}{1 + (\beta/2)\overline{\gamma}_{\rm MT}}}\right)$.

2) ASEP: Based on (7), the ASEP at the MT related to the SM downlink transmission can be formulated using the NN approximation as follows [9]:

$$
{\rm ASEP}_{\rm MT} \approx N_{\Delta_{\rm BS}^{\rm (min)}} {\rm APEP}\left(\Delta_{\rm BS}^{\rm (min)}\right) \tag{9}
$$

where $\Delta_{\rm BS}^{\rm (min)} = \min\limits_{\mathbf{x}_{\rm BS}, \widetilde{\mathbf{x}}_{\rm BS}} \left\{\beta\left(\mathbf{x}_{\rm BS}, \widetilde{\mathbf{x}}_{\rm BS}\right)\right\}$ is the minimum value of $\beta(\cdot)$ among all possible pairs of SM constellation points, and $N_{\Delta_{\rm MT}^{\rm (min)}}$ is the average number of nearest neighbours of $\mathbf{x}_{\rm BS}$ in the SM constellation diagram, *i.e.*, having the metric $\Delta_{\rm BS}^{\rm (min)}$.

3) Diversity Order: By following the arguments in [10], and by using the first–order Taylor expansion, (7) can be approximated as follows:

$$
{\rm APEP}\left(\mathbf{x}_{\rm BS} \to \widetilde{\mathbf{x}}_{\rm BS}\right) \approx \mathcal{G}_b \left(\frac{\kappa_b \left(E_0/N_0\right)}{1 + \kappa_m^{1-\lambda_{\rm MT}} \left(E_0/N_0\right)^{1-\lambda_{\rm MT}}} \right)^{-\overline{N}_{\rm MT}} \tag{10}
$$

where $\mathcal{G}_b = \frac{2^{\overline{N}_{\rm MT}-1} \beta^{-\overline{N}_{\rm MT}} \Gamma\left(0.5 + \overline{N}_{\rm MT}\right)}{\sqrt{\pi}\,\Gamma\left(1 + \overline{N}_{\rm MT}\right)}$, $E_{\rm BS} = \kappa_b E_0$, $E_{\rm MT} = \kappa_m E_0$ and $\sigma_u^2 = 1/2$.

From (10), it is apparent that the diversity order depends heavily on the value of $\lambda_{\rm MT}$, which in turn depends on the quality of the self–interference cancellation. More specifically, we note that the diversity order is equal to $\lambda_{\rm MT}\overline{N}_{\rm MT}$. Therefore, if $\lambda_{\rm MT} \approx 1$, a full–diversity of $\overline{N}_{\rm MT}$, which is the number of antennas that are in the receive mode at the MT, can be achieved. On the other hand, if $\lambda_{\rm MT} \approx 0$, the diversity order is approximately zero.

B. Uplink Transmission

1) APEP: By following a similar approach to that used in Section III-A, the APEP of the symbol $x_{1,\rm MT}$ for the downlink transmission can be formulated as follows:

$$
\begin{aligned}
&{\rm APEP}\left(x_{1,\rm MT} \to \widetilde{x}_{1,\rm MT}\right) \approx \frac{1}{M_{\rm MT}} \sum_{x_{2,\rm MT} \in \mathcal{A}_{\rm MT}} \sum_{\widetilde{x}_{2,\rm MT} \in \mathcal{A}_{\rm MT}} \mathcal{T}_{1,2} \\
&\mathcal{T}_{1,2} = {\rm E}_{\mathbf{h}_u^{\rm (BS)}}\left\{ Q\left(\sqrt{ \frac{\left|E_{\rm MT}\right|^2 \left(\mathcal{D}_1 \sum\limits_{b \in \Omega_{\rm BS}^{\rm (Rx,1)}} \left|h_{m'b}\right|^2 + \mathcal{D}_2 \sum\limits_{b \in \Omega_{\rm BS}^{\rm (Rx,2)}} \left|h_{m''b}\right|^2 \right)}{2\left(N_0 + 2E_{\rm BS}\sigma_{\rm BS}^2\right)} } \right)\right\} \\
&= \frac{1}{\pi} \int_0^{\pi/2} \left(\frac{\sin^2(\theta)}{\sin^2(\theta) + (\mathcal{D}_1/4)\overline{\gamma}_{\rm BS}} \right)^{\overline{N}_{\rm BS}} \left(\frac{\sin^2(\theta)}{\sin^2(\theta) + (\mathcal{D}_2/4)\overline{\gamma}_{\rm BS}} \right)^{\overline{N}_{\rm BS}} d\theta
\end{aligned} \tag{11}
$$

where: i) $\mathcal{D}_1 = \left|d_{1I}\right|^2 + \left|d_{2Q}\right|^2$ and $\mathcal{D}_2 = \left|d_{2I}\right|^2 + \left|d_{1Q}\right|^2$, with $d_{kI} = x_{kI,\rm MT} - \widetilde{x}_{kI,\rm MT}$ and $d_{kQ} = x_{kQ,\rm MT} - \widetilde{x}_{kQ,\rm MT}$, and $k = \{1, 2\}$; and iii) $\overline{\gamma}_{\rm BS} = \frac{2E_{\rm MT}\sigma_u^2}{N_0 + 2E_{\rm BS}\sigma_{\rm BS}^2}$.

Similarly to Section III-A1, if $\mathcal{D}_1 = \mathcal{D}_2$, the integral in (11) can be solved by using [8, 5A.4b]. On the other hand, if $\mathcal{D}_1 \neq \mathcal{D}_2$, the integral can be solved using [8, 5A.58]. These expressions are not explicitly provided here due to space limitations. It is also easy to show that the APEP for $x_{2,\rm MT}$ is given by an analogous expression.

2) ASEP: By using the NN approximation, the ASEP at the BS related to the CIOD uplink transmission can be formulated as follows:

$$
{\rm ASEP}_{\rm BS} \approx N_{\Delta_{\rm MT}^{\rm (min)}} {\rm APEP}\left(\Delta_{\rm MT}^{\rm (min)}\right) \tag{12}
$$

where: i) $\Delta_{\rm MT}^{\rm (min)} = \min\limits_{x_{1,\rm MT}, \widetilde{x}_{1,\rm MT}} \left\{\left|x_{1,\rm MT} - \widetilde{x}_{1,\rm MT}\right|\right\}$ is the minimum Euclidean distance among all pairs $\left(x_{1,\rm MT}, \widetilde{x}_{1,\rm MT}\right)$, with $x_{1,\rm MT} \neq \widetilde{x}_{1,\rm MT}$, of the constellation diagram; and ii) $N_{\Delta_{\rm MT}^{\rm (min)}}$ is the number of nearest neighbours of $x_{1,\rm MT}$.

3) Diversity Order: By following similar considerations as in Section III-A3, (11) can be approximated as follows:

$$
{\rm APEP}\left(x_{1,\rm MT} \to \widetilde{x}_{1,\rm MT}\right) \approx \mathcal{G}_m \left(\frac{\kappa_m \left(E_0/N_0\right)}{1 + \kappa_b^{1-\lambda_{\rm BS}} \left(E_0/N_0\right)^{1-\lambda_{\rm BS}}} \right)^{-2\overline{N}_{\rm BS}} \tag{13}
$$

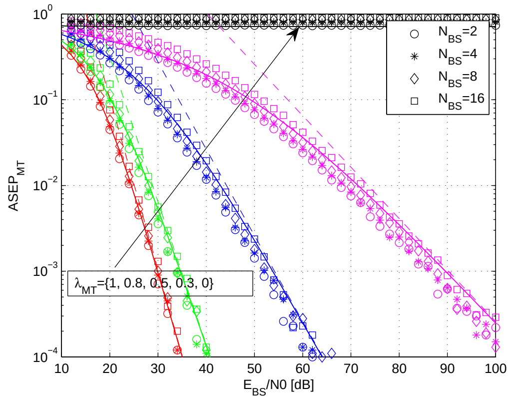

Fig. 1: ASEP at MT related to the SM downlink transmission, with $M_{\rm BS} = 16$ and $\overline{N}_{\rm MT} = 2$. The solid lines are obtained using the expressions (7) and (9), the dashed lines using (10) and (9) (asymptotic), and the markers using Monte Carlo simulations.

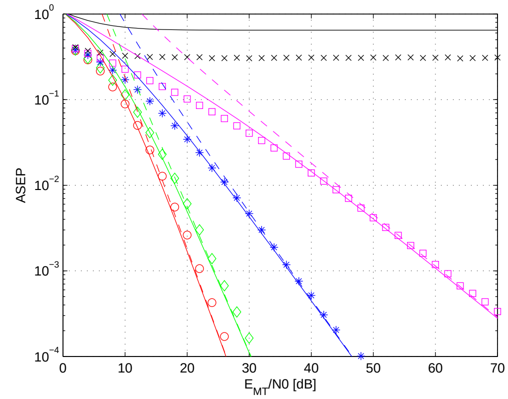

Fig. 2: ASEP at BS related to the CIOD uplink transmission, with $M_{\rm MT} = 4$ and $\overline{N}_{\rm BS} = 1$. The solid lines are obtained using the expressions (11) and (12), the dashed lines using (13) and (12) (asymptotic), and the markers using Monte Carlo simulations.

where $\mathcal{G}_m = \frac{2^{4\overline{N}_{\rm BS}-1}\mathcal{D}_1{}^{-\overline{N}_{\rm BS}}\mathcal{D}_2{}^{-\overline{N}_{\rm BS}}\Gamma\left(0.5+2\overline{N}_{\rm BS}\right)}{\sqrt{\pi}\Gamma\left(1+2\overline{N}_{\rm BS}\right)}$.

From (13), we note that the diversity order of the CIOD uplink transmission is equal to $2\lambda_{\rm BS}\overline{N}_{\rm BS}$. Therefore, for a sufficiently small value of $\lambda_{\rm BS}$ ($\lambda_{\rm BS} \approx 0$), the diversity order is zero, and for a large value of $\lambda_{\rm BS}$ ($\lambda_{\rm BS} \approx 1$), a full–diversity of $2\overline{N}_{\rm BS}$ can be achieved. It should further be noted that these diversity order predictions hold true if and only if $\mathcal{D}_1 \neq 0$ and $\mathcal{D}_2 \neq 0$.

C. Remarks

a) From (7) and (9), it is apparent that the ASEP of the downlink transmission is independent of $N_{\rm BS}$. Since SM is used for the downlink transmission, therefore, its rate can be increased significantly by increasing $N_{\rm BS}$, and more importantly, without much degradation in its error–performance. This makes SM a suitable candidate technology for full–duplex wireless networks, provided several antenna–elements are available at the transmitter.
b) As shown in Section III-B3, for the uplink transmission, CIOD can achieve a full–diversity (for large $\lambda_{\rm BS}$) if and only if $\mathcal{D}_1 \neq 0$ and $\mathcal{D}_1 \neq 0$. This requires rotation of the square QAM constellation. For this purpose, we adopt the value of $\arctan{(2)}/2$, which is the optimal rotation angle for the conventional CIOD.
c) Unlike conventional systems, SM and CIOD systems operating in the full–duplex mode do not require dedicated antenna–elements for reception. Therefore, SM and CIOD based single–RF MIMO systems are implicitly suitable for full–duplex operation. However, appropriate circuit–level designs are required at the BS and at the MT to enable the transmission and reception of data in agreement with the symbol time switching mechanism that is particular to SM and CIOD based transmissions.

IV. Numerical and Simulation Results

In Fig. 1 and Fig. 2, we present some numerical results to assess the performance of the proposed full–duplex wireless protocol, and to substantiate the accuracy of our mathematical framework. For illustrative purposes, we assume $\sigma_u^2 = 1/2$ and $E_{\rm MT} = E_{\rm BS}$. The following important observations can be made: i) The proposed mathematical frameworks are in good agreement with the Monte Carlo simulations; ii) as expected, the ASEP at the MT and at the BS increases with the decrease in $\lambda_{\rm MT}$ and $\lambda_{\rm BS}$, respectively. Furthermore, the diversity orders follow the arguments made in Section III-A3 and in Section III-B3; iii) as suggested by (7) and (9), and as remarked in Section III-C, the $\mathrm{ASEP}_{\rm MT}$ for the SM downlink transmission is almost independent of $N_{\rm BS}$.

V. Conclusion

In this Letter, we proposed a novel full–duplex protocol suitable for simultaneous transmission between the uplink and the downlink. The protocol makes the best use of the limited antenna resources by utilising the inactive antennas in the single–RF SM and CIOD MIMO systems. Furthermore, we provided mathematical expressions which accurately predicts the ASEP and the achievable diversity order at the BS and at the MT. Our mathematical analysis also highlighted certain interesting performance trends when SM and CIOD are used for full–duplex operation.

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