

1 **Inverse Analysis of Deep Excavation using Differential Evolution Algorithm**

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22 **Revision** submitted to: **International Journal for Numerical and Analytical**
23 **Methods in Geomechanics**

24 November 2013

25

26 7000 words, 14 figures, 4 tables

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29 **ABSTRACT**

30 This paper presents the applications of the Differential Evolution (DE) algorithm in
31 back analysis of soil parameters for deep excavation problems. A computer code,
32 named Python-based Differential Evolution (PyDE), is developed and incorporated
33 into the commercial finite element software ABAQUS, with a parallel computing
34 technique to run FE analysis for all trail vectors of one generation in DE in multiple
35 cores of a cluster, which dramatically reduces the computational time. A synthetic case
36 and a well instrumented real case, i.e., the Taipei National Enterprise Center (TNEC)
37 project, are used to demonstrate the capability of the proposed back-analysis
38 procedure. Results show that multiple soil parameters are well identified by
39 back-analysis using DE optimization algorithm for highly nonlinear problems. For the
40 synthetic excavation case, the back-analyzed parameters are basically identical to the
41 input parameters which are used to generate synthetic response of wall deflection. For
42 the TNEC case with totally nine parameters to be back-analyzed, the relative errors of
43 wall deflection for the last three stages are 2.2%, 1.1% and 1.0%, respectively.
44 Robustness of the back-estimated parameters is further illustrated by a forward
45 prediction. The wall deflection in the subsequent stages can be satisfactorily predicted
46 using the back analyzed soil parameters at early stages.

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48 **Keywords:** Excavations, Inverse analysis, Differential Evolution, Optimization
49 Algorithms, Deflection, Cam-clay model

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57 INTRODUCTION

58 In urban area, the construction safety of a deep excavation becomes crucial with the
59 increase of building density. It is critical important to predict and control the ground
60 movement and structure deformation of a deep excavation during the construction
61 procedure. Numerical modeling which is commonly used in the design and
62 construction process [1-3], can provide a better understanding of soil behavior during
63 the construction, verify performance of a highly complex excavation through
64 comparison with field observations and further predict future performance [4-6].
65 However, accurate prediction of deformations due to deep excavation using numerical
66 tools is still complicated for engineers. The accuracy of numerical simulation of deep
67 excavation depends greatly on the selection of constitutive models and determination
68 of appropriate soil parameters [7]. Although comprehensive laboratory and in situ
69 tests can be conducted, there are some difficulties in the precise determination of the
70 soils parameters due to sample disturbance and measurement error [8]. Furthermore,
71 even with well measured soil parameters from laboratory or in situ tests, the estimated
72 performance still deviate from the field observation due to inherent spatial variability
73 [9, 10] and inadequacy of the simulation model itself [11].

74 Detailed observations can provide better understanding of the behavior of a deep
75 excavation and make the design and construction procedure more rational [12-14]. To
76 determine the representative soil parameters based on field observations, back
77 analysis or inverse analysis approaches have been adopted in both deterministic and
78 probabilistic approaches [15-21]. The back analysis methods are basically based on
79 different kinds of optimization techniques, which allows the performance of inverse
80 analyses in a more rational and objective manner. The optimization methods for back
81 analysis can be divided into three groups based on algorithms, i.e., direct methods,
82 gradient based methods [17, 22] and artificial intelligence algorithm based methods
83 [23-24]. Direct methods only need evaluations of the function and do not require
84 gradient calculations of the error function, and therefore are easy to implement. Some

85 commonly-used direct optimization methods are the Univariate method, Pattern
86 Search, the Powell method and the ordinary Simplex method. Gradient-based methods,
87 such as the Steepest Descent, the Conjugate Gradient, the Newton methods, can
88 accelerate the optimization progress but need to evaluate derivatives. It has been
89 reported in the literature that direct and gradient based methods are not sufficient for
90 high dimensional, strongly non-convex or flat error function, conditions with multiple
91 correlated parameters. Among these, Finno and Calvello [17] conducted model
92 calibration for the parameters in the Hardening-Soil (H-S) model using a modified
93 Gauss–Newton method. Tang and Kung [22] applied the quasi-Newton method for
94 back analysis of excavation-induced wall deflection. Zentar et al. [25] combined the
95 steepest decent method with the Levenberg–Marquardt method and identified the
96 Modified Cam-Clay (MCC) parameters based on the pressuremeter test data. They all
97 found that simultaneously back analysis highly correlated parameters may not yield a
98 unique optimized solution. In addition, optimization results by the gradient based
99 method may depend on the choice of initial parameter values [17].

100 The artificial intelligence algorithm based methods, including neural network method,
101 annealing method, swarm intelligence algorithms, evolution strategies, genetic
102 algorithms, etc., are known to be robust and efficient to solve very complex problems
103 and are considered as global optimization methods compared with the local
104 optimization methods such as gradient based methods [26-29]. The applications of
105 artificial intelligence based methods, especially the Darwin’s theory of evolution
106 inspired algorithms such the genetic algorithms and the evolution algorithms, in
107 geotechnical problems, have been increased in recent years. These evolution-inspired
108 algorithms are stochastic global search techniques and are recognized to be highly
109 efficient in dealing with large, discrete, non-linear and poorly understood optimization
110 problems. As the most popular evolution-inspired algorithm applied in geomechanics,
111 the GA algorithm has been employed to solve various problems such as
112 characterization of discontinuity in fractured rocks [30-32], parameter identification

113 of constitutive models [24, 32, 33], identification of critical slip surfaces in slope
114 stability analysis [34-35], optimization of pile group design [36], and reliability
115 analysis [37]. When applied in the back analysis of excavation problems, the GA
116 algorithm is found to be able to identify the best solution of the problem even with a
117 flat or noisy error function [24, 30, 38].

118 There are several drawbacks of the classic GA algorithm by Holland (1982) [39]. First,
119 the parameters are represented by binary strings and the binary strings are decoding
120 into real numbers by interpolation within the ranges of each parameter. This choice
121 limits the resolution with which an optimum can be located to the precision set by the
122 number of bits in the integer. Second, only a portion of parents which have relatively
123 high fitness is selected for reproduction. The selected parents are mixed with their
124 generated children as a secondary generation and use random mutation for diversity.
125 However, there is no guarantee that the best-so-far solution will not be lost, although
126 cross-over and random mutation schemes are used [40].

127 The Differential Evolution (DE) algorithm [41-43], which is similar to GA, emerged
128 as a very competitive form of evolutionary computing in the late 90's. The DE
129 algorithm was sometimes considered as a variant of GA in literature because they are
130 identical with respect to their major optimization process such as selection, mutation
131 and recombination/crossover. However, it exhibit significant differences with respect
132 to the selection scheme, the encoding of the variables and especially the
133 self-adaptation of the parameters. Hence, DE and GA are considered as different
134 classes of evolution computing algorithms [44].

135 The major difference between DE and GA is the difference vector based mutation,
136 which is also the core idea and main strength of DE. DE generates new parameter
137 vectors by adding the scaled difference vector between two population members to a
138 third member (the base vector). While in GAs, mutation is treated as a random change
139 of some parameter. The advantage of the difference vector based mutation is that both
140 the step size and the orientation of a mutation can be automatically adapted to the

141 objective function landscape and hence it promotes basin-to-basin transfer, where
142 search points may move from one local minimum to another one [42, 44].

143 Secondly, most GAs use crossing two vectors to produce two trial vectors, often by
144 one-point crossover, which means randomly selecting a single crossover point such
145 that all parameters to the left of the crossover point are inherited from vector 1, while
146 those to the right are copied from the vector 2 and constructing a second trial vector
147 by reversing the roles of the two vectors. However, DE uses a binary n-point
148 crossover to generate one trial vector. Thirdly, DE and GA are different in selection
149 scheme. In parent selection scheme, unlike GAs that select parents based on their
150 fitness, DE gives all the individuals equal chance for being selected as parents and the
151 base vectors are randomly picked up without any regard for their fitness values. In
152 survivor selection scheme, DE employs a one to one competition where each parent
153 vector competes once only against its own offspring. Parent-offspring competition has
154 a superior ability to maintain population diversity when compared with ranking or
155 tournament selection where elites and their offspring may dominate the population
156 rapidly. DE algorithm also has flexibility to handle different kind of constraints on the
157 unknown parameters with a robust convergence [44]. In addition, DE uses
158 floating-point numbers that are more appropriate than decoded binary strings for
159 representing points in continuous space and uses computer resources efficiently.

160 Certainly, some real-coded GAs have also been developed to handle this problem.

161 Amorim et al. (2012) [45] compared the performance of GA and DE for solving a
162 history mapping inverse problem in oil reservoir engineering. GA performed better
163 than DE on the initial generations, i.e., the objective error function value dropped
164 much faster in the beginning of GA than DE. However, once close to the region of a
165 good solution, GA was not successful in refining this possible solution. On the other
166 hand, DE could refine very well the solutions and achieved impressive fitness values.

167 For the problems with different numbers of target parameters from 2 to 16, DE could
168 all produce smaller relative errors between the target parameter vector and the

169 optimized parameters obtained. When the number of parameters is greater five, the
170 relative error between the target parameter vector and the optimized parameters
171 obtained by DE is around 10% of that of GA.

172 The application of DE in geotechnical engineering emerges in recent years. Vardakos
173 et al. (2012) [46] conducted a back analysis of tunneling using the Differential
174 Evolution and showed that DE exhibits excellent convergence, repeatability
175 characteristics. Xing et al. (2013) [47] combined the support vector machine method
176 and DE algorithm for optimization of tunnel construction process.

177 Although comprehensive investigations have been conducted using the classic GA
178 algorithm in back analysis of excavation problems, the ability of DE in back
179 analyzing braced excavation problems has not been examined based on the authors'
180 knowledge. The application of evolutionary computing algorithms in excavation back
181 analysis should be further explored. In the previous back analyses of excavation using
182 GA or other artificial intelligence algorithm based methods, the scale of the problem
183 (in terms of number of optimization parameters) is often limited to less than five
184 parameters and relatively simple constitutive soil models. As the scale and the
185 characteristic of the problem may greatly influence the shape of objective function
186 and hence the performance of the optimization method [44, 45], real case studies with
187 more soil layers and more parameters to identify and complex constitutive law should
188 be conducted to reinforce the applicability of evolutionary based algorithms.

189 In this paper, we investigate the potential use of the Differential Evolution in the
190 back-analysis of braced excavation response. A computer code named Python-based
191 Differential Evolution (PyDE) is developed using the objective-oriented language
192 Python and implemented in the commercial FEM software ABAQUS. As the high
193 computational cost is a common drawback of evolutionary based algorithms [42, 44],
194 a parallel computing technique is used to run parallel FE analysis for one generation
195 in multiple cores of a computer cluster and dramatically reduce the computational
196 time. The implementation of the back analysis method based on DE and its

197 computational performance are demonstrated using both a synthetic excavation
198 problem and a complex real case, i.e., the Taipei National Enterprise Center (TNEC)
199 project, in which nine parameters of Modified Cam-clay (MCC) are estimated. The
200 efficiency and convergence characteristics using DE are discussed. Furthermore, the
201 robustness of the back-estimated parameters is illustrated by a forward prediction,
202 where field observations during early stages of excavation to predict wall deflections
203 during later stages.

204

205 **METHODOLOGY OF BACK ANALYSIS FOR EXCAVATION**

206 **Objective Function of Back Analysis**

207 The match of simulated results to the observations is one of the most important
208 indicators of how well a model represents an actual system. The difference between
209 the simulated result and observed result is measured by an objective function. To
210 determine the optimized soil parameters by back analysis, the soil parameters are
211 updated repeatedly through iteration process until the simulated result is close enough
212 to the observed result. Some commonly-used objective functions are
213 least-squares/weighted least-squares objective function [23, 24], maximum
214 likelihood [15], and Bayesian posterior probability [18-21]. In this study, a normalized
215 form of least-square objective function is adopted [22]:

$$216 \quad f(\mathbf{X}) = \frac{1}{N} \sum_{i=1}^N \left(\frac{d_i(\mathbf{X})}{d_i^*} - 1 \right)^2 \quad (1)$$

217 where N is the number of measurement points that used to back-figure the target
218 parameters in the optimization procedure, d_i^* is the observed result and $d_i(\mathbf{X})$ is the
219 simulated result, $\mathbf{X} = [x_1, x_2, \dots, x_m]^T$ is the vector of m input parameters. In particular,
220 for wall deflection of a braced excavation, N is the number of measurement points for
221 a vertical cross section of the wall; d_i^* is the measured wall deflection at point i ; d_i is

222 the calculated wall deflection at point i based on numerical modeling. The advantage
223 of the normalized least-square or relative error objective function is that different
224 types of measurements such as surface settlement, bracing load and pore water
225 pressure response could be incorporated to extract the soil parameters. However, the
226 normalized objective function may be give preference to small values of experimental
227 data and is not defined by observed zero values. In addition, the uncertainty in the
228 field measurements is not considered. Some researchers used the scalar or weighted
229 least square function with the weights related with the uncertainty of measurement.
230 The uncertainty based weights however are selected based on engineering judgment
231 [24, 29]. For the least-square objective function, it is hard to know the appropriate
232 number of measured data to use specifically when different measurement types are to
233 be combined and the interval between measurement points may affect the objective
234 function. To make the objective function independent of the interval between the
235 measurement points, the area between the experimental and the computational curves
236 could be used as the objective function [33]. Effect of different types of objective
237 functions is out of the scope of this study and will be further investigated in another
238 paper.

239 **Optimization using Differential Evolution**

240 Evolutionary algorithms are stochastic optimization methods inspired on Darwin's
241 evolution theory and natural selection [44]. There are many different variants of
242 evolutionary algorithms and the common underlying idea behind all these techniques
243 is the same: given a population of individuals, recombination and/or mutation, and
244 natural selection (survival of the fittest), which causes a rise in the fitness of the
245 population. Storn and Price [41-42] proposed an evolutionary algorithm called the
246 Differential Evolution (DE). DE uses floating-point instead of bit-string encoding and

247 arithmetic operations instead of logical ones. No special routine is needed to define,
 248 input, manipulate or output the floating-point format because the most common
 249 programming languages support this format. In binary system, a parameter was
 250 changed by inverting or “flipping” a bit. For example, if the initial parameter value is
 251 15 (01111 binary) and the optimal value is 16 (10000 binary), the probability of this
 252 improving move happen is p^5 , where p is the probability of a bit was inverted.
 253 Generally speaking, floating-point numbers and manipulating them with arithmetic
 254 operators offer advantages over the classic GA “bit flipping” approach including (a)
 255 easy to use, (b) efficient memory utilization, (c) lower computational complexity, (d)
 256 lower computational effort – fast convergence and (e) greater freedom in designing a
 257 mutation distribution [42].

258 A typical DE algorithm includes processes of initialization, mutation, crossover and
 259 selection. Details of the four processes are as follows:

260 (1) Initialization

261 Assume there are N_p vectors of input parameter in each generation as shown in Fig. 1.
 262 The i th parameter vector in g th generation is $\mathbf{X}_{i,g}$ as an individual that could produce
 263 a unique objective function value. The j th parameter value of
 264 $\mathbf{X}_{i,g} = [x_{0,i,g}, x_{1,i,g}, \dots, x_{D-1,i,g}]$ is denoted as $x_{j,i,g}$. D is the dimension of the parameter
 265 vector. Before the population is initialized, the upper and lower bounds for each
 266 parameter should be specified based on experience or tests. The j th parameter value of
 267 i th parameter vector in the first population, $x_{j,i,0}$, can be determined by a random
 268 number generator once the bounds are specified.

269
$$x_{j,i,0} = rand(0,1) \times (b_{j,U} - b_{j,L}) + b_{j,L} \quad (2)$$

270 where $rand(0,1)$ is a random number from an uniform distribution $U(0,1)$, $b_{j,U}$ and
 271 $b_{j,L}$ are the upper bound and the lower bound of the j th parameter, respectively.

272 Babu and Sastry [40] , Vardakos et al. [46] and Chakraborty [43] showed that DE
 273 algorithm can converge to the global optimum irrespective of the initial random
 274 generated population.

275 (2) Difference vector based mutation

276 Select $\mathbf{X}_{i,g}$ as a trial vector for genetic operation. Three other vectors, $\mathbf{X}_{r0,g}$, $\mathbf{X}_{r1,g}$
 277 and $\mathbf{X}_{r2,g}$, are randomly selected from the remaining vectors from the g th generation.

278 A mutant vector $\mathbf{V}_{i,g} = [v_{0,i,g}, v_{1,i,g} \dots, v_{D-1,i,g}]$ is generated as:

279
$$\mathbf{V}_{i,g} = \mathbf{X}_{r0,g} + F \times (\mathbf{X}_{r1,g} - \mathbf{X}_{r2,g}) \quad (3)$$

280 where F is the constant mutation scaling factor. According to Storn and Price [34], the
 281 optimum value of F can be taken from 0.4 to 1.0. F only affects the relative step size
 282 since the distribution of difference vectors is itself adaptive. For example, we have 5
 283 vectors in one generation (Figure 2a). The number of all possible non-zero difference
 284 vectors (pair to pair) is 4×5 . Figure 2(b) illustrates 20 non-zero difference vectors
 285 generated by the five vectors in Figure 2(a). In the graph, differentials have been
 286 scaled by half ($F = 0.5$), and transported to a common origin. Note that the
 287 distribution is symmetric about zero because every pair of vectors gives rise to two
 288 opposite but equal difference vectors. Thus, F can be kept constant during
 289 optimization without compromising DE's ability to generate steps of the required size
 290 [42-43]. With a scaling factor F and the difference vector together, it can ensure that
 291 the mutation vectors do not duplicate existing points. In addition, scaling can shift the
 292 focus of the search between local and global if the scaling factor is reasonable [42].

293 Difference vector based mutation is the major difference between DE and GA. In
 294 classic GA, some of the individuals are randomly mutated by inversion of one byte on

295 individual chain. The advantage of the difference vector based mutation is that both
 296 the step size and the orientation of a mutation can be automatically adapted to the
 297 objective function landscape and hence promising regions of the fitness landscape are
 298 investigated automatically once they are detected and it promotes basin-to-basin
 299 transfer, where search points may move from one local minimum to another one [42,
 300 44].

301 (3) Crossover

302 In order to increase the diversity of the perturbed parameter vectors, crossover is
 303 introduced. A new trial vector $\mathbf{U}_{i,g} = [u_{0,i,g}, u_{1,i,g}, \dots, u_{D-1,i,g}]$ is created from the
 304 original trial vector $\mathbf{X}_{i,g}$ and the mutant vector $\mathbf{V}_{i,g}$:

$$305 \quad u_{j,i,g} = \begin{cases} v_{j,i,g} & \text{if } (\text{rand}(0,1) \leq Cr \text{ or } j = j_{rand}) \\ x_{j,i,g} & \text{otherwise.} \end{cases} \quad (4)$$

306 where $Cr \in [0,1]$ is a crossover constant and j_{rand} is a random chosen index
 307 $\in [0,1,2,\dots,D]$. The crossover probability, Cr , is a user-defined value that controls the
 308 fraction of parameter values that are copied from the mutant. To determine which
 309 source contributes a given parameter, the crossover compares Cr to the output of a
 310 uniform random number generator, $\text{rand}(0,1)$. If the random number is less than or
 311 equal to Cr , the trial parameter is inherited from the mutant, $\mathbf{V}_{i,g}$; otherwise, the
 312 parameter is copied from the original trial vector vector, $\mathbf{X}_{i,g}$. It means that the
 313 j_{rand} th parameter will inherit from the mutant vector and each of the other $(D-1)$
 314 parameters will have Cr probability to inherit from the mutant vector. Figure 3
 315 illustrates the process of binary n-point crossover in DE.

316 In boundary-constrained problems, it is essential to ensure that the reproduced
 317 parameter values lie inside their allowed ranges otherwise the results could be
 318 unreasonable and the problem may not converge. In this study, parameter values in the
 319 new trial vector that violate boundary constraints are replaced with random numbers

320 generated within the feasible range [48]:

$$321 \quad u_{j,i,g} = \begin{cases} \text{rand}(0,1) \times (b_{j,U} - b_{j,L}) + b_{j,L} & \text{if } u_{j,i,g} \notin [b_{j,L}, b_{j,U}] \\ u_{j,i,g} & \text{otherwise} \end{cases} \quad (5)$$

322 (4) Selection. By comparing the objective function values of $\mathbf{U}_{i,g}$ with $\mathbf{X}_{i,g}$, the
323 individual with the lower objective function value is selected to next generation.

$$324 \quad \mathbf{X}_{i,g+1} = \begin{cases} \mathbf{U}_{i,g} & \text{if } f(\mathbf{U}_{i,g}) \leq f(\mathbf{X}_{i,g}) \\ \mathbf{X}_{i,g} & \text{otherwise} \end{cases} \quad (6)$$

325 The same procedure is followed for all N_p parameter vectors in each generation and
326 new N_p parameter vectors are formed for the next generation. The above evolution of
327 generation is repeated until a maximum of generation is reached or the minimum
328 objective function for one generation is less than a given error. The parameter vector
329 of the last generation is then considered as the optimal or back estimated parameter
330 vector.

331 **Development of PyDE Code in ABAQUS**

332 In this study, a computer code named Python-based Differential Evolution (PyDE) is
333 developed using an open-source objective-oriented language Python to implement
334 back analysis with the DE algorithm in the well-known commercial FEM package
335 ABAQUS. Python language is a high-level programming language that supports
336 object-oriented programming and allows easy integration with software programmed
337 in Fortran, C, C++, and other languages. It provides a clear and readable syntax and is
338 highly stable and can be easily transformed to different operating systems. It also
339 includes a large set of standard libraries and its open-source license enables the
340 modification and distribution of Python-based applications with almost no restrictions.
341 Moreover, as the standard programming language of ABAQUS, Python can easily
342 change the model data, such as material parameters and structure geometry, and read
343 the resulting output databases. The interaction between PyDE code and ABAQUS in
344 the back analysis is shown in the flowchart in Fig. 4. In the PyDE code, the trial vector,

345 $\mathbf{X}_{i,g}$, is selected and the new trial vector $\mathbf{U}_{i,g}$ is developed through mutation and
346 crossover. After that, the two parameter vectors are imported into ABAQUS as input
347 parameters for the FEM analysis. The results of numerical analysis are used to
348 calculate the objective function and the selection of trail vector for the next generation
349 is made. If the objective function is less than a given value or the maximum number of
350 generations is reached, the inverse analysis procedure is terminated.

351 As the high computational cost is a common drawback of evolutionary based
352 algorithms, a parallel computing technique is used to run parallel FE analysis for each
353 parameter vector of one generation in multiple cores of a computer cluster and this
354 can dramatically reduce the computational time. Assume one generation with 90
355 individual parameter vectors. If each FEM calculation takes 1 minute of CPU time,
356 then 9000 minutes (about 6 days) are needed for 100 generations. In this study, the 90
357 FEM simulations in one generation were submitted at the same time to 90 CPUs of
358 the computer cluster. Therefore, the back analysis can be finished in several hours.

359

360 **EXAMPLE APPLICATION: SYNTHETIC EXCAVATION CASE**

361 For a complex excavation site, there are usually multiple soil layers and a number of
362 soil parameters should be considered in simulation of the excavation. Multiple soil
363 parameters may greatly increase the difficulty of back analysis and affect its efficacy.
364 On the other hand, a synthetic case instead of real complex excavation problems can
365 reduce the difficulty of back analysis for a highly nonlinear problem and avoid the
366 unknown errors from the measurement data [24]. Hence, to examine the efficacy and
367 robustness of the back analysis approach for excavation problems, we start with a
368 synthetic excavation case with only one soil layer.

369 **Numerical Model of the Synthetic Case**

370 The synthetic case is a 6 m deep and 20 m wide excavation zone with only one layer
371 of clayey soil, as shown in Fig. 5. The retaining wall is 0.6 m thick and 12 m deep.

372 The excavation is conducted in one step and a strut is installed at the level of ground
373 surface to support the retaining wall. In the finite element simulation, due to the
374 geometric symmetry, only a half of the excavation is modeled under plain strain
375 condition. The overall model size is 50 m long and 25 m high, which is considered
376 large enough to avoid boundary constraints. The left boundary is x-symmetric, which
377 means the vertical displacement is allowed however the horizontal direction
378 displacement and rotation is fixed. The boundary on the right side is also fixed along
379 horizontal direction and the vertical direction is free. The bottom boundary is fixed
380 along both horizontal and vertical directions. The Modified Cam-clay (MCC)
381 constitutive model is adopted for the clayey soil in this excavation site. The retaining
382 structure including the retaining wall and the strut are assumed to be linear-elastic. The
383 element type for the soil is 4-node bilinear rectangular element. Spring element and
384 2-node linear beam element are adopted for the strut and the retaining wall,
385 respectively.

386 **Sensitivity Study of Soil Parameters**

387 Prior to the back analysis, a sensitivity study is conducted to find out the most
388 important soil parameters which can affect the wall deflection of the excavation. Four
389 soil parameters of the MCC model, including the slope of the critical state line in the
390 $q-p'$ plane (M), the slope of isotropic normal consolidation line in the $e-\ln(p')$ diagram
391 (λ), the slope of isotropic swelling/recompression line in the $e-\ln(p')$ diagram (κ), and
392 the Poisson's ratio (ν), are selected for sensitivity analysis. In addition, as the soil-wall
393 interface may also affect wall deflection [49], the friction coefficient μ for the
394 interface between the wall and surrounding soil is also chosen for the sensitivity
395 analysis. In the benchmark case, the values of ν , κ , λ , M and μ are 0.2, 0.014, 0.14,
396 1.2 and 0.2, respectively. Then, each parameter is varied according to the values listed
397 in Table 1, while the other four parameters keep the values in the benchmark case. All
398 values vary within the representative ranges for clayey soils [50, 51]. The soil unit

399 weight γ of the clay soil layer is 18.0 kN/m³. The void ratio e and the coefficient of
400 earth pressure at rest K_0 are assumed to be 1.0 and 1.0, respectively. The Young's
401 modulus and the Poisson's ratio of the retaining wall are assumed to be 12 GPa and
402 0.2, respectively. The stiffness of the strut is assumed to be 10,000 KN/m. Numerical
403 simulations with these values of input parameters can produce a reasonable shape of
404 wall deflection according to the previous studies of numerical simulations and field
405 measurement [52].

406 Fig. 6 shows the sensitivity of the wall deflection on the five parameters. It can be
407 seen that the variation of λ and μ have little effect on the wall deflection. However,
408 the variation of ν , κ and M have significant effects on the deformation of the wall.
409 During the excavation, the soil in front of the wall is in the unloading condition which
410 is directly related to the swelling index κ [11]. The Poisson's ratio ν has a significant
411 effect on the wall deflection because it affects the shear modulus (G) of the soil. The
412 slope of the critical state line in the q - p' plane (M) has the most significant effects
413 especially when it has a small value 0.8 since it affects the plastic surface shape and
414 the soil can enter the plastic zone easily with a small value of M . It is also considered
415 as an important parameter to be identified in back analysis of pressuremeter test data
416 [25] or triaxial test data [51]. A quantitative sensitivity analysis is conducted for the
417 five parameters in terms of the variation of objective function with respect to the
418 variation of each parameter. The horizontal wall deflection simulated using the
419 parameters of the benchmark case in the sensitivity study were used as the synthetic
420 observation data. The relative error objective function is calculated based on the

421 results in Figure 6. The dimensionless sensitivity coefficients $S_i = \frac{\Delta f / \Delta x_i}{f / x_i}$ [53], of ν ,

422 κ , λ , M and μ with respect to the benchmark case are 3.45, 0.97, 0.57, 1.02, 0.56,
423 respectively. Therefore, in this paper, the three parameters, ν , κ and M , are selected as
424 the target parameters to be identified in the back analysis of excavation. It should be
425 noted that different kinds of quantitative parameter sensitivity analysis could be used

426 to further describe the effects of these parameters on the wall deflection. However,
427 due to the limited length of this paper they are not implemented in this paper and the
428 back analyzed results shows that using these three parameters are enough to reach
429 satisfactory wall deflections.

430 **Back Analysis of Soil Parameters Using Synthetic Data**

431 The horizontal wall deflection simulated using the parameters of the benchmark case
432 in the sensitivity study were used as the synthetic observation data. The upper and
433 lower bounds of the three target soil parameters in the back analysis are listed in Table
434 2. According to [41-42], three DE's control variables, namely population size N_p ,
435 mutation scaling factor F and crossover probability Cr , are set as 30 ($10 \times m$), 0.5 and
436 0.9, respectively. The target value for the objective function is 0.01% and the
437 maximum generation number is 50. The back analysis is terminated at the 47th
438 generation with the minimum objective function value of 4.4×10^{-5} . As shown in Table
439 2, the optimized parameters are basically the same as the input parameters, which
440 illustrates that the exact values of the three parameters of the MCC model can be
441 accurately back-analyzed.

442 Fig. 7 shows the contour of the objective function and the iteration path of the two
443 parameters (ν and M). The contour of the objective function is constructed using all
444 calculation results of the 47 generations. Although, the objective function value is
445 actually determined by the three parameters used in back analysis, this graph clearly
446 shows the convergence of objective function. If the objective functions on other space
447 (M and κ) or (ν and κ) illustrate the similar contours and iteration paths. Fig. 8
448 presents the evolution process of the objective function value and the soil parameters
449 using the boxplots of the objective functions and back estimated parameters in 1st, 10th,
450 20th, 30th, 40th and 47th generations. The top and bottom of the box represent the 25th
451 and 75th percentiles of the back-estimated parameter. A horizontal line near the
452 middle of the box identifies the median value. The minimum and maximum values of
453 the parameter are also shown in the boxplot. As can be seen, the objective function

454 value experienced a significant decrease during the first ten generations and are
455 reducing at a lower speed for the following generations. The soil parameters in the
456 first generation are widely distributed, while in the 47th generation the distributions of
457 the parameters are much narrower and converge to the exact input soil parameters,
458 especially for M and ν . It seems that the estimation of M and ν was well established
459 first and the values of κ in the 47th generation still have large divergence. Fig. 9 shows
460 the comparison between the calculated wall deflection using the back-estimated
461 optimal parameters with the lowest objective function value in the last generation and
462 the synthetic wall deflection of the benchmark case. As expected there is excellent
463 agreement between the simulations and the synthetic measurements. In general,
464 according to the results of back analyses using the synthetic case, the efficacy and
465 accuracy of the back analysis are satisfactory. It is desirable to incorporate the real
466 excavation case histories to further examine the applicability of the method.

467

468 **EXAMPLE APPLICATION: TAIPEI NATIONAL ENTERPRISE** 469 **CENTER**

470 In this study, the excavation of Taipei National Enterprise Center (TNEC) is employed
471 to back analyses soil parameters based on the wall deflection at each excavation stage.
472 The Taipei National Enterprise Center (TNEC) excavation is selected as a
473 comprehensive field monitoring system and large amount of laboratory tests were
474 conducted in this site [12, 54].

475 **Numerical Model of the TNEC Excavation**

476 The TNEC case is located in the Taipei Basin and its typical soil profile of the site is
477 shown in Fig. 10. The ground consists of six alternating silty sand (SM) and silty clay
478 (CL) layers overlying a thick gravel formation. The silty clay layers are mainly
479 low-plasticity and slightly over-consolidated soft to medium clay. The gravel layer is
480 located 46 m below the ground surface. The excavation was conducted using the

481 top-down construction method in seven stages. The final excavation depth was 19.7 m.
482 A 0.9 m thick and 35 m deep diaphragm wall which was supported by two levels of
483 horizontal struts and five levels of 150 mm thick concrete floor slabs was used as the
484 retaining structure. Fig. 10 also illustrates the sequences of excavation and the
485 horizontal supporting system. Detailed information of TNEC case can be referred to
486 [12, 54].

487 Fig. 11 shows the finite element mesh used in this analysis. To reduce the
488 computational load, only half of the excavation is modeled according to symmetry.
489 The right boundary is 100 m away from the excavation center and is well beyond the
490 influence zone of excavation, which is normally around four times of the excavation
491 depth. The bottom boundary is set at the surface of the gravel soil layer, i.e., 46 m
492 below the surface. The boundary conditions in the synthetic case were implemented
493 here, which means that the horizontal and rotational freedom was fixed on the left side,
494 the horizontal and vertical freedom was fixed on the bottom and only the horizontal
495 freedom was fixed on the right side.

496 In the FEM model, the clayey soil layers (CL) and the sandy soil layers (SM) were
497 simulated by Modified Cam-clay (MCC) model and Mohr-Coulomb (MC) model,
498 respectively. In this study, drained condition is assumed and effective stress
499 constitutive models are for both sand and clay soils. Table 3 presents the input
500 parameters for the sandy soil layers. For the sandy soil layers, the effective frictional
501 angle ϕ' is obtained from direct shear tests. According to Ou and Lai [2], the Young's
502 modulus E of sandy soils can be estimated from the following equation:

$$503 \quad E = 2\beta\rho V_s^2(1+\nu) \quad (7)$$

504 where ρ is the soil density; V_s is the shear wave velocity and can be estimated using
505 an empirical equation with the SPT N value; β is a reduction factor which considers
506 the difference between small strain and the strain at the normal condition and the β
507 value can be set equal to 0.5 based on experience [52]. The soil cohesion c and the

508 dilatancy angle ψ were both set as zero for the SM layers. Based on the sensitivity
509 analysis in the previous section, the three soil parameters, i.e., ν , κ and M , are
510 identified as the most important parameters of the MCC model. Therefore, in this case
511 study, total nine soil parameters of the three clay layers are chosen to be estimated in
512 the back analysis. Table 4 lists the upper and lower bounds of these three parameters
513 with the consideration to cover most possible values of MCC model parameters for
514 clayey soils [25, 51, 55]. For the clayey soil layers, the logarithmic compression index,
515 λ , is equal to $C_c/2.303$, where C_c is the compression index. The in situ at rest earth
516 pressure coefficient (K_0) is estimated using the Jacky's equation: $K_0 = 1 - \sin(\phi')$.

517 The Young's modulus of the diaphragm wall used in the analysis is 12 GPa and the
518 Poisson's ratio is assumed to be 0.2 based on Ou et al. [54]. The average nominal axial
519 stiffness per unit width for the first strut level and the second strut level are 14,980
520 KN/m/m and 64,363 KN/m/m respectively. As preloading forces were applied to the
521 struts during the installation procedure, the strut forces are taken as 98.1KN/m and
522 392KN/m for the first and second temporal horizontal struts, respectively. The axial
523 stiffness of the concrete floor slabs is assumed to be 146,512 KN/m/m, which is about
524 80% of its nominal value, considering the reduction of the concrete stiffness due to
525 large bending moment of the diaphragm wall and the occurrence of cracks in the
526 concrete. Since the wall friction coefficient has a relatively small effect on the wall
527 deflection as shown in the previous sensitivity analysis, the friction coefficient of the
528 wall-soil interface, μ , is assumed to be a constant value of 0.273, which represents the
529 weighted average of μ along the whole length of the diaphragm wall.

530 **Identification of Soil Parameters for Each Excavation Stage**

531 The same control parameters of differential evolution as in the synthetic case are
532 implemented in TNEC case with 90 ($10 \times m$) individual vectors in each generation.
533 The field measured wall deflection at each stage of excavation is used to back analyze
534 the nine soil parameters. When the number of generation reaches 100 or when the

535 value objective function is less than 1.0%, the back analysis process is terminated.
536 The computation load for this case study is very heavy if we use a normal laptop
537 computer. For each excavation stage, maximum 9,000 simulations (100 generations)
538 have to be calculated in FE analysis program. For back analysis for all the seven
539 stages, the computer time could be months in a normal computer. In this study, the
540 simulations for 90 parameter vectors in one generation are submitted at the same time
541 in a super computer cluster with 500 2.3GHz cores, so the calculation time is
542 dramatically reduced to only a few hours. Fig. 12 shows the boxplots for back
543 estimated parameters, ν , κ and M , of three clayey layers. The x axis of the graph, S1
544 to S7, denotes the seven stages of the excavation. It can be seen that the ranges for all
545 nine parameters except M of the fifth soil layer, are significantly narrowed. For the
546 first soil layer, the back estimated soil parameters for the seven excavation stages are
547 very close. For the third soil layer which influences the wall deflection, the back
548 estimated parameters vary with the excavation stages and gradually reach steady
549 values at the final stage. The parameters of the fifth soil layer varied significantly with
550 excavation stage and the parameter M has a wide distribution. It is mainly because the
551 fifth layer is below the excavation base and has limited influence on the wall
552 deflection. Therefore, it is hard to determine the parameters for the fifth layer through
553 back analysis using the observed wall deflection.

554 The estimated wall deflections using the back-analyzed soil parameters with the
555 optimal objective function in each stage are compared with the field observation as
556 shown in Fig. 13. The values of objective function in terms of the relative error of the
557 estimated wall deflection for the seven stages are 9.7%, 16.5%, 12.8%, 5.2%, 2.2%,
558 1.1% and 1.0%, respectively. It can be seen that the estimated wall deflections are
559 very close to the field measurements especially for the last three stages. For the first
560 three stages, the estimated wall deflection seems not agree well with the field
561 measured values, especially for the first stage. This is because the objective function
562 is the relative error while the measured wall deflections of the first three stages are

563 relatively small. In addition, the wall deflections in the first three stages are
564 significantly affected by the second soil layer, which is a sandy soil layer and the soil
565 properties of the sand soil layers are constant values and are not tuned in the back
566 analysis. The results obtained in this study for stage 5-7 are comparable to the relative
567 errors (2.70%, 1.07%, and 2.84% for stage 5-7, respectively) obtained by NOT
568 algorithm (nonlinear optimization technique with a quasi-Newton method) in Tang
569 and Kung [22]. In Tang and Kung [22], parameters of all six soil layers are back
570 estimated, while in this study only the parameters of clayey layers were back
571 analyzed.

572 **Forward Prediction using Identified Parameters**

573 The practice goal of incorporating the back analysis in excavation is to predict the
574 response of the excavation based on the early stage observations. Therefore, in this
575 example, the input parameters optimized in the early stages of the excavation are used
576 to predict the performance of the excavation at subsequent stages to further examine
577 the capacity of the back analysis methodology and the developed code. The relative
578 errors of prediction for stages 4-7 calculated with the optimized parameters in the
579 previous stages are presented in Fig. 14. The numbers in y-axis represents the stage
580 numbers for prediction. For example, the wall deflection in stage 4 is predicted using
581 the optimized parameters obtained from back analysis of stage 1 to 3 and the relative
582 errors of the prediction are presented in the first row of bullets in y-axis. As shown in
583 the graphs, the wall deflection can be generally satisfactorily predicted using the back
584 analyzed soil parameters at previous stages. A better prediction can be obtained when
585 using the optimized parameters from back analysis of the most recent previous stage
586 than using those from much earlier stages. The reason is that the performance of
587 excavation is not drastically changed between two adjacent stages. The relative errors
588 of prediction by the optimized parameters from stage 1-3 are greater than those
589 obtained using the optimized parameters from stage 4-6. This may due to the large
590 uncertainties of back estimated parameters in the first three stages.

591 **LIMITATIONS**

592 As illustrated in the two examples, the back analysis using the Differential Evolution
593 Genetic Algorithm is a powerful tool for model calibration using field measurement.
594 It can locate the optimized multiple soil parameters within limited computation load
595 and can provide acceptable predictions for subsequent responses of an excavation.
596 However, it is important to note that the process of parameter estimation is carried out
597 in the context of a specified numerical model that includes geometry, boundary
598 conditions and constitutive models for soils. Hence, it is unable to overcome inherent
599 limitations in the selected material constitutive model such as the inability to capture
600 small strain stiffness.

601 Selecting a proper objective function is very important in inverse analysis and the one
602 implemented in this study gives preference to small values and it may be affected by
603 the number of measurement points. The uncertainty in the field measurements is not
604 considered in this study. Effect of different types of objective functions is out of the
605 scope of this study and will be further investigated in another paper.

606 In this study, the soils in the excavation are assumed to be drained. As the construction
607 of an excavation usually takes several months, the soil behaviors are more likely
608 partially drained [52]. A coupled hydro-mechanical analysis is more appropriate for
609 such condition. Coupled analysis is out of the scope of this study and will be
610 considered in further research studies.

611

612 **CONCLUSIONS**

613 The paper demonstrated the potential application of a global optimization method,
614 Differential Evolution (DE) algorithm, in the back-analysis of the deep excavation
615 problems. Wall deflection data are used as observations in an inverse analysis that
616 calibrates the parameters of soil model. The performance of the developed Differential
617 Evolution algorithm code PyDE which is integrated in the commercial software

618 ABAQUS is illustrated using a synthetic case and a real case of excavation. The
619 following conclusions can be made:

620 (1) The distributions of multiple input soil parameters are well identified after the
621 back-analysis using DE optimization algorithm. For the synthetic excavation case, the
622 back-analyzed parameters are basically identical to the input parameters which are
623 used to generate synthetic response of wall deflection.

624 (2) For the TNEC case with totally nine parameters to be back-analyzed, a
625 satisfactory agreement between simulated response using the back estimated
626 parameters and field monitored performance is obtained. The relative errors of wall
627 deflection for the last three stages are 2.2%, 1.1% and 1.0%, respectively. This
628 illustrates good performance of convergence and efficiency for complex and nonlinear
629 deep excavation problem.

630 (3) In addition, with the back estimated soil parameters at given stage, the wall
631 deflection at subsequent stages can be well predicted. The parameters estimated make
632 a better prediction for the most recent previous stage than for much earlier stages,
633 since similar performance can be observed for two adjacent stages.

634

635 **ACKNOWLEDGEMENTS**

636 The work in this paper was substantially supported by grants from the Natural Science
637 Foundation of China (Project No. 41172252). The authors are grateful for the support
638 from Shanghai Rising-Star Program of Science and Technology Commission of
639 Shanghai Municipality (Project No. 12QA1401800).

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836 Table 1. Soil parameters used in the parametric study of the synthetic case

Soil Parameters	ν	κ	λ	M	μ
	0.1	0.007	0.07	0.8	0.1
Benchmark value	0.2	0.014	0.14	1.2	0.2
	0.3	0.021	0.21	1.6	0.3

837

838

839 Table 2. Back-analyzed soil parameters for the synthetic case

Parameter	True value	Min	Max	Optimal value
ν	0.2	0.1	0.3	0.2026
κ	0.014	0.007	0.021	0.0137
M	1.2	0.8	1.6	1.1997

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856 Table 3. Soil properties of SM soil layers in the TNEC case

Soil Layer	Depth (m)	γ (kN/m ³)	c (kPa)	ϕ' (°)	Dilatancy angle ψ (°)	E (kPa)	K_0	ν
2	5.6-8	18.9	0	30	0	68351	0.49	0.3
4	33-35	19.6	0	33	0	265473	0.49	0.3
6	37.5-46	19.6	0	35	0	300247	0.47	0.3

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859 Table 4. Soil parameters of CL soil layers in the TNEC case

Soil Layer	Depth (m)	γ (kN/m ³)	λ	K_0	e_0	κ	ν	M
1	0-5.6	18.3	0.152	1.0	1.067	[0.01, 0.05]	[0.1, 0.35]	[0.8, 1.6]
3	8-33	18.9	0.152	0.51	0.820	[0.01, 0.05]	[0.1, 0.35]	[0.8, 1.6]
5	35-37.5	18.2	0.152	0.52	0.824	[0.01, 0.05]	[0.1, 0.35]	[0.8, 1.6]

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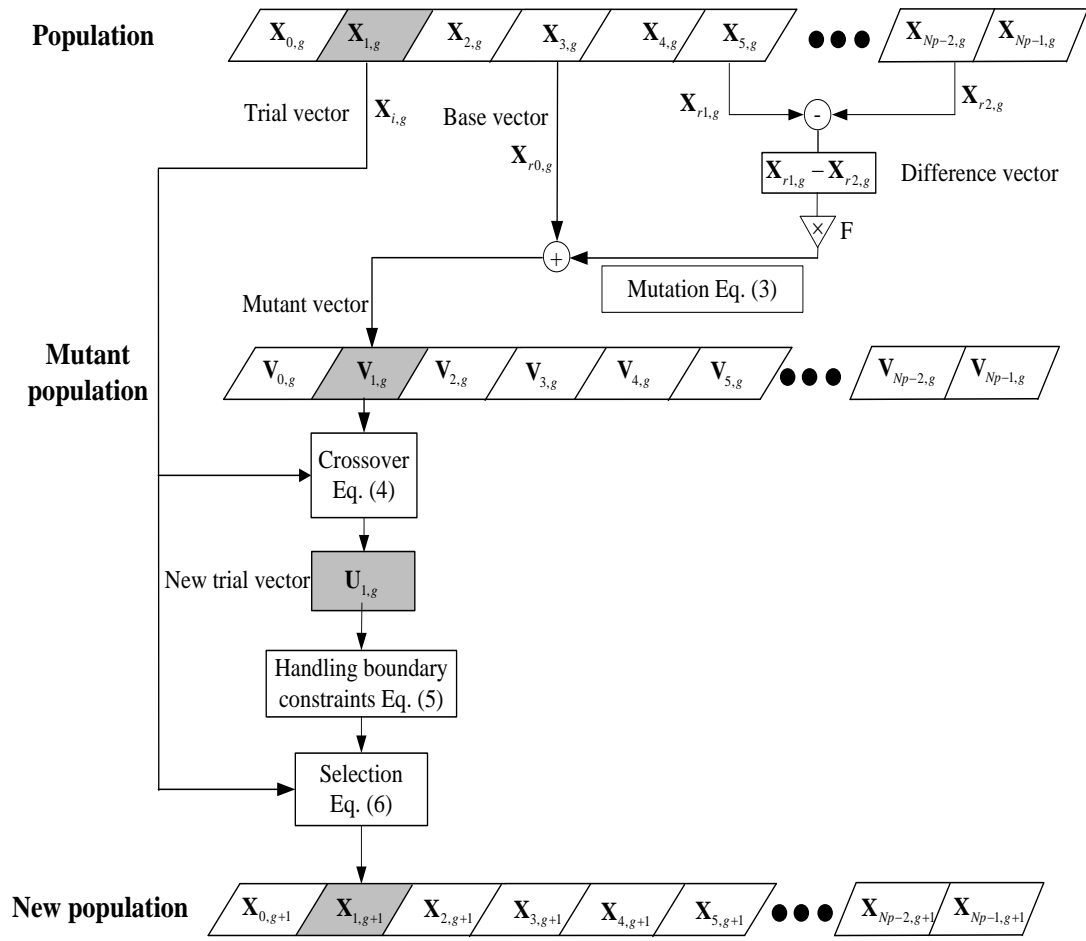
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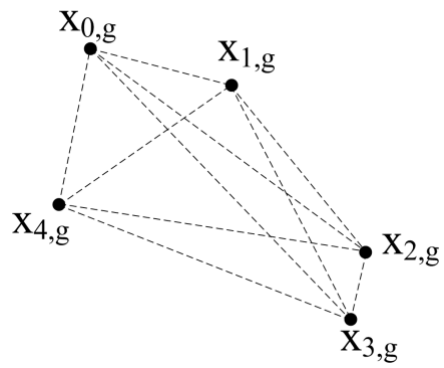
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Figure 1. Flow chart of Differential Evolution

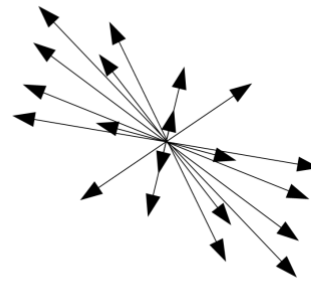
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(a)



(b)



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884 Figure 2. Illustration of difference vectors in a population of 5 vectors: (a) five vectors;

885 (b) non-zero difference vectors scaled by half ($F = 0.5$), and transported to a common

886 origin (adopted from Price et al. [42])

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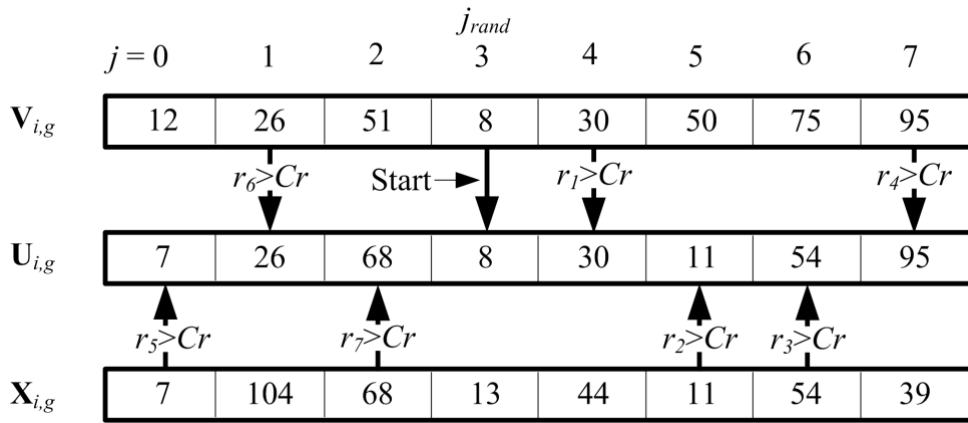
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904 Figure 3. Schematic plot of binary n-point crossover in DE (adopted from Price et al.

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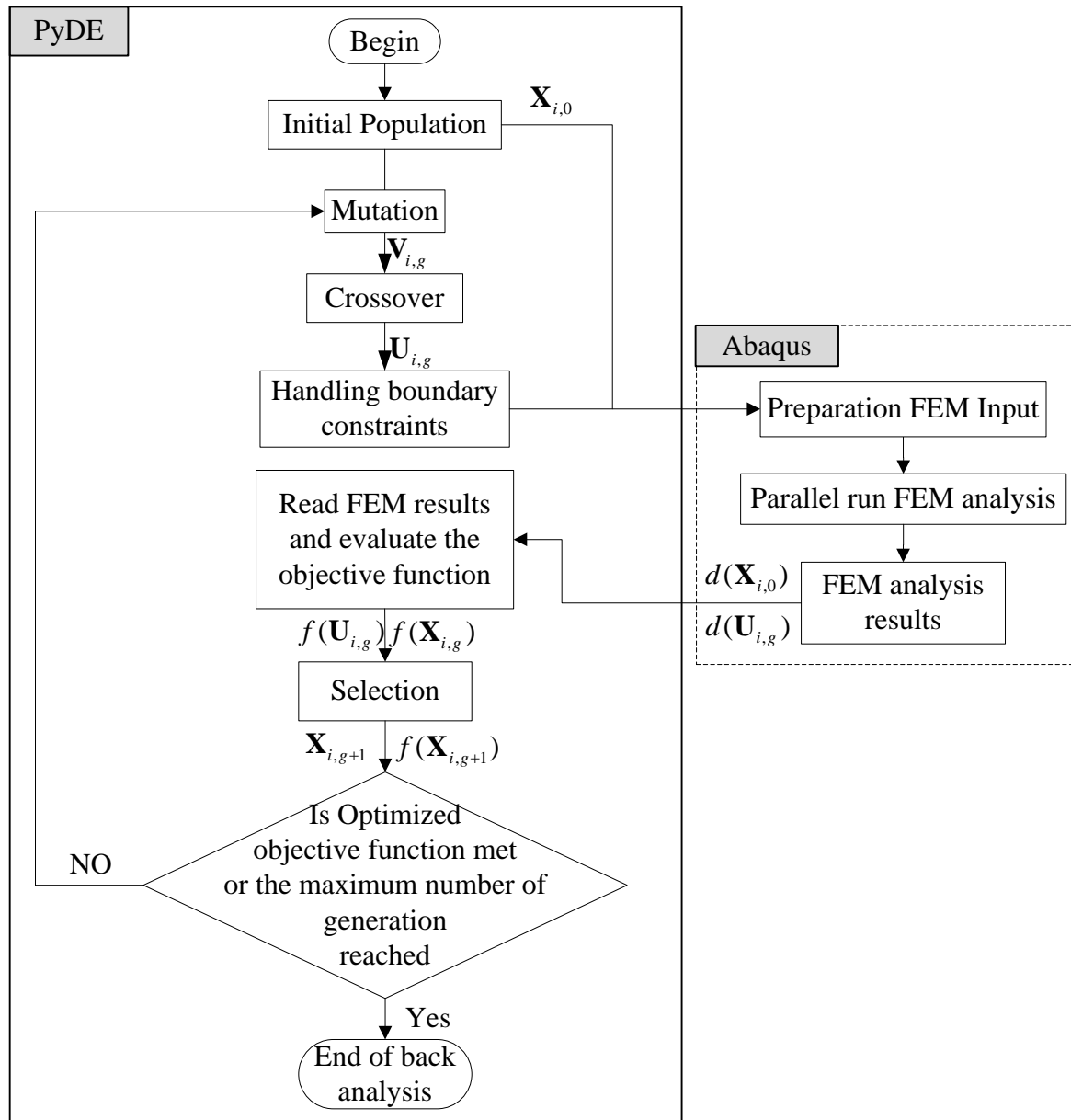
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916 Figure 4. Back analysis with interaction between PyDE code and ABAQUS

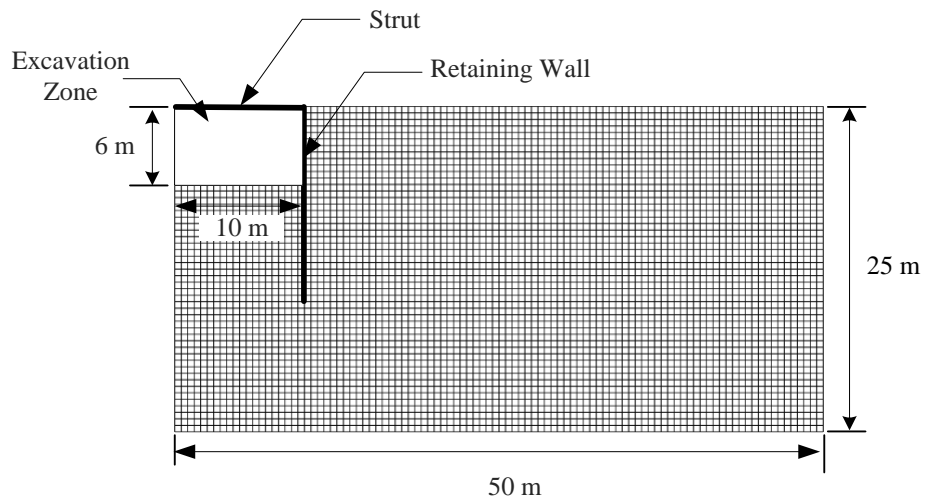
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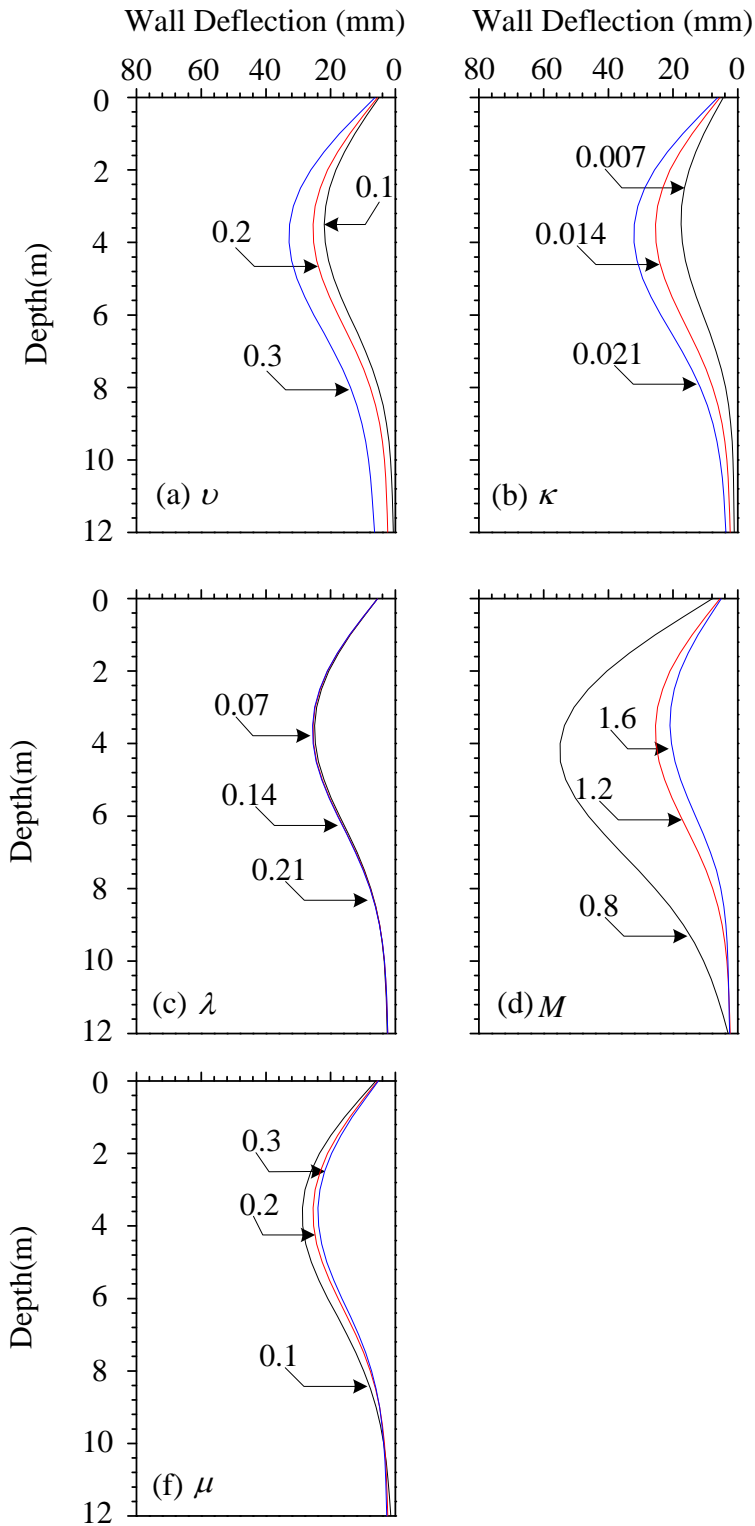
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923 Figure 5. Geometry and finite element mesh of the synthetic excavation case

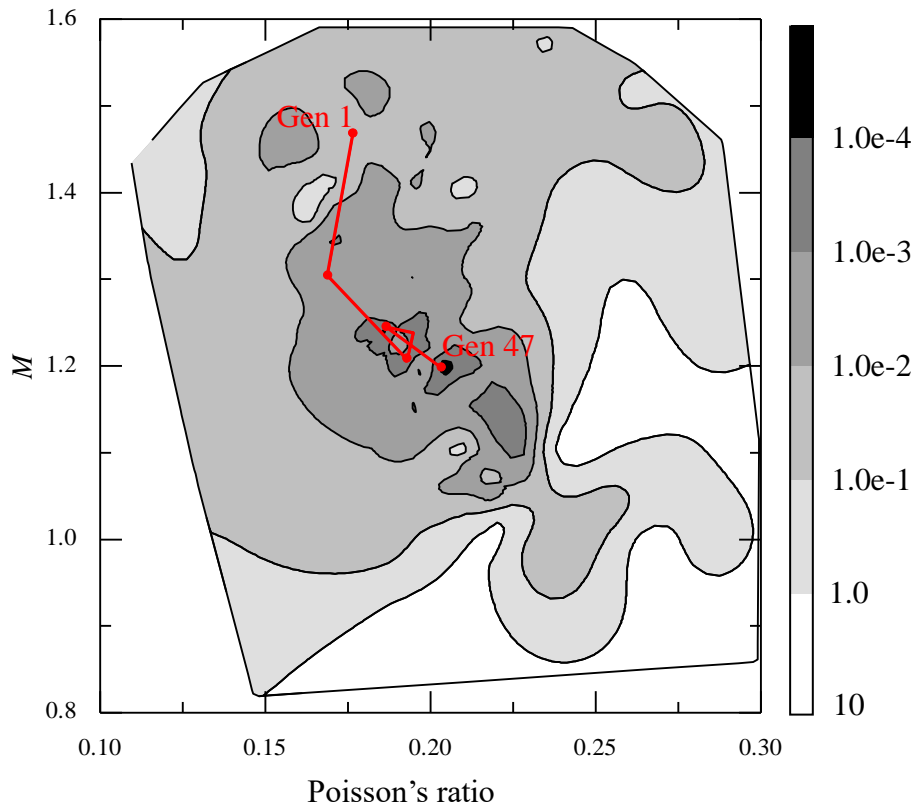
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926 Figure 6. Effects of soil parameters on wall deflection of the synthetic excavation.

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929 Figure 7. Contours of the objective function and iteration paths for ν and M

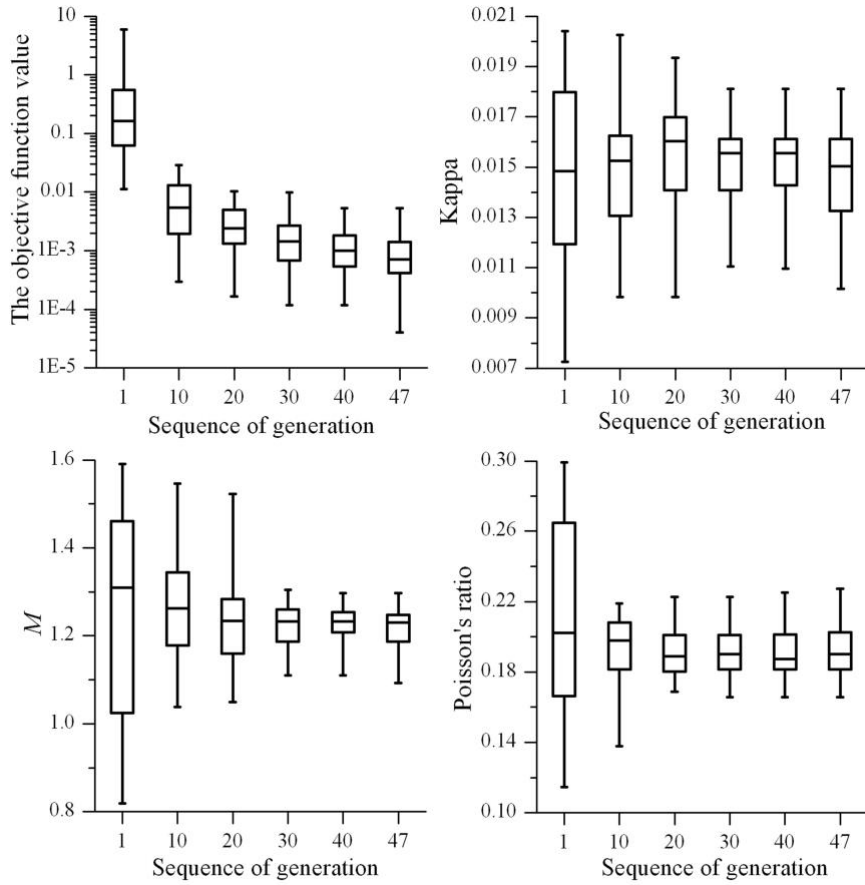
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936 Figure 8. Evolution process of objective function values and soil parameters

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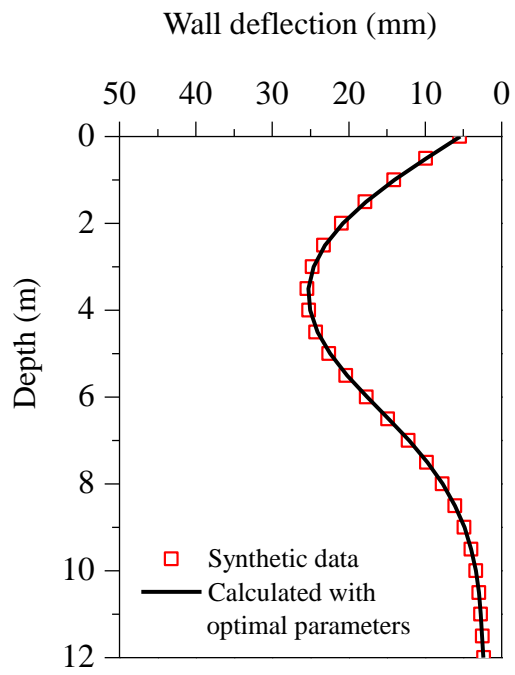
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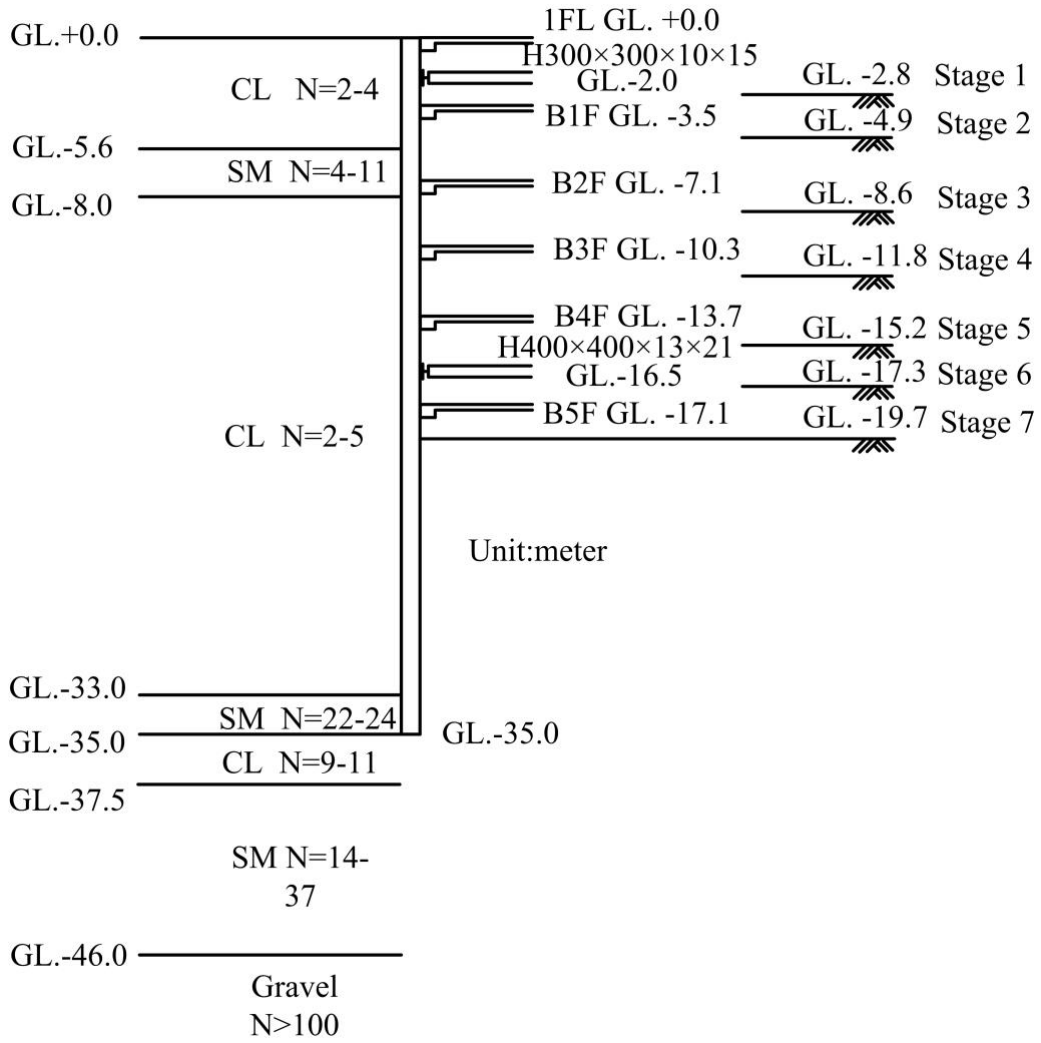


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952 Figure 9. Comparison between the synthetic measurement and the calculated wall

953 deflection based on optimal soil parameters for the synthetic excavation case

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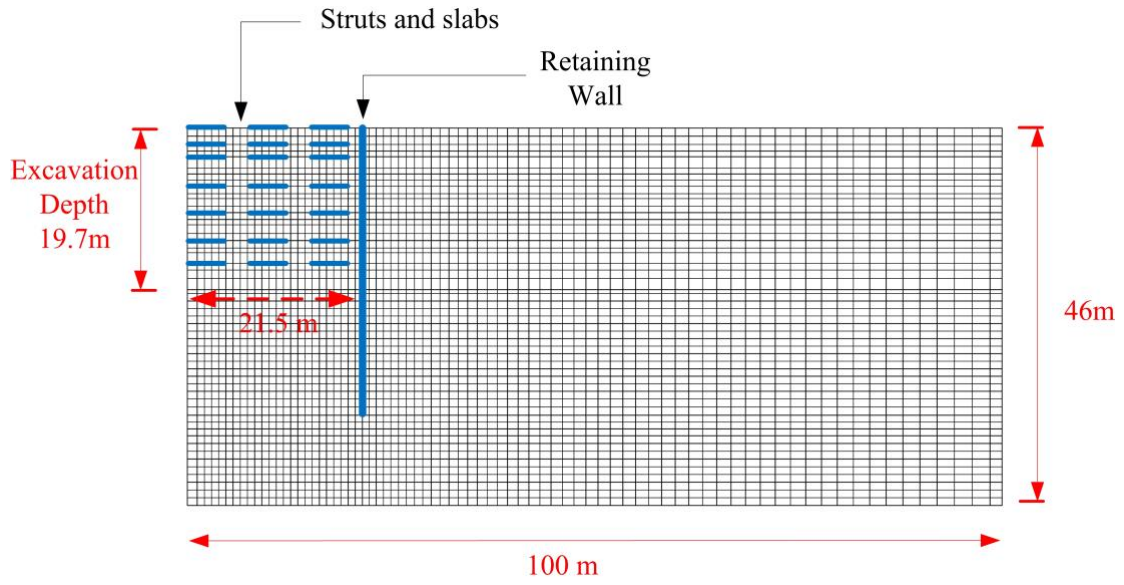
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956 Figure 10. Construction sequence of excavation and geology profile of TNEC

957 excavation project

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961 Figure 11. Finite element mesh of the TNEC case

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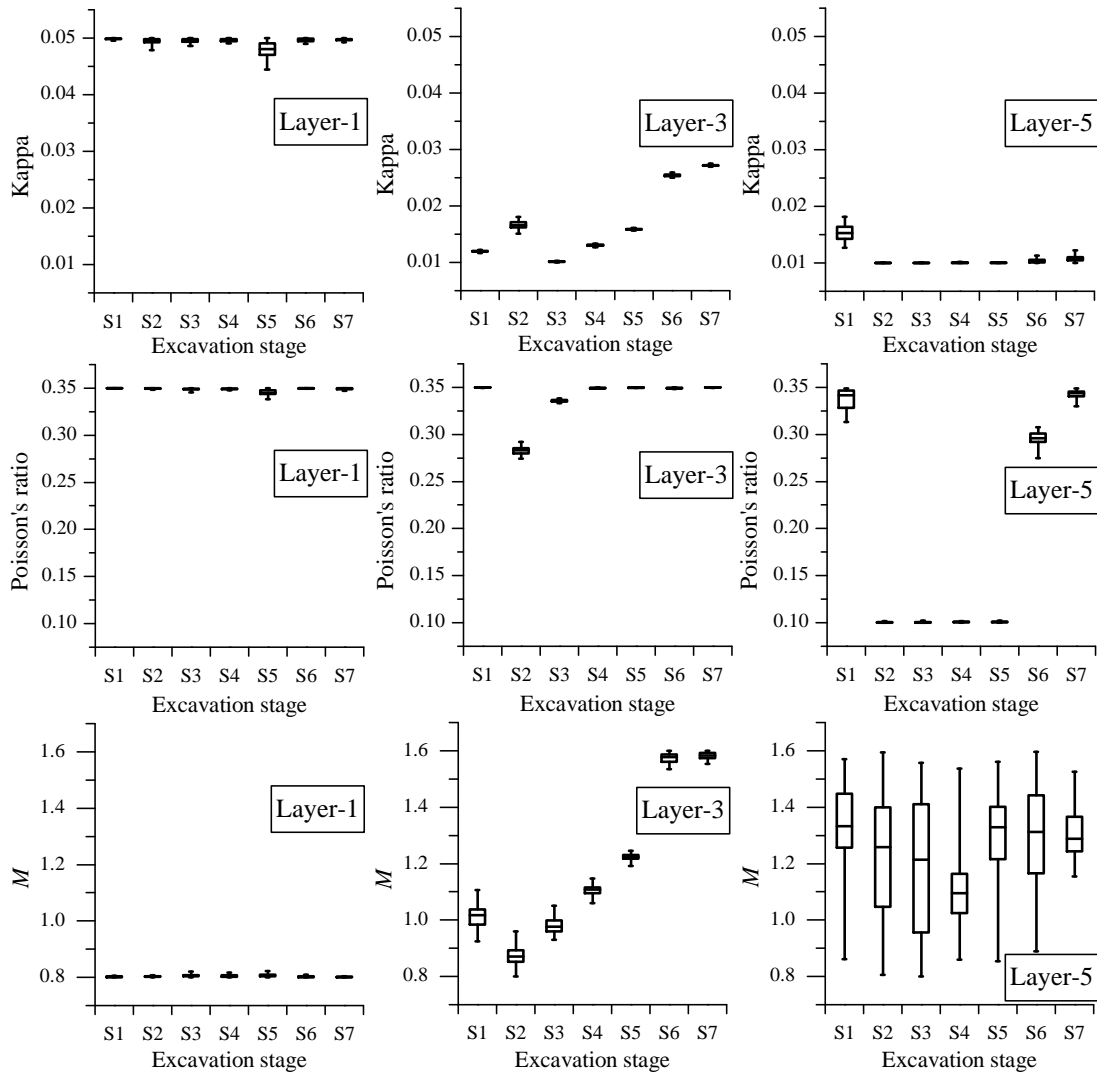
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974 Figure 12. Boxplots of the back-analyzed parameters of TNEC site

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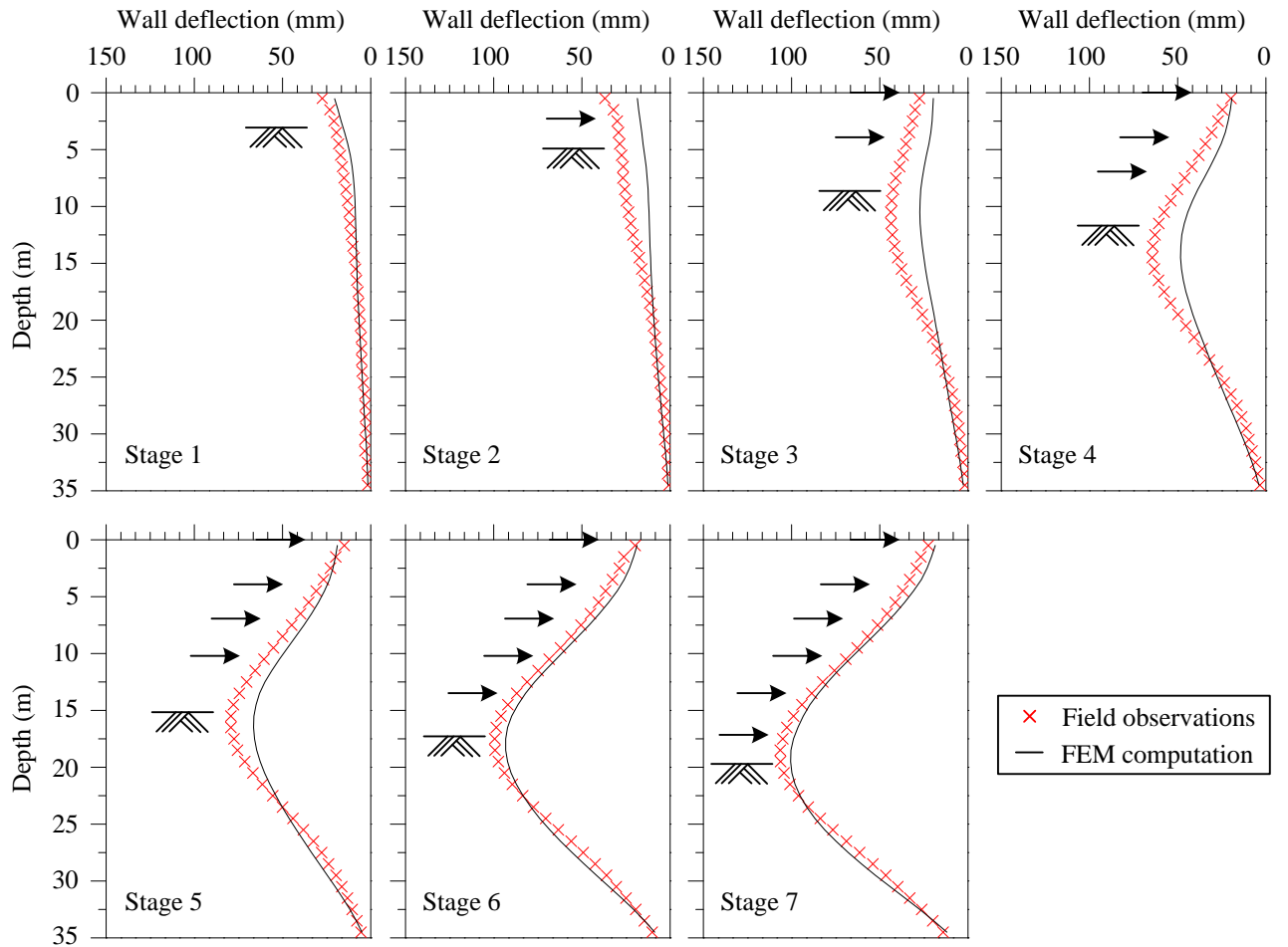
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985 Figure 13. Comparison of observed and calculated wall deflections at various

986 excavation stages

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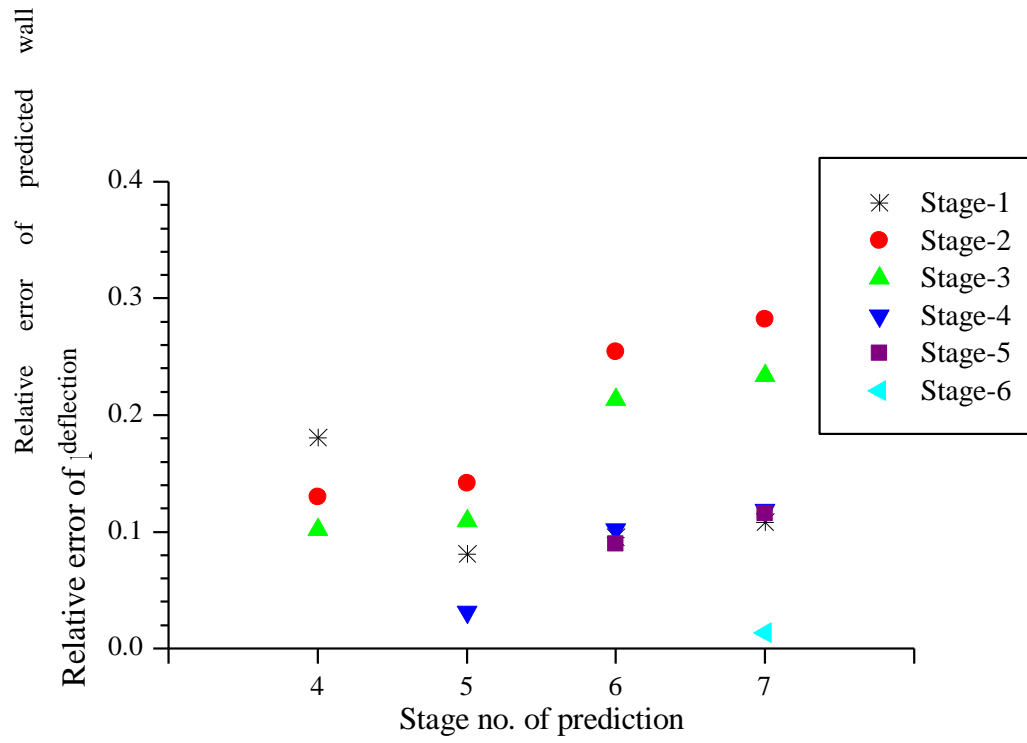
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999 Figure 14. Relative error of prediction at each stage with optimized parameters

1000 obtained from back analysis of previous stages

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