Fast Adaptive Minorization-Maximization Procedure for Beamforming Design of Downlink NOMA Systems

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***Abstract*—We develop a novel technique to accelerate minorization-maximization (MM) procedure for the non-orthogonal multiple access (NOMA) weighted sum rate maximization problem. Specifically, we exploit the Lipschitz continuity of the gradient of the objective function to adaptively update the MM algorithm. With fewer additional analysis variables and low complexity second-order cone program (SOCP) to solve in each iteration of the MM algorithm, the proposed approach converges quickly at a small computational cost. By numerical simulation results, our algorithm is shown to greatly outperform known solutions in terms of achieved sum rates and computational complexity.**

***Index Terms*—5G, NOMA, majorization-minimization, algorithm acceleration.**

I. Introduction

Fifth generation (5G) networks are expected to provide a complete paradigm shift in contemporary wireless services. The minimum technical requirements of 5G systems have been approved in International Mobile Telecommunications-2020 (IMT-2020) [1]. Specifically, IMT-2020 promises to provide superior performance in the three cases of utilization including mobile broadband, low latency communications, and massive machine type communications [1]. 3rd Generation Partnership Project (3GPP) has been working on 5G standardization following the time guidelines of International Telecommunication Union Radiocommunication Sector (ITU-R) with Release 16 scheduled in late 2019 [2]. The concept of non-orthogonal multiple access (NOMA) is being considered as a promising multiple access technique for 5G new radio (NR) to meet the benchmarks set in IMT-2020 [3].

Extensive research has been carried out on various aspects of 5G and non-orthogonal multiple access (NOMA) systems [4], [5]. Power allocation for a two user NOMA system with fairness under consideration was studied in [6]. Noticeably, Hanif *et al.* [7] used minorization-maximization (MM)[1] strategy to solve sum rate maximization in the downlink of multiple-input single-output (MISO) NOMA. The approach in [7] relies heavily on introduction of additional variables thus making it slow and computationally expensive. The secrecy sum rate maximization problem for a NOMA network where each user is equipped with a single antenna was considered in [9]. Yang *et al.* conducted a performance analysis and optimization theoretic study of space division multiple access NOMA with limited feedback in [10]. From the perspective of optimal channel assignment, power allocation for sum rate maximization with two user NOMA was studied in [11]. In [12], Zhang *et al.* used geographical information to perform multi-group multicast operation with NOMA. In their recent work, Zhu *et al.* [13] proposed beamforming design to maximize the weighted sum rate for MISO NOMA based on a rank constrained formulation.

Contributions: In this correspondence, we consider the weighted sum rate maximization problem for downlink MISO NOMA systems. We note that the known solutions are highly complex and the algorithmic efficiency has not been the primary focus of these designs. To this end, we address the issue of devising a computationally efficient algorithm for the considered problem. Our major contributions include: a) obtaining an equivalent formulation of the main problem without introducing a large number of additional analysis variables; b) exploiting the MM approach to produce a simplified convex problem solved in the n^{th} step of the proposed algorithm; c) accelerating the iterative algorithm by prudently using the Lipschitz continuous gradient property of the objective function; and d) last but not least, conducting numerical experiments to demonstrate the superiority of the proposed approach.

The rest of the paper is organized as follows. Section II presents the system model and problem formulation. Proposed solution and numerical results are discussed in Section III and Section IV, respectively. Finally, conclusions are drawn in Section V.

Notation: We will use boldface letters to denote vectors. The k^{th} element of the vector $\mathbf{x}$ is denoted by x_k. $\mathbf{x}^T$ is the transpose of $\mathbf{x}$, while $\mathbf{x}^H$ is the Hermitian transpose of $\mathbf{x}$. $\mathbf{1}_{m\times n}$ denotes an $m\times n$ matrix of all 1s. The Euclidean norm of $\mathbf{x}$ is denoted by $||\mathbf{x}||_2$. The value of a variable y after the n^{th} step of an iterative process is $y^{(n)}$. We denote the absolute

M. Juntti's work has been supported in part by 6Genesis Flagship (grant 318927) funded by the Academy of Finland. This publication has emanated from research supported in part by a Grant from Science Foundation Ireland under Grant number 17/CDA/4786.

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[1]Here we remark that the terminology of minorization-maximization is standard for maximization problems in the context of MM algorithms [8].

value of a complex number c by $|c|$. $\mathrm{Re(c)}$ and $\mathrm{Im(c)}$ are the real and imaginary components of c, respectively.

II. System Model and Problem Formulation

A. System Model

We consider the downlink of a single cell system, where the base station (BS) equipped with N antennas adopts beamforming to serve K single-antenna users. To facilitate NOMA, each user k is assumed to be capable of performing successive interference cancellation (SIC). The equivalent baseband channel for user k is denoted by $\mathbf{h}_k \in \mathbb{C}^{N\times 1}$ and assumed to be perfectly known to the BS. The BS multiplies the data symbol s_k intended for user k by a beamforming vector $\mathbf{w}_k$, and transmits a superposition of all weighted symbols. The received signal at user k is

$$y_k = \sum_{j=1}^{k-1} \mathbf{h}_k^H \mathbf{w}_j s_j + \mathbf{h}_k^H \mathbf{w}_k s_k + \sum_{j=k+1}^{K} \mathbf{h}_k^H \mathbf{w}_j s_j + n_k \quad (1)$$

where n_k represents the zero-mean Gaussian noise experienced by user k. Without loss of generality, we assume that the noise variance, σ^2, is same at all K users. For SIC, a user first decodes the strongest signal and removes the decoded signal from its received signal. The process is repeated until the user is able to decode its own data. Without loss of generality, we assume that $||\mathbf{h}_1||_2 < ||\mathbf{h}_2||_2 < \cdots < ||\mathbf{h}_K||_2$.

Here we remark that finding the optimal decoding order remains an open problem. User k successively decodes and subtracts $s_1, ..., s_{k-1}$ from y_k, then decodes s_k treating $s_{k+1}, ..., s_N$ as noise. Thus, the achievable rate at the k^{th} user is

$$R_k = \log\left(1 + \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{\sum_{j=k+1}^{K} |\mathbf{h}_k^H \mathbf{w}_j|^2 + \sigma^2}\right) \quad \text{(nats/s/Hz)}. \quad (2)$$

It is important to note that to achieve (2), we implicitly assume the signal of user k can be successfully decoded at all subsequent users $j > k$. This is only possible if the data rate of user k is bounded by

$$R_k \le \log\left(1 + \frac{|\mathbf{h}_j^H \mathbf{w}_k|^2}{\sum_{m=k+1}^{K} |\mathbf{h}_j^H \mathbf{w}_m|^2 + \sigma^2}\right), j = k+1, \ldots, K \quad (3)$$

Combining (2) and (3) the achievable rate at the kth user is

$$R_k = \begin{cases} \log\left(1 + \frac{|\mathbf{h}_K^H \mathbf{w}_K|^2}{\sigma^2}\right) & k = K \\ \log\left(1 + \min\limits_{j\in[k,K]} \frac{|\mathbf{h}_j^H \mathbf{w}_k|^2}{\sum\limits_{m=k+1}^{K} |\mathbf{h}_j^H \mathbf{w}_m|^2 + \sigma^2}\right) & k = [1, K-1] \end{cases} \quad (4)$$

where the notation $j \in [k, K]$ denotes $j \in \{k, k+1, \ldots, K\}$. To perform SIC successfully in the stated user ordering, we also impose the following constraints

$$|\mathbf{h}_k^H \mathbf{w}_j|^2 \ge |\mathbf{h}_k^H \mathbf{w}_{j+1}|^2, \; k = 1, ..., K, \; j = 1, ..., K-1. \quad (5)$$

The constraints in (5) correspond to the power ordering used in the context of power domain single-input single-output downlink NOMA in [5, Sec. II]. Further to this, the constraints in (5) also ensure that the interfering signal strength is lower compared to that of the desired signal. Hence, (5) also facilitates in boosting the sum data rate of the system.

B. Problem Formulation

We are interested in the weighted sum rate maximization of the NOMA system considered in Sec. II-A subject to a sum power constraint. This problem is mathematically stated as

$$\underset{\{\mathbf{w}_j\}}{\text{maximize}} \quad \sum\nolimits_{k=1}^{K} \alpha_k R_k \quad (6a)$$

$$\text{subject to} \quad (5) \;\&\; \sum\nolimits_{j=1}^{K} ||\mathbf{w}_j||_2^2 \le P_{\text{tx}} \quad (6b)$$

where α_k, a positive constant associated with user k, maintains user fairness, R_k is given in (4) and P_{tx} is the maximum power available to the transmitter. In addition to providing trade-off between fairness and sum rate in resource allocation process, varying α_k in weighted sum rate maximization can be used to trace the entire boundary of the rate region of NOMA beamforming.

Note that to solve (6) the authors in [7] introduce several auxiliary variables to arrive at a second order cone program (SOCP) at each iteration of their algorithmic solution to the NOMA sum rate problem. In total, their method requires the introduction of at least quadratic in K additional analysis variables, which is not favourable in practice, for instance, in large-scale antenna schemes. Further, the effect of the large number of analysis variables in [7] is also manifested in the form of slow convergence, as we will see in the numerical results section.

III. Proposed Solution

The proposed solution is built upon the MM algorithm with only K additional analysis variables. First we reformulate (6) equivalently as

$$\underset{\mathbf{w},\boldsymbol{\gamma}\ge 0}{\text{maximize}} \quad \sum\nolimits_{k=1}^{K} \alpha_k \log(1 + \gamma_k) \quad (7a)$$

$$\text{subject to} \quad \sum\nolimits_{m=k+1}^{K} |\mathbf{h}_j^H \mathbf{w}_m|^2 + \sigma^2 \le \frac{|\mathbf{h}_j^H \mathbf{w}_k|^2}{\gamma_k}, \quad k = 1, ..., K, \; j = k, ..., K, \; \text{ and (6b)} \quad (7b)$$

where $\mathbf{w}$ and $\boldsymbol{\gamma}$ denote the vectors that contain all $\mathbf{w}_k$, stacked column-wise, and γ_k, respectively. Note that we have newly introduced K optimization variables i.e., $\gamma_k, k = 1 \ldots K$. The non-convexity of (7) is due to (7b) and (5). Let $\mathbf{w}_j^{(n)}$ denote the beamforming vector in the n^{th} iteration. Then, in light of the MM technique we can lower bound the left side of (5) as

$$\begin{aligned} |\mathbf{h}_k^H \mathbf{w}_j|^2 \ge& |\mathbf{h}_k^H \mathbf{w}_j^{(n)}|^2 + 2\mathrm{Re}(\mathbf{w}_j^{(n)H} \mathbf{H}_k (\mathbf{w}_j - \mathbf{w}_j^{(n)})) \\ \triangleq& f(\mathbf{h}_k, \mathbf{w}_j; \mathbf{w}_j^{(n)}) \end{aligned} \quad (8)$$

where $\mathbf{H}_k \triangleq \mathbf{h}_k \mathbf{h}_k^H$ and for ease of notation, we write $\mathbf{w}_j^{(n)H}$ instead of $(\mathbf{w}_j^{(n)})^H$. In fact, $f(\mathbf{h}_k, \mathbf{w}_j; \mathbf{w}_j^{(n)})$ is simply an affine approximation of $|\mathbf{h}_k^H \mathbf{w}_j|^2$ at $\mathbf{w}_j^{(n)}$.

We now turn our attention to (7b) and note that the right side of the inequality is *jointly* convex in $\mathbf{w}_k$ and γ_k. Thus, we can find a convex approximation to (7b) by linearizing its right side, producing the following lower bound

$$\frac{|\mathbf{h}_j^H\mathbf{w}_k|^2}{\gamma_k} \geq \frac{|\mathbf{h}_j^H\mathbf{w}_k^{(n)}|^2}{\gamma_k^{(n)}} + \frac{2}{\gamma_k^{(n)}}\mathrm{Re}(\mathbf{w}_k^{(n)H}\mathbf{H}_j(\mathbf{w}_k - \mathbf{w}_k^{(n)})) - \frac{|\mathbf{h}_j^H\mathbf{w}_k^{(n)}|^2}{(\gamma_k^{(n)})^2}(\gamma_k - \gamma_k^{(n)}) \triangleq g(\mathbf{h}_j, \mathbf{w}_k, \gamma_k; \mathbf{w}_k^{(n)}, \gamma_k^{(n)}) \quad (9)$$

where $\gamma_k^{(n)}$ represents γ_k in the n^{th} iteration of our algorithm to be proposed in the ensuing discussion. Using the bounds in (8) and (9), in the $(n+1)^{\text{th}}$ iteration of our algorithm, we solve the following convex problem

$$\underset{\mathbf{w},\boldsymbol{\gamma}\geq 0}{\text{maximize}} \ \sum\nolimits_{k=1}^{K} \alpha_k \log(1+\gamma_k) \quad (10a)$$

$$\text{subject to} \ \sum\nolimits_{m=k+1}^{K} |\mathbf{h}_j^H\mathbf{w}_m|^2 + \sigma^2 \leq g(\mathbf{h}_j, \mathbf{w}_k, \gamma_k; \mathbf{w}_k^{(n)}, \gamma_k^{(n)}), k = [1, K], j = [k, K] \quad (10b)$$

$$|\mathbf{h}_k^H\mathbf{w}_{j+1}|^2 \leq f(\mathbf{h}_k, \mathbf{w}_j; \mathbf{w}_j^{(n)}), k = [1, K], j = [1, K-1],$$

$$\sum\nolimits_{j=1}^{K} ||\mathbf{w}_j||_2^2 \leq P_{\text{tx}} \quad (10c)$$

Algorithm 1-CS/AS MM-based algorithm for solving (6)

Generate feasible $\mathbf{w}^{(0)}$ and $\boldsymbol{\gamma}^{(0)}$ and set $n := 0$
repeat
  Solve (10) with surrogate objective to obtain optimal $\mathbf{w}^*$ and $\boldsymbol{\gamma}^*$
  `// CS and AS correspond to constant and`
  `// adaptive surrogate functions,`
  `// respectively.`
  Update $\mathbf{w}^{(n+1)} := \mathbf{w}^*$, $\boldsymbol{\gamma}^{(n+1)} := \boldsymbol{\gamma}^*$, and $n := n+1$
until convergence is achieved

We remark that the objective in (10a) is concave and thus (10) is a convex problem, which can be solved optimally in polynomial time. That said, problem (10) is a mix of exponential (i.e. non-symmetric cone) and quadratic cones for which dedicated solvers[2] are limited, and the computational complexity is not completely known [14]. Thus it is natural to find some high-quality approximate representation of an exponential cone, like using a system of SOCs as done in [15], but at the cost of increased complexity. It is also possible to reformulate (10) as an SOCP for the special case of sum rate maximization considered in [7] (i.e., $\alpha_k = 1, k = 1, 2, \ldots, K$). More specifically, we can equivalently replace the objective of (10) with the geometric mean of $(1+\gamma_k)$'s which can be represented by a system of second-order cone (SOC) constrains [7]. Consequently, (10) reduces to an SOCP. For the general case, the trick introduced in [15] can be used to reformulate (10) as an SOCP. For both these scenarios, several additional variables need to be introduced.

To obtain a more efficient algorithm we further approximate the objective of (10) by a quadratic form, and reformulate the resulting problem as an SOCP with minimal overhead.[3] In the context of the MM algorithm, we need to find a conic surrogate function that minorizes the objective function. To this end, we recall that $\log(1+\gamma_k)$ is 1-Lipschitz continuous gradient[4] on the domain $\gamma_k \geq 0$, and the following inequality holds [17]

$$\log(1+\gamma_k) \geq \log\bigl(1+\gamma_k^{(n)}\bigr) + \frac{1}{1+\gamma_k^{(n)}}\bigl(\gamma_k - \gamma_k^{(n)}\bigr) - \theta\bigl(\gamma_k - \gamma_k^{(n)}\bigr)^2 \triangleq h_\theta(\gamma_k; \gamma_k^{(n)}) \quad (11)$$

for $\theta \geq \frac{1}{2}$. Hence, we employ $h_\theta(\gamma_k; \gamma_k^{(n)})$ as a surrogate of $\log(1+\gamma_k)$ in the objective function of (10). We note that second-order Taylor series expansion does not lead to the lower bound in (11). The lower bound (11) follows directly from [17, Eq. (24)] by first noting that a function f, whose gradient ∇f has Lipschitz constant L, can be upper bounded as follows

$$f(x) \leq f(y) + \nabla f(y)^T(x-y) + \frac{1}{2\lambda}\|x-y\|_2^2 \quad (12)$$

where $\lambda \in (0, 1/L]$ [18]. The gradient of $g(x) = -\log(1+x)$, $x > 0$ is Lipschitz continuous with parameter $L = 1$, since

$$\|\nabla g(x_1) - \nabla g(x_2)\|_2 = \left|\frac{x_1 - x_2}{(1+x_1)(1+x_2)}\right| \leq |x_1 - x_2| \quad (13)$$

where the last inequality follows from $(1+x_1)(1+x_2) > 1$ for $x_1, x_2 > 0$. Now a straightforward application of (12) leads to the bound in (11). Our proposed solution is outlined in **Algorithm 1-CS/AS**. By **Algorithm 1-CS** we mean the version in which the surrogate function with a fixed θ is employed. Likewise, **Algorithm 1-AS** is used to refer to the version where θ is made adaptive as we describe in the next section.

A. Making Surrogate Function Adaptive

In order to guarantee the monotonic convergence of **Algorithm 1-CS**, $h_\theta(\gamma_k; \gamma_k^{(n)})$ should be a lower bound on the objective function. Moreover, to make the bound as tight as possible, setting $\theta = 1/2$ is a natural choice. However, as mentioned by Nesterov in [19, pp. 71], monotonicity (which he refers to as relaxation) is never the key to deriving fast converging algorithms. Following this philosophy, we abandon the requirement of monotonicity in the above algorithm and allow the constant θ to be small and adaptive to further improve its convergence rate. To this end we remark that owing to adaptive $\theta_k^{(n)}$, we have a loose approximation of the objective permitting efficient implementation in some early iterations but ensuring the approximation is tight at convergence. This results in attaining the stabilized objective value quickly as the number of iterations increase. To study the behaviour of our approximating function, we plot $h_\theta(\gamma_k; \gamma_k^{(n)})$ for different values of $\gamma_k^{(n)}$ and θ in Fig. 1. It is seen that in the vicinity

[2]CVX may be involved to solve this type of problems but the accuracy is not guaranteed.

[3]The resulting problem is a quadratically constrained quadratic program which can be solved directly by some dedicated solvers. However, reformulating it as an SOCP can be computationally beneficial because quadratic cones are simpler than quadratic functions [16].

[4]A differentiable function $f(x)$ is said to have an L-Lipschitz continuous gradient if $|f'(x) - f'(y)| \leq L|x-y|$ for some $L > 0$.

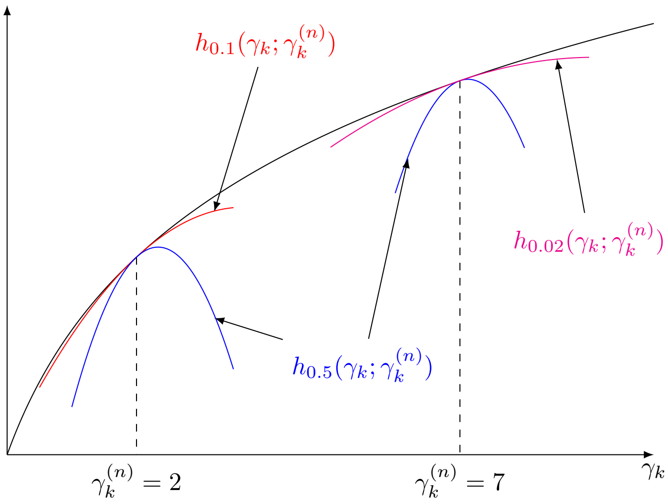

Fig. 1. Illustration of $\log(1+\gamma_k)$ and the surrogate function $h_\theta(\gamma_k;\gamma_k^{(n)})$ at $\gamma_k^{(n)}=2$ and at $\gamma_k^{(n)}=7$ for different θ.

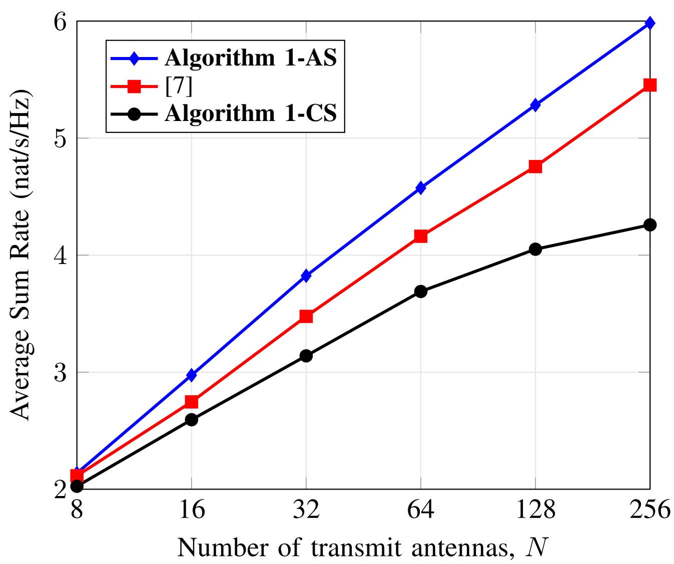

Fig. 2. Average sum rates of algorithms against the number of transmit antennas, N. The number of users is $K=3$.

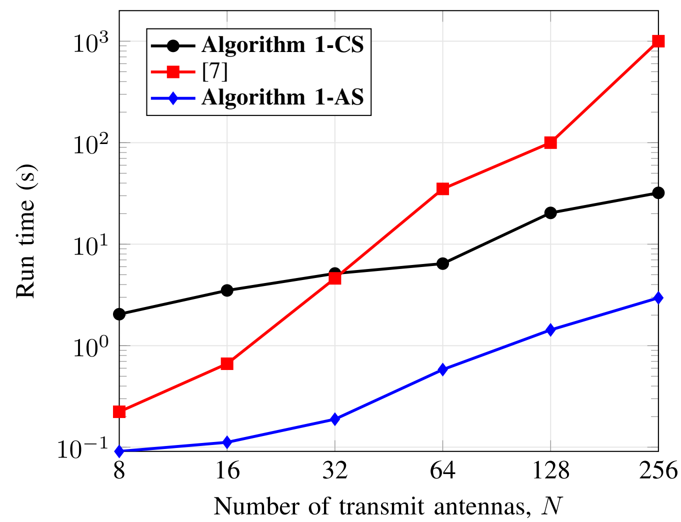

Fig. 3. Average run time against the number of transmit antennas, N. The number of users is $K=4$.

of a given $\gamma_k^{(n)}$, θ can be small for $h_\theta(\gamma_k;\gamma_k^{(n)})$ to be a good lower bound in practice. Furthermore, it is also observed that for the best fitting curves θ is roughly inversely proportional to $\gamma_k^{(n)}$. Motivated by these findings we propose the following rule to update θ in **Algorithm 1-CS/AS**

$$\theta_k^{(n)}=\frac{n}{nc+\alpha\left(1+\gamma_k^{(n)}\right)^2} \tag{14}$$

for some constants $\alpha>0$ and $0<c\leq 2$. The update rule in (14) is empirically motivated for our problem, and to the best of authors' knowledge, is not known. The update given in (14) is used in step 3 of **Algorithm 1-CS/AS**. Consequently, we have **Algorithm 1-AS**. The idea is to use a small value of $\theta_k^{(n)}$ for first few iterations to achieve a good approximation to the objective. When $\gamma_k^{(n)}$ is not large initially, small $\theta_k^{(n)}$ can be attained by appropriately choosing α and c. As the number of iterations becomes large, the impact of α reduces and by suitably choosing c within its defined range, $\theta_k^{(n)}$ is changed such that it is as close as possible to being greater than or equal to 0.5. Analytically determining optimal values of α and c remains open.

B. Convergence and KKT Point Attainment

We will consider the convergence of **Algorithm 1-AS**. The convergence of **Algorithm 1-CS** follows on similar lines. First notice that since $\gamma_k^{(n)}$ is bounded above due to finite transmit power, it is certain that $\theta_k^{(n)}>1/2$ for $n>n_0$ for some n_0. Now corresponding to the formulations in (7) and (10) we introduce some abstract notation for original functions and constraints and their approximations to comprehensively elaborate our arguments. Denote the i^{th} original function by $F_i(\mathcal{D}_i,\mathcal{V}_i)$, where $\mathcal{D}_i$ and $\mathcal{V}_i$ represent the sets of given data and optimization variables of the i^{th} function. Similarly, the i^{th} minorizing function corresponding to $F_i(\mathcal{D}_i,\mathcal{V}_i)$ is denoted by $MF_i(\mathcal{D}_i,\mathcal{V}_i;\mathcal{V}_i^{(n)})$, where $\mathcal{V}_i^{(n)}$ represents the set of optimization variables in the n^{th} iteration that act as parameters of the underlying minorization-maximization approach in our case. For $i=0$, $F_0(\mathcal{D}_0,\mathcal{V}_0)\triangleq\sum_{k=1}^{K}\alpha_k\log(1+\gamma_k)$ and $MF_0(\mathcal{D}_0,\mathcal{V}_0;\mathcal{V}_0^{(n)})\triangleq\sum_{k=1}^{K}\alpha_k h_\theta(\gamma_k;\gamma_k^{(n)})$ represent the objective and its minorizing function, respectively. Similarly, we denote the original and approximated constraints as $C_i(\mathcal{D}_i,\mathcal{V}_i)\triangleq H_i(\mathcal{D}_i,\mathcal{V}_i)-F_i(\mathcal{D}_i,\mathcal{V}_i)\leq 0$ and $AC_i(\mathcal{D}_i,\mathcal{V}_i;\mathcal{V}_i^{(n)})\triangleq H_i(\mathcal{D}_i,\mathcal{V}_i)-MF_i(\mathcal{D}_i,\mathcal{V}_i;\mathcal{V}_i^{(n)})\leq 0$, respectively, where $H_i(\mathcal{D}_i,\mathcal{V}_i)$ is not required to be approximated. It is easy to verify the following properties: (a) $F_i(\mathcal{D}_i,\mathcal{V}_i)\geq MF_i(\mathcal{D}_i,\mathcal{V}_i;\mathcal{V}_i^{(n)}),\forall\mathcal{V}_i$, (b) $F_i(\mathcal{D}_i,\mathcal{V}_i^{(n)})=MF_i(\mathcal{D}_i,\mathcal{V}_i^{(n)};\mathcal{V}_i^{(n)})$, (c) $\nabla F_i(\mathcal{D}_i,\mathcal{V}_i^{(n)})=\nabla MF_i(\mathcal{D}_i,\mathcal{V}_i^{(n)};\mathcal{V}_i^{(n)})$. If $\mathcal{V}_i^{(n)}$ are feasible for $C_i(\mathcal{D}_i,\mathcal{V}_i)\leq 0$, these are also feasible for $AC_i(\mathcal{D}_i,\mathcal{V}_i;\mathcal{V}_i^{(n)})\leq 0$ due to (b). We also observe that $AC_i(\mathcal{D}_i,\mathcal{V}_i^{(n+1)};\mathcal{V}_i^{(n)})\leq 0$. Now the feasibility of

$\mathcal{V}_i^{(n+1)}$ for $C_i(\mathcal{D}_i, \mathcal{V}_i) \leq 0$ follows from (a), because $C_i(\mathcal{D}_i, \mathcal{V}_i^{(n+1)}) \leq AC_i(\mathcal{D}_i, \mathcal{V}_i^{(n+1)}; \mathcal{V}_i^{(n)}) \leq 0$. Since $\mathcal{V}_i^{(0)}$ is feasible for the original problem, therefore, the sequence of variables will always satisfy $C_i(\mathcal{D}_i, \mathcal{V}_i) \leq 0$.
Now suppose the value of the objective function in the n^{th} run is $\mathcal{O}^{(n)}$. It is clear that

$$\begin{aligned}&\mathcal{O}^{(n+1)} = F_0(\mathcal{D}_0, \mathcal{V}_0^{(n+1)}) + MF_0(\mathcal{D}_0, \mathcal{V}_0^{(n+1)}; \mathcal{V}_0^{(n)}) - \\ &MF_0(\mathcal{D}_0, \mathcal{V}_0^{(n+1)}; \mathcal{V}_0^{(n)}) \geq F_0(\mathcal{D}_0, \mathcal{V}_0^{(n)}) + MF_0(\mathcal{D}_0, \mathcal{V}_0^{(n)}; \mathcal{V}_0^{(n)}) \\ &- MF_0(\mathcal{D}_0, \mathcal{V}_0^{(n)}; \mathcal{V}_0^{(n)}) = F_0(\mathcal{D}_0, \mathcal{V}_0^{(n)}) = \mathcal{O}^{(n)} \end{aligned} \tag{15}$$

where the inequality above follows due to (a) and (b). Hence, the objective function is monotonically non-decreasing and due to bounded power, it converges to a finite maximum value. Now we will show that at all points (and therefore at convergence as well) the algorithm satisfies the Karush-Kuhn-Tucker (KKT) conditions. For non-negative dual variables λ_i, ζ, the KKT conditions of (10) in terms of our compact notation can be written as:

$$\begin{aligned}&-\nabla MF_0(\mathcal{D}_0, \mathcal{V}_0^{(n)}; \mathcal{V}_0^{(n)}) + \sum_i \lambda_i \nabla AC_i(\mathcal{D}_i, \mathcal{V}_i^{(n)}; \mathcal{V}_i^{(n)}) + \\ &\zeta \nabla(\|\mathbf{w}_j^{(n)}\|_2^2 - P_{tx}) = 0 \end{aligned} \tag{16a}$$

$$\lambda_i AC_i(\mathcal{D}_i, \mathcal{V}_i^{(n)}; \mathcal{V}_i^{(n)}) = 0, \forall i, \zeta(\|\mathbf{w}_j^{(n)}\|_2^2 - P_{tx}) = 0 \tag{16b}$$

$$\lambda_i, \zeta \geq 0, \forall i, AC_i(\mathcal{D}_i, \mathcal{V}_i^{(n)}; \mathcal{V}_i^{(n)}) \leq 0. \tag{16c}$$

Due to the properties (b) and (c), it is straightforward to see that the KKT conditions in (16) are the same as that of the original problem.

C. Complexity Perspective

The number of analysis variables introduced in both **Algorithm 1-CS** and **Algorithm 1-AS** are K, and in [7] this number is $\mathcal{O}(K^2)$. Similarly, the number of constraints in both **Algorithm 1-CS** and **Algorithm 1-AS** are $\mathcal{O}(K^2)$, whereas, the number of constraints in the corresponding formulation of [7] are $\mathcal{O}(K^3)$. Specifically, the $\mathcal{O}(K^2)$ constraints in our formulation consist of $\mathcal{O}(KN)$ and $\mathcal{O}(N)$ dimensional SOC constraints. In comparison, the dimension of large SOCs in [7] is $\mathcal{O}(K^3)$. Owing to these estimates, our formulation is expected to have better complexity as we see in the numerical results section.

IV. Numerical Results

In this section, unless otherwise mentioned, the number of transmit antennas and transmit power are set as $N = 8$ and $P_{\text{tx}} = 43$ dBm, respectively. The channels are modeled as $\mathbf{h}_k = \sqrt{\text{PL}_k} \times \tilde{\mathbf{h}}_k$, where PL_k and $\tilde{\mathbf{h}}_k$ denote the path loss and small scale fading coefficients for each user k. The path loss is computed using the Stanford University Interim (SUI) path loss model in [20]. Entries of $\tilde{\mathbf{h}}_k$ are assumed to be uncorrelated and are drawn from a circularly complex Gaussian distribution with zero mean and unit variance. For **Algorithm 1-AS**, the parameters for updating $\theta_k^{(n)}$ in (14) are taken as $c = 2$ and $\alpha = 16$. The results are compared against the solution in [7]. We employ MOSEK [21] through the YALMIP toolbox [22] for numerical computations for all algorithms in comparison on the same PC to ensure fair and reliable results. We assume equal power allocation and generate $\mathbf{w}^{(0)}$ and $\boldsymbol{\gamma}^{(0)}$ as

$$\mathbf{w}_k^{(0)} = \sqrt{\frac{P_{\text{tx}}}{K \times N}} \mathbf{1}_{N \times 1}, \ \forall k \tag{17}$$

$$\gamma_k^{(0)} = \min_{j \in [k,K]} \left(\frac{1}{K-k} + \frac{\sigma^2}{|\mathbf{h}_j^H \mathbf{w}_j|^2} \right), \ \forall k. \tag{18}$$

In Fig. 2, the average sum rates of **Algorithm 1-CS**, **Algorithm 1-AS**, and the method in [7] are compared. A system with three users whose distances to the BS are 1000 m, 700 m and 400 m, respectively is considered. For wireless systems it is desirable that an optimization algorithm stabilizes quickly enough so that it responds to the changing data (wireless channels) and yields optimal solution corresponding to each data set. Thus, in our experiment, we set the maximum number of iterations for all algorithms under consideration to 5. If the difference between the objective values in two consecutive iterations is less than 10^{-4}, an algorithm is considered to have converged, and thus early termination occurs. Thus **Algorithm 1-CS/AS** can stop at 5 iterations even when full convergence is not achieved, and the same stopping criterion is also applied to the method of [7]. The sum rates obtained are averaged over 1000 sets of random channel realizations. We can see that **Algorithm 1-AS** achieves superior performance compared to [7] and **Algorithm 1-CS**. Since the objective function is adaptively updated in **Algorithm 1-AS**, the algorithm speeds up initially and stabilizes within 5 iterations for most channels. In contrast, the method presented in [7] needs more than 5 iterations to stabilize. While both [7] and **Algorithm 1-AS** converge to the same point if we do not limit the maximum number of iterations, the approach in [7] is slow to respond to varying channels. Moreover, the approach in [7] has higher complexity as shown in Sec. III-C, and is numerically sensitive to system parameters as discussed below. **Algorithm 1-CS** exhibits poor sum rate as the surrogate function is a quite crude approximation to the objective for $\theta = 1/2$.

In Fig. 3, we plot the average run time of the algorithms under consideration over 1000 channel realizations as a function of the number of transmit antennas. The number of users is set to $K = 4$ and their distances to the BS are 1000 m, 700 m, 400 m, 250 m, respectively. For all algorithms, the iterations cease when the difference between the objective values for four iterations is less than 10^{-4}. The codes are executed on a 64-bit desktop with 16 Gbyte RAM and Intel CORE i7 processor. As expected, **Algorithm 1-AS** outperforms the remaining algorithms, and **Algorithm 1-AS** is more efficient for large N. The algorithm in [7] works reasonably well for small N but its complexity increases quickly with N. **Algorithm 1-CS** is inferior as it requires many iterations to converge. It is pertinent to mention that beyond $N = 32$ MOSEK very often experiences numerical issues in one of the iterations when solving the convex subproblems in [7] and thus cannot converge. Among 1000 channel realizations, the method of [7] only returned a solution for 198 cases (i.e. around 20% of successful runs were observed). The run time for [7] in Fig. 3 is obtained by averaging the total required time for these successful runs.

Interestingly enough, this issue does not arise in Fig. 2 when the number of users is $K = 3$, meaning that numerical stability of the work of [7] is sensitive to large scale problems. We do not observe this behaviour in our proposed methods.

V. Conclusion

In this correspondence, we have derived a low complexity fast converging algorithm to maximize the downlink NOMA weighted sum rate. We have shown that by merely introducing K analysis variables and by exploiting the Lipschitz continuous gradient property of the objective function, our proposed algorithm shows a much improved convergence behaviour. The superiority of our approach is further ascertained by noting its low average computational times versus the number of transmit antennas.

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