A Comparison of the Uplink Performance of Cell-Free Massive MIMO using Three Linear Combining Schemes: Full-Pilot Zero Forcing with Access Point Selection, Matched-Filter and Local-Minimum-Mean-Square Error

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***Abstract*—In this paper, three types of linear receiver for the uplink of cell-free massive multiple-input-multiple-output (MIMO) will be studied to gain a clear comparison and understanding of their performance. In a cell-free massive MIMO system, a large number of randomly distributed access points (APs) cooperate to serve a much smaller number of users in the same time-frequency resource. The three receivers of interest are matched-filter (MF) combining, full-pilot zero forcing (fpZF) combining and the local-minimum-mean-squared error (L-MMSE) combining. The APs use locally obtained channel state information to perform the combining. Max-min fairness power control is utilised for the MF and fpZF combining to ensure uniformly good service for all users in the system. We note that max-min fairness power control is not required for the L-MMSE combining since the L-MMSE schem itself can provide the worst served users with the same spectral efficiency as the MF with max-min fairness. In this paper an AP selection scheme is proposed for the fpZF combining. In particular, the proposed AP selection scheme provides users with reasonably good spectral efficiency using a subset of APs rather than all APs serving all users, which proves to increase the overall energy efficiency of the system. The results show that the fpZF consistently outperforms the MF even while using only a subset of the APs and can outperform the L-MMSE with a suitable number of antennas per AP.**

***Index Terms*—Cell-free massive MIMO, linear receivers, spectral efficiency, uplink.**

I. Introduction

Cell-free massive MIMO refers to a wireless system where a large number of access points (APs) serve a much smaller number of users [1]. The concept of cell-free massive MIMO actually originates from previous mobile communication systems such as network MIMO [2] or coordinated multipoint with joint transmission (CoMP-JT) [3]. Cell-free massive MIMO has shown to provide significant improvements over the conventional small-cell system in terms of spectral and energy efficiency [4], [5].

Cell-free massive MIMO leverages two key properties - the increased macro-diversity and favourable propagation. The increased macro-diversity is caused by the AP subarray antennas being sufficiently separated. The channel gains to any randomly located user can be viewed as uncorrelated random variables. The favourable propagation means the directions of two user channels to be asymptotically orthogonal. This leads to little inter-user interference which improves spectral efficiency. Using multiple antennas at each AP has shown that cell-free massive MIMO gains the benefit from collocated MIMO of channel hardening [6].

Cell-free massive MIMO operates in time division duplex. In this way it can exploit channel reciprocity for the uplink and downlink data transmission to reduce the overhead [4]. More explicitly, the channels are estimated by the APs using an MMSE estimator in the uplink training phase [7]. These channel estimates are then used for the receive combining, of which three types are compared in this paper.

The three combining schemes have previously been investigated in [8]–[10], however no direct comparison of all three under identical conditions is available in the literature. In [11], four levels of receiver cooperation were implemented, comparing matched-filter (MF) to the MMSE receiver. Zero forcing was not considered however nor was the use of multiple antenna APs. The full-pilot zero forcing (fpZF) receiver was demonstrated in [12], however no AP selection scheme was used therein. An AP selection scheme has been proven to improve the performance of the system in terms of capacity and limitation of the backhaul loads and the hardware impairments [13]. The main contributions of this paper are as follows:

- A comparison is drawn between the three linear receivers considered in the paper, in terms of their spectral efficiency. The max-min fairness applied to the MF and fpZF combining ensures each user has a similar level of service. This is also the case for the L-MMSE without the power optimization as shown in [11]. We note that a comparison of the three linear receivers has not been available in the literature until now.
- An AP selection scheme is proposed for the uplink of cell-free massive MIMO using the fpZF combining scheme to facilitates a practical implementation. The goal is to provide users with a similar level of service as if all APs were used while using fewer system resources. To this end, location based pilot assignment is used to mitigate against the pilot contamination. Previous implementations of the fpZF were limited in the number of users that could be applied in the system. The proposed AP selection scheme provides a scalable implementation of the fpZF in which there are no limitations on the number of users that can be served.

II. Cell-free Massive MIMO System Model

We consider a cell-free massive MIMO system where M APs cooperate to serve K users in the same time-frequency resource under time division duplex (TDD) operation. Each AP is equipped with L antennas and each user transmits with a single antenna. The APs are randomly distributed in an area of size $D \times D$ km^2 with the users also being randomly located in the same area. The APs are all connected to a CPU via a backhaul network. The number of APs in the system is assumed to be much greater than the number of users ($M \gg K$). The coherence interval is divided into three phases: uplink training, uplink payload data transmission and downlink payload data transmission. The uplink training phase and uplink payload data transmission are considered in this paper.

Let τ_c represent the length of the coherence interval and τ_p the portion of the coherence interval dedicated to the uplink training phase. This is imposed with the condition that $\tau_p < \tau_c$. The remainder of the coherence interval which is of length $(\tau_c - \tau_p)$ is left for uplink and downlink payload data transmission. In the uplink training phase, users send pilot signals to the APs that are serving them in order for the APs to estimate the user's channels. We assume that the APs do not share their estimated CSI with each other to reduce the overhead. Downlink pilots are not normally considered for cell-free massive MIMO, meaning that users do not need to estimate their effective channel gain. They instead rely on the channel hardening property which is inherited from collocated massive MIMO. The channel between user k and AP m is given by

$$\mathbf{g}_{mk} = \beta_{mk}^{1/2} \mathbf{h}_{mk}, \tag{1}$$

where β_{mk} represents the large scale fading coefficient and is assumed to be known a priori. This changes very slowly and remains constant for several coherence intervals. In our case the large scale fading is independent of the antenna indexes at each AP. In (1), $\mathbf{h}_{mk}$, $k = 1, ..., K, m = 1, ..., M$, are $L \times 1$ vectors of small scale fading coefficients between the L antennas of AP m and user k. The small scale fading is assumed to be Rayleigh fading. This is assumed to be static during each coherence interval and changes from one coherence interval to the next. The elements of $\mathbf{h}_{mk}$ are i.i.d $\mathcal{CN}(0, 1)$ random variables (RV). Note that $\mathcal{CN}(0, \sigma^2)$ denotes a circularly symmetric complex Gaussian distribution with zero mean and variance σ^2. The APs in the system are assumed to be connected to the CPUs with perfect backhaul which negates the consideration of errors in communications between the APs and CPUs.

A. Uplink Training Phase

All K users transmit their pilot sequences of length τ_p simultaneously to all APs. The pilots are mutually orthogonal and normalized. Let $\sqrt{\tau_p}\boldsymbol{\varphi}_{i_k} \in \mathbb{C}^{\tau_p \times 1}$ where $||\boldsymbol{\varphi}_{i_k}||^2 = 1$ is the pilot sequence transmitted by user k, $k = 1, 2, ..., K$, where $\|\cdot\|$ is the Euclidean norm. Let $i_k \in \{1, ..., \tau_p\}$ be the index of the pilot used by user k. The $\tau_p \times 1$ received pilot vector at AP m is given by:

$$\mathbf{y}_{p,m} = \sqrt{\tau_p \rho_p} \sum_{k=1}^{K} \mathbf{g}_{mk} \boldsymbol{\varphi}_{i_k}^H + \mathbf{w}_{p,m} \tag{2}$$

where ρ_p is the normalized pilot transmit signal-to-noise ratio (SNR) of each pilot symbol, $\mathbf{w}_{p,m}$ is a $\mathbb{C}^{L \times \tau_p}$ noise matrix, $(\cdot)^H$ denotes conjugate transpose. The elements of the noise matrix are i.i.d $\mathcal{CN}(0, 1)$ RVs. The received pilot signal in (2) is then used by AP m to estimate the channel $\mathbf{g}_{mk}$, $k = 1, ..., K$. The symbol $\check{\mathbf{y}}_{p,mk}$ represents the projection of $\mathbf{y}_{p,m}$ onto $\boldsymbol{\varphi}_{i_k}$, given by

$$\check{\mathbf{y}}_{p,mk} = \sqrt{\tau_p \rho_p} \mathbf{g}_{mk} + \sqrt{\tau_p \rho_p} \sum_{k' \neq k}^{K} \mathbf{g}_{mk'} \boldsymbol{\varphi}_{i_{k'}}^H \boldsymbol{\varphi}_{i_k} + \boldsymbol{\varphi}_{i_k} \mathbf{w}_{p,m} \tag{3}$$

The minimum mean-square error (MMSE) channel estimate of $\hat{\mathbf{g}}_{mk}$ is

$$\begin{aligned} \hat{\mathbf{g}}_{mk} &= \frac{\mathbb{E}\{\mathbf{g}_{mk} \check{\mathbf{y}}_{p,mk}^H\}}{\mathbb{E}\{\check{\mathbf{y}}_{p,mk} \check{\mathbf{y}}_{p,mk}^H\}} \check{\mathbf{y}}_{p,mk} \\ &= c_{mk} \check{\mathbf{y}}_{p,mk}, \end{aligned} \tag{4}$$

where for the MF and L-MMSE combining:

$$c_{mk} = \frac{\sqrt{\tau_p \rho_p} \beta_{mk}}{\tau_p \rho_p \sum_{k'=1}^{K} \beta_{mk'} |\boldsymbol{\varphi}_{i_{k'}}^H \boldsymbol{\varphi}_{i_k}|^2 + 1} \tag{5}$$

and for the fpZF combining:

$$c_{mk} = \frac{\sqrt{\tau_p \rho_p} \beta_{mk}}{\tau_p \rho_p \beta_{mk} + 1}. \tag{6}$$

The channel estimate $\hat{\mathbf{g}}_{mk}$ has L independent and identical Gaussian components. The mean square estimate of component l of the channel vector $\hat{\mathbf{g}}_{mk}$, denoted by γ_{mk}, is calculated as:

$$\gamma_{mk} \triangleq \mathbb{E}\left\{ |[\hat{\mathbf{g}}_{mk}]_l|^2 \right\} \tag{7}$$

where for the MF and L-MMSE combining:

$$\gamma_{mk} = \frac{\tau_p \rho_p \beta_{mk}^2}{\tau_p \rho_p \sum_{k'=1}^{K} \beta_{mk'} |\boldsymbol{\varphi}_{i_{k'}}^H \boldsymbol{\varphi}_{i_k}|^2 + 1} \tag{8}$$

and for the fpZF combining:

$$\gamma_{mk} = \frac{\tau_p \rho_p \beta_{mk}^2}{\tau_p \rho_p \beta_{mk} + 1} \tag{9}$$

The channel estimation error is $\tilde{\mathbf{g}}_{mk} = \mathbf{g}_{mk} - \hat{\mathbf{g}}_{mk}$ and from the MMSE estimation property we can say that $\tilde{\mathbf{g}}_{mk}$ is independent of $\hat{\mathbf{g}}_{mk}$. Therefore the elements of the following three are i.i.d and can be written as:

$$\begin{aligned} &[\mathbf{g}_{mk}]_l \sim \mathcal{CN}(0, \beta_{mk}) \\ &[\hat{\mathbf{g}}_{mk}]_l \sim \mathcal{CN}(0, \gamma_{mk}) \\ &[\tilde{\mathbf{g}}_{mk}]_l \sim \mathcal{CN}(0, \beta_{mk} - \gamma_{mk}) \end{aligned} \tag{10}$$

B. Uplink Payload Data Transmission

The signal received at AP m is modelled as:

$$\mathbf{y}_{u,m} = \sqrt{\rho_u}\sum_{k=1}^{K}\mathbf{g}_{mk}\sqrt{\eta_k}q_k + \mathbf{w}_{u,m}. \tag{11}$$

The symbol sent by the k-th user is q_k, where $\mathbb{E}\{|q_k|^2\} = 1$. The normalized uplink SNR is denoted as ρ_u, the data power control coefficient for each user is η_k and is subject to the following constraint:

$$0 \leq \eta_k \leq 1, \forall k. \tag{12}$$

Finally $w_{u,m} \sim \mathcal{CN}(0,1)$ is the additive noise.

III. Spectral Efficiency Analysis

A. Matched Filter Combining

To detect the symbol user k sends, AP m sends $[\hat{\mathbf{g}}_{mk}]_l^* \mathbf{y}_{u,m}$ to the CPU via the backhaul. The signal received at the CPU is:

$$r_{u,k} = \sum_{m=1}^{M}\sum_{l=1}^{L}[\hat{\mathbf{g}}_{mk}]_l^*[\mathbf{y}_{u,m}]_l. \tag{13}$$

Here $(\cdot)^*$ denotes complex conjugate and $[\hat{\mathbf{g}}_{mk}]_l$ is element l of the vector $\hat{\mathbf{g}}_{mk}$. In a similar manner to [8], we decompose the received signal $r_{u,k}$ as follows:

$$r_{u,k} = \mathrm{DS}_k \cdot q_k + \mathrm{BU}_k \cdot q_k + \sum_{k'\neq k}^{K}\mathrm{UI}_{kk'}\cdot q_{k'} + \mathrm{N}, \tag{14}$$

where

$$\mathrm{DS}_k \triangleq \sqrt{\rho_u\eta_k}\mathbb{E}\left\{\sum_{m=1}^{M}\sum_{l=1}^{L}[\hat{\mathbf{g}}_{mk}]_l^*[\mathbf{g}_{mk}]_l\right\},$$
$$\mathrm{BU}_k \triangleq \sqrt{\rho_u\eta_k}\sum_{m=1}^{M}\sum_{l=1}^{L}[\hat{\mathbf{g}}_{mk}]_l^*[\mathbf{g}_{mk}]_l - \mathrm{DS}_k,$$
$$\mathrm{UI}_{kk'} \triangleq \sqrt{\rho_u\eta_k}\sum_{m=1}^{M}\sum_{l=1}^{L}[\hat{\mathbf{g}}_{mk}]_l^*[\mathbf{g}_{mk'}]_l,$$
$$\mathrm{N}_k \triangleq \sum_{m=1}^{M}\sum_{l=1}^{L}[\hat{\mathbf{g}}_{mk}]_l^*[\mathbf{w}_{u,m}]_l.$$

B. Full-Pilot Zero Forcing Combining

In the case of the fpZF combining, as $\tau_p < K$ some of the estimated channels will be parallel. This will result in $\hat{\mathbf{G}}_m = [\hat{\mathbf{g}}_{m1},...,\hat{\mathbf{g}}_{mK}] \in \mathbb{C}^{L\times K}$ being rank-deficient. Instead we construct the full rank matrix $\bar{\mathbf{G}}_m$, such that

$$\bar{\mathbf{G}}_m \triangleq \mathbf{y_{p,m}}\mathbf{\Phi} \in \mathbb{C}^{L\times\tau_p} \tag{15}$$

which is related to its actual channel estimate by

$$\hat{\mathbf{g}}_{mk} \triangleq c_{mk}\bar{\mathbf{G}}_m\mathbf{e}_{i_k} \tag{16}$$

where $\mathbf{\Phi} = \left[\varphi_1,...,\varphi_{\tau_p}\right]$. The fpZF combining vector can then be composed as:

$$\mathbf{a}_{mk} = \frac{\bar{\mathbf{G}}_m(\bar{\mathbf{G}}_m^H\bar{\mathbf{G}}_m)^{-1}\mathbf{e}_{i_k}}{\sqrt{\left\|\bar{\mathbf{G}}_m(\bar{\mathbf{G}}_m^H\bar{\mathbf{G}}_m)^{-1}\mathbf{e}_{i_k}\right\|^2}}, \tag{17}$$

where $\mathbf{e}_{i_k}$ is the i_kth column of $\mathbf{I}_{\tau_p}$. The signal received at the CPU is then:

$$r_{u,k} = \sum_{m=1}^{M}\mathbf{a}_{mk}^H\mathbf{y}_{u,m} \tag{18}$$

The components of (14) are then

$$\mathrm{DS}_k \triangleq \sqrt{\rho_u\eta_k}\sum_{m=1}^{M}\mathbb{E}\left\{\mathbf{a}_{mk}^H\mathbf{g}_{mk}\right\},$$
$$BU_k \triangleq \sqrt{\rho_u\eta_k}\sum_{m=1}^{M}\mathbf{a}_{mk}^H\mathbf{g}_{mk} - \mathrm{DS}_k,$$
$$\mathrm{UI}_{kk'} \triangleq \sqrt{\rho_u\eta_k}\sum_{m=1}^{M}\mathbf{a}_{mk}^H\mathbf{g}_{mk},$$
$$\mathrm{N}_k \triangleq \sum_{m=1}^{M}\mathbf{a}_{mk}^H\mathbf{w}_{u,m}.$$

The desired signal strength is represented by DS, the beamforming uncertainty gain by BU and the inter-user interference caused by user k' by UI respectively. Treating the second, third and fourth terms as noise, the following spectral efficiency for user k can be written as:

$$S_{u,k} = \log_2\left(1 + \frac{|\mathrm{DS}_k|^2}{\mathbb{E}\{|\mathrm{BU}_k|^2\} + \sum_{k'\neq k}^{K}\mathbb{E}\{|\mathrm{UI}_{kk'}|^2\} + \mathbb{E}|\mathrm{N}_k|^2}\right) \tag{19}$$

Closed form expressions can then be found for both the MF and fpZF combining. The expression in (20) for the MF combining was found using a similar method to that used in [8]. The expression in (21) for the fpZF combining was found using a similar method to that used in [12].

C. L-MMSE Combining

In the L-MMSE combining each AP preprocesses its signal by computing local estimates of the data that can then be passed to the CPU where final decoding takes place. The combining vector for the L-MMSE conbining is:

$$\mathbf{v}_{mk} = \rho_u\eta_k\left(\sum_{k'=1}^{K}\rho_u\eta_{k'}\left(\hat{\mathbf{g}}_{mk'}\hat{\mathbf{g}}_{mk'}^H + C_{mk'}\right) + \sigma^2\mathbf{I}_L\right)^{-1}\hat{\mathbf{g}}_{mk} \tag{23}$$

The local estimate at each AP is:

$$\check{q}_{mk} \triangleq \mathbf{v}_{mk}^H\mathbf{y}_{u,m} = \mathbf{v}_{mk}^H\mathbf{g}_{mk}q_k + \sum_{k'\neq k}^{K}\mathbf{v}_{mk}^H\mathbf{g}_{mk'}q_{k'} + \mathbf{v}_{mk}^H\mathbf{w}_{u,m} \tag{24}$$

$$S_{u,k}^{MF} = \frac{1-\frac{\tau_p}{\tau_c}}{2}\log_2\left(1+\frac{L^2\rho_u\eta_k\left(\sum_{m=1}^{M}\gamma_{mk}\right)^2}{L^2\rho_u\sum_{k'\neq k}^{K}\eta_{k'}|\boldsymbol{\varphi}_{i_{k'}}^H\boldsymbol{\varphi}_{i_k}|^2\left(\sum_{m=1}^{M}\gamma_{mk}\frac{\beta_{mk'}}{\beta_{mk}}\right)^2+L\rho_u\sum_{k'\neq 1}^{K}\eta_{k'}\sum_{m=1}^{M}\gamma_{mk}\beta_{mk'}+L\sum_{m=1}^{M}\gamma_{mk}}\right) \tag{20}$$

$$S_{u,k}^{fpZF} = \frac{1-\frac{\tau_p}{\tau_c}}{2}\log_2\left(1+\frac{\rho_u(L-\Theta_k)(\sum_{m=1}^{M}\sqrt{\gamma_{mk}})^2\eta_k}{\rho_u\sum_{m=1}^{M}\sum_{k\in\mathcal{U}_m}(\beta_{mk}-\gamma_{mk})\eta_k+M}\right) \tag{21}$$

$$S_{u,k}^{MMSE} = \frac{1-\frac{\tau_p}{\tau_c}}{2}\log_2\left(1+\frac{\rho_u\eta_k\left|\sum_{m=1}^{M}\mathbb{E}\{\mathbf{v}_{mk}^H\mathbf{g}_{mk}\}\right|^2}{\rho_u\sum_{k'=1}^{K}\eta_{k'}\mathbb{E}\left\{\left|\sum_{m=1}^{M}\mathbf{v}_{mk}^H\mathbf{g}_{mk'}\right|^2\right\}-\rho_u\eta_k\left|\sum_{m=1}^{M}\mathbb{E}\left\{\mathbf{v}_{mk}^H\mathbf{g}_{mk'}\right\}\right|^2+\sigma^2\sum_{m=1}^{M}\mathbb{E}\left\{\|\mathbf{v}_{mk}\|^2\right\}}\right) \tag{22}$$

The CPU can then create an estimate of the signal q_k from user k by taking an average of the local estimates similar to the method proposed in [11]:

$$\hat{q}_k = \frac{1}{M}\sum_{m=1}^{M}\breve{q}_{mk}. \tag{25}$$

Therefore $\hat{q}_k$ can be written as:

$$\hat{q}_k = \mathbf{b}_k^H\mathbf{v}_{mk}^H\mathbf{g}_{mk}q_k + \sum_{k'\neq k}^{K}\mathbf{b}_k^H\mathbf{v}_{mk}^H\mathbf{g}_{mk'}q_{k'} + \mathbf{b}_k^H\mathbf{v}_{mk}^H\mathbf{w}_{u,m} \tag{26}$$

where $\mathbf{b}_k = [1/M...1/M]^T$ and is the weighting coefficient. The expression for the spectral efficiency of the system using L-MMSE combining can then be obtained and is presented in (22).

IV. User Access Point Selection

The expression for the spectral efficiency of the fpZF differs from the other two combining schemes as it utilizes a user AP selection. This results in the term $\sum_{k\in\mathcal{A}_k}$ being present in the denominator rather than $\sum_{k=1}^{K}$. User AP selection schemes have previously been implemented for both the uplink and downlink in cell-free massive MIMO. However the AP selection in the literature is limited to the MF combining. The fpZF has been shown to provide higher per user spectral efficiencies but has been limited in its implementations as the number of antennas per AP must be greater than the number of independent users in the system. Using AP selection, we can now consider only the users that are also served by the subset of APs that serve user k when calculating user k's SINR. This allows for fewer antennas per AP than previously used in the literature and more users being served in the system.

The users are first assigned a pilot sequence using the location based pilot assignment detailed in [13]. This algorithm sets a radius around each user, inside which no two users can use the same pilot. This ensures that when a user selects which APs it will transmit to, it will not suffer pilot contamination in doing so. This allows us to use the expression in (21) which ignores any pilot contamination. Pilot contamination is one of the main limiting factors in cell-free massive MIMO. Two users sharing the same pilot will have channels in parallel which results in much higher levels of inter-user interference. The users can then select their subset of APs using the following equation:

$$\sum_{m=1}^{M_{0,k}}\frac{\bar{\beta}_{mk}}{\sum_{m'=1}^{M}\beta_{m'k}} \geq \delta\% \tag{27}$$

The proposed AP selection scheme is outlined in Algorithm 1. In (27), $\{\bar{\beta}_{1k},...,\bar{\beta}_{Mk}\}$ represents the sorted descending order

Algorithm 1 Largest Large-Scale Fading User AP Selection

1: User k calculates it large scale fading coefficients (β_{mk}) between it and all the APs.
2: The coefficients are then sorted in descending order and the user chooses all the APs necessary to statisfy the condition in (27)
3: The sets $\mathcal{U}_m$, $\mathcal{A}_k$ and Θ_k can then be determined.
4: Let $\hat{\gamma}_{mk} = \gamma_{mk}$ and $\hat{\beta}_{mk} = \beta_{mk}$ if $k \in \mathcal{U}_m$ or 0 otherwise, $k = 1,...,K, m = 1,...,M$

set of the set $\{\beta_{1k},...,\beta_{Mk}\}$ and $M_{0,k} \leq M$ is the number of values of $\bar{\beta}_{mk}$ that are required to satisfy the condition. δ corresponds to the total percentage contribution which is at the discretion of the CPU. $\mathcal{U}_m$ represents the group of users served by AP m, $\mathcal{A}_k$ represents the subset of APs that serve user k and Θ_k is the number of unique users that are also served by any AP in users k's $\mathcal{A}_k$ including user k themselves.

V. Numerical Results and Discussion

In order to ensure that all users receive an equal level of service, max-min fairness data power control is applied using the constraint presented in (12). The formulation of this problem is well documented in the literature and can be solved using the bisection method detailed in [15]. This method is applied to the MF and fpZF combining. It is not necessary to use this power control for the L-MMSE combining as it can provide similar performance as other combining schemes for the weakest users in the system, while providing much higher levels of spectral efficiency for all other users. In the figures

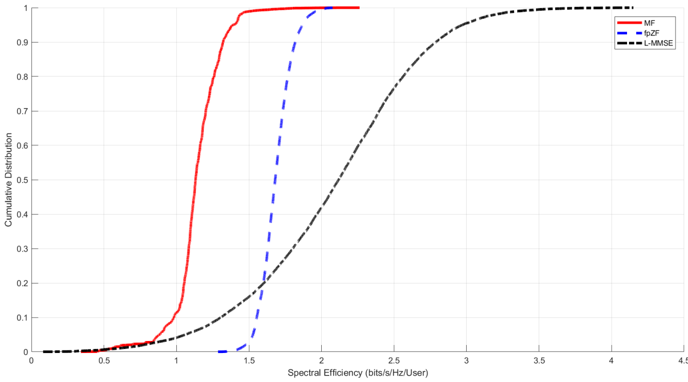

Fig. 1. Cumulative distribution of the per-user Spectral Efficiency for the three combining schemes. $M = 100$, $L = 8$, $\delta = 60\%$

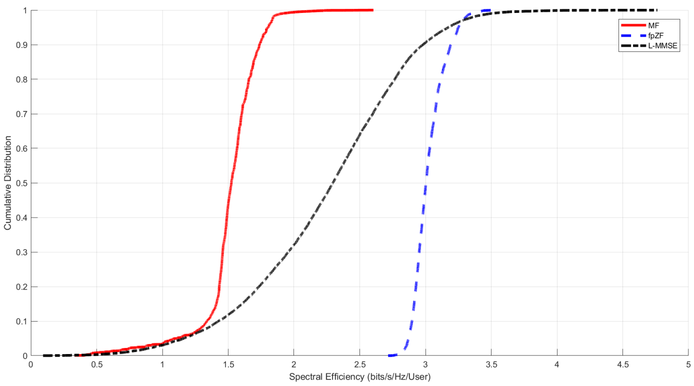

Fig. 2. Cumulative distribution of the per-user Spectral Efficiency for the three combining schemes. $M = 100$, $L = 16$, $\delta = 85\%$

we compare the per user overall spectral efficiency of three implementations of the cell-free massive MIMO system.

A. Parameters

In the three simulation setups below the following parameters are common: $\tau_c = 200$ (symbols), $\tau_p = 20$ (symbols), $K = 30$ and $\rho_p = \rho_u = 100$ mW. The area considered in the simulations is 1×1 km^2. All results are obtained by 400 random realizations of AP and user locations. The users are first assigned a pilot sequence, either randomly in the case of MF and L-MMSE or using the location-based pilot assignment for the fpZF. All pilots are transmitted with full power.

B. Analysis of Results

Figure 1 shows a system with $M = 100$, $L = 8$ and $\delta = 60\%$ for the condition (27) in order to select APs. As expected the L-MMSE matches the MF in its performance serving the users with the lowest spectral efficiencies. In comparing all three at the 90% likely mark the fpZF outperforms the MF by 58% and the L-MMSE by 19%. However the L-MMSE quickly overtakes the performance of the fpZF, providing 80% of users with higher values of spectral efficiency.
In Figure 2 we see the best performance of the fpZF in comparison to the other two combining schemes. Here $M = 100$,

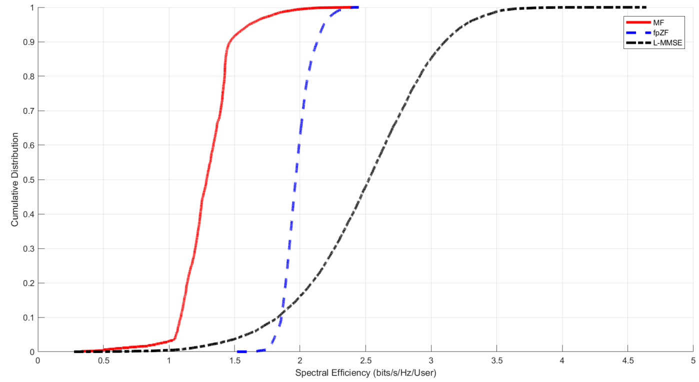

Fig. 3. Cumulative distribution of the per-user Spectral Efficiency for the three combining schemes. $M = 200$, $L = 8$, $\delta = 70\%$

$L = 16$ and $\delta = 85\%$ for the condition (27) in order to select APs. It is clear from the graph that fpZF vastly outperforms both other schemes for nearly all users in the system. The L-MMSE only outperforms the fpZF for less than 1% of the best served users in the systems. The increased performance of the fpZF can be attributed to two major factors. Firstly as the numerator of (21) depends directly on L, as the number of antennas per AP grows, the value L may be much larger than Θ_k. Secondly the value of δ can be higher. As stated before there can be more interfering users in a user k's $\mathcal{A}_k$ due to the greater number of antennas per AP. This allows users who receive the worst service to select more APs, and therefore increasing their spectral efficiency. The greater number of antennas per AP also allows users to receive a greater gain in spectral efficiency per AP and thus users with good quality of service require fewer APs to serve them.
In Figure 3 we trial a system with twice the number of APs as in the earlier two simulations. Here $M = 200$, $L = 8$ and $\delta = 70\%$ for the condition (27) in order to select APs. The system has the same number of overall antennas as in Figure 2, half the number of antennas per AP. All three combining schemes see improvements on the scheme in Figure 1 as expected, however only the L-MMSE outperforms its implementation in Figure 2. The fpZF shows an improvement of 70% on the MF at the 90% mark while the fpZF and L-MMSE have very similar performances with the fpZF being only 6% higher. The L-MMSE offers better quality of service for 88% of users.

VI. Conclusion

The majority of work conducted in researching cell-free massive MIMO has centred around the MF combining as it offers the least complexity. Though there are benefits to the simplicity of the MF combining, the higher spectral efficiency of the other linear receivers may offer more benefits. Previous implementations of the fpZF were limited by the number of users in the system and the number of antennas per AP. The fpZF with the AP selection proposed in this paper provides a practical and scalable implementation which consistently outperforms the MF combining while using fewer APs and therefore less of the systems resources. The fpZF shows very favourable performance when compared with the more complicated L-MMSE, which also uses all of the systems APs in the simulation. The results show that, with even just 16 antennas per AP, the fpZF can provide a better quality of service for the worst served users in comparison with the L-MMSE.

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