Assessment of the Condition of a Beam Using a Static Loading Test

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Abstract This paper presents a novel approach for structural damage detection. The cross-entropy method is used to estimate the structural parameters of a simply supported beam from its response to a static load consisting of a heavy vehicle. Various states of beam damage are modelled through local reductions in flexural rigidity. The proposed method estimates the distribution of stiffness throughout the beam using the deflections due to a heavy load located on the beam. Parameters such as initial distribution values, structural model, number of readings and sample size are varied to assess their influence on the prediction of the damage severity and its location.

Introduction

The monitoring of the condition of a bridge network is a vital part of any transport infrastructure management system. This paper investigates the use of the cross-entropy (CE) method for detecting and evaluating a change in the flexural stiffness of a bridge.

Damage Detection. The problem of estimating structural parameters from a limited number of measurements is an inverse problem with a non-unique solution. In recent years, many techniques have been developed to solve such problems as reviewed in [1,2]. Neural networks have been trained to recognise the structural response of healthy and damaged structures. Static deflection data have been used as input to neural networks to identify the structural parameters of a bridge truss [3]. Other techniques use spectral points in frequency response function data as input data to neural networks to identify damage to cantilever beams [4]. Probabilistic methods have also been proposed to detect damage to frame structures, where Bayesian analysis is used to estimate structural parameters [5, 6]. More frequently, changes in modal properties have been used to detect alterations in structural properties [7]. More recently, empirical modal decomposition and the Hilbert-Huang transform have been used to identify the instant at which damage occurs, by detecting spikes in the acceleration time history [8].

Static-Loading Test. Many of the existing damage detection strategies make use of dynamic data, based on forced or ambient vibrations. This paper uses the structural response derived from a static loading test. The conditions of this test could be those of a proof-loading test. In this paper a concentrated load, equivalent to two military tanks, is applied over 5m, centred at midspan [9]. Military tanks offer a relatively mobile concentrated load, causing bridges to be closed for only a short period of time for testing. By testing bridges in this way, it is possible to determine something of their actual load capacity, which may exceed that determined by conventional analysis. The current paper simulates beam deflections due to this static proof-loading test.

The Cross-Entropy Method. The CE method was first proposed by Rubenstein in 1997 [10]. The method can be summarised as an iterative procedure, with each iteration composed of two parts: 1. a random data sample is generated and 2. the method of generating the random data is altered, in order to produce a better sample in the next iteration [11].

The CE method has been applied to rare event simulations and combinatorial optimisation [12]. The current paper employs a CE algorithm for combinatorial optimisation to estimate beam stiffness. As a non-unique problem, a variety of stiffness combinations will give beams with similar deflection properties. The CE method is employed here to create distributions for stiffness values and to select various combinations from these distributions. As the algorithm progresses, the distributions are systematically altered to give selections with improved performance.

Characteristics of the Simulated Loading and Damaged Model

This section describes the modelling of the damage and the data employed for the beam and the load models. A finite element model of a simply supported beam, of span 15m, was created in Matlab [13]. Beam properties were selected to give a structure of stiffness comparable to a bridge of equivalent span. Static proof-loading of 1.4kN was spread over 5m, centred at midspan (Fig. 1).

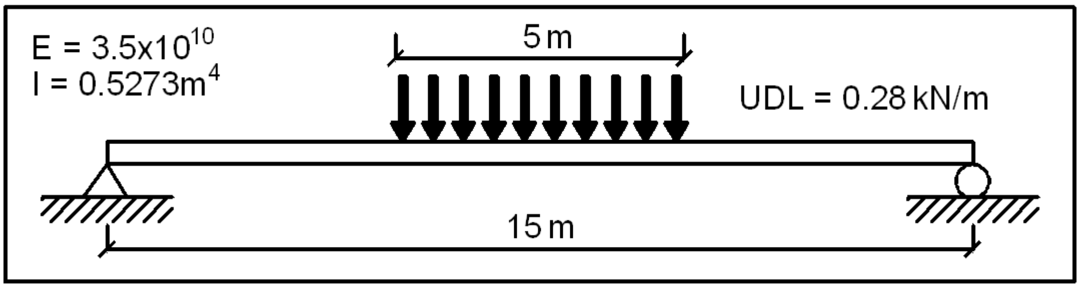

Figure 1 - Simply Supported Beam and Loading Test

All beam models were discretised into a mesh of 120 elements. Damage was modelled by a local reduction in beam stiffness. Fig. 2 shows the stiffness profile for a damaged beam, with cracking of 15% beam depth, located at 10m along the beam. Stiffness in elements close to the damage is also reduced to provide a more realistic model of cracking, as the material close to a crack may provide little contribution to stiffness [14, 15]. In the current paper, the CE algorithm estimates the stiffness of an equivalent beam, with at most 120 elements.

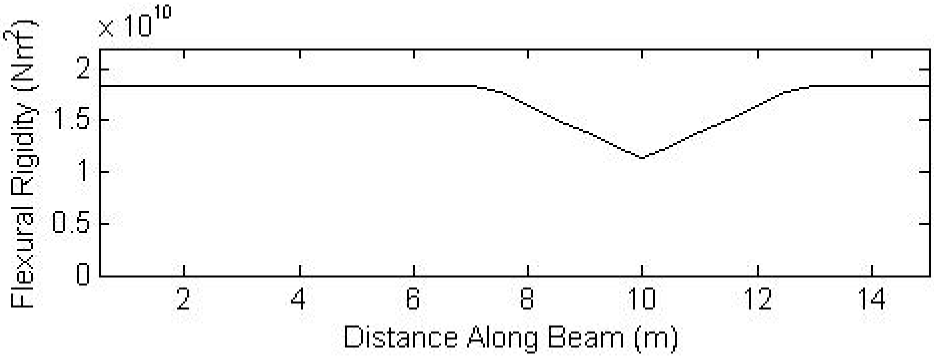

Figure 2 – Stiffness Profile for Damaged Beam

Description of the Cross Entropy Algorithm for Damage Detection

The CE algorithm attempts to identify the location and severity of damage to a simply supported beam given a number of deflection values resulting from a static loading test. The algorithm assembles stiffness values to create trial finite element beams. The structural response of each trial beam (TB) is then compared to that of the damaged beam. Each TB is composed of a number of elements, N_t, each with its own stiffness value.

At first, the stiffness of each element of the TB is represented by a series of values resulting from sampling an assumed normal distribution for each element. The distribution of stiffness for the element n is defined by a mean, μ_n, and a standard deviation, σ_n. The initial values, μ_{init} and σ_{init}, are chosen to give a broad range of possible values, with mean reasonably close to the known stiffness value for a healthy element. By randomly selecting a stiffness value from sampling the distribution of each element, a TB is created. At each iteration, the selection process is repeated a number of times, giving a list of sampled TBs. Each TB (with different stiffness throughout the beam length) is subjected to the same static loading test, and the resulting deflection is calculated. The algorithm attempts to choose combinations of stiffness values so as to minimise the discrepancy between the measured true deflection and the simulated deflection found from the TB.

If using R measured deflection readings ($u_{m,1}$, $u_{m,2}$ .... $u_{m,R}$), spread evenly along the beam, it is possible to calculate R simulated deflections for each TB, $u_{TB,1}$, $u_{TB,2}$ .... $u_{TB,R}$. An error objective function is defined here as the sum of the mean squared differences between the measured deflections and the TB deflections, over all measured nodes

At each iteration, only a portion of TBs giving a lowest error is retained. The mean and standard deviation of this portion form the mean and standard deviation for the distributions of the next iteration on an element-by-element basis. In this paper, 10% of the TBs are retained to generate the distributions for the subsequent iteration.

The system is said to converge when the error function stabilises, i.e., when the total error falls by less than 0.1% over 10 iterations. During the process, the standard deviation for all elements may be artificially increased, a technique known as 'injection'. If the system has become trapped in false minima, this 'injection' offers an opportunity to 'escape'. In this paper, injection is applied when convergence occurs for the first time. Once the system re-converges, the process is stopped.

Results

This section discusses the accuracy of the predictions and outlines the sensitivity of the algorithm to several parameters.

Distribution Development. Fig. 3 illustrates how the distributions for elemental stiffness values typically develop during the algorithm. The first row of distributions shows element 20, a healthy element, while the second row shows element 40, a damaged element. In each case, the solid black vertical line represents the target stiffness value for that element. The first column shows the broad distributions at iteration 1. At iteration 50, distributions for both elements have become narrower, and the means have shifted towards the exact stiffness value. Convergence was reached and the process was stopped at iteration 880, with a very narrow distribution for each element.

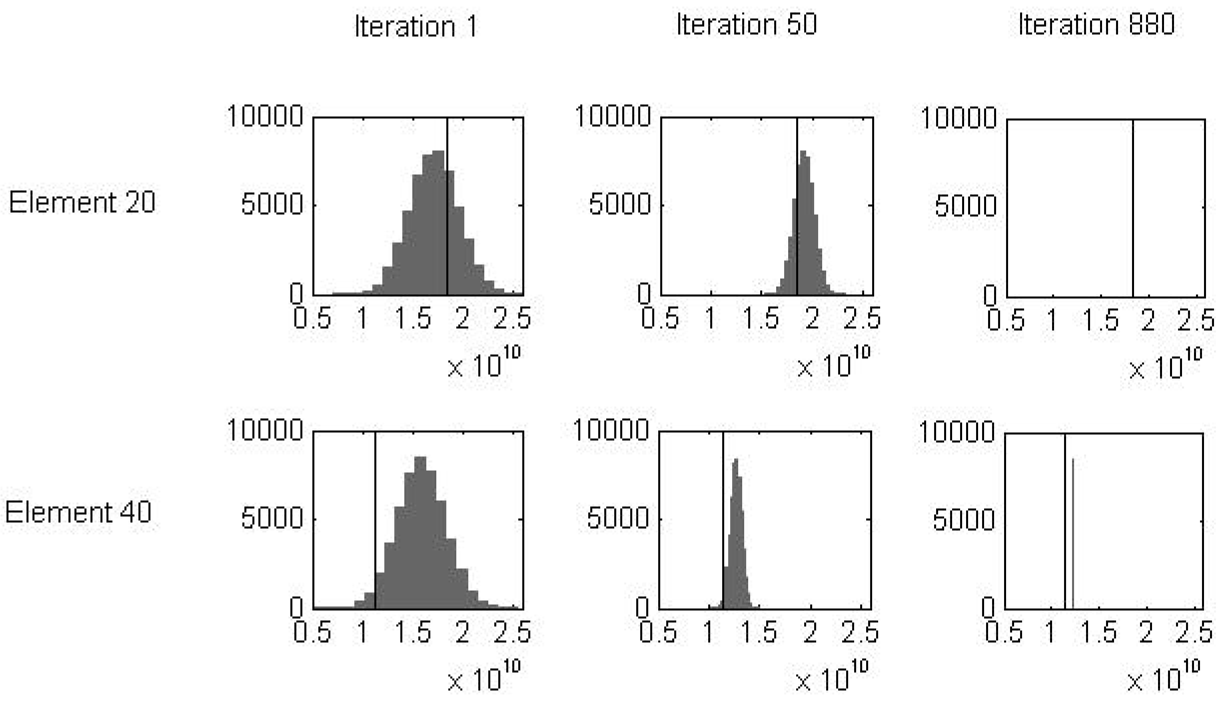

Figure 3 – Stiffness Distributions for Elements 20 and 40, at iterations 1, 50 and 880. Black vertical line represents target stiffness value.

Influence of the Initial Mean Values Assumed in Simulations. The CE algorithm was used to detect the damage shown in Fig. 2 for various values of initial mean, μ_{init}. For these simulations N_t = 60 and R = 29, corresponding to a deflection reading at every 0.5m. In each case, the same μ_{init} value was applied to all elements for the first iteration, as there is no prior knowledge of the damage location or severity. Fig. 4 shows the estimated stiffness profile for the beam with μ_{init} = 1.11x10^{10} Nm^2 and 1.66x10^{10} Nm^2 (equal to 60% and 90% the stiffness for a healthy beam element respectively). The process gives similar results for both values of μ_{init}. In both cases, the stiffness of damaged elements is estimated closely, while some inaccuracy is evident in elements close to the supports.

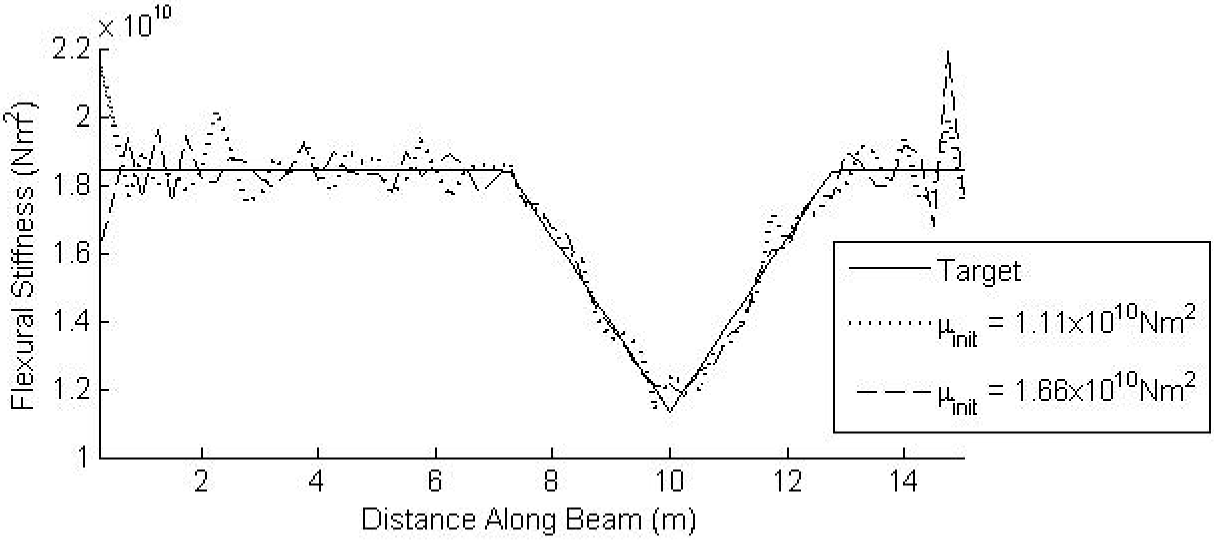

Figure 4 – Estimated and Target Stiffness Profiles for Different Values of μ_{init}

Influence of the Initial Standard Deviation Assumed in Simulations. The CE algorithm was used to detect the damage shown in Fig. 2 for various values of σ_{init}. For these simulations N_t = 60 and R = 29 deflection readings. Again, no prior knowledge was assumed and the same σ_{init} value was applied to all elements. Fig. 5 shows how the mean of the assumed distribution for element 11 (a healthy element) varies for each iteration. The continuous horizontal line represents the target value.

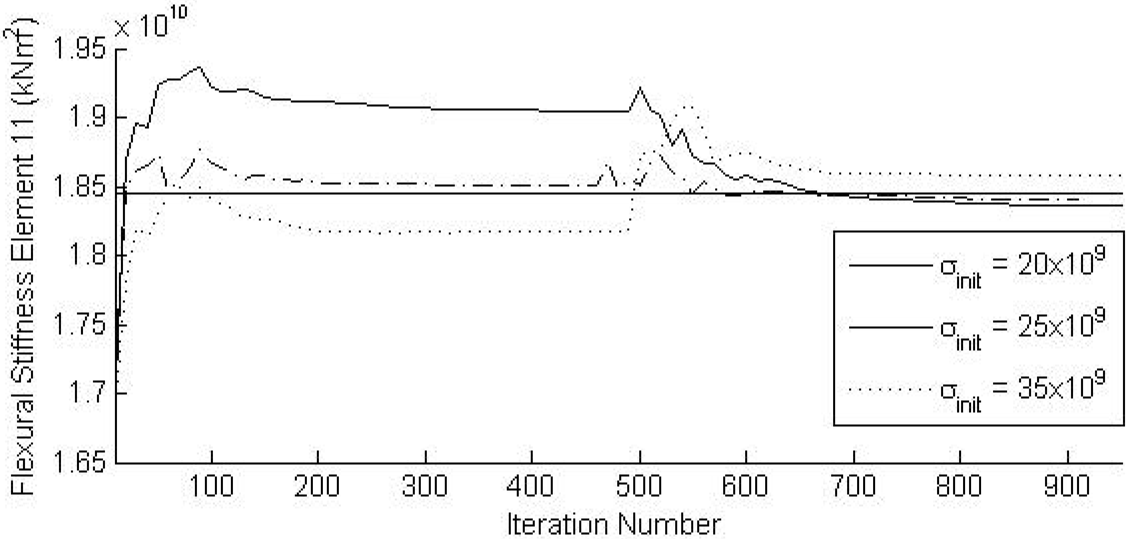

Figure 5 – Distribution Mean of Element 11 at each Iteration for Different Values of σ_{init}

For each σ_{init} value, convergence occurs for the first time between 450 and 500 iterations. Once convergence occurs, the σ_n value for each element is artificially increased to the initial value, σ_{init}. This injection had a positive effect on the mean value for each value of σ_{init}. The mean values have estimated the target more closely after injection. It is noted that this is not always the case; in many instances injection tends to drive the mean values away from the target value.

Influence of the Number of Trial Beams used in each Iteration. The number of TBs assembled at each iteration is denoted N_{TB}. Fig. 6 shows the output from CE for 2 values of N_{TB}, along with the target stiffness profile of Fig. 2. In each case the number of discretised beam elements is N_t =30. The algorithm estimates the stiffness of damaged elements quite well for both values of N_{TB}. However, with N_{TB} = 10,000 the estimated stiffness profile is less erratic, deviates less from the target profile and gives less error at the extreme elements.

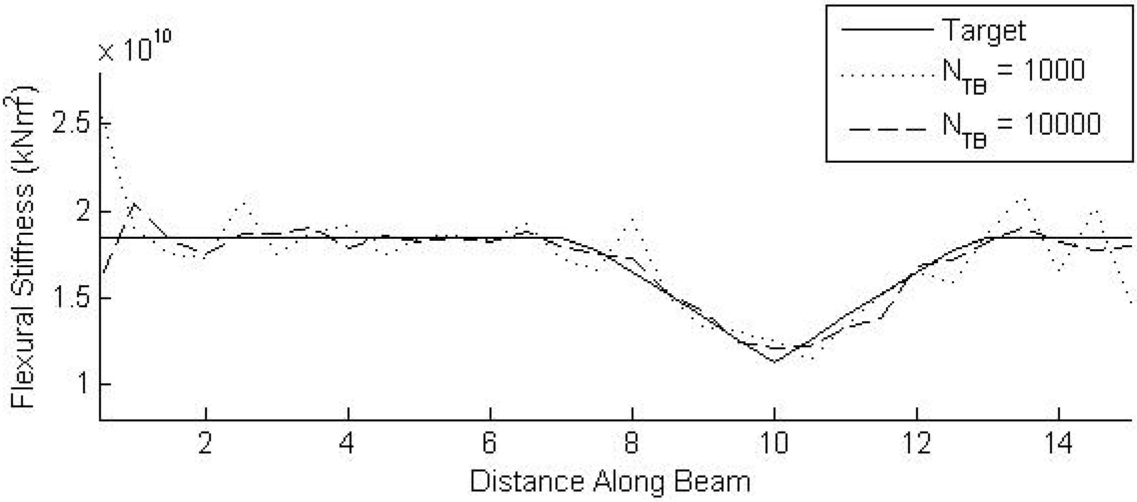

Figure 6 – Estimated and Target Stiffness Profiles for N_{TB} = 1000 and 1000

Influence of the Number of Elements Making the TB. The number of discretised elements making each trial beam is denoted by N_t. For larger values of N_t, the problem becomes more complex, with more parameters to estimate. Fig. 7 shows the output for the CE algorithm, identifying the profile of Fig. 2, with different number of elements in the TB. For these simulations R = 14, corresponding to a deflection reading at every metre. For both N_t = 30 and N_t = 120, the algorithm indicates damage location and severity reasonably well. With N_t = 120 the estimated profile is more erratic, with significant errors close to the supports.

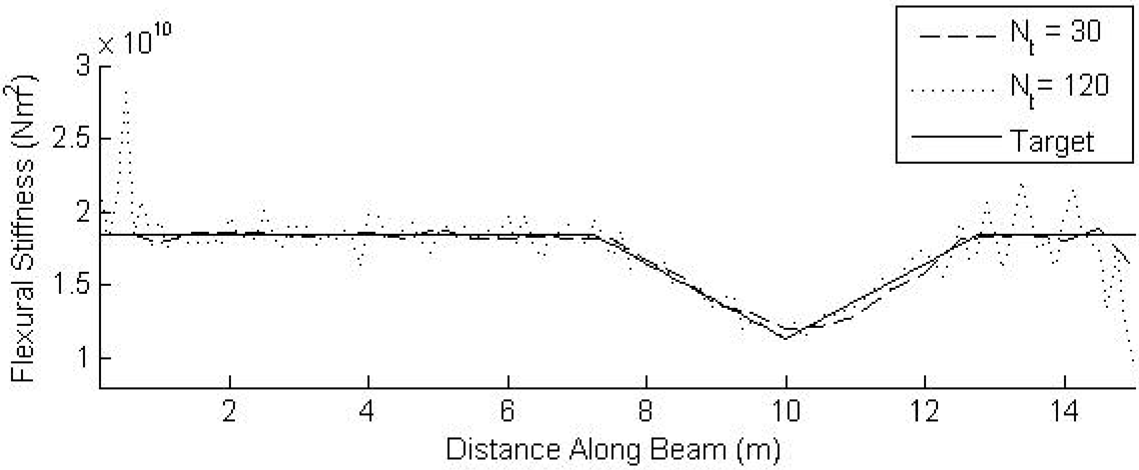

Figure 7 – Estimated and Target Stiffness Profiles for Various N_t

Error in the elements closest to the supports has been seen in a number of simulations. This phenomenon is investigated further in Fig. 8. Fig. 8 shows the development of the distribution mean for the healthy elements 1, 50 and 120, with target stiffness given by a horizontal line. It can be seen that the estimated stiffness of element 50 is relatively stable, while the mean for stiffness of elements 1 and 120 appears to fluctuate more freely. Beam deflection is least sensitive to the stiffness value of the extreme elements. As the algorithm minimises the deflection error, it is expected to have difficulty estimating the stiffness close to the supports.

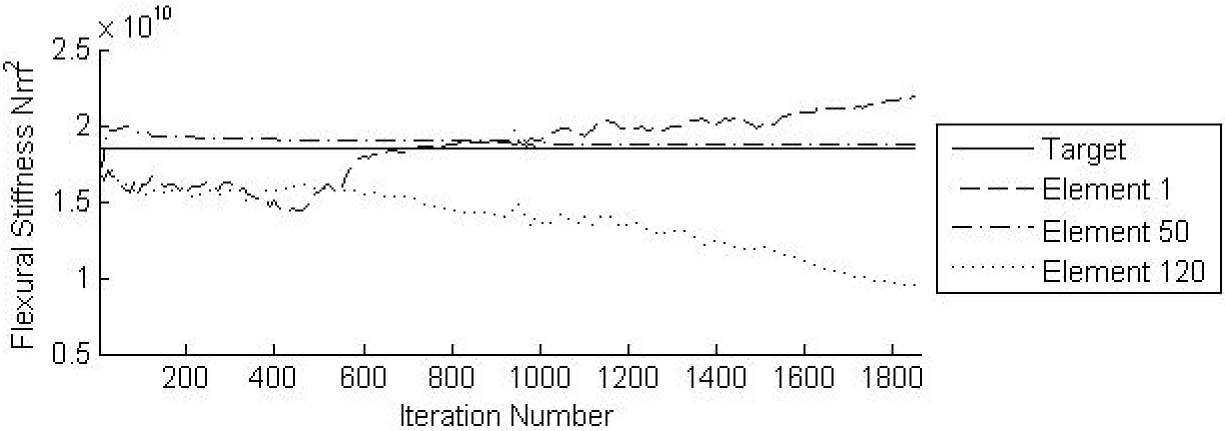

Figure 8 – Estimated and Target Stiffness Profiles for Elements 1, 50 and 120

Variation of the Location & Severity of the Beam Damage. A total of 9 damage events were modelled, corresponding to cracking of 5%, 10% and 15% beam depth located at 7m, 10m and 14m along the beam. It is expected that less severe damage, close to the supports would be most difficult to detect, as such damage would have the least pronounced effect on beam deflection. Fig. 9 shows the stiffness value for the most damaged element, for different damage events, with the target stiffness shown in white. For these simulations N_t = 60 and R = 29, corresponding to a deflection reading at every 0.5m. It can be seen that the output matches the stiffness of the most damaged element reasonably well, even where the damage is less severe and/or close to the supports.

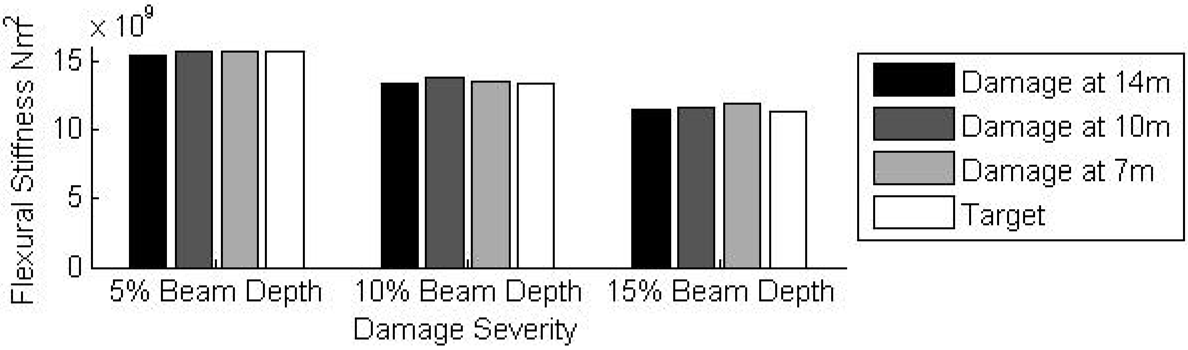

Figure 9 – Estimated and Target Stiffness for Various Damage Events

Influence of the Number of Deflection Readings Used in the Predictions. A greater number of deflection readings along the beam provides more detail about the deflection profile. By increasing the number of deflection values considered, an improvement in accuracy may be expected. Fig. 10 shows the estimated stiffness profile, along with the target profile of Fig. 2, with number of readings R = 29, 5 and 3. For each simulation, the number of discretised beam elements is N_t = 30. With R = 29 deflection points, the system attempts to match the deflection value at every internal node, and gives a good estimation of beam stiffness. But even with just 5 deflection values, the algorithm performs well. With just 3 deflection values, the damage location is less precise, and the stiffness of element 20 is overestimated by 17%, although damage can still be located reasonably well.

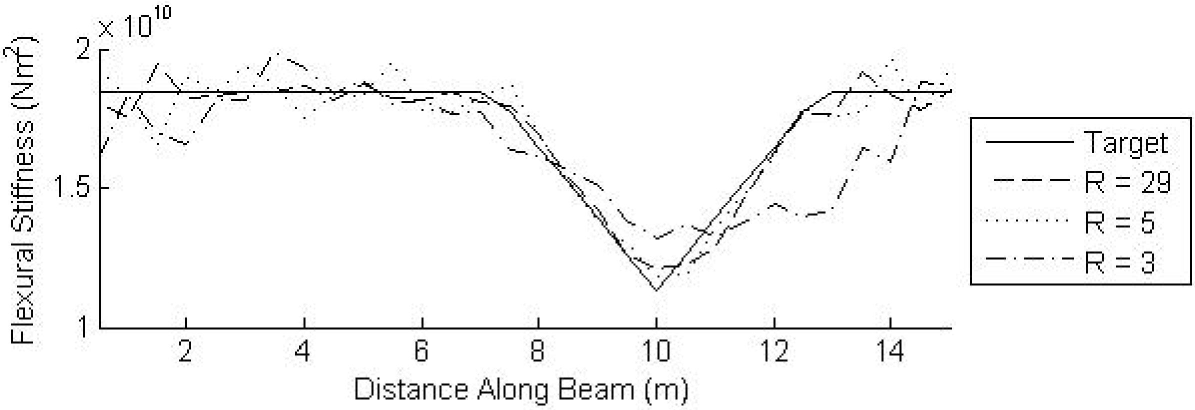

Figure 10 – Estimated and Target Stiffness Profiles for Various Number of Measurements, R

Testing of an Undamaged Beam. The deflection pattern of a healthy beam was also presented to the algorithm with $R = 14$. The undamaged case tests the ability of the algorithm to resist giving a false alarm, in indicating damage where none is present. Fig. 11 shows the estimated stiffness profile with $N_t = 30$. Fig. 11 shows inaccuracy in the estimation of element stiffness at the supports. The stiffness of element 1 is overestimated by 22%. Increased stiffness at the supports of a bridge could point to an increase in friction at the bearings, a common problem in beam bridges. The lowest stiffness value indicated by the algorithm is in element 3, with stiffness 94% of that for a healthy element, equivalent to a crack depth of 2% of beam depth.

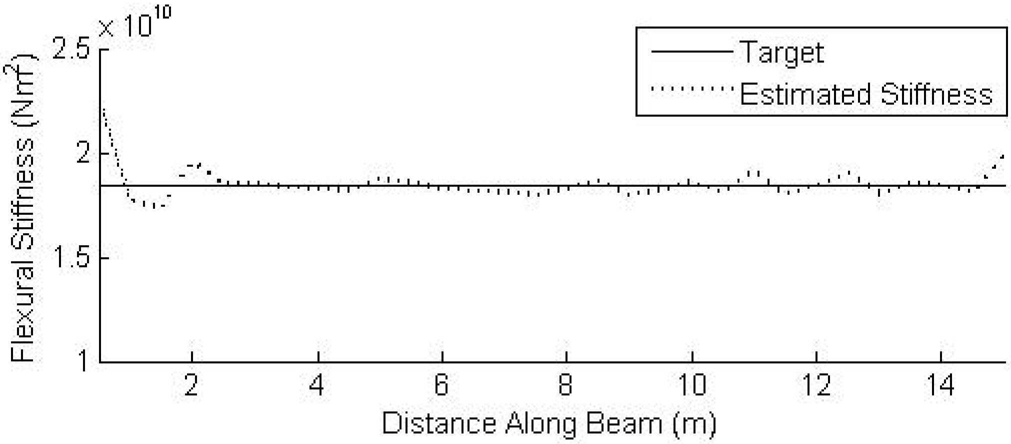

Figure 11 – Estimated and Target Stiffness Profiles for an Undamaged Beam

Summary & Conclusions

An algorithm that uses CE to detect damage has been presented. The method has been shown to have potential to estimate the structural parameters of a simply supported beam from the deflection due to a static loading test. Various beam damage states have been modelled by local reductions in flexural rigidity, and the ability of the CE algorithm to estimate the damage has been tested. The stiffness of the damaged elements has been closely identified in most of the cases. The results have shown that the algorithm is less accurate in estimating stiffness of undamaged elements close to the supports. It is possible that the use of rotation measurements close to the supports would give more accurate results than the use of the negligible displacements developing at these locations. This could be achieved in practice with the use of inclinometers. The method has also shown to be sensitive to the number of elements used to form each TB and the number of TBs formed per iteration.

The method proposed here has the advantage of operating without prior knowledge of system behaviour, other than a reasonable initial estimate of element stiffness values. This is in contrast to neural networks, which require extensive training in order to respond appropriately to stimulus. The ability of the method to identify damage in the presence of noise also requires attention, although large signal to noise ratios are expected, given the static nature of the test. The influence of noisy data may be reduced by using multiple measurements and more than one static loading case. The addition of a regularisation term to the error function may allow the algorithm to favour stiffness profiles with certain desirable properties, rather than minimising the deflection error alone. The method is also being extended to accept dynamic responses, or a combination of responses, again assembling stiffness values to match the measured response.

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