Proceedings of the Institution of Civil Engineers Bridge Engineering 162 June 2009 Issue BE2 Pages 75–85 doi: 10.1680/bren.2009.162.2.75

Paper 800015 Received 14/03/2008 Accepted 16/02/2009

Keywords: bridges/dynamics/stress analysis



Arturo González Lecturer, School of Architecture, Landscape and Civil Engineering, University College Dublin, Dublin, Ireland



Professor, School of Architecture, Landscape & Civil Engineering, University College Dublin, Dublin, Ireland



Maximum dynamic stress on bridges traversed by moving loads

D. Cantero MEng, A. González PhD and E. J. OBrien PhD

PhD Student, School of

Architecture, Landscape

& Civil Engineering, University College

Dublin, Dublin, Ireland

Most current research on dynamic effects due to traffic load on simply supported bridges is focused on the midspan section of the bridge because this location corresponds to the worst static bending moment. However, the maximum total moment allowing for dynamics may differ considerably from the maximum moment at midspan. This paper shows how the maximum can occur in a section relatively far from mid-span with a significant difference in magnitude.

I. INTRODUCTION

Bridge codes treat dynamic effects due to moving traffic differently. For example, the American Association of State Highway and Transportation Officials (AASHTO)¹ defines a factor called the dynamic load allowance (DLA), which is applied to static imposed loads. The DLA is the same for all spans, being 1.15 for fatigue and fracture and 1.33 for all other limit states.² However, different DLA values are specified for individual components of the bridge, such as deck joints (1.75). In the Eurocode EN 1991-2: 2003 (published as a British Standard³), different load models with built-in dynamic amplifications are defined, specifying optional factors to allow for site-specific situations such as the influence of expansion joints. These load models have been developed by the Eurocode working group based on experimental results from a number of countries.⁴ Dynamic effects are considered using dynamic factors obtained from numerical simulations and combined with static results to obtain characteristic values for each load model.⁵ Figure 1 presents the global dynamic factors used in the Eurocode for bending moment in the case of one loaded lane.

It is common practice to use a dynamic amplification factor (DAF) or similar parameter to allow for uncertainties associated with the structure, material and applied load. A more realistic characterisation of the total load effect would require experimental testing and/or the use of complex computer models. The DAF is defined here as the ratio of maximum total (including dynamics) to maximum static load effect.⁶ Other definitions⁷ specify that a ratio be defined for a given measurement point, while still others⁸ associate the factor with mid-span directly. In both simulations⁹ and experimental measurements,^{10,11} only the mid-span section (where, intuitively, the maximum stresses for a simply supported beam are expected to develop) is typically analysed. Furthermore, other parameters used to evaluate the

dynamic response of a bridge due to passing traffic, for example DLAs,¹² are also specifically used to evaluate maximum effects at mid-span.

This paper shows that DAFs based on stresses developed at midspan may lead to a significant underestimation of the maximum total stresses on a bridge. The differences between the maximum load effect at any point on a bridge and the mid-span load effect are quantified for theoretical simulations of a heavy five-axle articulated truck crossing a bridge. The influence of vehicle properties, road profile and bridge length on these differences is investigated.

Other load effects such as beam displacements and shear effects were analysed in a preliminary study by means of a simple constant-load model for a range of highway speeds. It was found that consideration of maximum mid-span deflection may lead to a small underestimation of the highest total displacement of less than 0.5%, whereas the same assumption for bending moments might give errors greater than 9%. Consideration of the maximum shear forces developed at supports leads to negligible differences when compared with the maximum shear across the entire beam length. If the solution is visualised in terms of a Fourier summation, the contribution of inertial bridge forces to displacements and bending moments reaches a maximum at a number of beam sections (i.e. mid-span for the first mode, or 1/4 and 3/4 span for the second mode) that vary with the mode of vibration and that interfere with each other. However, the modal contribution to shear stress has a maximum at the supports, regardless of the mode number. The highest total shear in a beam will thus typically develop at the supports. This paper focuses on analysis of the critical sections holding the maximum bending moment and how they compare with the maximum mid-span bending moment.

2. VEHICLE-BRIDGE INTERACTION MODEL

The crossing of a planar five-axle articulated truck at constant speed *c* over a simple supported Euler–Bernoulli beam was simulated based on the approach proposed by Frýba,¹³ El-Madany¹⁴ and Harris *et al.*⁸ The vehicle is composed of a two-axle tractor and a three-axle semi-trailer, linked by a hinge. The effect of vehicle roll on bridge dynamics is not considered; analysis is in the pitch plane only.

2.1. Vehicle model

The vehicle model allows for vertical displacements of the



tractor $y_{\rm T}$, semi-trailer $y_{\rm S}$ and suspensions y_i (i = 1, 2, 31, 32 and 33), and the pitch of the tractor $\theta_{\rm T}$ and semi-trailer $\theta_{\rm S}$ (see Figure 2). The geometrical relationship is

$$y_{\mathsf{S}} = y_{\mathsf{T}} + b_5 \theta_{\mathsf{T}} + b_4 \theta_{\mathsf{S}}$$

As a result, the vehicle model has eight independent degrees of freedom (dof). For linear suspension components, the equations of motion of the eight-dof vehicle model can be expressed in the form

$$\mathbf{M}\{\mathbf{\ddot{u}}\} + \mathbf{C}\{\mathbf{\ddot{u}}\} + \mathbf{K}\{\mathbf{u}\} = \{\mathbf{F}\}$$

where the mass M, damping C and stiffness K matrices are given in Appendix 1 and $\{u\}$ and $\{F\}$ are vectors of generalised coordinates (Equation 3) and forces (Equation 4) respectively.

$$\{\boldsymbol{u}\} = \{y_{\mathrm{T}} \ \theta_{\mathrm{T}} \ \theta_{\mathrm{S}} \ y_{1} \ y_{2} \ y_{31} \ y_{32} \ y_{33}\}^{\mathrm{T}}$$

4 {**F**} = { 0 0 0
$$-F_{t1}$$
 $-F_{t2}$ $-F_{t31}$ $-F_{t32}$ $-F_{t33}$ }^T

In the numerical simulation, the tyre is prevented from applying negative (uplift) forces to the bridge surface with the following condition

$$F_{ti} = k_{ti}[y_i(t) - y_{br}(x_i, t) + r_i(t)] \ge 0$$

i = 1, 2, 31, 32, 33

where $y_{br}(x_i, t)$ and $r_i(t)$ are the displacements of the beam and the road profile respectively, underneath the *i*th axle at instant *t*.

Typical parameters of a European five-axle truck configuration (Table 1) are employed in the simulations.¹⁵ Suspension parameters are chosen to represent the behaviour of air-sprung

systems with parallel viscous dampers.¹⁶ It is also assumed that the three axles of the tridem share the rear static load equally, as load-sharing mechanisms are common in multi-axle heavy vehicle suspension.¹⁷

2.2. Bridge model

The beam model is a simply supported Euler–Bernoulli beam of length *L* with modulus of elasticity *E*, second moment of area *J* and constant mass per unit length μ . The vertical displacements of the beam y(x, t) at section *x* and time *t*, due to *n* forces $F_{ti}(t)$ moving at speed *c*, are governed by Equation 6^{13}

$$EJ \frac{\partial^4 y(x,t)}{\partial x^4} + \mu \frac{\partial^2 y(x,t)}{\partial t^2} + 2\mu \omega_{\rm b} \frac{\partial y(x,t)}{\partial t}$$
$$= \sum_{i=1}^n \partial(x_i - ct) F_{ti}(t)$$

where δ is the Dirac function and $\omega_{\rm b}$ is the damped circular



Dimensional data: m $a_1 = -0.13$ $a_2 = 1.10$ $b_1 = 0.5$ $b_2 = 2.5$ $b_{31} = 1.30$	$b_{32} = 2.40$ $b_{33} = 3.50$ $b_4 = 4.15$ $b_5 = 2.15$
Mass and inertia data	
Tractor sprung mass, m_{T} : kg	4500
Tractor pitch moment of inertia, $I_{\rm T}$: kg m ²	4604
Semi-trailer sprung mass, <i>m</i> s: kg	31450
Semi-trailer pitch moment of inertia, $I_{\rm S}$: kg m ²	16302
Tractor front axle unsprung mass, m_1 : kg	700
Tractor back axle unsprung mass, m_2 : kg	1100
Semi-trailer axles unsprung masses, m_{31} , m_{32} , m_{33} : kg	750
Spring rates: kN/m	
$k_1 = 400$	$k_{t1} = 1750$
$k_2 = 1000$	$k_{t2} = 3500$
$k_{31} = k_{32} = k_{33} = 750$	$k_{t31} = k_{t32} = k_{t33} = 3500$
Viscous damping rates: kNs/m	$c_1 = c_2 = c_{31} = c_{32} = c_{33} = 10$
Table I. Five-axle truck model parameters	

frequency. For small damping ratios $\zeta,$ the damped frequency is given by

7
$$\omega_{\rm b} = \frac{\zeta}{\sqrt{1-\zeta^2}} \approx \zeta \omega_{(j)}$$

and the natural frequencies of the bridge $\omega_{(j)}$ are given by

8
$$\omega_{(j)}{}^2 = \frac{j^4 \pi^4}{L^4} \frac{EJ}{\mu}$$

9 $\omega_{(j)} = j^2 \omega_{(1)}$

Equation 6 is solved by the method of finite Fourier integral transformation, which considers the modal coordinates defined by Equations 10 and 11

10
$$q_{(j)}(t) = 2 \int_0^L y(x,t) \sin\left(\frac{j\pi x}{L}\right) dx$$

$$11 y(x,t) = \sum_{j=1}^{\infty} q_{(j)}(t) \sin\left(\frac{j\pi x}{L}\right)$$

Combining Equations 6, 10 and 11 yields

12
$$\begin{aligned} \ddot{q}_{(j)}(t) + 2j^2 \omega_{\rm b} \dot{q}_{(j)}(t) + j^4 \omega_{(1)} q_{(j)}(t) \\ &= \frac{2}{\mu L} \sum_{i=1}^n F_{ti}(t) \sin\left(\frac{j\pi ct}{L}\right) \end{aligned}$$

where x_i is the location of axle *i* at time *t*, $q_{(j)}(t)$ is the modal coordinate of the beam deflection and ε_i is a function described by

$$\varepsilon_{t} = \begin{cases} 1 & \text{for } 0 \le x_{i} \le L \\ 0 & \text{for } x_{i} < 0; x_{i} > L \end{cases}$$

Unless otherwise specified, the beam model parameters used in the simulations are as listed in Table 2.

2.3. Numerical solution

The aim of the simulation is to analyse the bending stresses developed in the beam as the vehicle passes over. Frýba¹³ suggests calculating the total bending moment in the beam as the sum of two bending moments

$$\mathbf{M}(x,t) = \mathbf{M}_{\mathbf{R}}(x,t) + \mathbf{M}_{\mu}(x,t)$$

where $M_R(x, t)$ is the quasi-static bending moment at *x* produced by all $F_{ti}(t)$ (Equation 15) and $M_{\mu}(x, t)$ is the bending moment produced by the inertial forces of the bridge (Equation 16)

15
$$M_{R}(x,t) = \begin{cases} \sum_{i=1}^{n} \varepsilon_{i} F_{ti}(L-x_{i}) \frac{x}{L} & \text{for } x_{i} \ge x \\ \sum_{i=1}^{n} \varepsilon_{i} F_{ti}(L-x) \frac{x_{i}}{L} & \text{for } x_{i} \le x \end{cases}$$

16
$$M_{\mu}(x, t) = -\frac{\mu L^2}{\pi^2} \sum_{j=1}^{\infty} \frac{1}{j^2} \ddot{q}_{(j)}(t) \sin j\pi x$$

These equations can then be solved using standard numerical integration techniques for a sufficient number of modes of vibration j to satisfactorily quantify the beam response. It is possible to assemble a system of differential equations, which consists of eight equations for the truck dof and another equation for each mode of vibration considered. For the vehicle-bridge interaction model employed, Equation 14 offers accurate results for relatively small numbers of modes of vibration.¹³ Two

Length, L: m	25
Mass per unit length, M: kg/m	18 358
Young's modulus, E: N/m ²	3·5 × 10 ¹⁰
Section inertia, J: m ⁴	I·3901
Damping, ζ: %	3
Table 2. Beam model parameters	

integration schemes, Wilson- θ^{18} and Runge–Kutta, were considered. The difference in accuracy between the two was negligible,¹⁹ so the Wilson- θ was adopted, this being the faster in carrying out the simulations. The Wilson- θ method is a variation of the Newmark method¹⁹ and is described in Appendix 2.

3. LOCATION OF MAXIMUM BENDING MOMENT DUE TO A MOVING VEHICLE

In order to identify the maximum load effect regardless of location on the bridge, a new parameter known as the fulllength dynamic amplification factor (FDAF) is introduced. FDAF is defined as the ratio of maximum total load effect along the full bridge length to the maximum static load effect at midspan. As described in Section 1, most of the existing research has focused on DAF (ratio of mid-span values), and here DAFs and FDAFs are compared for a typical heavy vehicle configuration and a range of vehicle speeds, road profiles and bridge lengths.

3.1. Maximum static moment

For any given observation point on a simply supported beam, he influence line for a static bending moment (due to a single moving load crossing the beam) has a triangular shape, with the maximum located when the moving load is at the observation point. The influence lines for all possible sections of a 25 m beam are represented in a contour plot in Figure 3. The axes represent the positions of the observation point and the moving load on the bridge (measured from the start of the beam in each case). In Figure 3, the overall maximum can be seen to be when the observation point is at mid-span and the moving load is passing that point. For other observation points, the maximum bending moment also occurs when the moving load is passing overhead. These points are identified by the dashed line in the figure.

The static bending moment due to a series of moving loads is obtained by superposing the individual effects due to each load for every observation point; this is illustrated in Figure 4 for the five-axle truck described in Section 2. It can be seen that the overall maximum static moment does not occur at mid-span.

The critical observation point (COP) is defined here as the observation point on the bridge where the maximum bending moment occurs. In the case of Figure 4, the COP is located at





approximately 11·45 m from the start of the bridge. For this particular case, the difference in maximum static bending moment during the vehicle crossing event between mid-span and the COP is 0·96%. This maximum moment occurs when the first axle of the rear tridem is located over the critical section. The value of the overall maximum moment and the location of the COP will depend on the magnitude of the loads, the spacing between them, the bridge length and the boundary conditions.

3.2. Maximum total moment on a bridge with a smooth road surface

The vehicle–bridge interaction model described in Section 2 is used to determine the response of a 25 m simply supported bridge to a vehicle travelling at 90 km/h on a perfectly smooth road profile. The resulting total bending moments are normalised by dividing by the maximum static moment at mid-span. These normalised bending moments (NBMs) are illustrated in Figure 5. The maximum NBM at mid-span is the DAF, which in this case is found to be 1.061. The maximum NBM for all



possible observation points is the FDAF, in this case 1.077. The COP corresponding to this value is 11.65 m from the start of the bridge—that is, 0.85 m from mid-span.

3.2.1. Speed influence. Results such as those illustrated in Figure 5 depend on a number of parameters, such as bridge and vehicle properties. Among these, vehicle speed is one of the most relevant.²⁰ The difference between the DAF and FDAF becomes more obvious when illustrated for a range of speeds (Figure 6a). While some DAFs oscillate around unity, especially for small speeds, the FDAF always remains greater than one. The crossing of a vehicle over the structure always results in an increase in bending moment over the static case.

Figure 6b shows that the COP varies significantly with vehicle speed and can be greater or less than mid-span. Comparison of Figures 6a and b shows that there is a clear relationship—sudden changes in the COP occur where there are local minima in the DAF graph. Where the COP falls close to mid-span, the DAF and FDAF graphs converge.

There are significant differences between DAF and FDAF. In terms of the dynamic increment (DI) (defined as variation with respect to the static value expressed in percentage), for a DAF of 0.999 (DI = -0.1%) and its corresponding FDAF of 1.024 (DI = 2.4%), the difference is up to 2.5%. As for the static case, the difference is related to the divergence of the COP from midspan.

3.2.2. Bridge damping influence. In structural dynamic analysis, damping is of great importance because it dissipates the system energy, although it is not easy to find an appropriate value for an actual structure. The influence on FDAF is presented in Figure 7a, showing that the higher the damping ratio the lower the dynamic response, reducing the magnitude of the dynamic amplification but preserving the shape. However,





the difference between amplification factors in terms of DI does not follow a simple relationship with damping, as can be seen in Figure 7b.

3.3. Maximum total moment on a bridge with a rough road surface

In addition to speed, the condition of the road profile is a major factor influencing the response of a bridge to a passing vehicle.²¹ Simulations were carried out to analyse the influence of three different road profiles (ISO classes A, B and C²²) and vehicle speeds on DAF. For each road type, 200 different profiles were generated randomly and, for each of these, a range of speeds from 50 to 150 km/h at 1 km/h intervals were considered to obtain DAFs and FDAFs. The sampling population was therefore $3 \times 200 \times 101 = 60$ 600.

Figure 8 shows that even for the smoothest roads, dispersion of the COP is considerable and becomes larger with rougher



profiles (i.e. class C). As shown in Sections 3.1 and 3.2, the deviation of the COP from mid-span is related to the magnitude of the difference between DAF and FDAF.

Figure 9 shows the DI differences between the DAF and FDAF for road classes A, B and C in 3D histogram form. For instance, a DAF of 1·2 on a class B profile (Figure 9b) can have a 20% greater DI for the FDAF in adverse conditions—that is, $1\cdot 2 + 0\cdot 2 = 1\cdot 4$. For this particular bridge, there is a trend towards higher differences for DAFs between 1 and 1·2.



Maximum DAFs and FDAFs are very high compared with the mean values, but the frequency of these maxima is low. Figure 10 compares the histograms of DAF and FDAF and shows how most events fall into a narrow range of values. The mean DAFs and FDAFs are given for each road class in Table 3. Whereas for the smoothest road profile the DI difference between DAF and FDAF is small, for rougher roads (class C) the difference can be considerable (> 6%). However, the differences are less at the 95% and 99% confidence levels.

3.4. Influence of bridge length

FDAFs were also calculated for beam lengths of 15, 35 and 70 m. The parameters of these beams are listed in Table 4. For smooth profiles, the variation of amplification factor with speed shows a similar pattern for all four bridge lengths (Figure 5a). The pattern is made of peaks and valleys, although they differ in location and magnitude. The maximum difference between DAF and FDAF increases as the span decreases. When considering vehicle speeds in the range 40–110 km/h, the maximum FDAF is smaller or equal to 1·1, regardless of the bridge length.



		Mean		95%	confidence i	nterval	99% confidence interval			
Road class	FDAF	DAF	DI differ- ence: %	FDAF	DAF	DI differ- ence: %	FDAF	DAF	DI differ- ence: %	
Smooth	I·058	I ·048	ا ∙00	I·098	I·085	1.30	1.106	I ·090	ا ∙60	
А	1.091	I·073	I · 80	1.181	1.167	I ·40	1.237	1.213	2.40	
В	1.143	1.113	3.00	I · 297	1.275	2.20	1.375	1.352	2.30	
С	1.265	I·203	6.20	1.221	I · 504	4.70	1.705	1.665	4.00	

		Beam lengt	h, <i>L</i> : m
	15	35	70
Mass per unit length, M: kø/m	28 125	21 752	30 752
Section moment of inertia, <i>J</i> : m ⁴	0.52	273 3.4	162 19.5313

Table 4. Model parameters for 15, 35 and 70 m beams (Young's modulus and damping ratios remain the same as the original 25 m bridge model)

The locations of the COP of 15, 35 and 70 m beams result in similar graphs to the 25 m bridge (Figure 6b) where the critical points oscillate around the COP associated with the static case, and sharp changes coincide with the valleys.

Similarly to the analysis carried out in Section 3.3, a range of road profiles and vehicle speeds were tested on the beam models described in Table 4. The results are presented in Tables 5–7. Comparison of the results from the four studied beam lengths (Tables 3, 5–7) shows that, in general, for smooth profiles, the shorter the bridge, the bigger the difference between amplification factors. For rough road profiles, there is no clear trend in differences between DAFs and FDAFs.

4. MAXIMUM MOMENT DUE TO A HEAVY VEHICLE FLEET

Both the DAF and the FDAF are strongly influenced by vehicle weight and speed. A range of typical weights and speeds were taken from weigh-in-motion (WIM) data collected on a highway in Auxerre, France. Normal (Gaussian) distributions were fitted to gross vehicle weight (GVW), axle load distribution and speeds for five-axle heavy goods vehicles (Table 8).¹⁵ Monte Carlo

	Mean			95% confidence interval			99% confidence interval			
Road class	FDAF	DAF	DI difference: %	FDAF	DAF	DI difference: %	FDAF	DAF	DI difference: %	
Smooth	1.093	1.056	3.70	1.231	1.180	5.10	I·240	1.195	4.50	
А	1.129	I ·087	4.20	1.301	I · 247	5.40	1.373	1.325	4.80	
В	1.173	1.125	4.80	1.392	1.330	6.20	1.503	I·454	4.90	
С	I·302	I·233	6.90	1.612	I·540	7.50	I · 797	1.718	7.90	

Table 5. Dynamic amplification factors and dynamic increment differences (L = 15)

	Mean			95% confidence interval			99% confidence interval			
Road class	FDAF	DAF	DI difference: %	FDAF	DAF	DI difference: %	FDAF	DAF	DI difference: %	
Smooth	I·037	1.031	0.60	I·072	I·067	0.50	I·073	I·070	0.30	
Α	1.090	1.069	2.10	1.178	1.162	I·60	I · 223	I · 203	2.00	
В	1.160	1.113	4 ·70	1.333	1.292	4.10	I·424	1.375	4.90	
С	I·331	I·226	10.20	I∙668	1.221	11.70	1.861	I·727	13.40	

Table 6. Dynamic amplification factors and dynamic increment differences (L = 35)

_	Mean			95%	95% confidence interval			99% confidence interval		
Road class	FDAF	DAF	DI differ- ence: %	FDAF	DAF	DI differ- ence: %	FDAF	DAF	DI difference: %	
Smooth	I·029	1.021	0.74	1.028	1.056	0.19	I·058	I·057	0.11	
А	1.118	1.090	2.83	1.246	1.551	2.58	I·304	I · 273	3.09	
В	1.223	1.186	3.70	I · 484	I·435	4.87	I · 584	1.242	4.21	
С	I·462	1.416	4.64	I·934	I·876	5.84	2.081	2.023	5∙85	

Table 7. Dynamic amplification factors and dynamic increment differences (L = 70)

Description	Mean	Standard deviation
GVW: kN Proportion of GVW carried by 1st axle: %	604·3 3·4	56∙4 I∙6
Proportion of GVW carried by 2nd axle: %	28.8	2.8
Proportion of GVW carried by tridem: %	57.8	3.6
Speed: m/s	19.5	2.7
Table 8. Normal distribution parame Auxerre, France	ters from `	WIM data at

simulations were used to randomly generate 100 000 vehicles and the maximum static bending moment at mid-span was calculated for each vehicle. The 500 events with the greatest static moment were analysed dynamically for three road classes and five bridge lengths. For each road type, 100 different profiles were generated, and a typical speed was generated. The vehicle–bridge interaction model of Section 2 was employed for each vehicle crossing. Values of m_S (and the associated I_S) in Table 1 were varied according to the sampled GVW.

Figure 11 shows the maximum values of DAF and FDAF with a 95% confidence interval. Amplification factors clearly increase for rougher road classes, and the difference between DAF and FDAF increases accordingly as can be seen in the figure. A change in beam length influenced the results only slightly. There is thus not a clear influence of bridge length, partially due to the narrow vehicle speed range measured and reproduced in the Monte Carlo simulations.

Figure 12 illustrates the average DI difference between DAF and FDAF for each beam length and road profile class. The mean increase in DAF depends mostly on the road condition; the influence of bridge length is relevant, although to a lesser extent.

It has been shown that the total maximum stress developed on a simply supported beam due to a vehicle or traffic fleet might not be at mid-span. The DAFs employed in design bridge codes are conservative, but adequate as the marginal cost of adding strength to a structure under construction is small and future





loading conditions are uncertain. It is when assessing an existing structure that engineers can gather a better knowledge of applied loads and structural response through the use of bridge measurements. The uncertainty in some parameters can then be reduced by defining material strength and loading conditions and the safety factors associated with them. For such an assessment, this paper has shown that measurements should be performed not only at the centre point but also at relevant lengths along the beam—that is, the second third of the bridge span. Therefore, if the mid-span strain was taken as reference when assessing maximum stresses due to the passage of a vehicle, this value should be factored by a safety coefficient. The safety coefficient will depend on the characteristics of the road, the bridge and the vehicle or traffic fleet being analysed.

5. CONCLUSIONS

It is common practice to use the DAF (or equivalent) to quantify the increase in bridge response due to the dynamics of vehicles and bridge–vehicle interaction. The DAF is the ratio of the maximum total load effect to the maximum static load effect at a given section. In the case of a simply supported beam model and bending moment, mid-span is traditionally selected as the assumed critical section that the DAF refers to. Since the maximum moment may not be necessarily located at mid-span, this paper introduced the FDAF, which is the ratio of the maximum total load effect across the full bridge length to the maximum static load effect at mid-span. Numerical simulations of bridge response due to the crossing of a five-axle truck were used to compare and quantify differences between DAF and FDAF.

It was found that the difference between FDAF and DAF generally increases as the separation between COP and mid-span increases. The COP depends on the magnitude of the loads, the inter-axle spacing, the bridge length and the boundary conditions.

Total (static plus dynamic) moments were obtained for a bridge with a smooth road profile and for bridges with class A, B and C profiles. Variation in the location of the critical section increases with rougher profiles, and the difference between DAF and FDAF increases. A Monte Carlo simulation was performed, varying vehicle weights and speeds according to measured WIM statistics. The differences between DAF and FDAF were found to be modest but significant, ranging from 1 to 5% difference in DI, depending on road roughness.

Due to the conservative nature of existing codes of practice, they are still considered to be adequate for highway bridges at normal traffic speeds. However, assessing an existing structure requires evaluation of the magnitude of maximum stresses due to moving traffic, which do not necessarily take place at the mid-span section. Clearly, the differences between stresses at mid-span and those at critical sections cannot be ignored in an accurate assessment that may save a bridge from replacement or strengthening. However, as this paper is the first known study to consider the entire bridge length, further investigations involving experimental tests and complex theoretical models are needed. Furthermore, in the case of railway bridges, where vehicle speeds are considerably higher, the consequences of fulllength analysis are as yet unknown.

APPENDIX I. MASS, DAMPING AND STIFFNESS MATRICES

Mass matrix M

		$(m_{\rm T} + m_{\rm S})$	$b_5m_{\sf S}$	$b_4m_{\sf S}$	0	0	0	0	0)
		$b_5m_{ m S}$	$\left(I_{\rm T} + b_5^2 m_{\rm S} + \frac{m_4 m_5}{m_4 + m_5} a_1^2\right)$	$\left(b_4b_5m_{S}-rac{m_4m_5}{m_4+m_5}a_1a_2 ight)$	0	0	0	0	0
17	М —	$b_4m_{\sf S}$	$\left(b_4 b_5 m_{S} - rac{m_4 m_5}{m_4 + m_5} a_1 a_2 ight)$	$\left(I_{\rm S} + b_4^{\ 2}m_{\rm S} + \frac{m_4m_5}{m_4 + m_5}a_2^{\ 2}\right)$	0	0	0	0	0
17	IVI —	0	0	0	m_1	0	0	0	0
		0	0	0	0	m_2	0	0	0
		0	0	0	0	0	m_{31}	0	0
		0	0	0	0	0	0	m_{32}	0
		(0	0	0	0	0	0	0	m_{32} /

Damping matrix C

		$ (c_1 + c_2 + c_{31} + c_3) [-b_1c_1 + b_2c_2 + b_5(c_{31})] $	$(c_{32} + c_{33}) + c_{32} + c_{33})$]	$\begin{bmatrix} -b_1c_1 + b_2 \\ b_1^2c_1 + b_2^2c_1 \end{bmatrix}$	$c_2 + b_5(c_{31} + c_{32})$ $c_2 + b_5^2(c_{31} + c_{32})$	$(c_{2}^{2} + c_{33})]$ $(c_{2}^{2} + c_{33})]$	$ \begin{bmatrix} (b_{31} + b_4)c_{31} + (b_{32} + b_4)c_{32} + (b_{33} + b_4)c_{33} \end{bmatrix} \\ b_5 \begin{bmatrix} (b_{31} + b_4)c_{31} + (b_{32} + b_4)c_{32} + (b_{33} + b_4)c_{33} \end{bmatrix} $
		$[(b_{31} + b_4)c_{31} + (b_{32} + b_4)c_{33}]$	$a_2 + (b_{33} + b_{33})$	$_{4})c_{33}]$	$b_5[(b_{31} + b_4)c_{31} +$	$(b_{32} + b_4)c_{32} +$	$(b_{33} + b_4)c_{33}]$	$\left[(b_{31} + b_4)^2 c_{31} + (b_{32} + b_4)^2 c_{32} + (b_{33} + b_4)^2 c_{33} \right]$
	C=	$-c_{1}$				b_1c_1		0
	1	$-c_{2}$				$-b_2c_2$		0
		$-c_{31}$				$-b_5c_{31}$		$-(b_{31} + b_4)c_{31}$
		$-c_{32}$				$-b_5c_{32}$		$-(b_{32} + b_4)c_{32}$
18	($-c_{33}$				$-b_5c_{33}$		$-(b_{33} + b_4)c_{33}$
			$-c_{1}$	$-c_{2}$	$-c_{31}$	$-c_{32}$	$-c_{33}$	
			b_1c_1	$-b_{2}c_{2}$	$-b_5c_{31}$	$-b_5c_{32}$	$-b_5c_{33}$	
			0	0	$-(b_{31} + b_4)c_{31}$	$-(b_{32} + b_4)c_{32}$	$-(b_{33} + b_4)c_{33}$	
			c_1	0	0	0	0	
			0	c_2	0	0	0	
			0	0	c_{31}	0	0	
			0	0	0	c_{32}	0	
			0	0	0	0	c_{33}	/

Stiffness matrix K

		$\binom{(k_1 + k_2 + k_{31} + k_{32} + k_3)}{[-b_1k_1 + b_2k_2 + b_5(k_{31} + k_{32} + b_5(k_{31} + k_{32} + b_4)k_{31} + (b_{32} + b_4)k_{32} + (b_{33} + b_4)k_{33} + (b_{33} + b_4)k_{33$	$\binom{3}{3} + k_{33} = k_{33}$	$(k_{33}] b_5$	$[-b_1k_1 + b_2k_2 \\ [b_1^2k_1 + b_2^2k_2 \\ [(b_{31} + b_4)k_{31} + (b_{31} + b_4)k_{31}] = (b_1^2k_1 + b_2^2k_2)$	$+ b_5(k_{31} + k_{32} + b_5^2(k_{31} + k_{32} - b_{32} + b_4)k_{32} + (b_5^2(k_{31} + k_{32} - b_{32} + b_4)k_{32} + (b_5^2(k_{31} + k_{32} + b_{32})k_{32})k_{33} + (b_5^2(k_{31} + k_{32} + b_{33})k_{33})k_{33} + (b_5^2(k_{31} + k_{33})k_{33})k_{33})k_{33} + (b_5^2(k_{31} + k_{33})k_{33})k_{33})k_{33} + (b_5^2(k_{31} + k_{33})k_{33})k_{33})k_{33} + (b_5^2(k_{31} + k_{33})k_{33})k_{33})k_{33})k_{33} + (b_5^2(k_{31} + k_{33})k_{$	$(k = k_{33})]$ + $k_{33})]$ $(k_{33} + b_4)k_{33}]$	$\begin{bmatrix} (b_{31} + b_4)k_{31} + (b_{32} + b_4)k_{32} + (b_{33} + b_4)k_{33} \end{bmatrix}$ $b_5[(b_{31} + b_4)k_{31} + (b_{32} + b_4)k_{32} + (b_{33} + b_4)k_{33}]$ $\begin{bmatrix} (b_{31} + b_4)^2k_{31} + (b_{32} + b_4)^2k_{32} + (b_{33} + b_4)^2k_{33} \end{bmatrix}$
	K=	$-k_1$,, .		$b_1 k_1$	00 17 001 [
		$-k_{2}$				$-b_{2}k_{2}$		0
		$-k_{31}$				$-b_5k_{31}$		$-(b_{31} + b_4)k_{31}$
		$-k_{32}$			$-b_5k_{32}$		$-(b_{32} + b_4)k_{32}$	
19		\				$-b_5k_{33}$		$-(b_{33} + b_4)k_{33}$
			$-k_1$	$-k_2$	$-k_{31}$	$-k_{32}$	$-k_{33}$	
			b_1k_1	$-b_{2}k_{2}$	$-b_5k_{31}$	$-b_5k_{32}$	$-b_5k_{33}$	
			0	0	$-(b_{31} + b_4)k_{31}$	$-(b_{32} + b_4)k_{32}$	$-(b_{33} + b_4)k$	33
			k_1	0	0	0	0	
			0	k_2	0	0	0	
			0	0	k_{31}	0	0	
			0	0	0	k_{32}	0	}
			0	0	0	0	k_{33})

 m_4 and m_5 are given by

20
$$m_4 = m_T + m_1 + m_2$$

21 $m_5 = m_S + m_{31} + m_{32} + m_{33}$

APPENDIX 2. THE WILSON- Θ METHOD

The Wilson- θ method is essentially an extension of the linear acceleration method in which a linear variation of acceleration from time *t* to time $t + \Delta t$ is assumed.²³ Equation 22 represents the equation of motion of a system subject to a forcing vector $\{F\}$ that must be satisfied at time $t_{n+\theta} = t_n + \theta \Delta t$ (with $\theta \ge 1$).

22
$$\mathbf{M}\{\mathbf{\ddot{y}}\}_{n+\theta} + \mathbf{C}\{\mathbf{\dot{y}}\}_{n+\theta} + \mathbf{K}\{\mathbf{y}\}_{n+\theta} = \{\mathbf{F}\}_{n+\theta}$$

The displacement and velocity at $t_{n+\theta}$ are related to $\{y\}_n n$, $\{y\}_n$ and $\{y\}_n$ by Equations 23 and 24

23
$$\{\boldsymbol{y}\}_{n+\theta} = \{\boldsymbol{y}\}_n + \theta \Delta t \{\boldsymbol{\dot{y}}\}_n + (\theta \Delta t)^2 [(1/2) - \beta] \{\boldsymbol{\ddot{y}}\}_n + (\theta \Delta t)^2 \beta \{\boldsymbol{\ddot{y}}\}_{n+\theta}$$

24
$$\{\dot{\mathbf{y}}\}_{n+\theta} = \{\dot{\mathbf{y}}\}_n + \theta \Delta t (1-\gamma) \{\dot{\mathbf{y}}\}_n + \theta \Delta t \gamma \{\ddot{\mathbf{y}}\}_{n+\theta}$$

By substituting Equations 23 and 24 into Equation 22, $\{\vec{y}\}_{n+\theta}$ can be found by solving the non-linear equation. The acceleration at t_{n+1} is then deduced from $\{\vec{y}\}_n$ and $\{\vec{y}\}_{n+\theta}$ by linear interpolation.

25
$$\{\vec{\mathbf{y}}\}_{n+1} = \left(1 - \frac{1}{\theta}\right)\{\vec{\mathbf{y}}\}_n + \left(\frac{1}{\theta}\right)\{\vec{\mathbf{y}}\}_{n+\theta}$$

From which the displacement and velocity at t_{n+1} can be obtained by using the standard Newmark formulae

26
$$\{\boldsymbol{y}\}_{n+1} = \{\boldsymbol{y}\}_n + \Delta t \{\boldsymbol{\dot{y}}\}_n + \Delta t^2 \left(\frac{1}{2} - \beta\right) \{\boldsymbol{\ddot{y}}\}_n + \Delta t^2 \beta \{\boldsymbol{\ddot{y}}\}_{n+1}$$

27
$$\{\mathbf{\dot{y}}\}_{n+1} = \{\mathbf{\dot{y}}\}_n + \Delta t(1-\gamma)\{\mathbf{\ddot{y}}\}_n + \Delta t\gamma\{\mathbf{\dot{y}}\}_{n+1}$$

In the Wilson- θ method, it is assumed $\beta = 1/6$ and $\gamma = 1/2$. The parameter θ is often chosen to be 1·4.

ACKNOWLEDGEMENT

The authors acknowledge financial support from ARCHES (assessment and rehabilitation of central European highway structures), a research project within the European 6th frame-work programme.

REFERENCES

- 1. AMERICAN ASSOCIATION OF STATE HIGHWAY AND TRANSPORTATION OFFICIALS. *Standard Specification for Highway Bridges*, 16th edn. AASHTO, Washington, DC, 1996.
- 2. O'CONNOR C. and SHAW P. A. Bridge Loads. An International Perspective. Spon Press, London, 2000.
- 3. BRITISH STANDARDS INSTITUTION. Eurocode 1: Actions on Structures, Part 2: Traffic Loads on Bridges. BSI, London, 2003, BS EN 1991-2.
- 4. BRULS A., CALGARO J. A., MATHIEU H. and PRAT M. ENV 1991– Part 3: The Main Models of Traffic Loads on Road Bridges (Report 74). IABSE, Delft, 1996.
- 5. DAWE P. *Research Perspectives: Traffic Loading on Highway Bridges.* Thomas Telford, London, 2003.
- 6. ZHANG Q., VROUWENVELDER A. and WARDENIER J. Dynamic amplification factors and EUDL of bridges under random traffic flows. *Engineering Structures*, 2001, 23, No. 1, 663–672.
- GONZALEZ A. Dynamic effects of a five-axle truck on a shortspan bridge. In *Bridge Engineering Research in Ireland* (MCNALLY C. and BRADY S. (eds)). University College Dublin Press, Dublin, 2002, pp. 3–10.
- HARRIS N. K., O'BRIEN E. J. and GONZALEZ A. Reduction of bridge dynamic amplification through adjustment of vehicle suspension damping. *Journal of Sound and Vibration*, 2007, 302, No. 3, 471–485.
- SAVIN E. Dynamic amplification factor and response spectrum for the evaluation of vibrations of beams under successive moving loads. *Journal of Sound and Vibration*, 2001, 248, No. 2, 267–288.
- 10. SETHILVASAN J., THAMBIRATNAM D. P. and BRAMELD G. H. Dynamic response of a curved bridge under moving truck load. *Engineering Structures*, 2002, 24, No. 1, 1283–1293.
- 11. CANTIENI R. *Dynamic Load Tests on Highway Bridges in Switzerland*. Swiss Federal Laboratories for Materials Testing and Research, Dübendorf, 1983.
- 12. NATIONAL COOPERATIVE HIGHWAY RESEARCH PROGRAM. Synthesis of Highway Practice 266: Dynamic Impact Factors for Bridges. National Academy, Washington, DC, 1998.
- FRÝBA L. Vibration of Solids and Structures under Moving Loads. Noordhoff, Groningen, 1972.
- EL-MADANY M. M. Design optimization of truck suspensions using covariance analysis. *Computers & Structures*, 1988, 28, No. 2, 241–246.
- 15. GRAVE S. Modelling of Site-specific Traffic Loading on Short to Medium Span Bridges. PhD thesis, Trinity College Dublin, Ireland, 2001.
- 16. FANCHER P. S., ERVIN R. D., WINKLER C. B. and GILLESPIE T. D. A Factbook of the Mechanical Properties of the Components for Single-unit and Articulated Heavy Trucks. University of Michigan Transportation Research Institute, Michigan, 1986, Technical report UMTRI-86-12.
- 17. CEBON D. Handbook of Vehicle-Road Interaction. Swets & Zeitlinger, Lisse, 1999.
- 18. BATHE K. J. and WILSON E. L. *Numerical Methods in Finite Element Analysis.* Prentice-Hall, Englewood Cliffs, NJ, 1976.
- XIE Y. M. An assessment of time integration schemes for non-linear dynamic equations. *Journal of Sound and Vibration*, 1996, 192, No. 1, 321–331.
- 20. BRADY S. P. *The Influence of Vehicle Velocity on Dynamic Amplification in Highway Bridges.* PhD thesis, University College Dublin, Ireland, 2003.

21. ORGANIZATION FOR ECONOMIC CO-OPERATION AND DEVELOPMENT. Dynamic Interaction between Vehicles and Infrastructure Experiment, 1998. See www.oecd.org/dataoecd/9/22/ 2754516.pdf for further details. (accessed 10/12/2008).

22. INTERNATIONAL ORGANIZATION FOR STANDARDIZATION. Mechanical

Vibration–Road Surface Profiles: Reporting of Measured Data. ISO, Geneva, 1995, ISO 8608.

23. TAN G. H., BRAMELD G. H. and THAMBIRATNAM D. P. Development of an analytical model for treating bridge–vehicle interaction. *Engineering Structures*, 1998, 20, No. 1, 54–61.

What do you think?

To comment on this paper, please email up to 500 words to the editor at journals@ice.org.uk *Proceedings* journals rely entirely on contributions sent in by civil engineers and related professionals, academics and students. Papers should be 2000–5000 words long, with adequate illustrations and references. Please visit www.thomastelford.com/journals for author guidelines and further details.