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Modelling the Vehicle in Vehicle-Infrastructure Dynamic Interaction Studies

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Abstract

This paper presents the equations of motion for a general articulated road vehicle, with variable numbers of wheels for the tractor and trailer. The equations are applicable to vehicle-infrastructure dynamic interaction problems for two- and three-dimensional systems, allowing for the definition of a wide variety of vehicle configurations with the same formulae.

Keywords: vehicle forces; equations of motion; dynamics; articulated truck; bridge; pavement

1. Introduction

The complexity of mathematical models used to describe vehicle dynamics varies with the purpose of the research and the expected level of accuracy. In the case of the dynamic interaction between a vehicle and the infrastructure, the equations of motion of the vehicle [1] and the equations governing the bridge or pavement response can be defined separately and then, combined together to guarantee equilibrium and compatibility of displacements at the contact points [2, 3]. For this purpose, a vehicle can be modelled as a single vertical force [4] or as a series of constant forces [5, 6] in its most simple form. However, with the advance in computational power, vehicle models are now commonly idealised as single Degrees Of Freedom (DOF) [7], two DOF's [8-10] or multiple DOF's [1, 11-13] made of linear, mass and rigid elements. Some researchers have included a hinge to model an articulation between tractor and trailer [14-17], or have even modelled vehicles with towed trailers [18]. Vehicle models made of plane and volume finite elements have been developed [19] allowing for a detailed representation of the vehicle aerodynamic forces, strains and deformations. However, it seems unlikely that vehicle deformations could significantly affect the levels of dynamic stresses in the infrastructure. So, models made of linear elements have been shown to offer sufficient accuracy to analyse the influence of vehicle forces on the infrastructure [1, 20]. The objective of this paper is to facilitate a general form of the vehicle equations that can be easily extended to any specified number of axles or wheels in the tractor and/or trailer, in planar or three-dimensional vehicle-infrastructure interaction problems.

This paper presents the equations of motion for a general road vehicle, condensed into one single system of second order differential equations. These equations can be easily implemented in a computer model and solved using any standard integration scheme, such as Runge-Kutta [21], Newmark-β [22], Wilson-θ [23] or the exponential method [24] among others. The proposed vehicle model consists of two major bodies, tractor and trailer, represented as lumped masses joined to the road or bridge surface by spring-dashpot systems, which model the suspension and tyre mechanisms (Figure 1). Each axle is represented as a rigid bar with lumped masses at both ends that correspond to the wheel and suspension masses (Figure 2). In addition, each wheel is connected to the road surface by another spring-dashpot system that imitates the tyre response. With the equations of motion presented here it is possible to define any number of axles for tractor and trailer, reduce the model to a tractor only vehicle, or downgrade from a three-dimensional to a planar vehicle model. Hence the equations can be used to model any vehicle configuration from a quarter-car to a multi-axle articulated truck.

2. Equations of motion of the vehicle

The model assumes vertical tyre-ground contact forces at single points, constant vehicle speed, driving path in a straight line, and negligible lateral and yaw motions. Horizontal forces are not considered and as result, the model is not applicable to the analysis of vehicles with varying speed or curved infrastructure where centrifugal forces could play an important role. Therefore, the equations have been derived for mechanical elements with linear stiffness and damping properties. The motion of the entire system is defined by the tractor sprung mass vertical displacement y_T , pitch θ_T and roll γ_T rotations, the trailer pitch θ_S and roll γ_S, and one additional vertical displacement for each considered wheel y_i. Note that, due to the articulation between the tractor and trailer, there is a geometric relationship given by equation (1), resulting in one dependent degree of freedom, i.e., the trailer vertical displacement y_S, which can be expressed in terms of the system rotations and tractor vertical displacement [25].

$$y_S = y_T + h_1\,\theta_T + h_3\,\theta_S \qquad (1)$$

The equations of motion can be derived from equilibrium of forces and moments acting on each mass and they are presented individually in equations (2 - 8). Applying Newton's second law of motion to the tractor sprung mass, the equilibrium of the inertial forces of both sprung masses due to their vertical acceleration, and damping and stiffness forces transmitted by the suspension can be established as in equation (2).

$$(m_T + m_S)\,\ddot{y}_T + m_S\,h_1\,\ddot{\theta}_T + m_T\,h_3\,\ddot{\theta}_S +$$

$$\sum_T \left[k_i\,(y_T + b_i\,\theta_T + a_i\,\beta_T - y_i) + c_i\left(\dot{y}_T + b_i\,\dot{\theta}_T + a_i\,\dot{\beta}_T - \dot{y}_i\right)\right] +$$

$$\sum_S [k_i\,(y_T + h_1\,\theta_T + (b_i + h_3)\,\theta_S + a_i\,\beta_S - y_i) +$$

$$c_i\left(\dot{y}_T + h_1\,\dot{\theta}_T + (b_i + h_3)\,\dot{\theta}_S + a_i\,\dot{\beta}_S - \dot{y}_i\right)\Big] = 0 \qquad (2)$$

Equations (3) and (4) are the result of applying the law of conservation of angular momentum to the tractor moments of inertia of the pitch and roll motions respectively.

$$m_S\,h_1\,\ddot{y}_T + \left(I_{Ty} + m_S\,{h_1}^2 + m\,{h_2}^2\right)\cdot\ddot{\theta}_T + (m_S\,h_1\,h_3 - m\,h_2\,h_4)\,\ddot{\theta}_S +$$

$$\sum_T b_i\left[k_i\,(y_T + b_i\,\theta_T + a_i\,\beta_T - y_i) + c_i\left(\dot{y}_T + b_i\,\dot{\theta}_T + a_i\,\dot{\beta}_T - \dot{y}_i\right)\right] +$$

$$\sum_S h_1\,[k_i\,(y_T + h_1\,\theta_T + (b_i + h_3)\,\theta_S + a_i\,\beta_S - y_i) +$$

$$c_i\left(\dot{y}_T + h_1\,\dot{\theta}_T + (b_i + h_3)\,\dot{\theta}_S + a_i\,\dot{\beta}_S - \dot{y}_i\right)\Big] = 0 \qquad (3)$$

$$I_{Tx}\,\ddot{\beta}_T + \textstyle\sum_T a_i\left[k_i\,(y_T + b_i\,\theta_T + a_i\,\beta_T - y_i) + c_i\left(\dot{y}_T + b_i\,\dot{\theta}_T + a_i\,\dot{\beta}_T - \dot{y}_i\right)\right] = 0$$

$$(4)$$

Similarly to the equilibrium of the tractor mass, and taking equation (1) into account, the equations of motion of the trailer are given by equations (5, 6).

$$m_S\, h_3\, \ddot{y}_T + (m_S\, h_1\, h_3 - m\, h_2\, h_4)\, \ddot{\theta}_T + \left(I_{Sy} + m_S\, {h_3}^2 + m\, {h_4}^2\right) \ddot{\theta}_S +$$

$$\sum_S (h_3 + b_i)\, [k_i(y_T + h_1\, \theta_T + (h_3 + b_i)\, \theta_S + a_i\, \beta_S - y_i) +$$

$$c_i \left(\dot{y}_T + h_1\, \dot{\theta}_T + (h_3 + b_i)\, \dot{\theta}_S + a_i\, \dot{\beta}_S - \dot{y}_i\right)] = 0 \qquad (5)$$

$$I_{Sx}\, \ddot{\beta}_S + \sum_S a_i\, [k_i\, (y_T + h_1\, \theta_T + (h_3 + b_i)\, \theta_S + a_i\, \beta_S - y_i) +$$

$$c_i \left(\dot{y}_T + h_1\, \dot{\theta}_T + (h_3 + b_i)\, \dot{\theta}_S + a_i\, \dot{\beta}_S - \dot{y}_i\right)] = 0 \qquad (6)$$

The equations for the unsprung masses have been derived considering that the axle is a rigid bar, the masses are lumped at both ends and suspensions are connected to the axle at distances d_i and d_{i0} from the wheels, as illustrated in Figure 2. Inertial, suspension and tyre forces acting on each unsprung mass must be in equilibrium. These dynamic equations of equilibrium are slightly different for the unsprung masses of the tractor (7) and unsprung masses of the trailer (8), since the trailer vertical displacement y_S is expressed in terms of other DOF of the sprung masses, as seen in equation (1).

$$m_i\, \ddot{y}_i + \frac{d_{io}}{wb}\left[k_{io}\left(\left(1 - \frac{d_{io}}{wb}\right) y_{io} + \frac{d_{io}}{wb}\, y_i - y_T - b_{io}\, \theta_T - a_{io}\, \beta_T\right) +\right.$$

$$\left. c_{io}\left(\left(1 - \frac{d_{io}}{wb}\right) \dot{y}_{io} + \frac{d_{io}}{wb}\, \dot{y}_i - \dot{y}_T - b_{io}\, \dot{\theta}_T - a_{io}\, \dot{\beta}_T\right)\right] +$$

$$\left(1 - \frac{d_i}{wb}\right)\left[k_i\left(\left(1 - \frac{d_i}{wb}\right) y_i + \frac{d_i}{wb}\, y_{io} - y_T - b_i\, \theta_T - a_i\, \beta_T\right) +\right.$$

$$\left. c_i\left(\left(1 - \frac{d_i}{wb}\right) \dot{y}_i + \frac{d_i}{wb}\, \dot{y}_{io} - \dot{y}_T - b_i\, \dot{\theta}_T - a_i\, \dot{\beta}_T\right)\right] + kt_i(y_i - r_i) + ct_i(\dot{y}_i - \dot{r}_i) = 0$$

(7)

$$m_i\,\ddot{y}_i + \frac{d_{io}}{wb}\left[k_{io}\left(\left(1-\frac{d_{io}}{wb}\right)\,y_{io} + \frac{d_{io}}{wb}\,y_i - y_T - h_1\,\theta_T - (h_3+b_{io})\theta_S - a_{io}\,\beta_S\right)\right.$$

$$\left. c_{io}\left(\left(1-\frac{d_{io}}{wb}\right)\,\dot{y}_{io} + \frac{d_{io}}{wb}\,\dot{y}_i - \dot{y}_T - h_1\,\dot{\theta}_T - (h_3+b_{io})\dot{\theta}_S - a_{io}\,\dot{\beta}_S\right)\right] +$$

$$\left(1-\frac{d_i}{wb}\right)\left[k_i\left(\left(1-\frac{d_i}{wb}\right)y_i + \frac{d_i}{wb}\,y_{io} - y_T - h_1\,\theta_{T-} - (h_3+b_i)\theta_S - a_i\,\beta_T\right)+\right.$$

$$\left. c_i\left(\left(1-\frac{d_i}{wb}\right)\dot{y}_i + \frac{d_i}{wb}\,\dot{y}_{io} - \dot{y}_T - h_1\,\dot{\theta}_T - (h_3+b_i)\dot{\theta}_S - a_i\,\dot{\beta}_T\right)\right] + kt_i(y_i - r_i) +$$

$$ct_i(\dot{y}_i - \dot{r}_i) = 0 \qquad (8)$$

2.1 Matrix formulation

Using the symbols given in the nomenclature appendix, the equations of motion (2-8) of the vehicle can be expressed in matrix form as:

$$\mathbf{M}\,\ddot{\boldsymbol{z}} + \mathbf{C}\,\dot{\boldsymbol{z}} + \mathbf{K}\,\boldsymbol{z} = \boldsymbol{f} \qquad (9)$$

where the DOF's are given by:

$$\boldsymbol{z} = (y_T \quad \theta_T \quad \beta_T \quad \theta_S \quad \beta_S \quad y_1 \quad \cdots \quad y_i)^T \text{ for i = 1,2,…, N} \qquad (10)$$

The symmetric mass matrix is given by:

$$\mathbf{M} = \begin{pmatrix} \mathbf{M1} & 0 \\ 0 & \mathbf{M2} \end{pmatrix} \qquad (11)$$

where the submatrices **M1** and **M2** represent the mass matrices associated to the DOFs of the sprung and unsprung masses respectively and they are defined in equations (12,13).

$$\mathbf{M1} = \begin{pmatrix} m_T + m_S & m_S\,h_1 & 0 & m_S\,h_3 & 0 \\ & I_{Ty} + m_S\,{h_1}^2 + m\,{h_2}^2 & 0 & m_S\,h_1\,h_3 - m\,h_2\,h_4 & 0 \\ & & I_{Tx} & 0 & 0 \\ & Symm. & & I_{Sy} + m_S\,{h_3}^2 + m\,{h_4}^2 & 0 \\ & & & & I_{Sx} \end{pmatrix} \quad (12)$$

$$\mathbf{M2} = \begin{pmatrix} m_1 & \cdots & 0 & \cdots & 0 \\ & \ddots & \vdots & & \vdots \\ & & m_i & \cdots & 0 \\ & Symm. & & \ddots & \vdots \\ & & & & m_N \end{pmatrix} \quad (13)$$

where m is an auxiliary variable defined in equation (14).

$$m = \frac{(m_T + \sum_T m_i)\,(m_S + \sum_S m_i)}{m_T + m_S + \sum_T m_i + \sum_S m_i} \quad (14)$$

Note that the summation of matrix elements with subscripts T and S indicates the addition of corresponding parameters for tractor and trailer respectively.

The stiffness matrix is given by:

$$\mathbf{K} = \begin{pmatrix} \mathbf{K1} & -\mathbf{K2} \\ -\mathbf{K2}^T & \mathbf{K3} \end{pmatrix} \quad (15)$$

where the submatrices **K1** and **K2** are defined in equations (16, 17).

$$\mathbf{K1} = \begin{pmatrix} \sum k_i & \sum_T b_i\,k_i + h_1 \sum_S k_i & \sum_T a_i\,k_i & \sum_S (h_3 + b_i)\,k_i & \sum_S a_i\,k_i \\ & \sum_T {b_i}^2\,k_i + {h_1}^2 \sum_S k_i & \sum_T a_i\,b_i\,k_i & h_i \sum_S (h_3 + b_i)\,k_i & h_1 \sum_S a_i\,k_j \\ & & \sum_T {a_i}^2\,k_i & 0 & 0 \\ & Symm. & & \sum_S (h_3 + b_i)^2\,k_i & \sum_S a_i\,(h_3 + b_i)\,k_i \\ & & & & \sum_S {a_i}^2\,k_i \end{pmatrix}$$

(16)

K2

$$= \begin{pmatrix} k_1 & \cdots & k_i & \cdots & k_T & k_{T+1} & \cdots & k_{T+j} & \cdots & k_N \\ b_1\, k_1 & \cdots & b_i\, k_i & \cdots & b_T\, k_T & h_{T+1}\, k_{T+1} & \cdots & h_{T+j}\, k_{T+j} & \cdots & h_N\, k_N \\ a_1\, k_1 & \cdots & a_i\, k_i & \cdots & a_t\, k_T & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & \cdots & 0 & (h_3 + b_{T+1})\, k_{T+1} & \cdots & \left(h_3 + b_{T+j}\right) k_{T+j} & \cdots & (h_3 + b_N)\, k_N \\ 0 & \cdots & 0 & \cdots & 0 & a_{T+1}\, k_{T+1} & \cdots & a_{T+j}\, k_{T+1} & \cdots & a_N\, k_N \end{pmatrix}$$

(17)

The **K3** submatrix is defined element by element obtaining a symmetric matrix using equations (18) and (19), where the subscript notation *io* is used for parameters that correspond to the i^{th} opposite wheel.

$$\mathbf{K3}(i,i) = \left(1 - \frac{d_i}{wb}\right)^2 k_i + \left(\frac{d_{io}}{wb}\right)^2 k_{io} + kt_i \qquad \text{For i = 1, 2, …, N} \qquad (18)$$

$$\mathbf{K3}(i,io) = \left(1 - \frac{d_i}{wb}\right)\frac{d_i}{wb}\, k_i + \left(1 - \frac{d_{io}}{wb}\right)\frac{d_i}{wb}\, k_{io} \qquad \text{For i = 1, 2, …, N} \qquad (19)$$

where *wb* is defined in equation (20) for a three-dimensional model, whereas for a planar model the unity value has to be adopted.

$$wb = a_i + a_{i0} \qquad (20)$$

The damping matrix, **C**, has an identical format to the stiffness matrix and can be found by substituting *k* for *c* in the components of equation (15), as can be seen in equations (21-25).

$$\mathbf{C} = \begin{pmatrix} \mathbf{C1} & -\mathbf{C2} \\ -\mathbf{C2}^{\boldsymbol{T}} & \mathbf{C3} \end{pmatrix} \qquad (21)$$

C1

$$
= \begin{pmatrix}
\sum c_i & \sum_T b_i\, c_i + h_1 \sum_S c_i & \sum_T a_i\, c_i & \sum_S (h_3 + b_i)\, c_i & \sum_S a_i\, c_i \\
 & \sum_T b_i^{\,2}\, c_i + h_1^{\,2} \sum_S c_i & \sum_T a_i\, b_i\, c_i & h_i \sum_S (h_3 + b_i)\, c_i & h_1 \sum_S a_i\, c_j \\
 & & \sum_T a_i^{\,2}\, c_i & 0 & 0 \\
 & Symm. & & \sum_S (h_3 + b_i)^2\, c_i & \sum_S a_i\, (h_3 + b_i)\, c_i \\
 & & & & \sum_S a_i^{\,2}\, c_i
\end{pmatrix}
$$

(22)

C2

$$
= \begin{pmatrix}
c_1 & \cdots & c_i & \cdots & c_T & c_{T+1} & \cdots & c_{T+j} & \cdots & c_N \\
b_1\, c_1 & \cdots & b_i\, c_i & \cdots & b_T\, c_T & h_{T+1}\, c_{T+1} & \cdots & h_{T+j}\, c_{T+j} & \cdots & h_N\, c_N \\
a_1\, c_1 & \cdots & a_i\, c_i & \cdots & a_t\, c_T & 0 & \cdots & 0 & \cdots & 0 \\
0 & \cdots & 0 & \cdots & 0 & (h_3 + b_{T+1})\, c_{T+1} & \cdots & \left(h_3 + b_{T+j}\right) c_{T+j} & \cdots & (h_3 + b_N)\, c_N \\
0 & \cdots & 0 & \cdots & 0 & a_{T+1}\, c_{T+1} & \cdots & a_{T+j}\, c_{T+1} & \cdots & a_N\, c_N
\end{pmatrix}
$$

(23)

$$\mathbf{C3}(i,i) = \left(1 - \frac{d_i}{wb}\right)^2 c_i + \left(\frac{d_{io}}{wb}\right)^2 c_{io} + ct_i \qquad \text{For i = 1, 2, …, N} \qquad (24)$$

$$\mathbf{C3}(i,io) = \left(1 - \frac{d_i}{wb}\right)\frac{d_i}{wb}\, c_i + \left(1 - \frac{d_{io}}{wb}\right)\frac{d_i}{wb}\, c_{io} \qquad \text{For i = 1, 2, …, N} \qquad (25)$$

The force vector, $\boldsymbol{f}$ is given by:

$$\boldsymbol{f} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & ct_1\, \dot{r}_1 + kt_1\, r_1 & \cdots & ct_i\, \dot{r}_i + kt_i\, r_i & \cdots & ct_N \dot{r}_N + kt_N\, r_N \end{pmatrix}^T$$

For i = 1, 2, …, N (26)

At time, t = 0, assuming the vehicle is at rest, the initial values for the vector $\boldsymbol{z}$ can be obtained by solving the static problem in equation (9), this is, multiplying the inverse stiffness matrix by $\boldsymbol{f}$ at t = 0.

3. Discussion

The equations above consider only vehicle dynamics. The static load can be added to the calculated contact force or included in the force vector $\boldsymbol{f}$, with some rearrangements of the initial conditions. Moreover, the equations can be extended to include a towed trailer. The towed element might be described, following the presented equations, as a truck without trailer moving at the same speed as the primary vehicle, assuming there is no relation between degrees of freedom (Figure 3). Tyre damping has been included in the equations with the intention to represent a more general vehicle model. However, tyre viscous damping is generally small and can be ignored in predictions of vehicle response to road roughness [26]. Recommended parameters values for the suspension can be found in [27], where the results of an extensive suspension database analysis are presented. Lehtonen et al. [28] show experimentally obtained values for heavy tyres, Wong [29] presents appropriate truck tyre parameters, and Kirkegaard et al. [15] provide values recommended by a heavy goods vehicle manufacturer. Other parameters sources are Harris et al. [14], Kim et al. [12], Gillespie et al. [30] for articulated trucks, Fafard et al. [18] for articulated truck with towed trailer, and Li [31] for crane suspensions.

Finally, these vehicle equations can be combined with the equations of the infrastructure model under investigation to analyse vehicle-infrastructure problems, i.e., impact factor due to traffic in roads and bridges [6-17,32], dynamics in railway bridges [2, 3], pavement deterioration due to the passage of heavy vehicles [33-34], performance of vehicle elements such as suspensions or tyres [1, 36-39], evaluation of ride quality and pavement unevenness [40-42], or weigh-in-motion applications [43-45] amongst others. In the case of simulating the interaction between a vehicle and a bridge, Lagrange multipliers [17], dynamic condensation [3] or iterative procedures [31] are some of the most popular approaches to combine the equations of motion of both models.

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Appendix 1. Nomenclature

a_i, b_i	X and Y coordinate for wheel i from its corresponding sprung mass centre of gravity	m_i	unsprung mass for wheel i
c_i	suspension viscous damping coefficient for wheel i	m_T, mS	tractor and trailer sprung mass
ct_i	tyre viscous damping coefficient for wheel i	$\mathbf{M}$	mass matrix
$\mathbf{C}$	damping matrix	$\mathbf{M1}$,$\mathbf{M2}$	submatrix of $\mathbf{M}$
$\mathbf{C1,C2,C3}$	submatrix of $\mathbf{C}$	nT,nS	number of tractor and trailer wheels
d_i	suspension offset for wheel i	N	total number of wheels
$\boldsymbol{f}$	force vector	r_i	road profile under wheel i
h_1,h_2	horizontal and vertical distance from articulation to tractor centre of gravity	t	time
h_3,h_4	horizontal and vertical distance from articulation to trailer centre of gravity	wb	wheelbase on Y direction
h_5	horizontal distance from tractor centre of gravity to the front wheel	y_i	vertical displacement of wheel i
I_{Sx},I_{Sy}	trailer moment of inertia about X and Y axis	y_T	tractor vertical displacement
I_{Tx},I_{Ty}	tractor moment of inertia about X an Y axis	y_S	trailer vertical displacement
k_i	suspension stiffness coefficient for wheel i	$\boldsymbol{z}$	degrees of freedom vector
kt_i	tyre stiffness coefficient for wheel i	X,Y,Z	coordinates
$\mathbf{K}$	stiffness matrix	β_T, β_S	tractor and trailer roll angle

K1, **K2**, **K3**	submatrix of **K**	θ_T, θ_S	tractor and trailer pitch angle
m	auxiliary mass		

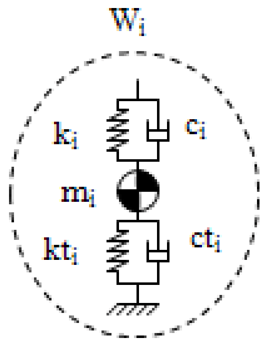

Figure 1(a) Suspension and tyre system.

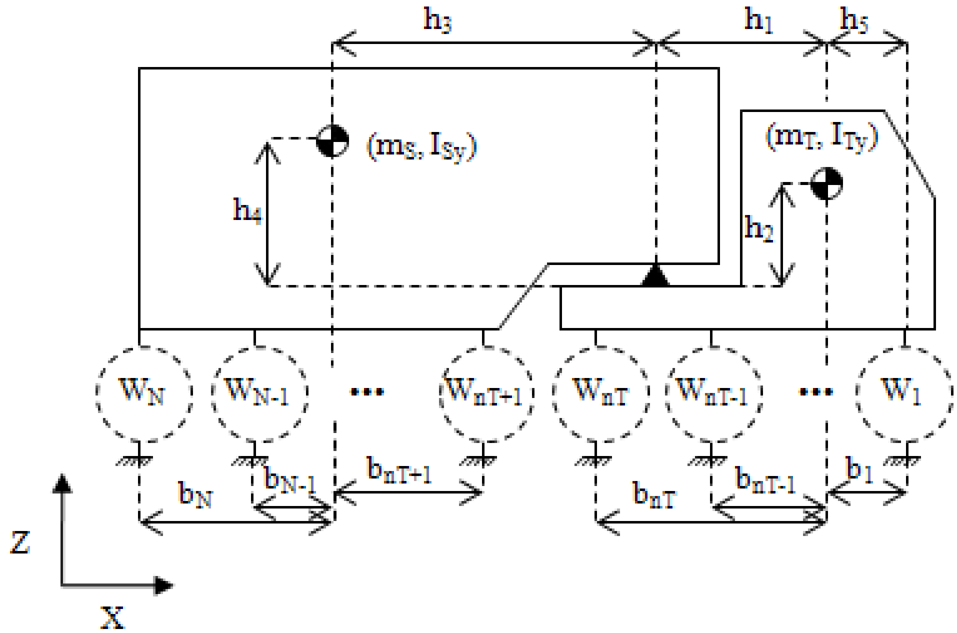

Figure 1(b) Side view sketch of Tractor + Trailer.

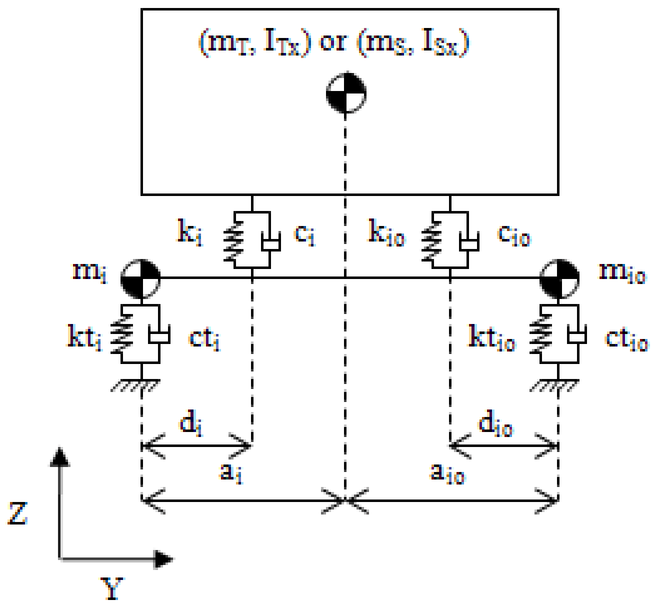

Figure 2. Front view for axle *i* (For 3 dimensional solution only).

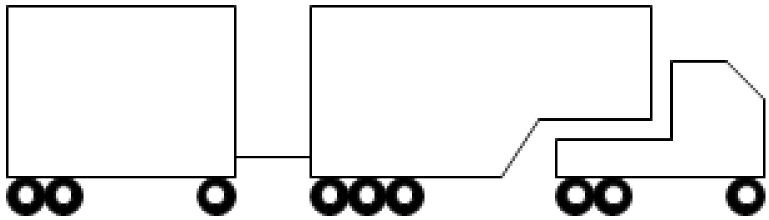
Figure 3. Addition of towed vehicle.