

# An Agent-based Modeling Approach to Study Price Impact

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**Abstract**—Price impact models are important for devising trade execution strategies. However, a proper characterization of price impacts is still lacking. This study models the price impact using an agent-based modeling approach. The purpose of this paper is to investigate whether agent intelligence is a necessary condition when seeking to construct realistic price impact with an artificial market simulation. We build a zero-intelligence based artificial limit order market model. Our model distinguishes limit orders according to their order aggressiveness and takes into account some observed facts including log-normal distributed order sizes and power-law distributed limit order placements. The model is calibrated using trades and orders data from the London Stock Exchange. The results indicate that agent intelligence is needed when simulating an artificial market where replicating price impact is a concern.

## I. INTRODUCTION

In financial markets, trades have impact on the market which is called *price/market impact*. A sell trade is always associated with a following fall in market price and a buy trade is always followed by a rise in market price. Also the price impact caused by a trade increases with the trade size. Thus, when trading a large amount of shares, the normal approach is to divide the shares into smaller chunks and spread them over time in order to reduce the price impact. One example from the real world is the recent purchase of IBM shares by Warren Buffet's company, which has invested \$10.7 billion in IBM stock since March 2011, now owning 64 million shares, equivalent of a 5.4% stake. The amount he has bought is about 16 times as large as the average daily volume<sup>1</sup> of IBM. It is more likely that he traded approximately 10.29%<sup>2</sup> of the average daily volume in each trading day since March, instead of purchasing 64 million shares in 16 trading days which otherwise should have caused the share price to go up dramatically.

Modeling price impact is an area of intense study both in academy and practice. A proper price impact model is important for devising many *Algorithmic Trading* strategies which are used to automate trade executions in order to achieve efficient execution price [1]. There are many models of price impact in the literature. The early work [2], [3], [4] adopt linear or nonlinear price impact functions of trade sizes. The recent work [5], [6] try to model the shape of the *limit order books* adopted in many financial markets. However,

their order book shape models are only approximations of the real market which do not capture the rich dynamics of a realistic order book.

Agent-based modeling has been applied to finance for many years [7]. It provides a microscopic approach to the study of the behaviors of financial markets. By modeling the interactions between the trader agents and the market agent under explicit trading rules, the agent-based artificial market can produce many features that exist in real markets. This method is very useful for research in financial market microstructure which focuses on the price formation and price discovery processes including how trading and market structure affect the behavior of market prices. Agent-based modeling methodology provides an alternative approach to model and study financial markets. It differs from the traditional equilibrium models which can not always provide analytical solutions [8].

The purpose of the paper is to examine whether agent intelligence is needed to replicate the realistic relationship between price impact and market order size with an artificial market simulation. This study builds a *zero-intelligence* based artificial stock market model. This model extends the previous work done by Smith et. al. [9] from three aspects: the first is that our model distinguishes limit orders according to their order aggressiveness; the second is that our model considers stochastic order sizes generated from log-normal distributions; the third is that our model takes into account power-law distributed limit order placements. Another contribution of this study is that the artificial market model with 18 parameters is calibrated using ultra high-frequency data from the London Stock Exchange. This research can help us to better understand the price formation process in stock market, and the artificial market can be used as a test bed for devising high-frequency trading strategies.

The paper is organized as follows: Section II gives a brief introduction to market mechanism and price impact; Section III reviews related work on agent-based artificial stock markets; Section IV describes our zero-intelligence based model; Section V presents the data used in this study and shows how to estimate the parameters of our model from the data; Section VI explains the experimental setup; Section VII analyzes and discusses the results; Section VIII concludes the study and suggests future work.

## II. BACKGROUND

In this section we provide an introduction to the market structure most commonly found in large international equity markets, and we also discuss the researches on price impact of a market order from the market microstructure literature.

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<sup>1</sup>Average daily volume (ADV), explained by Yahoo Finance, is the monthly average of the cumulative trading volume during the last 3 months divided by 22 days. The recent ADV of IBM stock is 6,135,410 shares.

<sup>2</sup>This is estimated by using the following values: shares outstanding (1.8 billion), average daily volume (6,135,410 shares) from Yahoo Finance at Nov. 15, 2011, and assuming that he traded from Mar. 1 to Sep. 30, 2011.

TABLE I  
ORDER BOOK 1

Bid		Ask	
Shares	Prices	Prices	Shares
300	50.19	50.22	200
<b>200</b>	50.18	50.23	300
400	50.17	50.24	100
500	50.16	50.25	300
300	50.15	50.26	200
100	50.14	50.27	600

TABLE II  
ORDER BOOK 2

Bid		Ask	
Shares	Prices	Prices	Shares
300	50.19	50.22	<b>200</b>
<b>500</b>	50.18	50.23	300
400	50.17	50.24	100
500	50.16	50.25	300
300	50.15	50.26	200
100	50.14	50.27	600

TABLE III  
ORDER BOOK 3

Bid		Ask	
Shares	Prices	Prices	Shares
300	50.19	50.22	<b>100</b>
500	50.18	50.23	300
400	50.17	50.24	100
500	50.16	50.25	300
300	50.15	50.26	200
100	50.14	50.27	600

TABLE IV  
ORDER BOOK 4

Bid		Ask	
Shares	Prices	Prices	Shares
300	50.19	<b>50.23</b>	<b>100</b>
500	50.18	50.24	100
400	50.17	50.25	300
500	50.16	50.26	200
300	50.15	50.27	600
100	50.14	50.28	300

### A. Limit Order Market

Today most equity markets operate an *electronic double auction limit order book*. Examples include the Electronic Communication Networks (ECNs) in the United States, the Toronto Stock Exchange, and the Hong Kong Stock Exchange. Electronic trading platforms in derivative markets have also gained popularity in recent years over the traditional open-outcry auctions, such as Chicago Mercantile Exchange's (CME) Globex platform and International Securities Exchange's electronic option trading platform. One advantage of an open limit-order book is the greater transparency offered by these systems when compared with dealer market settings. Price quotes and transactions are visible to all participants which generally improves the efficiency of price discovery, thus promoting market confidence. It also promotes competition as dealers/market makers are encouraged to post the best prices to attract order flow [10].

In a limit order market, traders can either submit a *limit order* or a *market order*. A market order is an order to buy or to sell a specified number of shares. It guarantees immediate execution but provides no control over its execution price. In contrast, a limit order is an order to buy or to sell a specified number of shares at a specified price. It provides control over its execution price but does not guarantee its execution.

Table I shows a sample order book, where all the buy and sell orders are visible/transparent to traders in the market. It consists of two queues which store buy and sell limit orders, respectively. Buy limit orders are called *bids*, and sell limit orders are called *offers* or *asks*. The highest bid price on the order book is called *best bid*, and the lowest ask price on the order book is called *best ask*. The difference between best bid and best ask is called *bid-ask spread*. Prices on the order book are not continuous, but rather change in discrete quanta called *ticks*.

In 2001, the US equity markets changed their minimum tick size from one sixteenth of a dollar to one cent. Since decimalisation, the average trade size has declined from 1,200 shares per transaction in 2000 to 300 shares today. This in turn has led to an explosion in the number of trades executed and a narrowing of spreads with large institutional orders taking longer to execute. Due to these changes, Wall Street firms (both buy-side and sell-side) have started to embrace AT for trade execution over the last few years [11]. The emergence of AT has resulted in a substantial increase in the speed of trade execution and a significant reduction in the average value of each trade [12].

In a limit order market, orders arrive randomly over time.

The price limit of a newly arrived order is compared to those of orders already held in the system to ascertain if there is a match. If so, the trade occurs at the price set by the first order. The set of unexecuted limit orders held by the system constitutes the dynamic order book, where limit orders can be cancelled or modified at any time prior to their execution. Limit orders on the order book are typically (depending on market rules) executed strictly according to (1) price priority and (2) time priority. Bid (ask) orders with higher (lower) prices get executed first with time of placement being used to break ties. A buy (sell) market order is executed at the best ask (bid) price. The limit order book is highly dynamic, because new limit orders will be added into the order book, and current limit orders will get executed or cancelled from the order book throughout the trading day. Table II shows the order book after a trader submits a buy limit order with 300 shares placed at price 50.18. Table III shows the order book after a trader submits a buy market order with 100 shares. Table IV shows the order book after a trader submits a buy market order with 300 shares.

Apart from market and limit orders, some stock exchanges also offer *hidden/iceberg orders* to allow traders to conceal the total size of a large limit order. Such orders consist of two components, a small component whose size is visible in the order book and a larger hidden component with a size known only to the order submitter. The hidden component is exposed to the market gradually through execution of the visible part of the order [13], [14]. Many electronic trading platforms have introduced this kind of order, including Euronext, the Toronto Stock Exchange, the London Stock Exchange, and XETRA. Hidden limit orders are often used by large liquidity traders to hide their intent to trade [15]. However, iceberg orders exhibit a less favorable time priority compared with pure limit orders [16], [17]. After the visible portion of an iceberg order is completely matched, other visible limit orders at the same limit price that were entered before the new portion is displayed take priority.

### B. Price Impact of a Market Order

Price impact is the average response of prices to trades [18]. It is related to the trade sign, trade size and market characteristics.

The relationship between price impact and trade volume has been extensively studied in the financial market microstructure literature. The conclusion of previous studies is that the price impact of a market order is a concave function of its order size, and has been validated using transaction-level data in [19], [20], [21], [22], [23]. Price

impact is measured as the difference of mid-quote price before the order arrives and the mid-quote price after it has been executed. The functional form takes the form

$$p = \gamma * v^\mu$$

where  $p$  is the price impact of a market order with order size  $v$ ,  $\gamma$  and  $\mu$  are constants of the function. The specific constants of the function vary in different markets and at different time periods. Lillo et. al. [19] find that the exponent  $\mu$  is 0.5 for small orders and 0.2 for large orders at NYSE. Farmer & Lillo [20] find that  $\mu$  is 0.26 for LSE data. Hopman [23] find  $\mu$  is 0.37 for Paris Bourse stocks.

Although a concave function is a good approximation of price impact of a market order, which has been empirically identified by many studies, it does not capture the full richness and complexity of the market dynamics [24]. The order book is found to play an important role in determining the price impact [20] [25] [23]. Farmer & Lillo [20] and Weber & Rosenow [26] both find that large price impacts of individual market orders are caused by a low density of limit orders in the order book, not by large trading volume. A proper empirical characterization of the price impact in stock market is still lacking [18].

### III. RELATED WORK: ZERO-INTELLIGENCE BASED ARTIFICIAL STOCK MARKET

Over the last decade, a number of agent-based artificial stock markets have been built in order to study and understand the stock market dynamics [27].

Our model is based on zero-intelligence (ZI) model. The notion of ZI was first introduced by Gode and Sunder [28]. The traders can randomly generate buying and selling orders. The impatient orders submitted by the traders are executed against those patient orders which are previously submitted to the market. The patient orders are placed in the order book according to the price and time priorities. Based on this idea, Smith et. al. [9] simulate a limit order market to study order flows and market dynamics. In Smith's model, the arrivals of the order flows composed of market orders, limit orders and order cancelations are modeled as poisson processes. Order sizes are produced from a half-normal distribution. One important finding of their results is that the price impact of a market order is a concave function of the size of the market order which is qualitatively consistent with empirical findings in previous studies.

Another research which simulates a limit order market in the spirit of ZI idea is conducted by [29]. In their model, there are 100 ZI traders endowed with the same cash and shares. The arrival of each trader is modeled as a poisson process. At each arriving time, the trader chooses to buy or sell with probability  $1/2$ , and decides to submit a market order, or a limit order, or cancel a previous submitted order with probabilities  $\pi_m$ ,  $\pi_l$  and  $1 - \pi_m - \pi_l$  respectively. This model distinguishes two types of limit orders according to order aggressiveness. Traders place a limit order uniformly distributed inside the bid-ask spread with probability  $\pi_{in}$ , or

power-law distributed away from the spread with probability  $1 - \pi_{in}$ . The volumes of incoming limit orders are produced from a log-normal distribution, and the sizes of market orders are the same as those of the best counterpart orders. Their model reproduces the principal stylized facts exhibited by real markets.

The most recent work is the paper by [30]. They build an agent-based stock market model consisting of ZI agents to mimic the Taiwan Stock Market (TWSE). Different from continuous matching in limit order market, the order matching at TWSE is organized every 25 seconds. In their model at each simulated time corresponding 0.01 second of real time, there are five possible events happening in the artificial market: limit order submission, market order submission, order cancelation, order matching and no activity. The probabilities of these events are estimated using real data from TWSE, which are then used for simulating the artificial market. Like [9], they use a stochastic order size which is generated randomly from a half-normal distribution. In the experiment, they compare the liquidity costs in real market and simulated market, which is measured as the difference of the virtual payment at the disclosed price when the order is entering the market and the actual transaction payment. The result shows that the liquidity cost generated by the simulation data are higher than those for the TWSE data which is possibly caused by their overestimated market order size.

### IV. AN ZERO-INTELLIGENCE BASED LIMIT ORDER MARKET MODEL

Our model tries to mimic a limit order market. The trading mechanism in our model follows the price/time priority described in Section II-A. Traders are allowed to submit orders at any time, and cancel their limit orders which are not executed. Our model extends Smith's model [9] by distinguishing different limit orders according to order aggressiveness, considering log-normal distributed order size and taking into account power-law distributed limit order placements in order to come as close as possible to a realistic order flow. An algorithm describing the artificial market simulation process is shown in Algorithm 1.

In our model, there are two agents: a buy agent and a sell agent. All the buy (sell) limit/market orders are placed by the buy (sell) agent. We assume that each hypothetical time in our artificial market corresponds to one millisecond in real market. At each time in the artificial market, a buy agent or a sell agent is chosen with probability  $1/2$ . The chosen agent at each time can perform one of the four actions as below to fulfill his investment object:

- do nothing,
- submit a market order,
- submit a limit order, or
- cancel an outstanding<sup>3</sup> limit order.

With probability  $\lambda_o$ , the agent will do nothing at all; with probability  $\lambda_m$ , the agent will submit a market order to the

<sup>3</sup>An outstanding limit order means that the limit order is still listed on the order book

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### Algorithm 1 Artificial Limit Order Market

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Generate a random number from a uniform distribution (0, 1) and determine the agent type according to the probability 0.5.

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switch Agent do
  case Buyer
    Generate a random number from a uniform distribution (0, 1) and determine the action type according to the probabilities  $\lambda_o$ ,  $\lambda_m$ ,  $\lambda_l$  and  $\lambda_c$ .
    switch Action do
      case Do nothing
        | Do nothing at all.
      endsw
      case Submit a market order
        | Execute a market order with the order size generated from a log-normal distribution with parameters  $\mu_{mo}$  and  $\sigma_{mo}$ 
      endsw
      case Submit a limit order
        Generate a random number from a uniform distribution (0, 1) and determine the limit order type according to probabilities  $\lambda_{crs}$ ,  $\lambda_{inspr}$ ,  $\lambda_{spr}$  and  $\lambda_{offspr}$ .
        switch LimitOrder do
          case Crossing Limit Order
            | Submit a limit order at the best ask price with the order size generated from a log-normal distribution with parameters  $\mu_{crs}$  and  $\sigma_{crs}$ .
            | Execute the crossing limit order. If it is not fully executed, place the rest of the limit order at the best ask price on the order book.
          endsw
          case Inside-spread Limit Order
            | Generate a random value denoted as  $P_{inspr}$  from a uniform distribution ( $BestBid$ ,  $BestAsk$ ).
            | Place a limit order at the price  $P_{inspr}$  on the order book with the order size generated from a log-normal distribution with parameters  $\mu_{inspr}$  and  $\sigma_{inspr}$ .
          endsw
          case Spread Limit Order
            | Place a limit order at the best bid price on the order book with the order size generated from a log-normal distribution with parameters  $\mu_{spr}$  and  $\sigma_{spr}$ .
          endsw
          case Off-spread Limit Order
            | Generate a random value denoted as  $RP_{offspr}$  from a power-law distribution with exponent  $\beta_{off-p}$ .
            | Place a limit order at the price ( $BestBid - RP_{offspr}$ ) on the order book with the order size generated from a log-normal distribution with parameters  $\mu_{offspr}$  and  $\sigma_{offspr}$ .
          endsw
        endsw
      endsw
      case Cancel an outstanding limit order
        | Cancels the oldest outstanding limit order that the buyer previously submitted.
      endsw
    endsw
  endsw
  case Seller
    | (similar to the case of buyer)
  endsw
endsw

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market; with probability  $\lambda_l$ , the agent will submit a limit order to the market; with probability  $\lambda_c$ , the agent will cancel a limit order which he previously placed and is not executed. The sum of these probabilities has to be one ( $\lambda_o + \lambda_m + \lambda_l + \lambda_c = 1$ ). Without loss of generality, we assume that the agent always cancels the oldest limit order which he previously placed.

If the agent wants to submit a limit order, he has four types of limit order to choose according to the order's aggressiveness. These four types of limit orders are:

- crossing limit order, which causes an immediate execution,
- inside-spread limit order, which is placed inside the bid-ask spread,
- spread limit order, which is placed at the best bid (ask) price if it is a buy (sell), and
- off-spread limit order, which is placed inside the order book with less attractive price than the best quote.

With probability  $\lambda_{crs}$ , the agent chooses a crossing limit order; with probability  $\lambda_{inspr}$ , the agent uses an inside-spread limit order; with probability  $\lambda_{spr}$ , the agent adopts a spread limit order; with probability  $\lambda_{offspr}$ , the agent places an off-spread limit order. The sum of these probabilities also

has to be one ( $\lambda_{crs} + \lambda_{inspr} + \lambda_{spr} + \lambda_{offspr} = 1$ ). We assume that crossing buy (sell) limit orders are always placed at the best ask (bid) price<sup>4</sup>. If the size of the crossing buy (sell) limit order exceeds the depth at the best ask (bid) quote, the unexecuted part of the buy (sell) limit order will be placed at the best ask (bid) quote. The inside-spread limit orders are uniformly placed between the best bid quote and the best ask quote. The placement of off-spread limit order follows a power-law distribution with exponent  $\beta_{off-p}$ <sup>5</sup>, and various order sizes are allowed in our model, which are generated from log-normal distributions<sup>6</sup>.

### V. MODEL CALIBRATION

In order to make our model more realistic, we estimate the 14 parameters (shown in Table VI) of our model using real data from London Stock Exchange (LSE). One important

<sup>4</sup>This assumption is made based on our observation of LSE market that crossing limit orders are rarely placed far away from the best quote on the opposite side of the market.

<sup>5</sup>Empirical work [31], [32] finds that the relative price of the off-spread limit order follows a power-law distribution. The relative price is the difference between the limit price of the order and the best quote.

<sup>6</sup>Empirical work [33] finds that order size is roughly distributed like a log-normal distribution with a power law tail.

TABLE V  
DESCRIPTIVE STATISTICS OF ROB DATA AT SEP. 6, 2010 FOR  
BARCLAYS CAPITAL

Descriptive Statistics	
Average midquote price:	323.04
Average bid-ask spread:	0.11
Average trade size (dollars):	2110606
Average trade size (shares):	6537
Number of trades:	4329
Number of order cancelations:	68783
Number of market orders:	3972
Number of limit orders:	142368

reason why we study LSE market is that it provides data that contain details of every order and every trade which enables us to calculate the parameters of our model. This section provides a brief introduction to the LSE data and describes how to estimate the parameters of our model using the data.

#### A. Data

LSE is one of the largest exchanges in the world. It provides an electronic order book platform, Stock Exchange Electronic Trading System (SETS), for trading indexed securities. This trading system has three sessions on each trading day:

- opening auction session, from 07:50:00 to 08:00:00, where buy and sell orders are matched at 08:00:00,
- continuous trading session, from 08:00:00 to 16:30:00, where buy and sell orders are continuously matched at price/time priority, and
- closing auction session, from 16:30:00 to 16:35:00, where buy and sell orders are matched at 16:35:00.

In this study, we use the Rebuild Order Book (ROB) data, which provide intraday information on all trades and orders. The precision of recording time in the data is one millisecond. ROB provides the information on all orders and trades at the end of each trading day in three files:

- ‘order details’ file, which contains details of every new persistent order<sup>7</sup> entering the electronic order book,
- ‘order history’ file, which contains information on changes to each persistent order, including order matching, order deletion, order expiration and order modification, and
- ‘trade report’ file, which contains details of every transaction.

Each order submitted to the electronic market has a unique code number which maps the new persistent orders listed on ‘order details’ file and their order trajectories (full execution, partial execution, deletion, expiration and modification) recorded on ‘order history’ file.

For each transaction, ‘order history’ file contains information on both sides of the transaction. The transaction record on ‘order history’ file is mapped to each transaction and has a unique trade code which is used to track the transaction details contained in ‘trade report’ file.

<sup>7</sup>Persistent orders are the orders which are stored on the electronic order book after they are submitted to the market.

The ROB data are raw data. In order to get the information we need to estimate the parameters of our model, we must infer non-persistent market orders and three missing events from the given data files. Market orders can be inferred from the ‘order history’ file as it records information on the matching side of each transaction. The first missing event in ROB data is the execution of iceberg limit order. LSE allows traders to place iceberg limit order, part of which is hidden in the order book and is not recorded in the ROB data. When the visible part of the iceberg limit order is matched by a market order with larger size, the hidden part will be executed against the rest of the market order. The traded hidden part of the iceberg limit order can be inferred from the records of the limit order whose transaction size is larger than its original size. The second missing event is the crossing limit order which causes immediate trade after submission. The ‘order details’ file only records the unexecuted part of the crossing limit orders. The traded part can be found from the ‘order history’ file in that each crossing limit order is matched by one or more persistent limit orders previously submitted to the market. The third missing event is the limit order which was submitted to the market a few days ago but is executed today. The information on details of these orders needs to be recovered from older data files.

For this study, we choose the stock Barclays Capital, which is one of the most frequently traded stocks in LSE. Our sample data covers details of all orders and trades from 1th April to 30th September 2010. We are only interested in the continuous trading session. The records before 8:00:00 and after 16:30:00 are ignored. Table V shows the descriptive statistics of the trades and orders data for Barclays Capital.

#### B. Parameter Setting of Agent-based Limit Order Market Model

Our data cover 126 trading days. During the continuous trading session, there are 30,600,000 milliseconds in each trading day. We calculate the numbers of market orders, limit orders and order cancelations<sup>8</sup> and the probabilities of these events on each day. As we assume only one event occurs at each millisecond in our model, the events occurring at the rest of the trading period are ‘do nothing’ events. The probability parameters for the four events (do nothing  $\lambda_o$ , limit order  $\lambda_l$ , cancelation  $\lambda_c$  and market order  $\lambda_m$ ) of our model are estimated as the average daily probabilities from the real data. We do the similar calculations for probabilities of the limit order types on each trading day using LSE data. The probability parameters for limit order types ( $\lambda_{crs}$ ,  $\lambda_{inspr}$ ,  $\lambda_{spr}$  and  $\lambda_{offspr}$ ) in our model are estimated as the average daily probabilities over the whole period weighted by the total number of limit orders on each trading day.

In our model, each value in order sizes is produced from log-normal distributions which take the functional form:

$$\exp(\mu + \sigma * r_{norm})$$

<sup>8</sup>We take order deletion and order expiration in ROB data as the same, both are counted as order cancelation events.

TABLE VI

PARAMETERS OF ARTIFICIAL LIMIT ORDER MARKET SIMULATION

Market Settings	Values
initial mid-quote price	300
initial bid-ask spread	0.5
tick size	0.01
Agent Type	Probabilities
buy or sell	0.5
Event Type	Probabilities
do nothing	$\lambda_o = 0.9847$
submit a market order	$\lambda_m = 0.0003$
submit a limit order	$\lambda_l = 0.0077$
cancel a limit order	$\lambda_c = 0.0073$
Limit Order Type	Probabilities
crossing limit order	$\lambda_{crs} = 0.0032$
inside-spread limit order	$\lambda_{inspr} = 0.0978$
spread limit order	$\lambda_{spr} = 0.1726$
off-spread limit order	$\lambda_{offspr} = 0.7264$
Order Size Type	Parameters of Log-normal Distribution
market order size	$\mu_{mo} = 7.5663$ $\sigma_{mo} = 1.3355$
crossing limit order size	$\mu_{crs} = 8.4701$ $\sigma_{crs} = 1.1982$
inside-spread limit order size	$\mu_{inspr} = 7.8709$ $\sigma_{inspr} = 0.9799$
spread limit order size	$\mu_{spr} = 7.8929$ $\sigma_{spr} = 0.8571$
off-spread limit order size	$\mu_{offspr} = 8.2166$ $\sigma_{offspr} = 0.9545$
Limit Price Type	Parameters of Power-law Distribution
off-spread relative limit price	$xmin_{off-p} = 0.05$ $\beta_{off-p} = 1.7248$

where  $\mu$  and  $\sigma$  are parameters,  $r_{norm}$  is a random number normally produced from (0,1), and  $exp$  is an exponential function of the natural number. Each value in relative limit prices is drawn from the power-law function as:

$$x_{min} \times (1 - r)^{-\frac{1}{1-\beta}}$$

where  $r$  is a random number uniformly generated from (0,1),  $x_{min}$  and  $\beta$  are the parameters which need to be estimated. The data in the whole period are used to estimate these parameters for all kinds of orders using the method of maximum likelihood<sup>9</sup>. In order to better fit the power-law distribution while keeping the reliability of the original data, we exclude the values  $x_t$  whose probabilities satisfy  $P(x > x_t) < 0.01$  or  $P(x < x_t) < 0.01$ . All the estimated parameters are shown in Table VI.

## VI. EXPERIMENTAL SETUP

Our model is simulated using Matlab. In our model, there are always at least three levels on each side of the order book in order to prevent the order book from being empty. The artificial market runs for 34,200,000 hypothetical milliseconds (correspond to 9 and a half hours) in each simulation. In some case, the artificial market may be unstable initially due to the stochastic order flows. Thus, the first 3,600,000 milliseconds (correspond to 1 hour) are used to warm up the market. We are only observing the later 30,600,000 milliseconds which correspond to the continuous trading session in one trading day. Our artificial market is run for 30 artificial trading days. Like LSE, our system also has a data recording function which writes down the details of every trade and every order.

<sup>9</sup>The matlab code for estimating the power-law distributions is developed by the researchers at Santa Fe Institute, which can be downloaded from the website <http://tuvalu.santafe.edu/aaronc/powerlaws/>.

TABLE VII

EVENTS AND LIMIT ORDER TYPES IN ARTIFICIAL MARKET

	Probabilities	Daily Occurrences
Order Sign		
buy	$0.5000 \pm 0.0001$	$15300854 \pm 2517$
sell	$0.5000 \pm 0.0001$	$15299145 \pm 2517$
Event Type		
do nothing	$0.9847 \pm 0.0000$	$30132401 \pm 722$
submit a market order	$0.0003 \pm 0.0000$	$9399 \pm 104$
submit a limit order	$0.0076 \pm 0.0001$	$233785 \pm 483$
cancel a limit order	$0.0073 \pm 0.0000$	$224413 \pm 472$
Limit Order Type		
crossing limit order	$0.0032 \pm 0.0001$	$736 \pm 29$
inside-spread limit order	$0.0977 \pm 0.0006$	$22829 \pm 152$
spread limit order	$0.1728 \pm 0.0010$	$40408 \pm 232$
off-spread limit order	$0.7262 \pm 0.0011$	$169810 \pm 444$

## VII. RESULTS AND DISCUSSIONS

After 30 runs of artificial market simulation, we calculated the numbers of events and orders and their occurrence probabilities in each day (equals each run). Table VII shows their means and standard deviations over the 30 simulated trading days. An observation from the table is that the events and orders probabilities in the simulated market are very close to those in the LSE market (shown in Table VI).

We compared the order sizes of different order types in the LSE market and those from the simulated market. Table VIII shows some summary statistics of order sizes and relative limit prices for the six-month period (126 trading days) in the LSE data and for the 30-artificial-day period in the simulated data respectively. One can observe that the average order sizes for all order types and the average relative limit prices in the simulated market are very close to those in the LSE market.

This study adopts the method used in [34] and [19] to measure the price impact of a market order. Letting the logarithm of midquote price be  $p$ , the price impact caused by a market order is calculated as

$$\Delta p = p_{after} - p_{before}$$

where  $p_{before}$  is the price just before the market order arrived the market and  $p_{after}$  is the price immediately after the order is executed. The methods used to measure the trade size are different in previous literature. Lillo et. al. [19] measure it as traded value in dollar divided by the stock value while Hopman [23] measure it as the number of shares in the order divided by the number of shares outstanding. In this study, the trade size is measured as the shares of the market order divided by the total trading volume in each trading day.

We investigate the average behavior of price impact by dividing the data based on trade size into 10 bins and compute the average price impact for the data in each bin. As many previous studies have demonstrated that the behavior of price impact is almost the same for both buy and sell trades, this study does not distinguish buy and sell trades. Figures 1 and 2 depict the relationship between the price impacts and trade sizes in the LSE and the simulated market respectively.

From Figures 1, one finds that the relationship between

TABLE VIII  
SUMMARY STATISTICS OF ORDER SIZES AND RELATIVE LIMIT PRICES

		Min	Max	Mean	S.D.	Observations
Market Order Size	LSE Data	1	789367	3916	6558	1184045
	Simulated Data	2	786637	4721	10481	281970
Crossing Limit Order Size	LSE Data	3	12500000	10562	46111	93200
	Simulated Data	36	543378	9676	16716	22080
Inside-spread Limit Order Size	LSE Data	1	2476197	3902	4761	2881792
	Simulated Data	16	351554	4237	5384	684870
Spread Limit Order Size	LSE Data	1	3398413	3723	4782	5087395
	Simulated Data	33	183278	3867	4028	1212240
Off-spread Limit Order Size	LSE Data	1	3666101	5966	11910	21412078
	Simulated Data	16	564201	5839	7123	5094300
Off-spread Limit Order's Relative Price	LSE Data	0.05	100.00	1.29	2.76	21412078
	Simulated Data	0.05	99.99	0.93	4.72	5107980

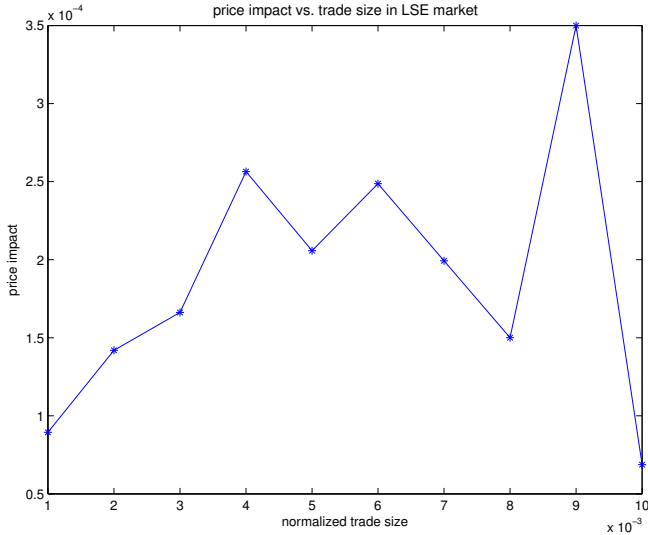


Fig. 1. Price impact vs. trade size in the LSE market

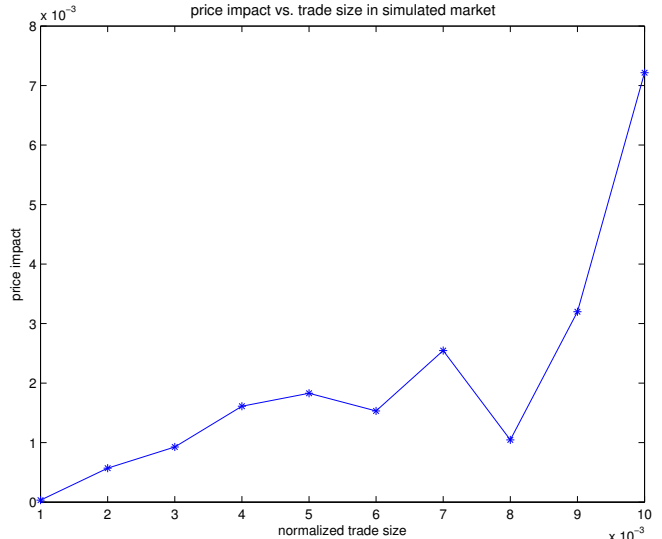


Fig. 2. Price impact vs. trade size in the simulated market

the price impact and trade size is nonlinear and concave. Larger trades do not always have higher price impact than smaller trades. This finding is consistent with the conclusion in previous literature. There are three explanations for this concavity in literature. The first explanation is that large-order trades have private information about the market price. The second explanation is that large-order traders are patient traders who wait for periods of high liquidity [35]. The third explanation is that the order book has deeper liquidities away from the best bid or ask [36].

By comparing the price impacts in the two markets, we find that generally the price impact in simulated market is larger than that in the LSE market. The price impacts for the smallest trade sizes are very close in the two markets, but the price impacts for larger trade sizes (ranging from  $2 \times 10^{-3}$  to  $10 \times 10^{-3}$ ) in simulated market are bigger than those in the LSE market. One possible reason is that limit price gaps in the simulated market are larger than those in the LSE market. However, this can be excluded by the fact that the average relative limit price in simulated market is smaller than that in the LSE market (showed in Table VIII). Another possible reason is that traders in the LSE market execute their orders intelligently, for example, by observing the market conditions. Farmer et al. [35] and Hopman [23] both find that large orders are traded when the market liquidity is

deep. Because the trader agents in our simulated market are ‘zero intelligent’ who do not care about the market liquidity when trading their orders, the price impacts caused by trading large orders in the simulated market are higher than the price impacts in the LSE market. This suggests that agent intelligence is needed when simulating an artificial market which tries to replicate the relationship between price impact and market order size.

## VIII. CONCLUSIONS

Trading affects price in financial markets. It is empirically demonstrated that a buy trade pushes the price up and a sell trade pushes it down. Modeling price impact is an active domain of research in trading firms. Many price impact models were also developed in the literature, but they are just approximations. This study adopts an agent-based modeling approach to model price impacts of market orders. Our model is based on Smith’s ZI model [9] in which agents have no intelligence and randomly submit orders to the market. In this study, we extend their model by distinguishing limit order with different order aggressiveness and considering power-law distributed limit order placements and log-normal distributed order sizes. After calibrated using trades and orders data from the LSE, the model simulates the continuous trading session in the LSE market. The

simulated market in this study can be used for devising high-frequency trade execution strategies and for the research of price formation and price discovery. The results show that the price impacts caused by large market orders in the simulated market are higher than the price impacts in the LSE market. Previous literature [35] has found that large-order traders wait for periods of high liquidity. In contrast, the agents in our model randomly trade orders without monitoring the market conditions. This suggests that agent intelligence is a necessary condition when simulating an artificial market where replicating realistic price impact is a concern.

Future work will focus on how to improve our model. The first task is to adopt the evolutionary methods to optimize the estimations of the parameters in our models in order to achieve a more realistic simulated market. The second task is to add some intelligent agents into our current model in order to better replicate the price impacts of market orders. Lastly but not the least, as Lyden's study [37] finds that price impact shows intraday seasonality, we will extend current research to investigate the intraday behavior of price impact.

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#### REFERENCES

- [1] R. Kissell and M. Glantz, *Optimal Trading Strategies: Quantitative Approaches for Managing Market Impact and Trading Risk*. AMA-COM, 2003.
- [2] D. Bertsimas and A. Lo, "Optimal control of execution costs," *Journal of Financial Markets*, vol. 1, no. 1, pp. 1–50, 1998.
- [3] R. Almgren and N. Chriss, "Optimal execution of portfolio transactions," *Journal of Risk*, vol. 3, no. 2, pp. 5–39, 2000.
- [4] R. Almgren, "Optimal execution with nonlinear impact functions and trading-enhanced risk," *Applied Mathematical Finance*, vol. 10, no. 1, pp. 1–18, 2003.
- [5] A. Obizhaeva and J. Wang, (2006) Optimal trading strategy and supply/demand dynamics. [Online]. Available: <http://www.nber.org/papers/w11444>.
- [6] A. Alfonsi, A. Fruth, and A. Schied, "Optimal execution strategies in limit order books with general shape functions," *Quantitative Finance*, vol. 10, no. 2, pp. 143–157, 2010.
- [7] S. H. Chen, "Varieties of agents in agent-based computational economics: A historical and an interdisciplinary perspective," *Journal of Economic Dynamics and Control*, vol. 36, no. 1, pp. 1–25, 2012.
- [8] C. Parlour and D. Seppi, "Limit order markets: A survey," in *Handbook of Financial Intermediation and Banking*, A. V. Thakor and A. W. Boot, Eds. Amsterdam: Elsevier, 2008, pp. 63–96.
- [9] E. Smith, J. Farmer, L. Gillemot, and S. Krishnamurthy, "Statistical theory of the continuous double auction," *Quantitative Finance*, vol. 3, no. 1, pp. 481–514, 2003.
- [10] CFA, "Market microstructure: The impact of fragmentation under the markets in financial instruments directive," *CFA Institute Publications*, vol. 2009, no. 13, pp. 1–60, 2009.
- [11] R. Kissell, "Algorithmic trading strategies," Ph.D. dissertation, Fordham University, US, 2006.
- [12] ASX, (2010) Algorithmic trading and market access arrangements. [Online]. Available: <http://www.asx.com.au/about/pdf/20100211>.
- [13] M. Aitken, H. Berkman, and D. Mak, "The use of undisclosed limit orders on the Australian stock exchange," *Journal of Banking and Finance*, vol. 25, no. 8, pp. 1589–1603, 2001.
- [14] B. Biais, P. Hillion, and C. Spatt, "An empirical analysis of the limit order book and the order flow in the Paris bourse," *Journal of Finance*, vol. 50, no. 3, pp. 1655–1689, 1995.
- [15] S. Bongiovanni, M. Borkovec, and R. Sinclair, "Let's play hide-and-seek: The location and size of undisclosed limit order volume," *Journal of Trading*, vol. 1, no. 3, pp. 34–46, 2006.
- [16] H. Bessembinder, M. Panayides, and K. Venkataraman, "Hidden liquidity: An analysis of order exposure strategies in electronic stock markets," *Journal of Financial Economics*, vol. 94, no. 3, pp. 361–383, 2009.
- [17] A. Esser and B. Monch, "The navigation of an iceberg: The optimal use of hidden orders," *Finance Research Letters*, vol. 4, no. 1, pp. 68–81, 2007.
- [18] J. Lillo, J. Farmer, and F. Lillo, "How markets slowly digest changes in supply and demand," in *Handbook of Financial Markets: Dynamics and Evolution*, T. Hens and K. Schenk-Hoppe, Eds. US: North Holland, 2009, pp. 57–160.
- [19] F. Lillo, J. Farmer, and R. Mantegna, "Master curve for price impact function," *Nature*, vol. 421, no. 3, pp. 129–130, 2003.
- [20] J. Farmer and F. Lillo, "On the origin of power-law tails in price fluctuations," *Quantitative Finance*, vol. 4, no. 1, pp. 7–10, 2004.
- [21] J. D. Farmer, P. Patelli, and I. Zovko, "The predictive power of zero intelligence models in financial markets," in *Proceedings of the National Academy of Sciences of the United States of America*, vol. 102, pp. 2254–2259, 2005.
- [22] M. Lim and R. Coggins, "The immediate price impact of trades on the Australian stock exchange," *Quantitative Finance*, vol. 5, no. 4, pp. 365–377, 2005.
- [23] C. Hopman, "Do supply and demand drive stock prices?" *Quantitative Finance*, vol. 7, no. 1, pp. 37–53, 2007.
- [24] C. Stephens, H. Waelbroeck, and A. Mendoza, (2010) Non-linear dynamics of order flow and its effects on price.
- [25] P. Weber and B. Rosenow, "Order book approach to price impact," *Quantitative Finance*, vol. 5, no. 1, pp. 357–364, 2005.
- [26] —, "Large stock price changes: volume or liquidity?" *Quantitative Finance*, vol. 6, no. 1, pp. 7–14, 2006.
- [27] A. Brabazon and M. O'Neill, *Biologically Inspired Algorithms for Financial Modeling*. Berlin: Springer-Verlag, 2006.
- [28] D. Gode and S. Sunder, "Allocative efficiency of markets with zero-intelligence traders," *Journal of Political Economy*, vol. 101, no. 1, pp. 119–137, 1993.
- [29] G. Daniel, "Asynchronous simulations of a limit order book," Ph.D. dissertation, University of Manchester, UK, 2006.
- [30] Y. Huang, S. Chen, M. Hung, and T. Yu, "An order-driven agent-based artificial stock market to analyze liquidity costs of market orders in the Taiwan stock market," in *Natural Computing in Computational Finance (vol. 4)*, A. Brabazon, M. O'Neill, and D. Maringer, Eds. Berlin Heidelberg: Springer-Verlag, 2012.
- [31] I. Zovko and J. Farmer, "The power of patience: a behavioural regularity in limit-order placement," *Quantitative Finance*, vol. 2, no. 5, pp. 387–392, 2002.
- [32] M. Potters and J. Bouchaud, "More statistical properties of order books and price impact," *Physica A: Statistical Mechanics and its Applications*, vol. 324, no. 1, pp. 133–140, 2003.
- [33] S. Maslov and M. Millis, "Price fluctuations from the order book perspective - empirical facts and a simple model," *Physica A*, vol. 299, no. 1, p. 234, 2001.
- [34] F. Lillo, J. Farmer, and R. Mantegna, (2002) Single curve collapse of the price impact function for the New York stock exchange. [Online]. Available: <http://arxiv.org/abs/cond-mat/0207428>.
- [35] J. D. Farmer, L. Gillemot, F. Lillo, S. Mike, and A. Sen, "What really causes large price changes?" *Quantitative Finance*, vol. 4, no. 4, pp. 383–397, 2004.
- [36] J. Bouchaud, M. Mezard, and M. Potters, "Statistical properties of stock order books: empirical results and models," *Quantitative Finance*, vol. 2, no. 4, pp. 251–256, 2002.
- [37] S. Lyden, "Time of Day and Market Impact," *Journal of Trading*, vol. 2, no. 3, pp. 76–81, 2007.