Upstand Finite Element Analysis of Bridge Decks

by

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Abstract

For slab bridge decks with wide transverse cantilevers, the plane grillage analogy is shown to be an inaccurate method of linear elastic analysis due to variations in the vertical position of the neutral axis. The upstand grillage analogy is also shown to give inaccurate results, this time due to inappropriate modelling of in-plane distortions. A new method, known as upstand finite element analysis, is proposed which is sufficiently simple to be used on an everyday basis in the design office. The method is shown to give much better agreement than the others when compared with an elaborate three-dimensional solid finite element model. Single- and two-span bridge decks with solid and voided sections are considered for both longitudinal and transverse bending stresses.

1 Introduction

The Plane Grillage Analogy is a popular method among bridge designers of modelling slab bridge decks in two dimensions. It involves the idealisation of the bridge slab as a mesh of longitudinal and transverse beams all located in the same plane. Implicit in the method is an assumption that all parts of the bridge cross section bend about a single neutral axis which is generally taken to be the centroid of the deck. The depth of this neutral axis below a datum (usually horizontal) is assumed not to vary throughout the bridge. Finite element analysis (FEA) is used extensively by designers for everyday bridge design, but is most often limited to planar analysis using plate bending elements, which, like the plane grillage method, assumes a constant neutral axis depth.

Slab bridge decks with wide edge cantilevers are known to have neutral axes of variable depth. Figure 1 illustrates a bridge deck in which the neutral axis rises significantly towards the edges as the cantilevers attempt to flex about their own centroids. It has been reported (O'Brien and Keogh 1996) that the neutral axis depth may also vary in the longitudinal direction in some cases.

Variability of neutral axis depth, when significant, results in the incorrect representation by a planar analysis of the behaviour of the bridge deck. The problem can be overcome by the use of a three-dimensional analysis method. Such methods do not require all members representing parts of the deck to be located in the one plane. Consequently this approach does not require a pre-assumed neutral axis position, and allows for a rational handling of cases in which the neutral axis depth varies. Many

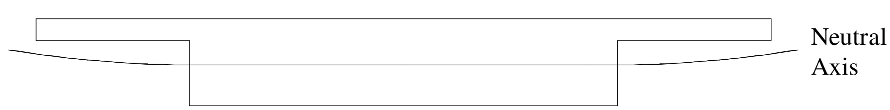


Figure 1 Slab bridge deck with wide edge cantilevers showing non-uniform neutral axis

such three-dimensional techniques have been suggested, ranging from space frame methods to Finite Element Analysis (FEA) using solid, brick-type elements. While these methods can be successfully used to model a variable-depth neutral axis, they are generally excessively complex for everyday use in design offices. This paper addresses this problem by proposing a simplified but accurate three-dimensional (3d) analysis method, referred to here as upstand finite element analysis (upstand FEA). The method is simple enough to be used in design offices for everyday design and is significantly more accurate than plane grillage or plane finite element analysis, particularly for bridge slabs with wide cantilevers.

2 Three Dimensional FEA

In order to compare the accuracy's of alternative simplified methods of analysis, it was necessary to apply a considerably more accurate method to a range of bridge decks. A program from the DYNA3D package, NIKE3D (Maker et al 1991), was used for this purpose to construct three-dimensional solid models from solid 'brick' elements. The program was tested by comparing analysis results for simply supported beam models with exact theoretical values. In addition, a model bridge deck, similar to the type of bridge considered here, was constructed from Glass Reinforced Cement (GRC). Results from this experiment were found to be generally similar to those from the 3d FEA analysis. However, repeated runs of the same experiment showed variations in results of up to 5% and it was concluded that such an experiment was only sufficient to confirm the approximate accuracy of the FEA program.

3 Grillage Analysis Methods

The theory governing grillage analysis is well established (Cusens & Pama 1975, Bhatt 1986) and the method is widely used among practitioners. Results have been compared to models and full-size bridges in the past (Best 1974, West 1973) and the method has been found to be reasonably accurate for many shapes of structure, loading conditions and support arrangements. However, plane grillage has been found by the authors (O'Brien and Keogh 1996) to be inaccurate for slab bridges with wide cantilevers.

Upstand grillage modelling is a simplified three-dimensional technique, which is essentially an extension of the plane grillage method. It has been suggested in the past for the analysis of bridges with variable-depth neutral axes (Hambly 1991). Figure 2(a) shows a single-span slab bridge deck with edge cantilevers. The length to depth ratio of the cantilevers is 6:1 which is the maximum considered practical. Figure 2(b) shows the equivalent upstand grillage model which was analysed using the STRAP computer program (ATIR 1991). This model is similar to a plane grillage except that each grillage member is located in the plane containing the centroid of the relevant portion of bridge. Therefore it is not necessary to assume an overall neutral axis position. For the example illustrated, all members are located on two planes, one for the edge cantilevers and one for the main part of the deck. The grillage members in each plane are connected by vertical beams with large flexural rigidities. Trial and error established a value which was effectively infinite while not being so large as to cause round-off errors. As has been suggested by other authors (Hambly 1991), all of the nodes were restrained against in-plane rotation.

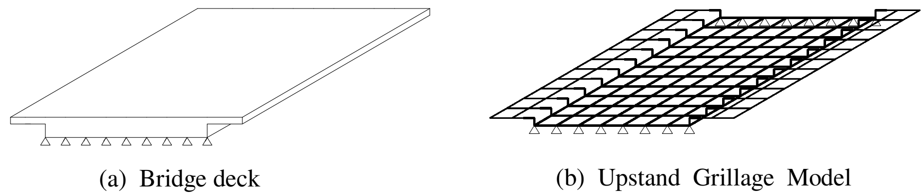


Figure 2 (a) Single span bridge deck and (b) upstand grillage model

The upstand grillage model of figure 2(b) was analysed under the action of opposing uniform moment, applied at either end of the bridge deck. This applied loading on a slab with cantilevers has the effect of generating in-plane forces. Unfortunately a grillage is ineffective at providing a realistic representation of a slab subjected to in-plane forces due to the tendency for the beams to bend individually in a similar manner to a Vierendeel girder. This behaviour is clearly not consistent with that of a slab.

A plane grillage analysis, using the STRAP program, and a 3d FEA were also carried out on the bridge of figure 2(a). Figures 3(a) and 3(b) show the longitudinal bending stress at the top fibre of the bridge deck at 1/8 span and 1/2 span respectively for each of these analyses. Only half of the bridge deck is shown as it is symmetrical; the cross section is included in these figures for clarity. Further details and results from these analyses have been presented elsewhere (Keogh and O'Brien 1996).

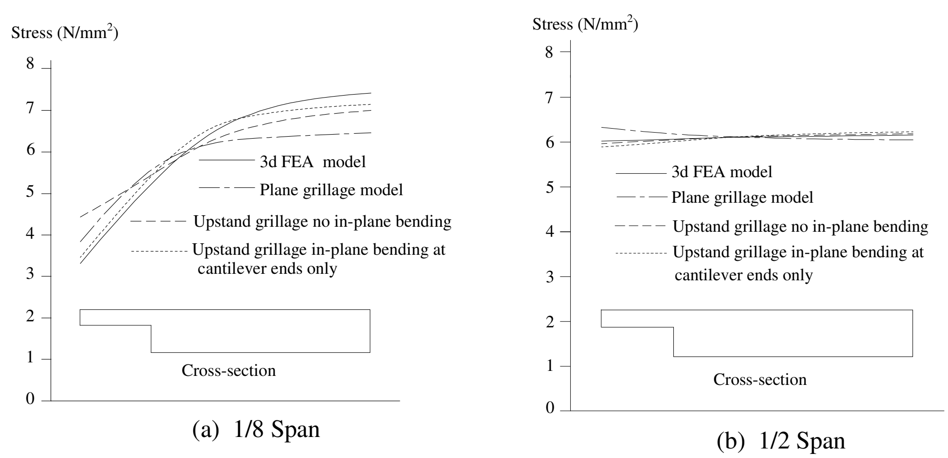


Figure 3 Top fibre longitudinal bending stress from various analyses at (a) 1/8 span and (b) 1/2 span

Assuming that the results from the 3d FEA model are correct, it can be seen from figure 3 that that the upstand grillage is generally more accurate than the plane grillage. This is most evident in the edge cantilever at 1/2 span and in the main part of the deck at 1/8 span. An exception to the general trend occurs in the edge cantilever at 1/8 span. In this region, the upstand grillage makes a poor prediction of stress, which is in fact worse than the plane grillage. Investigation of the 3d FEA model revealed that significant in-plane shear distortion occurs in this region. The restraining of nodes against in-plane rotation is counter-productive in such a case and is the reason for the poor prediction.

4 Upstand FEA

The finite element method has been used extensively in the analysis of bridge decks for some time. It is regarded by many bridge designers to be the most versatile method available and indeed the only one capable of dealing with certain bridge forms (Hambly 1991 and Cusens and Pama 1975). Although the method is used by bridge designers primarily for plate bending problems, it was pioneered by Turner et al. (1956), for in-plane analysis of two-dimensional elastic structures. The method was extended to the bending of slabs by Zienkiewicz and Cheung (1964) who also demonstrated the ability of the method to deal with various boundary conditions, variable slab thickness and slab orthotropy. In recent years, software have become available for personal computers which allows the use of plane finite element models for everyday analysis of bridges in the design office.

4.1 Upstand Finite Element Analysis

The upstand FEA method which is developed here is similar to the upstand grillage method discussed in the previous section, but with the grillage beams replaced by finite elements. The elements are located on different planes, according to the geometry of the bridge deck, and these planes of elements are connected, once again, by vertical beams with infinite flexural rigidity. As the model is three-dimensional, the elements will have to resist in-plane forces and deformations as well as out-of-plane bending. The method is introduced here by means of a simple example. It will then be further developed to cover other forms of bridge deck.

An upstand FEA model of the single-span bridge deck of figure 2 has been formulated and analysed using the STRAP computer program. The model can be seen in figure 4 along with some typical dimensions in mm. It is simply supported at each end and is acted on by opposing uniform moment. As with the upstand grillage model, the vertical members have effectively infinite flexural rigidity. The finite elements are capable of modelling both in-plane distortion and out-of-plane bending. The elements were isotropic and hence only the depth was required to define their stiffness. The depth of the elements was taken to be equal to the depth of the portion of slab which they idealised. All the elements were assigned typical material properties for concrete.

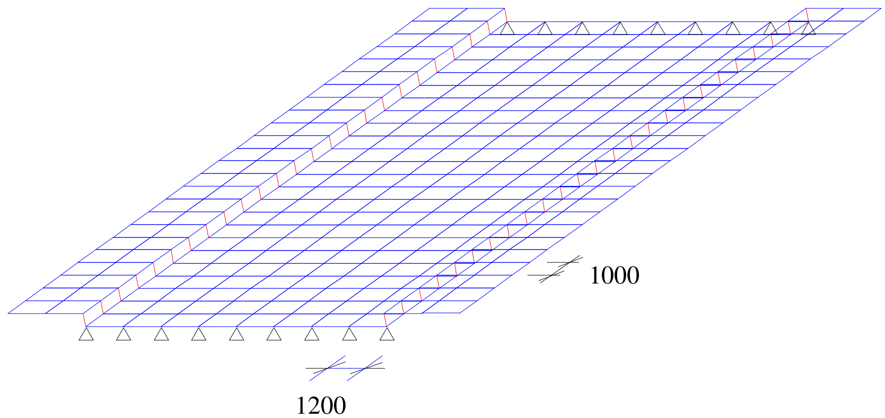


Figure 4 Upstand FEA of Single Span Bridge Deck with Wide Edge Cantilevers

The model geometry, material properties and loading were identical to those used in the upstand grillage model of the previous section. It is therefore possible to compare the results of this analysis directly with those from the previous section. Figures 5 (a) and (b) show the top stresses at 1/8 and 1/2 span locations. The upstand FEA analysis program gave bending moments and axial forces in each element and the stresses due to these were added. The results from the 3d FEA, the plane grillage analysis and the upstand grillage analyses of the previous section are also shown in this figure.

The upstand FEA is seen to compare almost exactly with the 3d FEA and the lines in the figures are almost indistinguishable in most cases. This provides a great improvement over the upstand grillage and the plane grillage with little increase in complexity. The technique will be extended to bridge decks with more complex geometry's in the following sections.

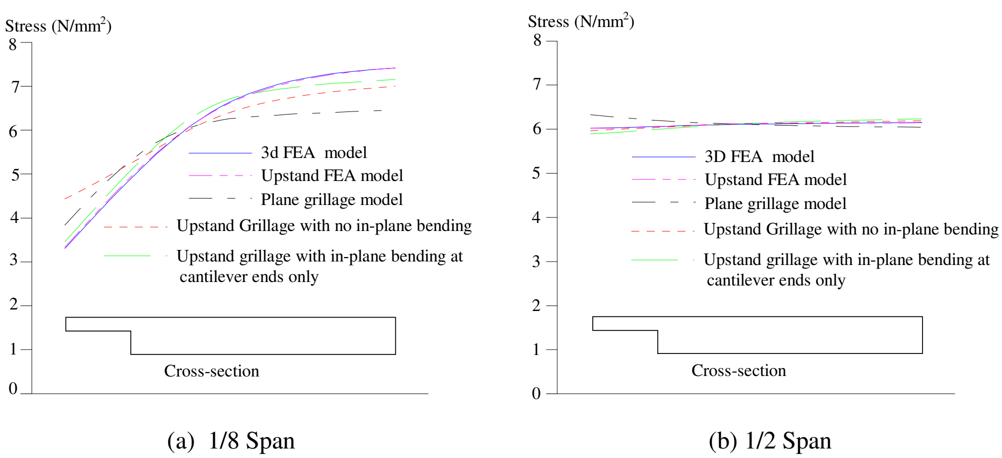


Figure 5 (a) Top Longitudinal Stress from Various Analyses at 1/8 Span (b) at 1/2 Span

4.2 Upstand FEA of Two-Span Bridge Deck

To further develop the upstand FEA method, a two-span bridge deck with wide edge cantilevers was analysed under the action of its self-weight. The formulation of the upstand FEA model for the two-span case follows directly from the single-span model of section 4.1, and uses the same element dimensions and properties, as the cross-sections of these two models are the same. The upstand FEA model is illustrated in Figure 6.

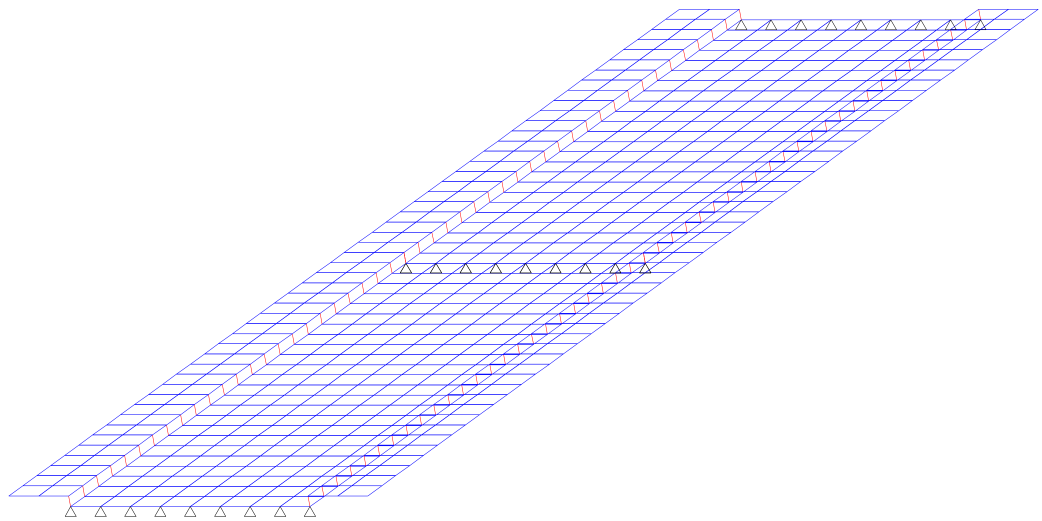
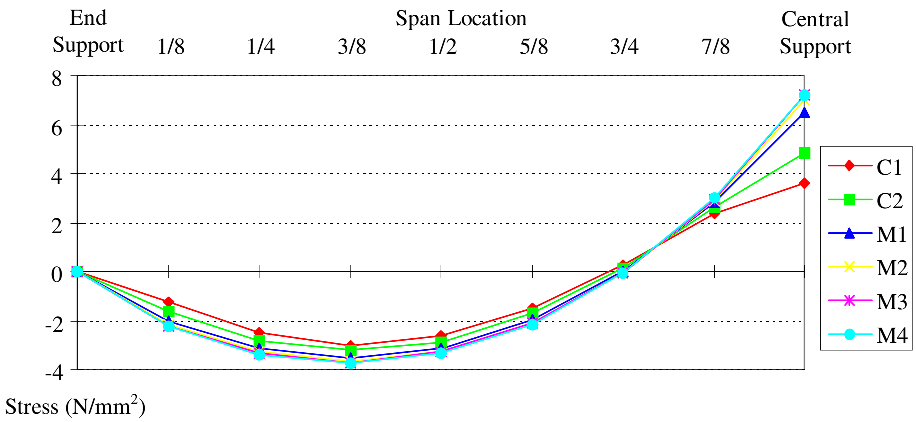


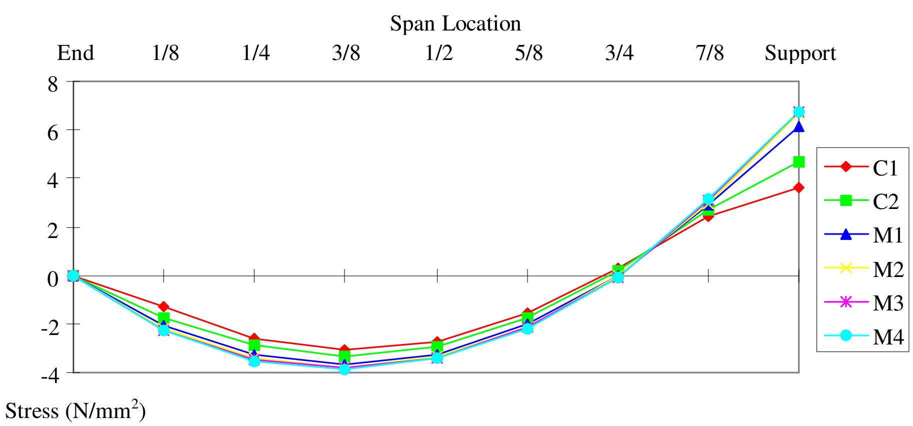
Figure 6 Upstand FEA of Two-Span Bridge Subjected to Self-Weight

The top stresses from this model were compared with those from a 3d FEA using the NIKE3D program. The top stresses were taken this time at 0.1m below the top surface of the bridge as this location was convenient for data processing purposes. In order to simplify the presentation of results, the bridge deck was notionally divided transversely into 12 longitudinal strips of equal width and labelled as shown in figure 7. The average top stress over each longitudinal strip predicted by the 3d FEA model, is presented in figure 8(a). As the bridge deck is symmetrical, results are only presented for one half (transversely) and for one span. Figure 8(b) shows the same quantity as predicted by the upstand FEA analysis. A plane FEA of this bridge deck was also carried out the results of which are shown in figure 8(c). It was considered more appropriate to compare the results of the upstand FEA with a plane FEA rather than a plane grillage. This shows that the improved accuracy is not due to the use of finite elements, but rather due to the three-dimensional nature of the model.

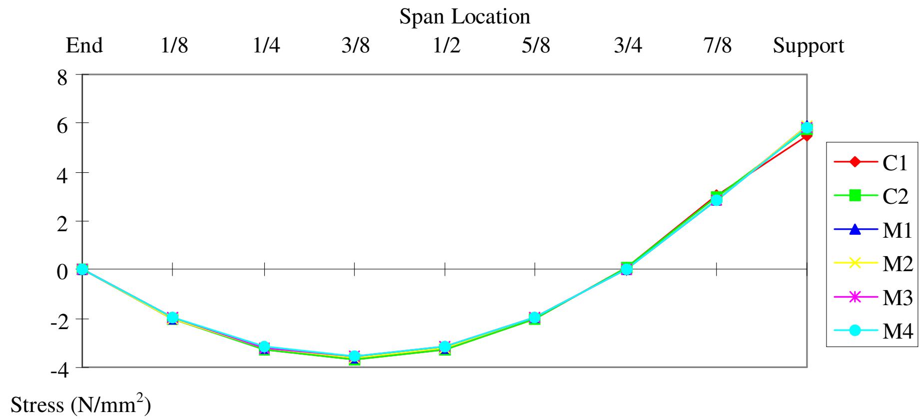
Figure 7 Transverse Subdivision of Two-Span Bridge Deck into Strips ('C' indicates cantilever strip, 'M' indicates main deck strip)



(a) 3d FEA



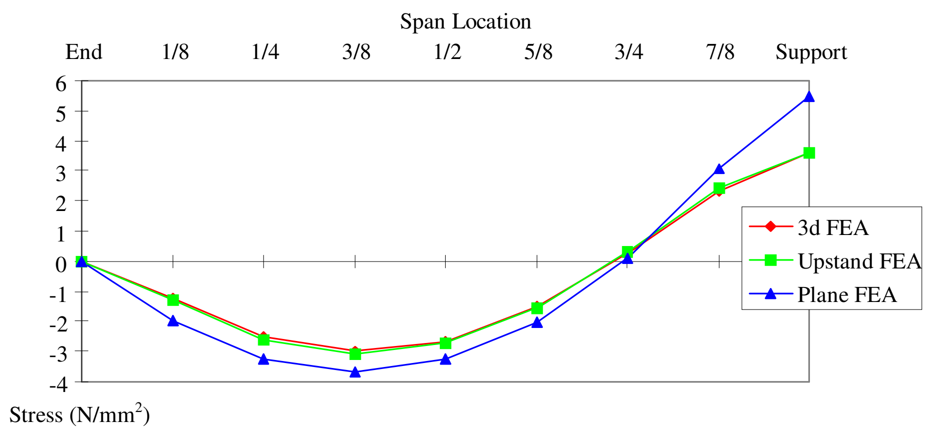
(b) Upstand FEA



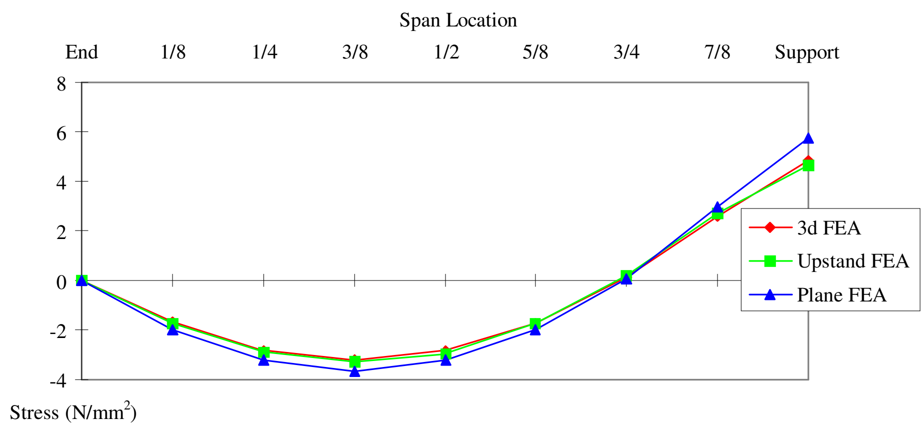
(c) Plane FEA

Figure 8 Predicted Top Longitudinal Stress in Two-Span Bridge Deck

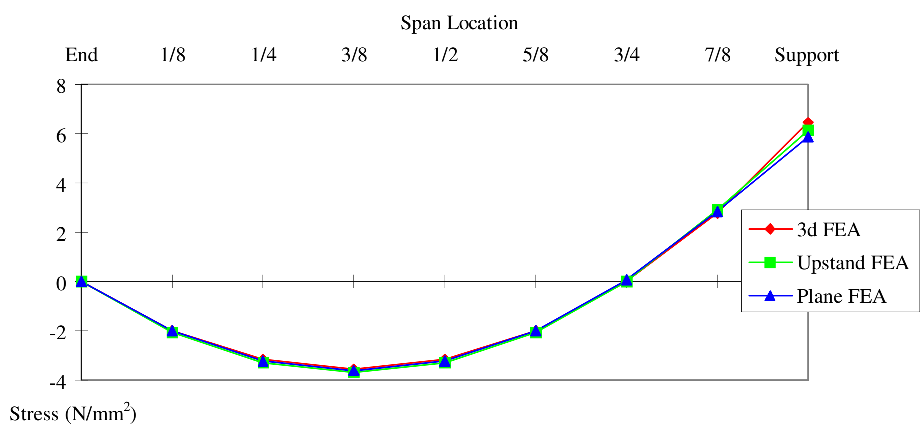
Figure 9 shows the predicted longitudinal top stresses from each of the methods for each longitudinal strip (C1 to M4) in turn.



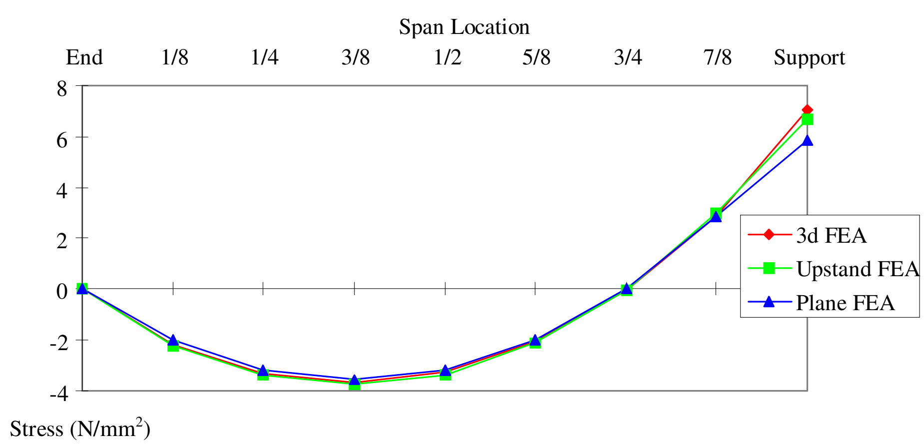
(a) Cantilever Strip C1



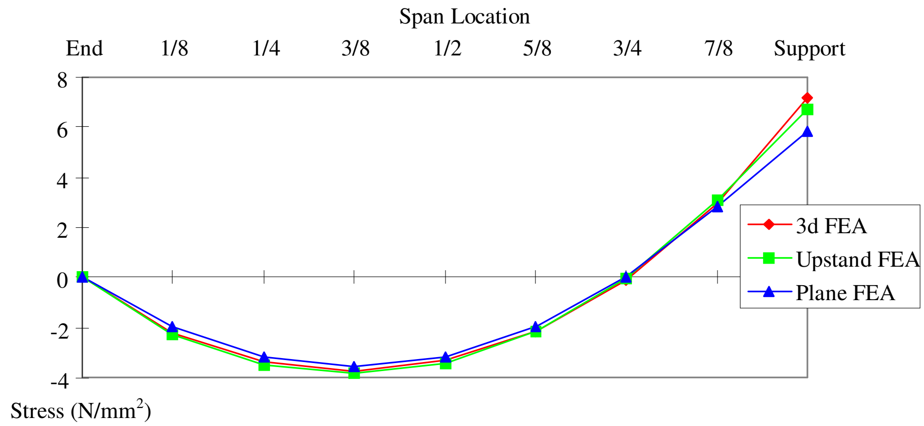
(b) Cantilever Strip C2



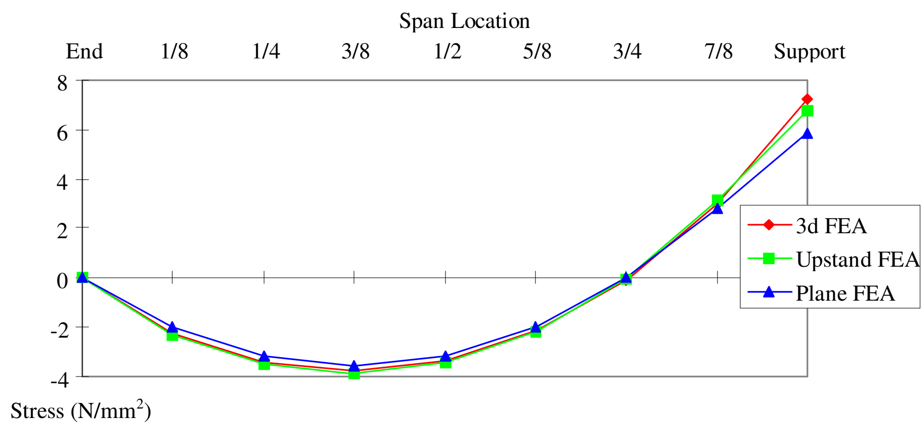
(c) Main Deck Strip M1



(d) Main Deck Strip M2



(e) Main Deck Strip M3



(f) Main Deck Strip M4

Figure 9 Top Longitudinal Stresses in Two-Span Bridge Deck

The predictions of stress from 3d FEA and the upstand FEA follow the expected pattern, with zero stress at the end support, maximum compressive stress (here shown negative) at 3/8 span, zero stress near 3/4 span and maximum tensile stress at the central support. In addition to this, the figures show how the stress varies transversely across the bridge deck. The maximum stress occurs towards the centre of the bridge and diminishes towards the edge cantilever. The predictions of stress from the plane FEA also follow the expected pattern along the span, but do not show the stress varying transversely to the extent that it does for the other analysis methods. In fact the plane FEA predicts an almost uniform (longitudinal) stress across the width of the bridge deck, except for the edge of the cantilever (C1) where it drops slightly over the support. This is due to the inability of the plane FEA to model the variation in neutral axis location.

By comparing the stresses at C1 and C2 predicted by each of the three analyses (figures 9 (a) and (b)), the 3d FEA and upstand FEA predictions are seen to be in very close agreement along the entire span. The stress predicted by the plane FEA does not agree as well with these. This disagreement is more pronounced in the edge of the cantilever at C1 (figure 9 (a)) than at the inside of the cantilever at C2 (figure 9 (b)). The plane FEA predicts significantly higher stresses in the cantilever than the other two analyses. This is due to the fact that the plane FEA assumes a neutral axis which does not rise in the edge cantilever. This causes the second moment of area of the cantilever to be too large, and thus to attract too much moment. Secondly the moment which it attracts induces a higher stress because the distance from the assumed neutral axis to the top fibre is too large.

By comparing the stresses at other transverse locations (figures 9 (c) to (f)) it can be seen that all three analyses predict stresses which are generally in close agreement, but that those predicted by the plane FEA deviate a little from the other two analyses, especially above the support. This is due to the fact that in the plane FEA, the incorrect neutral axis location in the cantilever caused an increased moment to be attracted to the cantilever. As the total moment at a section must remain constant, the moment, and hence stress, in the remainder of the deck must be lower that the true value.

A comparison has also been made between the transverse stresses predicted by both the 3d FEA model and the upstand FEA model. In the upstand FEA model it was necessary to consider both bending moments and axial forces in the transverse direction to arrive at the stresses in this direction. The stress in the transverse direction, again at 0.1m below the top surface of the bridge deck, can be seen at various span locations, as predicted by the two analyses, in figure 10.

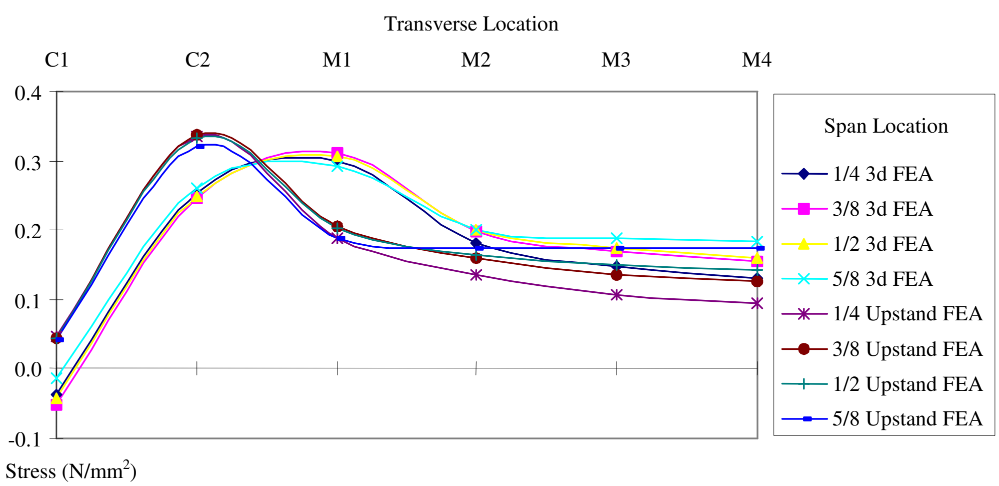


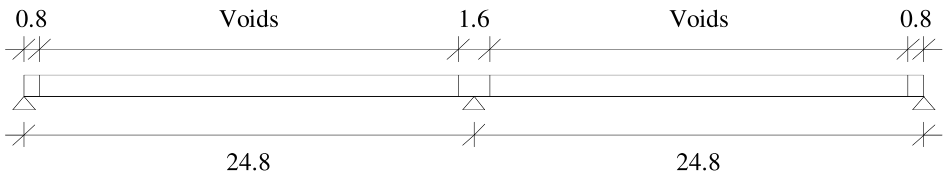
Figure 10 Top Transverse Stress in Two-Span Bridge Deck at Various Locations

It can be seen from this figure that for each analysis method, similar transverse stresses are predicted at C1, C2, M1 and M2 (refer to figure 7) for all span locations, i.e., in the cantilever and near it, the transverse stress is substantially independent of location longitudinally along the deck. At M3 and M4, there is a significant variation in transverse stress at different span locations. This is predicted by both analyses which are in reasonable agreement here. Although a significant variation in stress is predicted by both methods of analysis from C1 to M2, it can be seen that both predict a similar maximum value, and both predict a similar distribution of stress across the width of the deck. The main variation appears to be in the prediction of the location of the maximum transverse stress. The 3d FEA predicts this to occur further into the deck than the upstand FEA does. It is felt by the authors that this discrepancy is due to the rapid transition from the cantilever to the main part of the deck. This would be expected to have a more pronounced effect on the transverse stresses than on the longitudinal stresses. The comparison of shear stress between the two models is not made here as it is the subject of ongoing research at Trinity College Dublin (O'Brien et al. 1997).

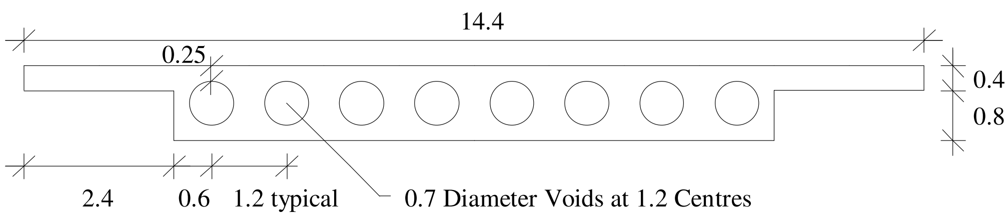
4.3 Upstand FEA of Voided Slab Bridge Deck

Longitudinal voids are often incorporated into slab bridge decks in the United Kingdom and Ireland in order to reduce their self-weight while maintaining a relatively high second moment of area. In order to determine the accuracy of upstand FEA for this form of construction, the two-span bridge deck of section 4.1 was modified to incorporate longitudinal voids in the main part of the deck. Solid sections were maintained above the end and central supports to simulate the presence of diaphragm beams. This structure was then analysed using a 3d FEA model under the action of its own self-weight. Figure 11 (a) shows the longitudinal elevation of the bridge indicating the position of the solid sections above the supports. Figure 11 (b) shows a cross-section through the voided part of the bridge deck and gives dimensions in metres. A cross-section through the voided part of the 3d FEA model can be seen in figure 12. As the 3d FEA program, uses finite elements with straight sides, the circular voids were approximated by octagonal voids. The diaphragms were also incorporated into the 3d FEA model.

This bridge deck was also analysed using an upstand FEA model. The modelling procedure was similar to that used in previous sections except for the addition of the voids. The process generally adopted for modelling voided slabs with isotropic elements is to calculate an 'equivalent depth' of element. This depth is that which gives a second moment of area for the element equal to the second moment of area of the voided section which it represents. It is clear that the area of such an equivalent element is not equal to that of the corresponding voided section. This is not normally a problem in a two-dimensional plane analysis, as in-plane axial forces are not modelled. However, in an upstand FEA, in-plane axial forces do exist. The stiffness of each portion of the deck results from a combination of the element stiffness, its area and its distance from the bridge neutral axis at that point. Consequently it is essential that the upstand FEA idealisation of a voided slab has both the correct area and second moment of area.



(a) Longitudinal Elevation



(b) Cross-Section Through Voided Deck

Figure 11 Voided Bridge Deck with Wide Edge Cantilevers (dimensions in metres)

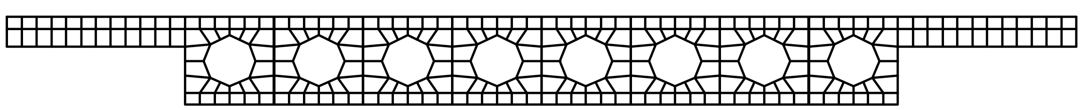
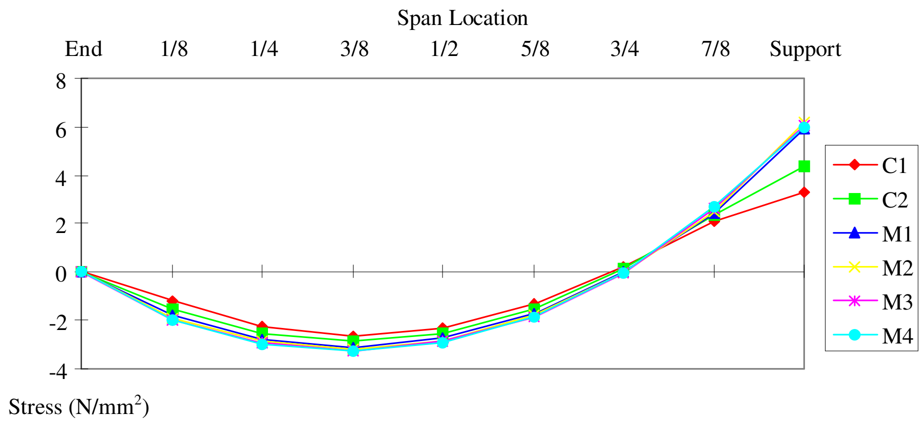


Figure 12 Cross-Section Through 3d FEA Model of
Two-Span Voided Bridge Deck

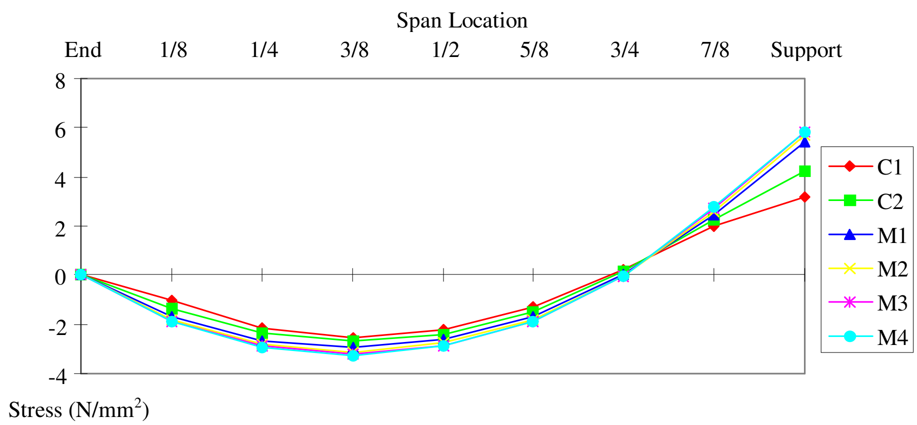
This may be achieved by the addition of extra longitudinal beams to make up the shortfall of second moment of area or by the use of a program which allows flexibility in the definition of these two properties. For this study, the depth of the elements were chosen to give the correct area and the shortfall in second moment of area was assigned to longitudinal beams.

Figure 13(a) shows the top longitudinal stress predicted by the 3d FEA for each of the transverse locations C1 to M4. Figure 13(b) shows the same quantity predicted by the Upstand FEA (with additional beams). Both of these graphs show the same pattern as was observed for the solid bridge deck. That is zero stress at the end support, maximum compressive stress at 3/8 span, zero stress near 3/4 span and maximum tensile stress at the central support. The variation of stress transversely across the bridge from M4 to C1 which was observed in the solid bridge deck is again observed in both of these figures.

Figure 14 provides comparisons of the stresses of the preceding figure for two typical transverse locations, C1 and M1. The predictions from the two analyses compare well with each other. Some disagreement is evident at the central support in M1 but this is quite small and localised. One possible cause for this disagreement is the complexity incurred by the rapid change from voided to solid section at this location.

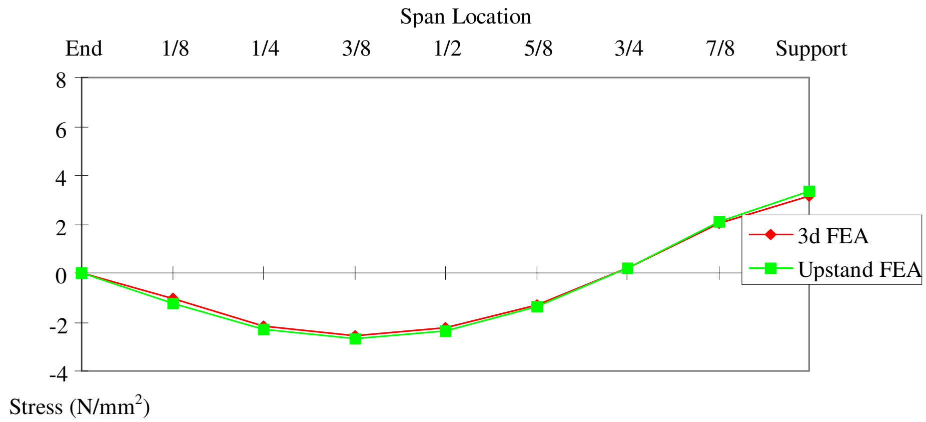


(a) 3d FEA

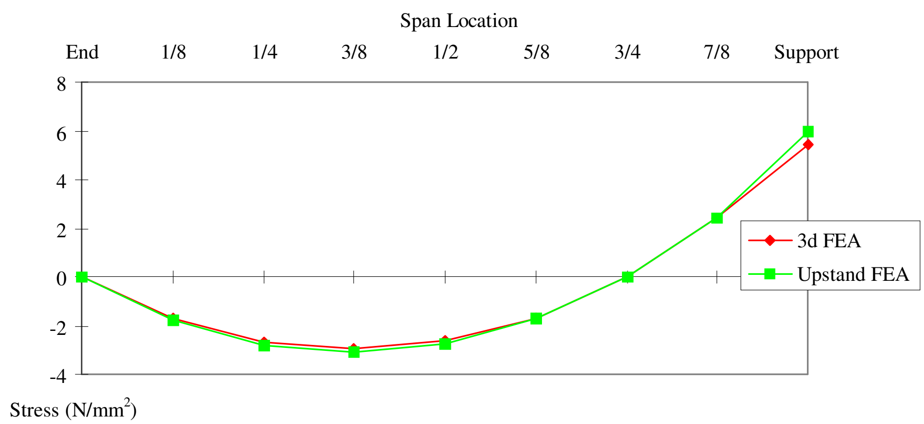


(b) Upstand FEA

Figure 13 Top Longitudinal Stress in Voided Bridge Deck



(a) Cantilever Strip C1



(b) Main Deck Strip M1

Figure 14 Top Longitudinal Stress in Voided Bridge Deck

5 Conclusions

An upstand FEA technique has been proposed for the analysis of slab bridge decks with edge cantilevers. In this technique, the edge cantilever and the main bridge deck are idealised by separate layers of simple finite elements, each located at the centroid of that part of the deck which they represent and connected by vertical beams of infinite flexural rigidity. The finite elements must be capable of in-plane distortion and out-of-plane bending.

For a single-span bridge deck with wide edge cantilevers, both the plane grillage and the upstand grillage analysis methods are shown to be inaccurate in predicting longitudinal bending stresses when compared to a 3d FEA. The proposed upstand FEA method on the other hand gives excellent results. The differences in accuracy are attributed to variations in the depth of the neutral axis and in associated in-plane distortions. Similar results are reported for longitudinal stresses in a two-span deck.

Predictions of transverse stresses from the upstand FEA and the 3d FEA were found to compare reasonably well. Both analyses predicted a similar maximum transverse stress and a similar distribution across the width of the deck. However, a discrepancy was noted in the location of the point of maximum stress.

A voided slab bridge deck, again with wide edge cantilevers, was also analysed using the upstand FEA technique. The presence of the voids complicated the modelling and, for the software used, required the addition of extra beams in order to maintain both the correct area and second moment of area of the voided sections. The model was analysed under the action of self-weight and a 3d FEA was carried out for comparison. The predictions of top longitudinal stress from the upstand FEA once again compared very well with those from the 3d FEA.

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