A Modified Muskingum Routing Approach for Floodplain Flows: Theory and Practice

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Abstract

Hydrological or hydraulic flood routing methods can be used to predict the floodplain influences on a flood wave as it passes along a river reach. While hydraulic routing uses both the equation of continuity and the equation of momentum to describe the dynamics of river flows, the simpler data requirements of hydrological routing makes it useful for preliminary estimates of the time and shape of a flood wave at successive points along a river. This paper presents a modified linear Muskingum hydrological routing method where the floodplain effects on flood peak attenuation and flood wave travel time are included in routing parameters. Developing the routing parameters initially involved routing hydrographs of different flood peak and duration through a 1-dimensional model of a generalised river reach in which a range of geometrical and resistance properties were varied. Comparison of upstream and simulated downstream hydrographs for each condition investigated, allowed the attenuation and travel time (storage constant, K, in standard Muskingum routing) of the flood

wave to be estimated. Standard Muskingum routing was then used to develop downstream hydrographs for each K value together with assumed storage weighting factors (x) ranging from 0 to 0.5. Flood peak attenuations were again determined through comparison of the upstream and routed downstream hydrographs and with these, linear relationships between x and these attenuations were developed. Actual weighting factors, corresponding to storage constants, were subsequently determined using these relationships for all attenuations determined from the 1-dimensional model simulations. Using multi-variate regression analysis, the computed values of K and x were correlated to catchment and hydrograph properties and expressions for determining both K and x in terms of these properties were developed. The modified Muskingum routing method based on these regressed expressions for K and x was applied to a case study of the River Suir in Ireland where good agreement between measured and routed hydrographs was observed.

Keywords: Overbank flow; Flood routing; Muskingum routing; Hydraulic and hydrological methods; floodplains; Modelling and Saint-Venant equations.

1 Introduction

Hydraulic or hydrological flood routing techniques are commonly used by engineers and hydrologists to predict the temporal and spatial variations of a flood wave through a river reach (Choudhury et al., 2002). The methodologies that have been developed vary in their complexity with more analytically rigorous methods having increased capacity to better accommodate the dynamics and influences of floodplain behaviour on the propagation of a flood wave in a natural channel. The Muskingum method of hydrological flood routing is one approach. The popularity of the Muskingum method derives primarily from its minimal data requirements. Knowledge of topographical catchment conditions are not required to understand the propagation of a flood wave as these dynamics are represented in a calibration carried out using observed data (Gallati and Maione, 1977). The Muskingum approach represents a hydrological flood routing technique and is based on the equation of continuity and a relationship that describes storage in the system. At the opposite end of the spectrum, full-scale dynamic wave models based on the Saint-Venant equations provide a more sophisticated means of hydraulic flood routing (Tung, 1985). Hydraulic routing uses the equation of continuity and a momentum balance equation (the Saint Venant equations) and involves their numerical solution using finite difference or characteristic methods. Simplification of the momentum equation produces approximate solutions (e.g. monoclinal wave, convective diffusion) that are easier to calculate and may be adequate in specific cases.

Advantages and disadvantages exist for using both hydrological and hydraulic flood routing techniques. Although hydraulic routing techniques can more adequately describe the dynamics of unsteady flows in canals and rivers, these methods are more demanding in their information inputs and require data to accurately represent the geometrical and resistance characteristics of the main channel and floodplain. Initial and boundary conditions are also required. Conversely, the data inputs and computational procedures for hydrological routing techniques are considerably simpler (Singh, 1988) and these methods are useful where preliminary estimates of the time and shape of a flood wave at successive points along a river are required, or where budgetary constraints may not facilitate full hydraulic routing.

A modified linear Muskingum hydrological routing method suitable for floodplain flows is presented in this paper. The method is based on the standard Muskingum method. However, rather than determining the routing parameters through analyses of observed upstream and downstream hydrographs for given flows, the proposed method estimates its routing parameters from empirical relationships describing floodplain effects on flood peak attenuation and flood wave travel time. The method therefore has similarities with the

Muskingum-Cunge method (Cunge, 1969) in that flood routing parameters are determined from geometrical and resistance properties of the channel, thus avoiding the need for a calibration process. Development of the method involved a combination of 1-dimensional hydraulic modelling and standard Muskingum flood routing to determine storage constants, *K* and storage weighting factors, *x*, for a range of catchment and hydrograph properties. Multi-variate regression analysis was used to correlate these computed values of *K* and *x* to these properties and expressions for determining both *K* and *x* were developed. The modified Muskingum routing method based on these regressed expressions for *K* and *x* was applied to a case study of the River Suir in Ireland where good agreement between measured and routed hydrographs was observed. The method offers a simple and inexpensive method of estimating the time and shape of a overbank flood wave as it progresses along a river channel of low to moderate sinuosity and in which backwater and inertia influences are likely to be small.

2 Muskingum Flood Routing

The Muskingum method of flood routing (McCarthy, 1938), based on a simple storage – discharge relationship in river systems, is extensively used in river engineering (Gill, 1979). The method performs best in river systems where inertia effects and backwater influences are small and where model parameters are appropriately chosen to represent the hydraulic behaviour of the system (Chang et al., 1983). The linear Muskingum model uses continuity and storage relationships expressed as:

Continuity: $$\frac{dS_t}{dt} = I_t - O_t \qquad (1)$$

Storage: $$S_t = K\left[xI_t + (1 - x)O_t\right] \qquad (2)$$

where S_t, I_t and O_t are simultaneous amounts of storage, inflow and outflow respectively at a given time t, K is a storage constant expressing the ratio between storage and discharge in a river reach and x is a dimensionless weighing factor that varies between 0 and 0.5 for natural rivers. This weighting factor describes the relative importance of inflow and outflow to storage. The storage time constant, K, equates closely to the flow travel time through the river reach (McCuen, 1998). If K and x are known, routing is performed using:

$$O_t = C_1 I_t + C_2 I_{t-\Delta t} + C_3 O_{t-\Delta t} \tag{3}$$

in which, C_1, C_2 and C_3 are routing coefficients given by:

$$C_1 = \frac{0.5\Delta t - Kx}{(1-x)K + 0.5\Delta t} \tag{4}$$

$$C_2 = \frac{Kx + 0.5\Delta t}{(1-x)K + 0.5\Delta t} \tag{5}$$

$$C_2 = \frac{-0.5\Delta t + (1-x)K}{(1-x)K + 0.5\Delta t} \tag{6}$$

where parameters are as defined for Eq. 2 and in which Δt is the time step and $I_{t-\Delta t}$ and $O_{t-\Delta t}$ are the inflow and outflow discharges at time $t\text{-}\Delta t$. Once C coefficients (which sum to unity) are determined, Eq. 3 is used repeatedly to determine outflow discharges, O_t, at any time.

Values for K and x that describe the storage characteristics of a river reach are usually derived from observed upstream and downstream hydrographs extracted from historical flow records. These methods are well reviewed and are broadly represented in five classifications: (a) graphical method; (b) least squares method; (c) method of moments and method of cumulants; (d) direct optimisation; and (e) those based on the Saint-Venant equations (Singh and McCann, 1980). More recently, Yoon and Padmanabhan (1993), identified a further three methods for linear model parameter estimation. These algorithms included forward and backward optimisation using t-statistics, an outliers filtering estimation method and a quadratic programming algorithm.

Graphical methods are commonly applied. The standard trial-and-error graphical approach used by McCarthy (1938) for the linear Muskingum model involves plotting $[xI_t + (1-x)O_t]$, known as the weighted discharge term, against the accumulated storage for different assumed values of x. Different values of x in Eq. 2 produce a family of curves that vary from being heavily looped to being reasonably linear. The particular value that generates the narrowest loop and can be best fitted with a straight line is considered the best estimate of x. The inverse slope of this line gives the required value of K. Although the graphical method is generally satisfactory (Chow, 1964; Linsley et al., 1975; Viessman et al., 1972; Wilson, 1990), it is time consuming to apply. Furthermore, no objective-selection criteria exists for choosing the appropriate value of x and the method therefore, requires a level of subjective interpretation to determine a value that optimises the linear relationship (Gelegenis and Serrano, 2000; Yoon and Padmanabhan, 1993; Chang et al., 1983). Muskingum routing parameters have also been estimated using a least-squares scheme based on minimising the sum of squares of the deviations between observed storage and computed storage for given inflow and outflow hydrographs (Gill, 1978; Birkhead and James, 1997; Al-Humoud and Esen, 2006). The underlying principles of the graphical and the least-square methods are the same and both methods should produce similar parameter values. The method of moments and the method of cumulants are similar and are based on relating the first and second moments or cumulants of the instantaneous unit hydrograph (IUH) of the Muskingum reach to the Muskingum routing parameters, K and x (Dooge, 1973). The method of direct optimisation is based on minimising the difference between observed and computed hydrographs to determine directly the routing coefficients of the Muskingum model without explicitly estimating K and x (Gelegenis and Serrano, 2000).

More recent advances in computer technologies have allowed the traditional Muskingum routing methods to be linked with hydrodynamic software packages for the analysis of surface water drainage in natural catchments (see for example the HEC-HMS and TOPMODEL hydrological models). These are based on the Saint-Venant equations that are derived from the principles of conservation of mass and momentum and can be written in their 1-dimensional form as:

$$\frac{\partial y}{\partial t} + y\frac{\partial V}{\partial x} + V\frac{\partial y}{\partial x} = 0 \tag{7}$$

$$\frac{\partial V}{\partial t} + V\frac{\partial}{\partial x_c} + gA\frac{\partial y}{\partial x} + g\left(S_f - S_0\right) = 0 \tag{8}$$

(I) *(II)* *(III)* *(IV)*

where y is the flow depth, V is the flow velocity, g is the acceleration due to gravity, S_o is the river bed slope, S_f is the slope of the energy line, x is the longitudinal distance and t is the time. These equations are simultaneous, quasi-linear, first-order partial differential equations of the hyperbolic type and are not amenable to general analytical solutions. The first term *(I)* in Eq. 8 is the local inertia term, the second *(II)* is the convective inertia term, the third *(III)* is the pressure differential term and the fourth *(IV)* accounts for the friction and bed slopes. Numerical methods for solving the Saint-Venant equations are broadly classified in two categories: (a) approximate methods; and (b) complete numerical methods. Approximate methods are based on the equation of continuity only or on a significantly curtailed equation of momentum.

Kinematic and diffusion wave models can be constructed to solve Eq. 7 and Eq. 8 by assuming that the significance of some terms is negligible compared to that of others (Moussa and Bocquillon, 1996). Models that neglect inertial terms (I and II) are known as diffusion wave models (Cunge, 1969; Bajracharya and Barry, 1997; Moussa and Bocquillon, 2009) and models that neglect both inertial and pressure terms (I, II, III) are referred to as kinematic models (Smith, 1979). The full Saint-Venant equations have also been solved for Muskingum routing applications using numerical techniques in channels with cylindrical and irregular cross-sectional geometries without the need for simplifying assumptions (Amein and Fang, 1970; Dooge et al., 1982; Wang et al., 2006).

Cunge (1969) included the effects of geometrical and resistance properties of a river reach in the original Muskingum method to develop the Muskingum-Cunge (M-C) model. Cunge showed that the Muskingum formula for solving flood routing problems is identical to a finite-difference approximation of the linearised diffusion wave equation, this equation being derived from the Saint-Venant equations by neglecting the inertial terms. In the M-C flood routing procedure, the necessity of calibration that characterises the Muskingum method is not required and the routing parameters, *K* and *x,* are obtained from hydraulic properties of the reach using:

$$K = \frac{\Delta x}{c} \tag{9}$$

and

$$x = \frac{1}{2}\left(1 - \frac{Q}{BS_0 c \Delta x}\right) \tag{10}$$

where parameters are as described above and where c is the flood wave celerity, Δx is the longitudinal channel distance increment, Q is the discharge and B is the average bed width of the channel.

The time step, Δt, used in the M-C routing procedure is appropriately chosen to fully define the shape of the inflow hydrograph. A dependency exists between Δt and Δx in M-C routing procedures and the choice of increments is therefore important to ensure that Δx is not significantly smaller than the distance travelled by the flood wave in a single time step, Δt. This Δx interval chosen in the M-C routing method is therefore based on Δt, c, Q, the channel top width and the longitudinal channel slope, S_o. The flood wave celerity, c, is obtained from the slope of the discharge - area curve for a given discharge, Q. Details of the M-C method are discussed in Volume III of the Flood Studies Report (FSR) (NERC, 1975). Tang et al., (1999a) investigated the properties of the M-C method for flood routing, using several hypothetical flood hydrographs in a prismatic channel with significant floodplains. Results indicated that the M-C method suffers from a loss of outflow volume which depends on bed slope and floodplain roughness. Furthermore, it was observed that an initial leading edge dip and trailing edge oscillation occur in the rising and recession limbs of the hydrograph respectively. These oscillations become more significant as the roughness of the floodplain increases, but gradually disappear with decreasing bed slope.

The standard linear Muskingum routing method assumes that K and x remain constant and these are determined by analysis of measured inflow and outflow hydrographs. The method therefore, does not accommodate changes in these parameters that would reflect more accurately the routing of storm sequences in the river reach beyond the calibration range (Kundzewicz and Strupczewski, 1982). More recent developments to the method however, do allow for parameter variability with changing characteristics of the inflow hydrograph (see for example Perumal, 1992a; Guang-Te and Singh, 1992; Al-Humoud and Esen, 2006). Perumal (1992a) developed a multi-linear Muskingum flood routing method based on a time distribution scheme. The physically based Muskingum method is used as the linear sub-model in this method and the parameters are varied at each time step for the routing of prescribed flow zones in the inflow hydrograph. Guang-Te and Singh (1992) proposed three versions of the linear Muskingum method with variable parameters. In these methods, the reach travel time (which depends on the storage, channel characteristics and discharge) is obtained from a simplification of the Saint-Venant equations. Al-Humoud and Esen (2006) proposed two approximate methods for the estimation of linear Muskingum flood routing parameters. The first method requires the computation of the slopes of the inflow and outflow hydrographs at their point of intersection, and the computation of the maximum storage within the reach. The second method requires the computation of the inflow and outflow hydrographs at two specific points.

Although the current paper is based on a linear Muskingum method, the relationship between $[xI_t + (1-x)O_t]$ and S_t is usually nonlinear and Gill (1978) proposed the following storage relationship for the Muskingum model:

$$S_t = K[xI_t + (1-x)O_t]^m \quad (11)$$

where *m* is an exponent that defines the nonlinear relationship between accumulated storage and weighted flow. This exponent cannot be directly determined from inflow and outflow hydrographs and therefore, alternative parameter estimation methods have been presented (see for example Gill, 1978; Tung, 1985; Yoon and Padmanabhan, 1993; Mohan, 1997; Kim et al., 2001; Geem, 2006; Chu, 2009). Gill (1978) proposed a routing scheme for the nonlinear model based on the segmented curve method where coefficients are determined by a least squares method. However, the technique is somewhat arbitrary in its process of selecting three points on the segmented curve for solving simultaneously the continuity (Eq. 1) and storage (Eq. 11) equations of the nonlinear method (Tung, 1985). Tung (1985) proposed procedures using the Hook-Jeeve (HJ) pattern technique in combination with simple linear regression (LR), the conjugate gradient (CG), and the Davidon-Fletcher Powell (DFP) techniques and used the state variables technique for routing. The approaches were compared with Gill's methodology which showed that HJ+CG and HJ+DFP techniques produced better estimations of the routing parameters. Yoon and Padmanabhan (1993) proposed a nonlinear least-squares regression technique which directly fits the nonlinear model. The estimation proceeds iteratively from an initial assumption of the parameters using the Marquardt algorithm (Marquardt, 1963). In addition, this method has internal logic that analyses the data and computes reasonably accurate values of the parameters to be estimated. This serves to expedite the optimisation process. Mohan (1997) suggested a calibration technique for determining K, x and m based on a genetic algorithm that avoids the need to make initial assumptions. Kim et al. (2001) proposed a harmony search algorithm for estimation of the same parameters and observed that the technique outperformed other genetic, heuristic and mathematical algorithms, evolutionary programming and non linear programming. Geem (2006) presented a Broyden-Fletcher-Goldfarb-Shanno (BFGS) technique for parameter estimation in nonlinear Muskingum models. The BFGS algorithm is a branch of the quasi-Newton method based on mathematical gradients that searches for an optimised solution of the unconstrained nonlinear equations. Chu (2009) combined a Fuzzy Inference System (FIS), implemented in an adaptive network framework with a nonlinear Muskingum model to estimate the outflow hydrograph. However, the calibration procedure for finding the correct values of the three parameters K, x and m to determine this outflow hydrograph is complicated (Kim et al., 2001).

3 Methods

A multi-stage process (Fig. 1) that included 1-dimensional HEC-RAS modelling of a generalised river reach, standard Muskingum routing and regression analysis was adopted for developing expressions for storage constants (K) and weighting factors (x) for use in the modified Muskingum method that is presented. The process begins with implementing a generalised HEC-RAS model using input flow hydrographs and geometric properties to determine travel times and relative attenuations which are then input into a regression model to develop equations for estimating Muskingum model parameters. The following paragraphs explain the process in detail.

Fig. 1

3.1 HEC-RAS Model of Generalised River Reach

The HEC-RAS model of the generalised river reach was executed in dynamic mode for an extensive range of geometrical and resistance properties. HEC-RAS (Hydraulic Engineering Centre – River Analysis System) is a 1-dimensional link and node river model developed by the US Army Corps of Engineers that discretises and solves the dynamic Saint-Venant equations using an implicit, finite difference method. Theoretical hydrographs of varying peak and also, hydrographs of varying duration were key inputs to the model.

3.1.1 Hydrographs of Varying Peak Flow

Hydrographs for a range of peak flows were developed using a methodology (Fig. 2) and associated software from Work-Package 3.1 of the Irish Flood Studies Update (FSU) programme (O'Connor and Goswami, 2010) that utilises a historical record of flow data.

Fig. 2

Any gauged location where a record of good quality data was available would therefore be suitable and one such site with a long and continuous flow record was chosen. Hydrograph development involved the following:

(1) The annual exceedence series of the maximum flood events for the selected site was identified from recorded data (shown for a single event in Fig. 2 (a)).

(2) Annual exceedence flood hydrographs were isolated from the flow record by discarding the complex segments on each side of the peak, leaving the single-peak hydrograph component (Fig. 2 (b)).

(3) The isolated flood hydrographs were then standardised to have a peak value of unity by

dividing all its flow ordinates by the peak flow.

(4) This unit peak is assumed to represent the 100 percentile flow and hydrograph widths were determined at percentiles of 98, 95, 90 …. 10 and 5.

(5) Widths corresponding to these flow percentiles were averaged over the annual exceedence series and the rising limb of the theoretical flood hydrograph was approximated by fitting a modified form of the Gamma curve to these widths. The shape of the full unit hydrograph was obtained by the addition of an exponential recession curve drawn from the point of inflection of the modified Gamma curve (Fig. 2 (c)).

(6) The required hydrographs were generated by scaling up the derived unit hydrograph for peak flows of different return periods (2, 5, 25, 50, 100, 500 and 1000 years were used). A base flow for the particular return periods was added at each ordinate (Fig. 2 (d)).

Fig. 3

Annual maximum flow series in most Irish catchments follow a Generalised Extreme Value (GEV) Type I distribution (NERC, 1975) and based on this distribution, flood quantiles for the hydrographs in Fig. 2 (Panel D) vary from 91.41 m^3/s for the 2-year event to 153.90 m^3/s for the 1000-year flood.

3.1.2 Hydrographs of Varying Duration

The hydrographs developed using this methodology have the same base length and therefore flood volume is determined solely by the flood peak. The relationship therefore, between flood volume and flood peak is not fully defined. Floods result from natural processes and to fully account for the random nature of their generation, duration needs to be included if flood volume is to be accurately related to peak. Although attempts to develop relationships between volume and peak of direct runoff for catchments elsewhere in the world have been made (see for example Rogers, 1980, Mimikou, 1983, Singh and Aminian, 1986), no validated relationship exists for Irish catchments and an attempt to develop such a relationship in the FSU was inconclusive (O'Connor and Goswami, 2010).

In the absence of a validated method that reflects the increased flood volumes that would usually be associated with floods of longer duration, a simple approach where flood duration is included independently of flood peak was implemented. The approach involved developing a triangular hydrograph of the same volume as the 1000-year hydrograph in Fig. 2 (Panel D) and linking this volume to the hydrograph characteristics by:

$$\text{Volume} = \frac{1}{2} \times T_B \times Q_P \tag{12}$$

where T_B is the hydrograph base width and Q_P is the flood peak (153.90 m^3/s). The duration of the 1000-year hydrograph corresponding to this volume is approximately 265 hours. This duration (T_B) was linked to the time to peak, T_P, by the Flood Studies Report (NERC, 1975) relationship:

$$T_B = 2.52 T_P \tag{13}$$

By further scaling the 1000-year hydrograph, the approach facilitated the development of a second set of hydrographs of different flood durations (Fig. 2 (e)).

3.1.3 Geometrical and Resistance Properties in HEC-RAS Model

The basic HEC-RAS model geometry (Fig. 3) included basic bankfull (B_{bf}) and floodplain (b_{fp}) widths of 25 m and a bankfull depth (h) of 2.5 m. This bankfull depth produced a bankfull flow for the flood hydrograph of the median flow having a 2-year return period and ensured that floodplains in the generalised model would be active for larger floods. The main channel side slopes and the floodplain boundaries were inclined at 45^o giving trapezoidal geometries in both the inbank and overbank zones. The basic hydraulic resistance of the main channel (n_{mc}) and floodplains (n_{fp}) was expressed in terms of Manning's n and assigned values of 0.03 and 0.25 respectively. The high base value of n_{fp} was chosen to ensure that measurable attenuations were observed in model simulations. The basic model length (L) was 50 km and its longitudinal slope of the floodplain (S_{fp}) was set at 1 m/km. In total, 65 variations of these basic properties were tested in eight sets of simulations, denoted by A to H and in which one property was varied at a time (Table A1). Case A investigated the effect of channel length (L), Case B investigated the effect of the longitudinal floodplain slope (S_{fp}), Case C varied the floodplain hydraulic resistance (n_{fp}), Case D the floodplain width (b_{fp}), Case E, the transverse floodplain slope (α) and Case F, the main channel hydraulic resistance (n_{mc}). The influence of flood peak (Q_P) and flood duration (T_B) was explored by routing the two sets of hydrographs (Fig. 2 Panel D and Fig. 2 Panel E) through the generalised model in the Case G and Case H simulations respectively. For each set of simulations, the effect of changes in a specific property on flood attenuation and travel time was examined by comparing input and output hydrographs.

3.2 Standard Muskingum Routing Model

The travel time of the peak of the flood wave determined for each simulation from the input and output hydrographs in the HEC-RAS model was assumed equal to the storage constant (K) in the standard Muskingum routing method (Fig. 1 Panel A). Estimation of corresponding weighting factors (x) was more involved. For each value of K, in combination with assumed values of x (increasing incrementally from 0 to 0.5 where low values reflect high attenuation and vice versa), standard Muskingum flood routing of the inflow hydrographs was repeatedly performed using Eq. 3, together with Eqns. 4, 5 and 6 to generate a series of outflow hydrographs. Peak outflows were determined and comparison of these with the peaks of the inflow hydrographs (Fig. 3 (b)) allowed a series of relative attenuations to be determined, from:

$$\% \text{ Relative attenuation } = \frac{Q_{P1} - Q_{P2}}{Q_{P1}} \times 100 \qquad (14)$$

where Q_{P1} and Q_{P2} are the peaks of the inflow and outflow hydrographs (Fig. 1 Panel A). Linear relationships between these relative attenuations from the Muskingum routing and assumed weighting factors (x) were developed. These relationships were used to determine actual weighting factors for the relative attenuations calculated by comparison of the inflow and outflow hydrographs for each HEC-RAS simulation (Fig. 1 Panel B). This produced a weighting factor for each of the 65 simulations for which corresponding storage constants were directly determined, covering the full range of geometrical and resistance properties that were assessed for the different inflow hydrographs.

3.3 Regression Analysis

Using multi-variate regression analysis, the computed values of the storage constants and weighting factors were correlated to catchment and hydrograph properties and expressions for determining both K and x in terms of these properties were developed.

4 Results

Estimated values of K and x from both the HEC-RAS hydraulic modelling and the standard Muskingum routing are shown in Table A1 and variations of these routing parameters with catchment and hydrograph properties are shown in Fig. 4 and Fig. 5 respectively. Variations in both flood peak (Q_P) and transverse floodplain slope (α) were shown to have only a small influence on storage weighting factors and were excluded from the analysis.

Fig. 4

Fig. 5

Results confirm that increasing floodplain length (Case A) and width (Case D), as noted by Wolff and Burges (1994), increases the capacity of the overbank zone to attenuate and delay the propagation of a flood wave along a channel. Longitudinal slope (Case B) is also important. Steep catchments have the capacity to convey floods more rapidly than those that are more mildly graded and the increased conveyance is reflected in reduced travel time (Fig. 4 (b)) and attenuation (Fig. 5 (b)). These trends are consistent with the findings of Wolff and Burges (1994); Tang *et al*., (1999b) and Ghavasieh et al. (2006) where large attenuations with sharp reductions in the variability of the cumulative distribution were observed in low gradient catchments. Tang *et al*., (1999b) reported that the Muskingum Cunge flood routing method suffers a certain amount of volume loss that depends on the longitudinal slope of the channel. Increasing the lateral slope of floodplains (Case E) results in a geometry in which overbank flow is continually redirected back towards the main channel. Channels therefore, with steep lateral slopes will convey an increased proportion of the flood volume in the main channel. Furthermore, floodplain resistance in the generalised river model was higher than that in the main channel. The reduced attenuations and higher wave speeds with reduced travel times (Fig. 4 (e)) in geometries with increasing transverse slopes are exacerbated given that a diminishing proportion of the total flow is being influenced by the higher floodplain roughness values. Increases in main channel (Case F) and floodplain resistance values (Case C) produce increased flood attenuation and travel times.

The full influence of floodplains on flood wave attenuation is complex and is also influenced by flow peak magnitude (Case G) and the resulting overbank depth. At low return periods (typically less that 2 years), flows will not significantly inundate the floodplain and will not be affected by the additional attenuation associated with the floodplain. Fig. 4 (g) indicates that the wave speed is relatively high for these in-bank events. At flows that produce low overbank depths (return periods of less than 5 years), floodplain influences increase attenuation and travel times. Case H simulations assessed flood duration on relative attenuation and flood wave travel time. When combined, flood peak (Q_P) and duration (T_B) define the flood volume. Floods with low volume hydrographs but sharp peaks and thus short durations experience significantly higher attenuation than those with higher volumes. Floods that are characterised by high volumes on the rising limb of the hydrograph will tend to occupy floodplain storage that is available and once occupied; this storage is no longer available for the remainder of the flood. The attenuation provided by the floodplain in these cases is thus limited. In contrast, hydrographs with low rising limb volumes, disperse most of the flood volume to storage and contribute therefore, to comparatively high relative downstream attenuations.

5 Development of Muskingum Model Parameters

The influences of the geometrical, resistance and hydrograph properties on K and x in Fig. 4 and Fig. 5 were included in a multi-variate regression analysis to generate expressions for these routing parameters. The floodplain width (b_{fp}) and the bankfull width (B_{bf}) were expressed as a single parameter defined by (b_{fp}/B_{bf}) that is consistent with other overbank flow research (for example Knight and Shiono, 1996). The expressions are:

$$K = 0.794 \frac{L\ n_{fp}^{0.24}\ n_{mc}^{0.42} \left(\frac{b_{fp}}{B_{bf}}\right)^{0.60} T_B^{0.07}}{S_{fp}^{0.53}\ \alpha^{0.09} \left(\frac{Q_P}{2b_{fp} + B_{bf}}\right)^{0.06}} \qquad (15)$$

$$x = 0.035 \frac{L^{0.03}\ S_{fp}^{0.16}\ T_B^{0.39}}{\left(\frac{b_{fp}}{B_{bf}}\right)^{0.05} \left(n_{mc}\right)^{0.06} n_{fp}^{0.008}} \qquad (16)$$

Application of Eq. 15 requires that $\alpha > 0$ and therefore horizontal floodplains are represented by a near-zero value of α. Similarly, the equation assesses floodplain effects and therefore peak flows should exceed bankfull discharge capacities in a given river reach. It should be noted that Eq. 15 and Eq. 16 are based solely on the influences of the assessed parameters on relative attenuations and delays of flood peaks determined by the HEC-RAS modelling of the variations in the generalised river reach. The values of parameter exponents are therefore based on the simulated data only and as with regression models of this type, parameters that may intuitively be considered to be important do not necessarily come to the fore in the analysis. The negative influence of floodplain roughness (n_{fp}) in Eq. 16 is a case in point. It would be expected that significant floodplain roughness would result in reasonably large storage and yield low storage weighting factors. However, in the HEC-RAS modelling, increasing floodplain resistance increased the proportion of flow being conveyed in the main channel for all flows investigated, with the result that simulated attenuations were low. The performance of these indices is shown in Fig. 6 where the routing parameters, K and x, are plotted on linear scales against those calculated using Eq. 15 and Eq. 16.

Fig. 6

Fig. 6 indicates that Eq. 15 and Eq. 16 reproduce reasonably well the simulated data for most of the geometrical, resistance and hydrograph properties. However, some limitations exist. Simulated weighting factors (x) are shown to vary most significantly with flood duration (T_B). The poor fit in this case may result from the assumption of independence between the flood peak (Q_P) and the flood duration (T_B) that was made when including duration as a parameter in the regression model. In addition, the clustering of n_{fp} values in Fig. 6 indicates the low influence of this property on the routing parameters determined using the equations.

6 Illustration of Modified Muskingum Flood Routing Method

The routing method presented was applied to the River Suir in Co. Tipperary, Ireland. The River Suir is typical of most Irish rivers in terms of its scale and its low main channel and floodplain sinuosity. A 16.8 km reach between the New Bridge (Station No. 16008) and Caher Park (Station 16009) gauging stations was tested (Fig. 7). Both stations, in addition to a third station at Killardry (Station 16007), where the flow of the main tributary (River Aherlowe) that joins the Suir between New Bridge and Caher Park is measured, are characterised by good quality, digitised 15-minute flow records from 1954 to 2007. Other less significant tributaries also join the river between these stations but are not gauged. The catchment areas to the New Bridge and Caher Park gauging stations are 1120 km^2 and 1602 km^2 respectively and the area to Killardry on the Aherlowe River is 273 km^2. The flood history of the river in this area also indicates that significant floodplain inundation is frequent.

Fig. 7

Illustrating the method involved application of the modified Muskingum method using Eq. 15 and Eq. 16 to route a selection of measured hydrographs at New Bridge through the River Suir reach and comparing these at Caher Park to those obtained from both measured data and from a HEC-RAS model of the river system. The HEC-RAS model was developed from 35 recently obtained cross-sections between New Bridge and Caher Park that defined the main channel geometry and floodplains to a width of approximately 30 m from the channel banks of the Suir and Aherlowe rivers. This data was augmented by LIDAR data to further define the floodplain topography to widths of approximately 500 m on each side of the main channel. Longitudinal distances between measured cross-sections in the reach were approximately 400 m and this resolution in the model was increased through cross-section interpolation. The lower reach of the River Aherlowe was included in this model. At present, 85% and 70% of the Suir and Aherlowe catchments respectively are covered by grassland pasture and this land use is dominant in the floodplains. A Manning's *n* value of 0.05 for grassland pasture with areas of brush described the hydraulic resistance of the floodplain and a coefficient of 0.04 that is typical for a reasonably straight, clean channel at full stage with some obstructions and marginal vegetation defined the main channel resistance (Cowan, 1956; Chow, 1959; Hollinrake and Millington, 1994).

The majority of natural hydrographs are complex and are characterised by kinks and multiple peaks that reflect both the temporal variability of the storm and the spatial heterogeneity of the catchment. Although it is theoretically possible to resolve a complex hydrograph into a series of simple hydrographs, the routing of simple hydrographs from isolated storm events extracted from long flow records was used in this study. Isolating these events was a laborious task and was assisted by FSU hydrograph processing software that facilitated the identification of simple hydrographs at the three gauging stations for three specific storm events. These events related to periods in December 1954/ January 1955, August 1986/ September 1986 and October 2004/ November 2004 (Fig. 8).

Fig. 8

Measured outflow hydrographs at Caher Park can potentially be influenced by a number of tributary inflows (Fig. 7) for which no flow data is available. Their impacts on the peak and timing of the flood hydrographs in the main River Suir, therefore is uncertain. To ensure that the measured outflow hydrographs at Caher Park were not unduly influenced by these tributaries and that measured data offers a basis for testing the modified method, measured hydrographs at New Bridge and Killardry (Fig. 8) were routed through the HEC-RAS model of the river system (that excludes the tributary network for which no data is available) and compared to observed hydrographs at Caher Park. The good agreement between the hydrographs routed in the HEC-RAS model and those measured at Caher Park for the three events investigated (Fig. 9), indicates that the contribution of the tributary flows, other than that from the Aherlowe River, is not significant.

Fig. 9

Parameters in Eq. 15 and Eq. 16 apply only to a single reach and cannot easily be extrapolated to a river system that includes a tributary network. For validation of the modified method therefore, Caher Park hydrographs for the three storm events that exclude the contribution from the Aherlowe River were required. These were approximated by routing the observed hydrographs at Killardry through a HEC-RAS model of the river from this location to Caher Park and subtracting these hydrographs from those measured at Caher

Park. The resulting hydrographs are referred to as 'adjusted' hydrographs. Although this process is somewhat artificial and backwater effects from interactions between the shared floodplain of the main channel and tributary are not included in the analysis, the effects are likely to be local and in the context of a 16.8 km reach, the approach was considered acceptable. Comparison of the adjusted hydrographs with those determined by routing the measured hydrographs at New Bridge using the modified Muskingum procedure allowed the performance of the approach to be illustrated.

The testing of the Muskingum method using Eq. 15 and Eq. 16 was based on assigning appropriate values to parameters that describe the geometry and hydraulic resistance of the channel and floodplains together with the characteristics of the flood hydrographs. Geometrical properties of the main channel and floodplain were determined from survey data and where necessary, averaged over the river reach. The floodplain slope (S_{fp}) and length (L) are FSU catchment descriptors for which numerical values are readily available. Main channel and floodplain Manning's resistances were estimated to be 0.04 and 0.05 respectively and flood peaks and durations were determined from measured inflow hydrographs at New Bridge. This data is summarised in Table 1 for the 1954/ '55, 1986 and 2004 flood events. For the initial testing of the method, the floodplain widths (b_{fp}) in Table 1 were computed by averaging (for 400 m intervals over the 16.8 km reach) the floodplain widths predicted from the HEC-RAS model when routing the measured hydrographs at New Bridge to Caher Park.

Table 1

Caher Park hydrographs using the modified Muskingum method (referred to as 'Muskingum') with these averaged floodplain widths are shown in Fig. 10 with the adjusted

Caher Park hydrographs. The Caher Park hydrographs from the HEC-RAS model (referred to as 'HEC-RAS') are also shown for comparative purposes.

Fig. 10

Although strong correlations between the Muskingum and HEC-RAS routed hydrographs are evident in Fig. 10, the usefulness of the proposed method as presented is limited in that the floodplain widths are derived from hydraulic routing. To improve the predictive capacity of the method, floodplain attenuation indicators (FAIs) that were developed for the Irish Flood Studies Update (FSU) were utilised. FAIs are flood polygons that define the active river floodplain in Irish catchments for the 10-year (Q_{10}), the 100-year (Q_{100}) and the 1000-year (Q_{1000}) floods from normal depth modelling at FSU nodes (approximate intervals of 500 m) on the main river network. The approach is based on the assumption that the median flood, Q_{med} with a return period of 2 years is equivalent to the bankfull flow in all rivers. Given that bankfull recurrence intervals in many rivers are in the order of 1 – 3 years (see for example Richards, 1982; Petit and Paquet, 1997; Castro and Jackson, 2001), this simplifying assumption is reasonable. The median flood is determined using an FSU relationship (Sweeney and Murphy, 2010) for ungauged catchments given by:

$$Q_{\mathrm{med}} = 1.237\times10^{-5}\,\mathrm{AREA}^{0.937}\,\mathrm{BFI}^{-0.922}\,\mathrm{SAAR}^{1.306}\,\mathrm{FARL}^{2.217}\,\mathrm{DRAIND}^{0.341}\,\mathrm{S1085}^{0.185}\,(1+\mathrm{ARTDRAIN})^{0.408} \qquad (17)$$

where AREA (km^2) is the catchment area of the river to the outlet point being considered, S1085 (m/km) is the average slope of the river between 10% and 85% of its length from the outlet, SAAR (mm) is the annual average rainfall on the catchment, FARL is a flood attenuation factor for reservoirs and lakes, BFI is the baseflow index, DRAIND (km^{-1}) is a simple index that relates the length of the upstream hydrological network (km) to the area of the gauged catchment (km^2) and ARTDRAIN is an index of the arterial drainage extent defined as the percentage area of benefiting lands with respect to the total catchment area.

Simple multiplication of Q_{med} by appropriate growth curve factors defined the magnitudes of Q_{10}, Q_{100} and Q_{1000}. Floodplain flows were determined by subtracting Q_{med} values from these flood quantiles and corresponding floodplain flow depths were determined iteratively at all nodes using the Manning equation based on the geometry at that node and a resistance coefficient that was consistent with the land use at the node. Incorporating these depths into a Digital Terrain Model (DTM) facilitated the production of flood polygons for Q_{10}, Q_{100} and Q_{1000} for the Irish river network.

The return periods for the 1954/ '55, 1986 and 2004 floods in the River Suir varied between 5 and 10 years. The Q_{10} FAI was therefore the most relevant polygon from which to estimate floodplain widths and an averaged value of 100 m was determined from the flood extent at all nodes between New Bridge and Caher Park. Hydrographs developed from the modified Muskingum approach using this floodplain width of 100 m are shown in Fig. 11 with those generated from hydraulic routing and those developed from observed data.

Fig. 11

6.1 Discussion of Results

Visual comparisons of the hydrographs in Fig. 10 and Fig. 11, although somewhat subjective, provide a quick and simple means of assessing the performance of the modified Muskingum routing method presented in this paper. Unsurprisingly, adjusted Caher Park hydrographs in Fig. 10, correlate closely with those of the modified Muskingum method in which floodplain widths were extracted from results of the HEC-RAS routing. However, given that floodplain widths are based on outputs from a hydraulic model, this approach is of limited use. More meaningful assessments of the method as a predictor of outflow hydrographs are shown in Fig. 11. Here, good agreement is again observed between hydrographs from the modified Muskingum method in which floodplain widths are determined from the FAI catchment descriptor, and the HEC-RAS and adjusted hydrographs. The goodness-of-fit between the Muskingum and adjusted hydrographs was qualitatively and less subjectively tested using the range of goodness-of-fit criteria recommended by Jewitt and Schulze (1999). Goodness-of-fit statistical tests measure the deviation of a simulated output from an observed input data set and different tests are applied to assess different hydrograph components. Root mean square errors (RMSE) determine the magnitude of error in the computed hydrographs and were estimated using the relationship by Schulze et al. (1995) given by:

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(Q_{comp(t)} - Q_{obs(t)}\right)^2}{n}} \quad \text{for } i = 1, 2, 3, \ldots\ldots n \tag{18}$$

where $Q_{\mathrm{comp(t)}}$ and $Q_{\mathrm{obs(t)}}$ are the computed and observed discharges at a number of different time steps n. Given that peak outflow is important in a single event hydrograph model, percentage errors in computed and observed peak flow rates, peak timing and volume were determined using the following equations (Green and Stephenson, 1985):

$$E_{peak} = \frac{Q_{p-comp} - Q_{p-obs}}{Q_{p-obs}} \times 100 \tag{19}$$

$$E_{time} = \frac{t_{p-comp} - t_{p-obs}}{t_{p-obs}} \times 100 \tag{20}$$

$$E_{volume} = \frac{V_{comp} - V_{obs}}{V_{obs}} \times 100 \tag{21}$$

where E_{peak}, E_{time} and E_{volume} are percentage errors in peak flow, timing and hydrograph volumes respectively, $Q_{p\text{-}comp}$ and $Q_{p\text{-}obs}$ are computed and observed peak flows, $t_{p\text{-}comp}$ and $t_{p\text{-}obs}$ are computed and observed times to peak flow and V_{comp} and V_{obs} are computed and observed hydrograph volumes.

Even though the RMSE, E_{peak}, E_{time} and E_{volume} statistics may assess model performance effectively, differences in the shape of computed and observed hydrographs may not be accounted for fully. To overcome this, Nash and Sutcliffe (1970) proposed a dimensionless coefficient of model efficiency (E), given as:

$$E = \frac{F_o^2 - F^2}{F_o^2} \tag{22}$$

in which $F^2 = \sum_{i=1}^{n}\left[Q_{obs(t)} - Q_{comp(t)}\right]^2$ and $F_o^2 = \sum_{i=1}^{n}\left[Q_{obs(t)} - Q_m\right]^2$.

The coefficient of efficiency in Eq. 22 provides a well accepted measure of fit between computed and observed hydrographs, its value increasing toward unity as the fit of the simulated hydrograph progressively improves. Values of E that exceed 0.8 (Green and Stephenson, 1986) are considered to reflect a good correlation between the adjusted and the Muskingum and HEC-RAS hydrographs in this study. Results of these statistical tests where the Muskingum and HEC-RAS hydrographs at Caher Park in Fig. 11 are compared to adjusted hydrographs developed from observed data are shown in Table 2.

Table 2

Table 2 shows that the modified Muskingum method produces outflow hydrographs that compare favourably with the adjusted hydrographs and are comparable to those developed through HEC-RAS modelling. The differences that exist between both the Muskingum and HEC-RAS hydrographs and the adjusted hydrographs are likely to result from the simplifying assumptions inherent in both methods. In reality, main channel and floodplain momentum exchanges in the River Suir may increase flood wave attenuation and decrease travel time. However, the modified routing parameters were developed from a multi-variate analysis of data from 1-D modelling of a generalised channel in which the energy losses from these interactions are not included. Furthermore, the influences of geometrical, resistance and hydrograph properties in this analysis were assessed independently in these analyses. Similarly, main channel and floodplain interactions are unaccounted for in the HEC-RAS hydrographs developed from 1-D modelling of the River Suir study reach. The impacts of these interactions however, appear low and the relationships in Eq. 15 and Eq. 16 are considered to provide reasonable estimates for K and x in the presented method. The satisfactory performance of the modified method further implies that the storage equation in Muskingum routing methods is a substitute for the momentum equation in hydraulic routing approaches in typical Irish rivers and therefore it is reasonable to relate the routing parameters to channel and flow characteristics (Perumal, 1992b).

It should be noted however that the limitations of the approach are the same as in most hydrological or Muskingum flood routing methods and therefore, the results from this method should be confirmed if applied to river reaches where backwater and inertia effects are significant, where floodplain sinuosity is excessively high or where significant lateral momentum exchanges between main channel and floodplain zones are influential. The method however, does provide a simple and inexpensive method for obtaining preliminary estimates of the time and shape of a flood hydrograph as it travels overbank along a river reach.

7 Conclusions

A modified linear Muskingum routing method suitable for floodplain flows is presented in this paper. Muskingum flood routing methods are based on storage – discharge relationships in river systems and can satisfactorily produce outflow hydrographs in river systems where inertia effects and backwater influences are small. However, values of the routing parameters, *x* and *K*, that describe the storage characteristics of a river reach are usually derived analytically from observed upstream and downstream hydrographs extracted from historical flow records. The proposed method uses explicit relationships for *K* and *x* described in terms of standard geometrical and resistance properties of channels with floodplains together with properties of standard inflow hydrographs. The relationships were based on regression analysis of computational data generated through 1-dimensional modelling of a generalised river reach. The expressions are:

$$K = 0.794 \frac{L\, n_{fp}^{0.24}\, n_{mc}^{0.42} \left(\frac{b_{fp}}{B_{bf}}\right)^{0.60} T_B^{0.07}}{S_{fp}^{0.53}\, \alpha^{0.09} \left(\frac{Q_P}{2b_{fp}+B_{bf}}\right)^{0.06}} \qquad x = 0.035 \frac{L^{0.03}\, S_{fp}^{0.16}\, T_B^{0.39}}{\left(\frac{b_{fp}}{B_{bf}}\right)^{0.05} \left(n_{mc}\right)^{0.06} n_{fp}^{0.008}}$$

Application of the method requires that $\alpha > 0$ and therefore horizontal floodplains are represented by a near-zero value of α. Furthermore, the modified method assesses floodplain effects and therefore flows must be sufficient to produce out of bank conditions. Application of the method in a reach of the River Suir, Co. Tipperary, Ireland was shown to produce outflow hydrographs that compared favourably to those developed from measured flow records.

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Appendix

Table A1

List of Figures

List of Tables