Hybrid Bayesian Approach for Fusing Range-based and Sourceless Localization Estimates Under Non-Stationary Observability

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Abstract—The paper proposes a hybrid Bayesian approach for multi-sensor data fusion for 3D localization. The approach addresses the problem of fusing range-based and sourceless localization estimates under conditions of varying observability in the range-based sub-system. The proposed localization approach uses a mixture of Single-Hypothesis-Tracking (e.g. Kalman filter) and Multi-Hypothesis-Tracking (MHT) (e.g. Particle Filters) Bayesian filtering to improve tracking accuracy under conditions of varying observability. Under conditions of sufficient (or no) range measurements a single hypothesis approach is used. Under the condition of insufficient range measurements (i.e, 1 or 2 ranges), MHT is used, since it more accurately models the distribution of real error in the estimated positions by means of Gaussian mixtures rather that a single Gaussian. The results show up to 10% improvement in 3D position estimation as compared to Single-Constraint-at-a-Time (SCAAT) approach and upto 24% improvement compared to an Extended Kalman Filter approach for intermittent 3 second partial range occlusions when tracking human arm movements.

I. INTRODUCTION

Motion capture (Mocap) and motion tracking are of interest in applications ranging from entertainment industry to healthcare [1]. Range-based Mocap systems employ multiple range measurements at wearable sensors to estimate 3D position [2]. Typically, range is measured between fixed transmitters and wearable sensors or vice versa. Range can either be measured directly using ultrasonic [3] or UltraWideBand [4] signals or estimated indirectly based on measurements from optical sensors [5]. These systems provide accurate absolute position estimates but generally suffer from Non-Line-of-Sight (NLOS) or occlusion problems. Anther class of Mocap systems are sourceless. Typically these self-contained systems rely on Inertial Measurement Units (IMUs) to measure linear and angular acceleration. While sourceless systems do not have NLOS problems, they do suffer from rapid growth in error when position is estimated by dead reckoning. This is due to IMU bias instability and inaccurate gravity compensation. Hybrid Mocap systems offer a more accurate and robust solution [6], [7]. Hybrid systems solve the NLOS, drift and calibration problems by incorporating range-based and sourceless subsystems and fusing the data. The conventional approach to data fusion is through a series of prediction-correction steps. The sourceless system parameters form the basis of the motion prediction model and the range measurements are used in the correction step. During the correction step, the measured ranges can be incorporated sequentially or as a single vector. The accuracy of the correction step depends on the number of transmitter-receiver pairs that have Line Of Sight (LOS). The nature of the transmitter-receiver occlusion pattern for a fixed infrastructure based system depends on the type of movements being performed.

Herein, we are interested in occlusions occurring in applications in which rehabilitation, sports or fitness exercises are monitored. The work presented in this paper addresses the problem of location estimation in cases of temporary partial occlusion of the ranging subsystem in hybrid Mocap systems. By temporary partial occlusion, we mean that for short time periods only 1 or 2 range estimates are available due to NLOS conditions between the other transmitters and receivers. We assume that 3 or more range estimates are available before partial occlusion occurs. Herein we propose a novel method that operates via selective transition between a Multiple-Hypothesis-tracking (MHT) and Single-Hypothesis-Tracking (SHT). The proposed method was found to provide higher accuracy than previously published algorithms, in terms of 3D position estimation, for the considered occlusion scenarios. In addition, the computational complexity of the proposed filter is lower than a conventional particle filter because it uses a lesser number of particles.

The remainder of the paper is organized as follows. Section II gives a mathematical description of the problem and also details the envisioned occlusion scenarios which the mobile device encounters during dynamics of the motion. Section III outlines previous work reported on 3D localization using range and inertial measurements, particularly emphasizing methods that use a mixture of two or more bayesian techniques. Section IV introduces the proposed approach and discusses the shortcomings of conventional approaches under particular scenarios. Section V presents 3D localization results using the proposed method and compares the results with other candidate approaches. Finally Section VI draws conclusions and discusses potential future work.

II. PROBLEM DESCRIPTION

Mocap systems determine the Position and Orientation (P&O) of multiple sensors units placed on landmarks on the

human body. Herein, we focus on 3D position estimation of a single sensor unit. A single sensor unit in our case is composed of an ultrasonic receiver and an IMU unit. The problem of localization of the sensor unit is modeled in a dynamic nonlinear state space model. The initial state is sampled from the Gaussian distribution given by:

$$X_0 = \mathcal{N}\{\mu_0, P_0\} \tag{1}$$

where μ_0 is the initial best estimate of the state and P_0 is the error covariance matrix associated with the state $(x \in \Re^n)$. The state transition function (motion model) and observation model are

$$\mathcal{P}(X_t|X_{t-1}) = \int \mathcal{F}(\mathcal{P}(X_{t-1}|z_{0:t-1}), u_t, \nu) dx \qquad (2)$$

$$z_t = \mathcal{H}(E(X_t|X_{t-1}), \omega) \tag{3}$$

where,

$$E(X_t|X_{t-1}) = \int X_t \mathcal{P}(X_t|X_{t-1}) dx \qquad (4)$$

where the observations $(z_t \in \Re^m)$ are a set of *m* ranges. The functions \mathcal{F}, \mathcal{H} are non linear functions. ν and ω are the process noise and measurement noise covariance respectively and u_t is the input from Inertial Measurement Units. The aim is to evaluate the posterior $\mathcal{P}(X_t|z_{0:t})$. The Bayes formula is used to track the posterior:

$$\mathcal{P}(X_t|z_{0:t}) = \int C\mathcal{P}(X_t|X_{t-1}, z_{0:t-1})\mathcal{P}(z_t|X_t)dx \quad (5)$$

Prior probability and likelihood are given by (2) and (3) respectively.

A hybrid motion tracking system based on ultrasonic and inertial measurements is described in [8]. Such a motion capture setup consists of at least three fixed ultrasonic transmitters with known position co-ordinates w.r.t. the fixed frame of reference. Figure 1 shows the motion capture setup for home based rehabilitation setup [9], [10]. The sensor unit (S_i) is equipped an ultrasonic receiver and undergoes motion within the volume covered by the set of transmitters (Tx_i) . During the movement, the sensor unit may have a direct Line Of Sight (LOS) to all the transmitters. A transmitter-receiver can suffer NLOS due to presence of blocking objects (e.g. limbs). It is possible to have a complete occlusion wherein none of the transmitter receiver pairs have direct LOS. In this case, the position of the MD must be estimated using dead reckoning based on the IMU data only. In other cases only a subset of the transmitter-receiver pairs have direct LOS. We refer to presence of 3 or more ranges as *sufficient* and the presence of lesser number of ranges are referred to as insufficient for the remainder of this paper. The authors use the term partial observability in this text to refer to whether the 3D position can be completely triangulated from the measured ranges. Sufficient ranges imply complete observability of the state and insufficient ranges imply partial observability.

Table I summarizes a few of the occlusion patterns (not exhaustive) that are possible when the sensor unit undergoes motion. In the first scenario (4s - 4s), 4 ranges are available



Fig. 1: Localization setup

throughout the motion at each sample. In the second scenario (4s -3s), 4 ranges are available for *s* samples and 3 ranges are available for next *s* samples, and the sequence is repeated for the duration of the motion. Other occlusion scenarios follow the same notation.

Scenario	Occlusion Pattern	Comment
1	4s - 4s	Full LOS
2	4s - 3s	Full Los
3	4s - 3s - 2s	Intermittent Single NLOS events
4	4s - 3s - 2s - 1s	Intermittent Double NLOS events
5	4s - 3s - 2s - 1s - 4s	Intermittent Complete Occlusions

TABLE I: Envisioned Occlusion Scenarios

III. BACKGROUND

Several derivatives of the Bayesian filter have been used for tracking the posterior probability of a given state space model. The Kalman filter is an optimal estimator for a linear system subject to Gaussian noise [11]. Several variants of the Kalman filter, such as the Extended Kalman Filter (EKF) and the Unscented Kalman Filter have been proposed for non linear systems with non gaussian noise, but these can be unstable and often diverge when the non-linearity in the system is high [12], [13]. A different subclass of Bayesian filters known as Sequential Importance Sampling (SIS) or Particle Filters track the posterior of a system by using a number of particles to approximate the probability distribution [14]. A sequential Monte Carlo method with marginalization using a Kalman filter was proposed for estimation of a partially observed gaussian state space in [15]. In [8], an EKF is used for fusion of ultrasonic and inertial measurements. The ultrasonic ranges were incorporated sequentially by using Single-Constraintat-a-Time (SCAAT) approach. The authors also proposed a fusion approach based on observation vector representation of ranges, but accurate modeling in the case of insufficient range measurements was not incorporated. The hybrid acousticinertial system in [16] is capable of reconstructing body

Filter	Multimodal	Gaussian Approximation	Non-Linear
Kalman	No	Yes	No
EKF	No	Yes	Yes
SIS	Yes	No	Yes
GPF	No	Yes	Yes
RB-SIS	Yes	No	Yes

TABLE II: Characteristics of Various Fusion Approaches

joint configuration. The system has non-stationary transmitter receiver pairs which limits its capability to provide absolute 3D positions and orientations. It also makes such a system unable to combine information from multiple ranges (method such as trilateration).

Hybrid estimation approaches have been successfully applied for approximating the posterior distribution. Rao Blackwellized Sequential Importance Sampling (RB-SIS) filters are computationally less complex than conventional SIS [17]. A Gaussian Particle Filter (GPF) applies the SIS filtering technique to state space systems with Gaussian additive noise [18]. A Kalman-particle kernel filter was applied to the terrain navigation problem, which is based on kernel representation of conditional density [19]. The dual layer particle filter estimates position in two steps [20]. The first step is a block level estimation and the second is a coarse estimation of position using a Particle Filter.

The solution to the envisioned problem requires an estimator which can track both unimodal and multimodal probability distribution. The estimator should also be capable of handling non-linearities in the systems and can be applied to both Gaussian and non-Gaussian error models. Table II summarizes the applicability of the above filters to the problem at hand. The EKF is optimal choice for non linear systems with unimodal probability distribution. Whereas, a SIS filter in optimal for tracking multimodal distributions with no underlying assumption of error model. Thus a hybrid estimator that incorporates the characteristics of an EKF and a SIS filter satisfies the requirements of our problem.

IV. APPROACH

A. Modeling Observation Likelihood

In the presence of sufficient ranges (each with Gaussian error), the uncertainty in estimated position can be modeled by a single Gaussian [21]. However, a Gaussian mixture is a more accurate representation of estimated position from insufficient ranges.

Herein, we represent the current estimate and its uncertainty as a M component mixture model with component weights (w^i) .

$$\mathcal{P}(z/X) = \sum_{i=0}^{M} w^{i} * \mathcal{F}(z, X^{i})$$
(6)

During the data fusion process, the measured ranges are used during the observation likelihood calculation. It is desirable that the observation likelihood model accurately models the real error function. Figure 2 compares the observation likelihood representation using a single Gaussian and a Gaussian mixture when 2 ranges are available. Using only two ranges, the best estimate for the current position is that it lies along the circumference of a circle. Clearly, the Gaussian mixture models the true error more accurately. The mixture of Gaussians tracks all possibilities of the current position being situated on the ring, whereas a single Gaussian has the mean at the center of the circle and needs a large variance to have the same solution coverage as the mixture representation.

B. Proposed Algorithm

The skeleton of the proposed algorithm is same as a basic Prediction-Correction estimator. Figure 3 shows a flowchart representing the steps involved in the proposed algorithm.

The estimation procedure starts with an initial estimate of the state, and associated uncertainty. The measurements form the IMU are used in the motion model for prediction of the next state. The prediction step uses standard state space transition model. The proposed method differs from other filters in the way it carries out the correction step. The correction step has three functional subunits. A single hypothesis tracking unit, a multiple hypothesis tracking unit,



Fig. 2: Using a Single Gaussian Vs Gaussian Mixture to represent Observation Likelihood.



Fig. 3: The Proposed Fusion Approach

and an information exchange unit. The correction step operates by switching between the SHT and MHT subunits. The decision for switching between the two subunits is based on the observability of the state. From the set of measured ranges, the potentially unreliable ranges are removed using a NLOS detection algorithm [22]. The set of reliable ranges forms the observation vector. The rank of the observation matrix is used as a parameter for determining whether the state is fully observable. The information exchange unit is used to maintain the state-space uncertainty information when switching between the two approaches. The pseudocode for the algorithm describes the mathematical equations involved during the prediction and correction steps. The notation used is as follows. X is the state vector and z is a set of measured ranges. A, H and P are state transition matrix, measurement matrix and error covariance matrix respectively. $W_{prior}^{i}(t)$, $W_{post}^{i}(t)$, L_{t} and C are prior weight on i^{th} particle, posterior weight, observation likelihood and normalization constant respectively.

The single hypothesis tracking subunit is based on design of an Extended Kalman Filter. The state vector (X_t) consists of the sensor unit's position and velocity along all three coordinate axes. The State transition matrix (A) is given by

$$A = \begin{bmatrix} 1 & t & \frac{t^2}{2} \\ 0 & 1 & t \\ 0 & 0 & 1 \end{bmatrix}$$

$$h = \sqrt{(x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2 + (z_i - \hat{z}_i)^2}$$

The observation matrix (H), is the Jacobian matrix of partial derivatives of the observation vector (h) with respect to X, i.e.

$$H_{(i,j)} = \frac{\partial h_i}{\partial X_j} \tag{7}$$

Where x_i is the predicted x co-ordinate and \hat{x}_i is the estimated x co-ordinate using the measured ranges. The y and z axis follow same notation.

The Multiple Hypothesis Tracking subunit is based on the design of a Sequential Importance Sampling Particle Filter. N particles are sampled using mean and variance parameter obtained from the information exchange unit. Each particle is assigned a prior weight $W_{prior}^{i}(t)$ based on its Euclidean distance from the prior mean. The observation likelihood calculation (L_t) uses the set of reliable ranges to triangulate the best estimate of the current state. The observation likelihood is calculated using the Euclidean distance between the current particle position and the triangulated position.

$$P(x^{i}) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x^{i} - \hat{x})^{2}/2\sigma^{2}}$$
(8)

Algorithm : Hybrid Bayesian Fusion (X_t, z_t, u_t)

$$\begin{split} I. \mathbf{Prediction} \\ \begin{cases} (i) X_{t+1} = AX_t + Bu_t + \omega_t \\ (ii) z_t = HX_t + v_k \\ (iii) P \equiv E[(\hat{X} - X)^2] \\ II. \mathbf{Correction} \\ \mathbf{if} \ (Rank(H_t) \leq \tau)) \\ \mathbf{then} \\ \begin{cases} \mathbf{Single Hypothesis Tracking} \\ (i) \hat{X}_t = X_t^- + K_t (\tilde{y}_t - H_t X_t^-) \\ (ii) K_t = P_t^- H_t^T (H_t P_t^- H_t^T + R_t)^{-1} \\ (iii) P_t = \frac{P_t^- H_t^-}{H_t^- H_t^T + R_t} \\ \end{cases} \\ \mathbf{for the transform for the tracking \\ (i) X_t^i = \mathcal{N} \{\mu_t, P_t\} \\ (ii) W_{prior}^i(t) = \mathcal{F}_1(X^i). \\ (iii) L_t = \mathcal{F}_2(X^i, z_t) \\ (iv) W_{post} = \frac{W_{prior} \times L_t}{C} \\ (v) \mu_t = \sum_{i=0}^N W_{post}(i) * X^i \\ (vi) P_t = \sum_{i=0}^N W_{post}(i) * ([X^i - \mu][X^i - \mu]^T) \\ \end{cases} \end{split}$$

function assigns weight to the particles. The weight is assigned to the particles using the prior probability and the observation likelihood in the Baye's formula in eq(5). Under partial observability, the cost function assigns equal weights to a number of particles that lie on the the higher dimensional space (i.e., circular or spherical). For instance when 2 ranges are available, the particles lying on the circumference of the circular region are given equal weights, which is higher than the weights of the particles outside the circumference.

The information exchange unit ensures that the probability distribution parameters are passed between the two subunits.



(a) Linear Motion

(b) Random Roto-Translation

(c) Flexion-Extension of Arm

Fig. 4: Trajectory estimated by different approaches



Fig. 5: Effect of ranging accuracy on filter performance

Such an information exchange maintains the continuity in state space uncertainty while the switching action is performed. This also helps to reduce the computational complexity of the Multiple Hypothesis Tracking subunit, as the particles can be sampled from and concentrated in a smaller space using the mean and variance information from the SHT subunit.

V. RESULTS AND DISCUSSION

The proposed method was tested for 3D localization for simulated linear motion, random roto-translational motion and for flexion and extension of the upper arm. Matlab was used to simulate 10 seconds of these movements with varying occlusion scenarios during the course of the motion. The motion was subjected to intermittent 3 second occlusions. During the occlusion period, different occlusion patterns were tried. IMU data was available at 50Hz, and the range estimates were available at 30Hz. A standard deviation of $0.05m/sec^2$ and an offset error of $0.1m/sec^2$ was added to the accelerometer data. These values were determined experimentally from a Shimmer research platform, containing the Freescale MMA7361 MEMs accelerometer. Two differet level of errors (1cm and 5cm) were added to the range estimates, to test the robustness of the algorithm. The authors selected 4 approaches to compare based on the literature survey and 3D position estimation was done using each of these approaches. The selected methods are Extended Kalman Filter (EKF) [12], Sequential-ImportanceResampling (SIS) or Particle Filter [14], Single-Constraint-ata-Time (SCAAT) [8] and Dual Layer Particle Filter (referred to as DPF in results) [20].

A. Estimation Accuracy

The candidate methods were applied to the simulated motion. The motion was exposed to the occlusion scenarios discussed in Section II. Table III summarizes results for flexion-extension of the upper arm. The table presents RMS error of the candidate approaches over the whole trajectory. Under Scenario 1, i.e., when all 4 ranges are continuously available, the proposed approach performs same as the SCAAT approach. But under occluded motions (i.e., scenarios 3, 4 and 5) the proposed approach performs better than the SCCAT approach. The results show up to 10% improvement compared to SCAAT approach.

Figure 4 compares the trajectory of the linear motion, random roto-translation and flexion-extension motion as estimated by different approaches. The shaded regions in the graph represent the occluded periods during the motion. It is evident in occluded periods, the proposed approach estimates the true trajectory more accurately as compared to SCAAT approach. Figure 5 depicts the effect of range estimation accuracy on 3D localization for all scenarios. In all scenarios the proposed

Scenario	Proposed	SCAAT	EKF	DPF	% Improvement
					(Proposed Vs SCAAT)
1	3.02	3.02	3.06	5.07	0
2	3.05	3.05	3.12	5.05	0
3	3.2	3.33	3.41	5.3	4
4	3.24	3.4	4.18	5.4	4.8
5	3.7	4.1	4.83	5.9	10

TABLE III: RMS error of different approaches for Flexion-Extension movement (in cm)

approach is most accurate estimator. Table IV shows the difference in maximum errors (Peak to Peak difference) for the trajectory estimation for SCAAT and the proposed approach. The Proposed approach shows up to 21% improvement in maximum error.

Scenario	Difference (mm)	% improvement
3	1.5	12
4	2.2	15
5	5	21

TABLE IV: Peak to Peak absolute error difference (in cm) and percentage improvement of Proposed Approach over SCAAT approach

B. Computational Complexity

The computational complexity of the proposed algorithm was compared to a conventional SIS filter. Table V shows the effect of varying the number of particles on the accuracy of the position estimate. It can be seen that the number of particles required to achieve the same accuracy by the hybrid approach is significantly less than that required by the conventional SIS approach. The proposed filter requires a lesser number of particles to approximate the probability distribution due to the vital information (mean and variance) that is exchanged from the SHT estimator.

No. Particles	SIS (mm)	Proposed (mm)
50	4.79	3.30
100	4.4	3.06
500	4.4	2.86
1000	3.74	2.57

TABLE V: Effect of no. of particles on estimation accuracy (RMS error)

VI. CONCLUSION AND FUTURE WORK

A hybrid Bayesian approach for fusion of range-based and sourceless localization subsystems is proposed. The proposed system selectively uses Multihypothesis tracking or Single hypothesis tracking based on the observation likelihood error model. The proposed approach is applicable to a broad range of partial range occlusion patterns. The results show up to 10% improvement as compared to SCAAT approach and upto 24% improvement compared to an Extended Kalman Filter approach. The proposed system is also computationally more efficient compared to a conventional particle filter. Work in near future will be on applying the proposed method to estimation of both position and orientation.

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