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Information Gap Decision Theory Approach to Deal with Wind Power Uncertainty in Unit Commitment

Alireza Soroudi*, Abbas Rabiee, Andrew Keane

Abstract

The renewable energy sources (RES) integration in the electricity supply utilities can reduce the energy procurement costs as well as the environmental concerns. Wind power is the most popular form of RES which is vastly utilized worldwide. This paper proposes a robust model for unit commitment (UC) problem, minimizing the operating costs considering uncertainty of wind power generation. In order to handle the uncertainties arising from volatile nature of wind power, information gap decision theory (IGDT) is utilized, where risk averse (RA) and opportunity seeker (OS) strategies are developed. RA strategy gives a robust decision making tool for handling the severe uncertainty of wind power, whereas the OS strategy makes benefit of possible uncertainties by adjusting the decision variables in a right way. Besides, the impact of demand flexibility (or demand response) on the operation costs is also investigated. The proposed model is examined on the IEEE 118-bus test system, and its benefits over the existing stochastic programming technique is examined. The obtained results demonstrate the applicability of the proposed method to deal with the UC problem with uncertain wind power generation. It is also observed that demand flexibility has positive impacts in both RA and OS strategies.

Index Terms

Demand response (DR), information gap decision theory (IGDT), uncertainty, unit commitment (UC), wind power.

NOMENCLATURE

Indices & Sets

\( g \)

Index for thermal generation units.

A. Rabiee is with the Department of Electrical Engineering, Faculty of Engineering, University of Zanjan, Zanjan, Iran, (e-mail: rabiee@znu.ac.ir)

*Alireza Soroudi and Andrew Keane are with the School of Electrical, Electronic and Communications Engineering, University College Dublin, (e-mail: alireza.soroudi@ucd.ie, Andrew.keane@ucd.ie)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$b, j$</td>
<td>Index for network buses.</td>
</tr>
<tr>
<td>$t$</td>
<td>Index for time intervals.</td>
</tr>
<tr>
<td>$l$</td>
<td>Index for transmission lines.</td>
</tr>
<tr>
<td>$ND$</td>
<td>Number of discrete intervals of the start-up cost function.</td>
</tr>
<tr>
<td>$NL$</td>
<td>Number of blocks of the piecewise linearization of the variable cost function.</td>
</tr>
<tr>
<td>$\Omega_G$</td>
<td>Set of generating units.</td>
</tr>
<tr>
<td>$\Omega_{G_b}$</td>
<td>Set of generating units installed at bus $b$.</td>
</tr>
<tr>
<td>$\Omega_{DR}$</td>
<td>Set of buses participating in DR program.</td>
</tr>
<tr>
<td>$\Omega_T$</td>
<td>Set of operating periods.</td>
</tr>
<tr>
<td>$\Omega_B$</td>
<td>Set of network buses.</td>
</tr>
<tr>
<td>$\Omega_{B_w}$</td>
<td>Set of network buses connected to wind farms.</td>
</tr>
<tr>
<td>$\Omega_L$</td>
<td>Set of transmission lines.</td>
</tr>
<tr>
<td>$\Omega_{I}$</td>
<td>Set of discrete intervals of the start-up cost function.</td>
</tr>
<tr>
<td>$\Omega_{\ell}$</td>
<td>Set of blocks of the piecewise linearization of the variable cost function.</td>
</tr>
<tr>
<td>$X$</td>
<td>Set of decision variables.</td>
</tr>
<tr>
<td>$T$</td>
<td>Total number of time intervals</td>
</tr>
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</table>

**Parameters**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{W}^{b}$</td>
<td>Auxiliary binary parameter which denotes the connection status of wind turbine $W$ to bus $b$.</td>
</tr>
<tr>
<td>$I_{b}^{D}$</td>
<td>Auxiliary binary parameter which denotes if demand in bus $b$ is participating in DR program or not.</td>
</tr>
<tr>
<td>$\Delta_{W}^{b}$</td>
<td>Capacity of wind turbines installed at bus $b$ (MW).</td>
</tr>
<tr>
<td>$\beta_{c/o}$</td>
<td>Critical/opportunistic percentage of increase/decrease of base cost in RA/OS approach (%).</td>
</tr>
<tr>
<td>$\Delta_{c/o}$</td>
<td>Critical/opportunistic increase/decrease of base cost in RA/OS approach ($$).</td>
</tr>
<tr>
<td>$D_{b}^{d/d}$</td>
<td>Demand increase/decrease rate in bus $b$ (MW/h).</td>
</tr>
<tr>
<td>$\zeta_{b,t}$</td>
<td>Demand response cost in bus $b$ at time $t$ ($$/MW h).</td>
</tr>
<tr>
<td>$\Delta_{t}$</td>
<td>Duration of time period $t$.</td>
</tr>
<tr>
<td>$B_{bi}$</td>
<td>Element $bi$ of susceptance matrix of transmission network.</td>
</tr>
<tr>
<td>$FC_{g}$</td>
<td>Fixed cost of unit $g$ ($$/h).</td>
</tr>
<tr>
<td>$P_{b,t}^{La}$</td>
<td>Initial active power demand in bus $b$ at time $t$ (before DR activation).</td>
</tr>
<tr>
<td>$P_{g}^{max/min}$</td>
<td>Maximum/minimum limit of power generation of $g$ unit.</td>
</tr>
<tr>
<td>$\theta_{b}^{max/min}$</td>
<td>Maximum/minimum limit of voltage angle in bus $b$ (Rad).</td>
</tr>
<tr>
<td>$P_{l}^{max}$</td>
<td>Maximum allowed power limit of transmission line $l$.</td>
</tr>
<tr>
<td>$\gamma_{b,t}^{max/min}$</td>
<td>Maximum/minimum of demand response limits in bus $b$ at time $t$ (%).</td>
</tr>
<tr>
<td>$P_{b,t}^{W}$</td>
<td>Predicted power produced by wind turbine unit at bus $b$ at time $t$ (MW).</td>
</tr>
<tr>
<td>$RU_{g}/RD_{g}$</td>
<td>Ramp-up/down limit of generation unit $g$, (MW/h).</td>
</tr>
<tr>
<td>$P_{b,t}^{W}$</td>
<td>Uncertain power produced by wind turbine unit at bus $b$ at time $t$ (MW).</td>
</tr>
<tr>
<td>$SDC_{g}/SUC_{g}$</td>
<td>Shut down/Start-up cost of unit $g$, ($$/h).</td>
</tr>
<tr>
<td>$DT/UT_{g}$</td>
<td>Minimum down/up time of unit $g$, (h).</td>
</tr>
<tr>
<td>$\lambda_{\ell,g}$</td>
<td>Slope of $\ell$-th block of the variable cost of unit $g$, ($$/MWh).</td>
</tr>
<tr>
<td>$SD/SU_{g}$</td>
<td>Shut-down/start-up ramp limit of unit $g$, (MW/h).</td>
</tr>
</tbody>
</table>
\( U_g^0 \)  Time periods unit has been on at the beginning of the planning horizon (end of hour 0) (h).
\( T_{\ell,g} \)  Upper limit of block \( \ell \) of the variable cost of unit \( g \), (MW).

**Variables**

\( \bar{P}_{b,t}^{W} \)  Actual injected power produced by wind turbine unit at bus \( b \) at time \( t \) (MW).
\( P_{b,t}^{L} \)  Active power demand in bus \( b \) at time \( t \) after DR activation.
\( \delta_{\ell,g}(t) \)  Auxiliary variable used for the linearization of the variable cost function of unit \( g \) at hour \( t \), it represents the \( \ell \)-th power block (MW).
\( \Gamma_b \)  Base cost when no uncertainty exists in problem ($).
\( u_{\ell,g}(t) \)  Binary variable which is equal to 1 if the power output of unit \( g \) at hour \( t \) has exceeded block \( \ell \).
\( v, y, z_{g,t} \)  Binary variable denotes on/off, start-up /shut-down status of generation unit \( g \) at time \( t \).
\( P^{C_w}_{b,t} \)  Curtailed power of wind turbine unit \( w \) connected to bus \( b \) at time \( t \) (MW).
\( \gamma_{b,t} \)  Demand response decision in bus \( b \) at time \( t \) (%).
\( DRC_{b,t} \)  Demand response cost in bus \( b \) at time \( t \) ($).
\( \Delta^{W+/-}_{b,t} \)  Excess/shortage Power produced by wind turbine (compared to predicted one) connected to bus \( b \) at time \( t \) (MW).
\( R_c/R_o \)  Maximum/minimum radius of uncertainty in RA/OS strategy (%).
\( T_{g,t}^{\text{max}} \)  Maximum power output of unit \( g \) at hour \( t \), (MW).
\( OF \)  Objective function.
\( P_{g,t} \)  Power produced by thermal unit \( g \) at time \( t \) (MW).
\( P^{W}_{b,t} \)  Power produced by wind turbine unit \( w \) connected to bus \( b \) at time \( t \) (MW).
\( \Delta^{\pm}_{\ell,t} \)  Power increased/decreased in bus \( b \) in time \( t \) due to demand response (MW).
\( P_{l,t} \)  Power flow in line \( \ell \) at time \( t \) (MW).
\( \alpha \)  Radius of uncertainty (%).
\( SUC_{g,t} \)  Start-up cost of unit \( g \) at hour \( t \), ($).
\( TC \)  Total costs ($).
\( \theta_{b,t} \)  Voltage angle in bus \( b \) at time \( t \) (Rad).
\( VC_{g,t} \)  Variable cost of unit \( g \) at hour \( t \), ($).

### I. INTRODUCTION

#### A. Background and motivations

The future electricity systems will face various challenges originating from both supply and demand sides. In one hand, the increasing concerns about the environmental and economic impacts of fossil-fueled electric energy resources has led to more attention to RES. On the other hand, the growth of electricity demand along with economical/environmental restrictions for expansion of transmission systems necessitate development of proper demand side management programs, in order to supply the customers’ demand adequately.

By increasing penetration of wind power, significant uncertainty is faced in the short-term and real-time operation of power systems, which should be considered properly in day-ahead unit
commitment (UC), optimal power flow (OPF) and even real-time economic dispatch (ED) problems. If this uncertainty is not properly handled, several operational problems such as insufficient ramping capabilities of thermal generating units, insufficient spinning reserve, transmission congestion [1] and involuntarily customer demand interruptions are likely to occur.

One dependable option for reduction of wind power generation uncertainty impacts, is the load flexibility or demand response (DR) program, which gained significant attentions in recent years [2]. DR provides an opportunity for consumers to play a special role in the operation of the electricity grid by reducing or shifting their electricity demand during peak periods in response to the financial incentives [3]. The role of Incentive based and price based DR programms have been discussed in [4]. DR programs are being used by electric system planners and operators as an option for balancing supply and demand. Methods of engaging customers in DR programs include offering time-based rates such as time-of-use pricing, critical peak pricing, variable peak pricing, real-time pricing, and critical peak rebates. It also includes direct load control programs [2].

B. Literature Review

In the presence of uncertain wind power, appropriate tools are necessary for system planners or operators to properly handle its impacts in their decisions. Traditionally, the uncertainty of wind power generation is handled by scheduling additional generation capacity as operating reserve. It is usually set by some deterministic methods such as a fixed percentage of load or, by probabilistic approaches which consider the wind power forecasting error similar that of load [5]. With the integration of large amount of wind power generation to the current systems, new approaches are being developed that precisely consider the characteristics of wind power [6]–[12].

DR programs have been employed as elasticities [13] to improve power system reliability and security. The effect of DR on the enhancement of frequency stability is studied in [14]. Also, thermal generation capacity investment in the presence of DR and high wind power penetration levels is studied in [15].

Due to the intermittent nature of wind power, in addition to the traditional reserves provided by thermal, hydro, and gas generators, DR has attracted more attention as another reserve resource to manage the uncertainty of wind power generation [16]. For example a robust optimization approach is proposed in [17] to solve the UC problem under the joint worst-case wind power output and demand response scenario. In [18] the proper amount of DR to be shifted from peak hours to off peaks, for alleviation of transmission congestion and enhancement of wind power utilization is determined through a network constrained UC (NCUC) problem. But, the wind power generation uncertainty is ignored in [18]. The approach proposed in [17] requires to have the bounds that
wind values falls into and it can not be used in opportunity seeker (OS) strategy. The IGDT [19] is a decision making method that has been utilized in various power system applications like generation asset allocation [20], day-ahead scheduling of electric vehicle aggregators [21], transmission planning [22], bidding strategy [23], restoration of distribution networks [24] and distribution network congestion management [25].

The work presented in this paper utilizes the IGDT to deal with wind power generation uncertainty in the UC problem in transmission level. The proposed framework is basically different with the work described in [25] which refers to distribution level. The proposed UC model uses DC power flow constraints along with the constraints such as generators’ ramp rates, cost coefficients, minimum up/down times, min/max operating limits while AC power flow is implemented in [25]. Additionally, the objective function in [25] is minimizing the congestion problem in distribution system, but in the proposed model the objective function to be minimized is total operating costs in transmission level. Finally, the proposed model is a mixed integer linear programming (MILP) which can be efficiently solved using commercial solvers like CPLEX [26] to find the global optimal solution. The model described in [25] is a mixed integer non-linear problem (MINLP) which is computationally expensive, and only a local optimal solution could be obtained.

The comparison between the existing methods and the proposed method is given in Table I. These methods are compared based on three criteria namely: uncertain input data, uncertainty modeling technique and decision variables and constraints. It should be noted that there is no magic method for modeling the uncertainties. Each method has its own cons and pros. Additionally the best method is determined based on the availability of the input data or severity of the uncertainty. The comparison between the data requirement of different uncertainty modeling techniques are depicted in Fig. 1. From Fig. 1, it can be seen that probabilistic methods need precise information about the probability density function of uncertain input data. Fig. 1-a), shows a multi-stage stochastic optimization. The nature of scenario based stochastic optimization is the combinatorial growth of computation burden. Fig. 1-b), shows a membership function which is required for fuzzy optimization. This method requires solving the problem for multiple values of $\alpha$-cuts and is also computationally expensive. Fig. 1-c), shows the interval optimization or robust optimization method. These methods need exact uncertainty set which uncertain inputs belong to them. The interval optimization usually needs two optimization for each objective function. The robust optimization methods usually requires solving bi-level optimization which is usually difficult to solve. Fig. 1-d), states that IGDT method needs uncertainty set but in contrast to other techniques this uncertainty set is not needed to be exactly known. The IGDT method receives the uncertain uncertainty set and tries to make the objective function resilient against the uncertainty of input parameters. The
Input-output model of IGDT technique is given in Fig. 2.

C. Contributions

Under the context of UC problem, a number of works have been reported in the literature to find the optimal generation schedule. However, to the best of our knowledge, no work in the literature considers the demand flexibility as a tool for dealing with severe uncertainties of wind power generation in deriving an optimal and robust operation schedule for thermal generation units, specially when no probability density function (PDF) is available for wind power generation. Accordingly, the main contributions of this paper are threefold:

1) To find the risk averse (RA) and opportunity seeker (OS) regions of wind power generation for decision maker.
2) To propose a robust UC model (through a RA IGDT model) for deriving the optimal DR to deal with wind power generation uncertainties.
3) To develop a framework which can be used in severe uncertainty cases when no bound, PDF or membership function is available for wind power generation.

D. Paper Organization

The rest of this paper is outlined as follows: Section II presents the basic concept of IGDT technique. The formulation of the proposed (UC) problem considering the demand flexibility and wind power is developed in Section III. In Section IV the IGDT is applied on the (UC) problem. Simulation results are presented in Section V and finally, Section VI summarizes the findings and concludes the paper.

II. Uncertainty Modeling Via IGDT

The IGDT was proposed in [19] for the first time. It is intended to find the optimal decisions in order to maximize the robustness of the objective function against the uncertainty of input parameters. In this section, the IGDT concept is explained as a general framework. Every engineering problem is a kind of decision making problem which can be formulated as follows: Without loss of generality, the minimization problem is adopted here.

\[
\min_X \text{OF} = f(X, \Psi) \quad (1a)
\]

\[
H(X, \Psi) \leq 0, G(X, \Psi) = 0 \quad (1b)
\]

The objective function in (1a) can be defined as cost or risk which is needed to be minimized. This minimization should be performed subject to some inequality and equality constraints (1b).
The decision variables are specified by \((X)\) and the input parameters are represented by \((\Psi)\). The common difficulty with solving the problems such as what described in (1), is the uncertainty of input parameters \((\Psi)\). There are some techniques to model these uncertainties such as stochastic programming (SP) \([27]\), fuzzy modeling \([28]\) and robust optimization (RO) \([29]\). Each technique needs some information about the uncertain parameter. For example, in SP the decision maker needs to know the PDF of uncertain parameter; Also, it is a computationally expensive approach.

The fuzzy modeling needs the membership function of the parameter \((\Psi)\) \([30]\). The RO also needs an uncertainty set to describe the behavior of the uncertain parameter \((\Psi)\). It is also incapable of modeling OS strategy. This justify the need for a powerful tool that would be able to overcome these shortcomings. In IGDT literature there are various types of uncertainty sets \([19]\) such as:

- **Energy-bound model**: This kind of uncertainty sets refer to phenomena which deviate from its nominal value in a transient manner.

  \[
  U(\alpha, \overline{\Psi}) = \left\{ \Psi(t) : \int_0^\infty |\Psi(t) - \overline{\Psi}(t)|^2 dt \leq \alpha^2 \right\}
  \]  
  \[(2)\]

- **Slope-bound model**: This kind of uncertainty sets refer to phenomena which the rate of deviation from its nominal value is constrained by a radius of uncertainty. It indicates the distance between what is known \((\overline{\Psi})\) and what is unknown \((\Psi)\).

  \[
  U(\alpha, \overline{\Psi}) = \left\{ \Psi(t) : \left| \frac{d(\Psi(t) - \overline{\Psi}(t))}{dt} \right| \leq \alpha \phi(t) \right\}
  \]  
  \[(3)\]

- **Envelope-bound model**

  \[
  U(\alpha, \overline{\Psi}) = \left\{ \Psi(t) : |\Psi(t) - \overline{\Psi}(t)| \leq \alpha |\overline{\Psi}(t)| \right\}
  \]  
  \[(4)\]

\(\phi(t)\) is a known function which dictates the shape of the envelope and the radius of uncertainty \((\alpha)\) determines the size of the envelope which is also uncertain.

In this work a special case of Envelope bound model is used where \(\phi(t)\) is equal to \(\overline{\Psi}(t)\) as given in (5):

\[
U(\alpha, \overline{\Psi}) = \left\{ \Psi : |\Psi(t) - \overline{\Psi}(t)| \leq \alpha |\overline{\Psi}(t)| \right\}
\]  
\[(5)\]

The forecasted value of input parameter \((\overline{\Psi})\) is assumed to be known. This is the only information available regarding the uncertain parameter \((\Psi)\).

The IGDT technique has two forms as follows:

- RA strategy
- OS strategy
III. UC Problem Formulation

This paper explores an approach in which the impact of demand side flexibility on the UC is investigated in the presence of uncertain wind power generation. In the studied UC problem, the network DC power flow constraints along with line flow and angle limits are considered. This UC model is adopted from [31] and is briefly expressed as follows. The objective function to be minimized, is defined as the total cost paid for energy balance and is calculated as follows:

\[ TC = \sum_{g,t} [v_{g,t} FC_g + z_{g,t} SDC_g + VC_{g,t} + y_{g,t} SU_{g,t}] + \sum_{t,b} DRC_{t,b} \] (6)

Variable costs of power generation can be approximated by a piecewise linear curve using binary variables as stated in [31]. As it is mentioned in [31], the start-up cost is a nonlinear function of the number of hours a unit has been off, which can be discretized. Hence, in this paper the start-up cost function and the the minimum up/down time constraints are considered based on the model given by [31]. Also, the following constraints are considered for maximum and minimum power generation limits of thermal units [31].

\[ \forall g \in \Omega_G \text{ and } \forall t \in \Omega_T: \]

\[ P_{g,t} \leq P_{g}^{\text{max}} (v_{g,t} - z_{g,t+1}) + z_{g,t+1} SD_g, \] (7)

\[ P_{g,t} \leq P_{g,t-1} + RU_g v_{g,t-1} + SU_g y_{g,t}, \] (8)

\[ P_{g,t} \geq 0, \] (9)

\[ P_{g}^{\text{min}} v_{g,t} \leq P_{g,t} \leq P_{g,t}, \] (10)

\[ P_{g,t-1} - P_{g,t} \leq RD_g v_{g,t} + SD_g z_{g,t} \] (11)

The DC power flow equations are as follows (\( \forall t \in \Omega_T, \forall b \in \Omega_B \)):

\[ \sum_{g \in \Omega_{G_b}} P_{g,t} + I_{b,t} W_{b,t} - P_{b,t}^{L} = \sum_{j \in \Omega_{B_b}} B_{bj} (\theta_{b,t} - \theta_{j,t}) \] (12)

Besides, the following limits are considered for line flows and angles.

\[ \theta_b^{\text{min}} \leq \theta_{b,t} \leq \theta_b^{\text{max}} \quad \forall b \in \Omega_B, \forall t \in \Omega_T \] (13)

\[ P_{l,t} = B_{bj} (\theta_{b,t} - \theta_{j,t}) \quad \forall l \in \Omega_L, \forall t \in \Omega_T \] (14)

\[ -P_{l}^{\text{max}} \leq P_{l,t} \leq P_{l}^{\text{max}} \forall l \in \Omega_L, \forall t \in \Omega_T \] (15)
Also, the DR constraints $\forall b \in \Omega_{DR}, \forall t \in \Omega_T$ are as follows. The original value of demand in bus $b$ at time $t$ is known and denoted as $(P^L_{b,t})$. If a node is participating in DR program, then this value can be changed (it is flexible). The new value of demand after the DR activation would become $(P^L_{b,t})$ according to (16).

$$P^L_{b,t} = P^L_{b,t} \times \gamma_{b,t}, \forall b \in \Omega_{DR}, \forall t \in \Omega_T$$ (16)

The value of $\gamma_{b,t}$ demonstrates the amount of change in demand compared to the original value $(P^L_{b,t})$. The increase and decrease in demand are limited by demand flexibility as forced by (17). $I^D_b$ is a binary parameter in (17) which indicates the participation ($I^D_b = 1$) and non-participation ($I^D_b = 0$) of node $b$ in demand response program. The $\gamma_{b,t}^{max}$ and $\gamma_{b,t}^{min}$ are the upper and lower flexibility limits of demand in bus $b$ and time $t$, respectively.

$$1 - \gamma_{b,t}^{min} I^D_b \leq \gamma_{b,t} \leq 1 + \gamma_{b,t}^{max} I^D_b, \forall b \in \Omega_{DR}, \forall t \in \Omega_T$$ (17)

In realistic applications, the maximum rate of change in demand for consecutive time periods is limited. These limits are modeled as follows:

$$P^L_{b,t} - P^L_{b,t-1} \leq D^u_{b,t}, \forall b \in \Omega_B, \forall t \in \Omega_T$$ (18)

$$P^L_{b,t-1} - P^L_{b,t} \leq D^d_{b,t}, \forall b \in \Omega_B, \forall t \in \Omega_T$$ (19)

$D^u_{b,t}$ and $D^d_{b,t}$ specify the ramp-up/down of demand in bus $b$, respectively. The total energy of demand in each bus should be kept constant during the operating horizon. This is achieved as follows:

$$\sum_t P^L_{b,t} \Delta_t = \sum_t P^L_{b,t} \Delta_t, \forall b \in \Omega_B, \forall t \in \Omega_T$$ (20)

The demand response cost is calculated as follows:

$$\zeta_{b,t}(P^L_{b,t} - P^L_{b,t}) \leq DR C_{t,b}, \forall b \in \Omega_{DR}, \forall t \in \Omega_T$$ (21)

$$\zeta_{b,t}(-P^L_{b,t} + P^L_{b,t}) \leq DR C_{t,b}, \forall b \in \Omega_{DR}, \forall t \in \Omega_T$$ (22)

In this paper, it is assumed that wind power generation is subject to severe uncertainty. This means that there is no probability density function, bounded known uncertainty set or membership function is available. The only available information is the wind forecast ($\bar{P}^W_{b,t}$). The uncertainty set described in (23) is the only reasonable assumption that the decision maker can make. This uncertainty set (in contrary to robust optimization technique) has one unknown variable ($\alpha$) which is also called “radius of uncertainty”. This radius of uncertainty specifies how much is the gap between what is
known \( \bar{P}_{b,t}^W \) and what is unknown and uncertain \( P_{b,t}^W \). Sometimes the decision maker needs to minimize this gap and sometimes it is desirable to maximize it.

\[
U(\alpha, \bar{P}_{b,t}^W) = \{ P_{b,t}^W : |P_{b,t}^W - \bar{P}_{b,t}^W| \leq \alpha \bar{P}_{b,t}^W \} \tag{23}
\]

The uncertain wind power generation \( P_{b,t}^W \) can be separated into two components: the actual injected wind power to the grid \( \tilde{P}_{b,t}^W \) (which is revealed after the actual realization of the uncertain wind power generation) and the curtailed wind power \( P_{C_w}^W \) as given in (24). The actual injected and curtailed wind powers cannot exceed the realized actual wind generation and similarly the realized actual wind generation cannot exceed the installed wind farm’s capacity at bus \( b \) \( (\Lambda_b^W) \) as given in (25) and (26).

\[
P_{b,t}^W = \tilde{P}_{b,t}^W + P_{C_w}^W \tag{24}
\]

\[
0 \leq P_{C_w}^W \leq P_{b,t}^W \leq \Lambda_b^W \tag{25}
\]

\[
0 \leq \tilde{P}_{b,t}^W \leq P_{b,t}^W \leq \Lambda_b^W \tag{26}
\]

The relation between the predicted wind power \( \tilde{P}_{b,t}^W \) and the uncertain wind power \( P_{b,t}^W \) is described in (27).

\[
P_{b,t}^W = \tilde{P}_{b,t}^W + \Delta_{b,t}^{W+} - \Delta_{b,t}^{W-} \tag{27}
\]

These imbalances are non-negative numbers and cannot be positive at the same time as forced by (28) to (29).

\[
0 \leq \Delta_{b,t}^{W\pm} \tag{28}
\]

\[
0 = \Delta_{b,t}^{W-} \Delta_{b,t}^{W+} \tag{29}
\]

The positive and negative imbalances are limited by a upper bound as specified in IGDT uncertainty set \( U(\alpha, P_{b,t}^W) \) so the following relations hold:

\[
\Delta_{b,t}^{W\pm} \leq \tilde{P}_{b,t}^W \alpha \tag{30}
\]

The excessive wind generation (compared to the predicted one) is limited by the installed capacity of wind in bus \( b \) \( (\Lambda_b^W) \) (31).

\[
\Delta_{b,t}^{W+} \leq \Lambda_b^W - \tilde{P}_{b,t}^W \tag{31}
\]

IV. IMPLEMENTATION OF IGDT ON THE UC PROBLEM

In this work, two strategies have been analyzed, namely RA IGDT-based UC and OS IGDT-based UC.
A. RA IGDT-based UC

In this strategy, the decision maker tries to increase the robustness of objective function (operating costs) against the wind power generation uncertainty. The uncertainty is an undesired phenomenon in this case and it is associated with the values lower than the predicted ones. The following steps should be taken to obtain the RA strategy.

1) Step 1: Solve the base case ($\alpha = 0$):

$$\Gamma_b = \min_X TC$$

Subject to

(6) to (31)

The optimization in (32) finds the base cost ($\Gamma_b$). It is assumed that there is no uncertainty exists in the model so the radius of uncertainty would become 0 and the hourly wind power generation would be the same as its predicted value.

2) Step 2: The decision maker specifies the $\beta_c$ and solves the following bi-level optimization problem:

$$R_c = \max_X \hat{\alpha}(X, \bar{P}_b^W)$$

$$\hat{\alpha}(X, \bar{P}_b^W) = \max_\alpha \left\{ \alpha : \max_{P_W^b \in U(\alpha, \bar{P}_b^W)} TC \leq \Delta_c \right\}$$

Subject to

$$\left\{ \begin{array}{l}
\Delta_c = (1 + \beta_c)\Gamma_b \\
(6) \text{ to } (31)
\end{array} \right.$$ 

The optimization problem described in (33b) is in the lower level of the bi-level problem. Fortunately, it can be solved for a fixed value of $X$ and then the solution is passed to the upper level. The decision variable in (33b) is $P_W^b$. The wind value that would maximize the $TC$, is in the uncertainty set $U(\alpha, \bar{P}_b^W)$ described in (23).

3) Step 3: Finish.

The decision variables ($X$), parameters ($\Psi$) and the sets are as follows:

$$X = \left\{ P_{g,t}^c, P_{c,w}^c, \bar{P}_b^W, \theta_{b,t}, P_{i,t}, P_{l,t} \right\}$$

$$\Gamma_b, \gamma_{b,t}, \Delta_{b,t}^\pm, \alpha$$

(34)

$$\Psi = \left\{ \beta_{c/o}, \Delta_{c/o}, I_b^W, \Lambda_b^W, P_{b,t}^W, \bar{P}_{b,t}^W, F_{g}^{\max/\min}, R_{U_g}, R_{D_g}, D_{b,t}^{\max/\min}, \lambda_b^D, B_{b,t}, \delta_{b,t}^{\max/\min}, P_{l,t}^{\max}, P_{L_0,b,t}^{\max/\min}, \bar{\gamma}_b^{\max/\min} \right\}$$

(35)

$$Set = \{ \Omega_{DR}, \Omega_T, \Omega_B, \Omega_L, \Omega_G, \Omega_{Gb} \}$$

(36)
It is interesting to know what the cost of robustness is. It is defined as follows: The decision maker tries to find the optimal values of $X_{\beta_c}$ based on the given $\beta_c$. The maximum obtained radius of uncertainty is found $R_c(\beta_c)$. What is the cost of applying $X_{\beta_c}$ if no uncertainty happens in reality? It is defined as follows:

$$Cost_r = TC(X_{\beta_c}, \alpha = 0) - \Gamma_b$$  \hfill (37)

The value of $Cost_r$ is called the cost of robustness.

B. OS IGDT-based UC

In this strategy, the decision maker is optimistic about the uncertain parameters. In other words, the uncertainty in wind power generation may act in a positive direction to reduce total costs compared to the base case. The OS IGDT tries to increase this chance. The uncertainty is a desired phenomenon in this case and it is associated with higher wind power values than the predicted ones. The following steps should be carried out to obtain the OS strategy.

1) Step 1: Solve the base case: In this step (32) is solve.

2) Step 2: The decision maker specifies the $\beta_o$ and solves the following bi-level optimization problem:

$$R_o = \min_X \alpha(X, \bar{P}_{b,t}^W)$$ \hfill (38a)

$$\alpha(X, \bar{P}_{b,t}^W) = \min_\alpha \left\{ \alpha : \min_{P_{b,t}^W \in U(\alpha, \bar{P}_{b,t}^W)} TC \leq \Delta_o \right\}$$ \hfill (38b)

Subject to \hfill (6) to (31)

The optimization problem described in (38b) is in the lower level of the bi-level problem. It can be solved for a fixed value of $X$ and then the solution is passed to the upper level. The decision variable in (38b) is $P_{b,t}^W$. The wind value that would maximize the $TC$, is in the following uncertainty set $U(\alpha, \bar{P}_{b,t}^W)$ as described in (23).

3) Step 3: Finish.

The decision maker tries to find the optimal values of $X_{\beta_o}$ based on the given $\beta_o$. The minimum obtained radius of uncertainty is found $R_o(\beta_o)$. What is the cost of applying $X_{\beta_o}$ if no uncertainty happens in reality? It is calculated as follows:

$$Cost_o = TC(X_{\beta_o}, \alpha = 0) - \Gamma_b$$ \hfill (39)

The value of $Cost_o$ is called the cost of opportuneness.
V. Simulation Results

A. Data

The proposed IGDT-based model for the UC is examined on the IEEE 118-bus system [32]. The mathematical model of UC is implemented in General Algebraic Modelling System (GAMS) [33] environment and solved by CPLEX solver [34] running on an Intel Xeon CPU E5-1620 3.6 GHz PC with 8 GB RAM. The peak value of demand in this system is 5880 MW and the 24 hours load curve of the system is given in Fig. 3. Also, it is assumed that 1650 MW wind power generation capacity is installed in this system. Table II gives the capacity and location of each wind farm in the system. It is also shown in Fig. 4 [35]. Besides, it is assumed that all wind farms are available in the entire horizon (i.e. \( I_{w,b,t}^B = 1, \forall b \in \Omega_B \)). The forecasted demand and wind power generation profiles are also depicted in Fig. 3 [36]. Moreover, it is assumed that all buses could participate in DR program (i.e. \( I_{b,t}^D = 1, \forall t, b \)). The demand response cost \( \zeta_{b,t} \) can be different for every bus \( b \) and time period \( t \). However for simplicity is is assumed to be 0.1 $/MW h \forall t, b$ [37]. In the following, two cases RA and OS strategies are studied.

B. RA Strategy

First, for a load flexibility of 10% the RA case is solved. In this case, the decision variables \( \mathbf{X} \) are optimally found in order to increase the robustness of the objective function. The base case cost (which is the same for both RA and OS strategies), is obtained as \( \Gamma_b = \$1120720.05 \), for the forecasted values of wind power generations. It is assumed that the tolerable deterioration in the base case TC (i.e. \( \beta_c \)) is 5%. Hence, the system operator is willing to find the maximum amount of actual wind power generation lack, to ensure that the TC will not increase beyond the

\[
\Delta_c = (1 + 0.05) \times \Gamma_b = \$1176756.05
\]

The variation of TC versus the robustness radius \( R_c \) is depicted in Fig. 5. It is observed from this figure that in RA case, maximum radius of uncertainty in wind power prediction (i.e. \( R_c \)) is 0.1214 (or 12.14%). In this case, the variation of participation from different energy procurement options versus robustness parameter, \( \beta_c \), are illustrated in Fig. 6. It could be observed from this figure that in RA strategy, by increasing \( \beta_c \), the share of wind power generation decreases, whereas contrarily thermal generations share increases to compensate the wind power deficiency.

C. OS Strategy

In this case, the opportuneness parameter \( \beta_o \) is set to be 5%. Hence, the variation of TC versus opportuneness radius \( R_o \) is depicted in Fig 5. It is observed from this figure that by increasing \( R_o \),
the TC decreases, and the maximum value of $R_o$ is obtained to be $0.2291$ (or $22.91\%$) with the minimum TC equal to $\Delta_o = (1 - 0.05) \times \Gamma_o = $1064684.05.

In this case, the variation of wind and thermal generation participation versus opportuneness parameter $\beta_o$ are illustrated in Fig. 6. It is observed from this figure that by increasing $\beta_o$, the participation of wind farms in energy procurement increase, while contrarily, the share of thermal generation units decreases. It is assumed that the decision maker will use the UC settings (i.e. the on/off states of thermal generation units) as the first stage decision variables. The second stage decision variables (i.e. schedule of thermal generation units) before and after the materialization of the uncertainty for both RA and OS strategies are depicted in Fig.7 and Fig.8, respectively. In order to materialize the uncertainty, it is assumed that in RA strategy an arbitrary wind power generation scenario is realized in the interval $[\bar{P}_{b,t}^W (1 - R_c), \bar{P}_{b,t}^W]$. For this realized scenario, the value of actual power generation of thermal units are depicted in Fig. 7, which is compared with the corresponding schedule obtained in the RA for the robustness radius of $R_c$. Similarly, in the OS case, an arbitrary realized wind power generation scenario is assumed in the interval $[\bar{P}_{b,t}^W (1 + R_o), \Lambda_b^W]$. Figure 8 gives the actual values of thermal units’ power production along with their corresponding scheduled values for the opportuneness radius, i.e. $R_o$. It is observed that in the RA case the deployment of expensive but agile technologies is occurred, while in the OS strategy the utilization of cheap technologies is favored.

D. Sensitivity analysis on the DR flexibility

In order to evaluate the effect of the demand flexibility (in the context of DR program), a sensitivity analysis is performed. In this analysis, demand flexibility is increased from 0 to $20\%$, for a $5\%$ tolerable increase (i.e. $\beta_c$) and desired decrease (i.e. $\beta_o$) of base case TC, and its impact on both the uncertainty radius and base case cost is obtained which is depicted in Fig. 9.

It is observed from this figure that by increasing the share of DR, the amount of required additional wind power generation (or the radius of uncertainty) in OS strategy is decreased negligibly from $23.27\%$ to $22.71\%$, which means that the desired TC is obtained with lower level of actual wind power generation.

But, in the RA strategy maximum radius of uncertainty in wind power prediction reduced slightly from $12.22\%$ to $12.12\%$. This shows the fact that by increasing the share of DR, the tolerable TC in RA is obtained with higher radius of wind power generation uncertainty. This sensitivity analysis indicates the positive impact of demand flexibility in both RA and OS strategies, since by increasing the DR flexibility in RA strategy the radius of uncertainty increases eventually, while contrarily in the OS strategy it is decreased. Also, by increasing the flexibility of load, the base case cost is decreased, which shows the ability of DR for reducing the operation costs.
E. Verification of the IGDT performance

In order to check the robustness and opportuneness of the proposed approach, Monte Carlo Simulations (MCS) are conducted. At first, the RA/OS strategy performed for $\beta_{c/o} = 5\%$. Then 2000 set of wind power generation patterns (or scenarios) are generated in three different regions. These regions are defined as follows:

- Region $S_1$: $[\bar{P}_{b,t}^W (1 - R_c), \bar{P}_{b,t}^W]$.
- Region $S_2$: $[\bar{P}_{b,t}^W, \bar{P}_{b,t}^W (1 + R_o)]$.
- Region $S_3$: $[\bar{P}_{b,t}^W (1 + R_o), \Lambda^W_b]$.

The DR and UC settings obtained by RA and OS have been applied to these patterns and the operating costs are calculated. It is inferred from Fig. 11 and Fig. 12 that using the decision variables obtained in each strategy guarantees that operating cost remain less than $(\Delta_{c/o})$. This is only true when the wind pattern remains in robust/opportunistic region. The robust regions for RA strategy are $S_1, S_2, S_3$ while the OS strategy’s success is guaranteed only in $S_3$.

F. Comparison of IGDT with stochastic programming technique

This section analyzes the benefits of the proposed IGDT approach over existing stochastic programming models for dealing with severe uncertainties. It is assumed that the wind uncertainty is modeled using $n$ scenarios ($n = |\Omega_s| = 3, 5, 10, 15, 20$) as shown in Fig. 13. The maximum forecast error in all these scenarios is 12.14 $\%$ which is equal to $R_c$ at $\beta_c = 5\%$ in RA strategy.

The total CPU time, number of variables and required iterations for different number of scenarios are reflected in Table III. Also, the cost of each scenario for all five cases are shown in Fig. 14. According to the observations supported by Table III, some shortcomings of stochastic technique are listed as follows:

- The results highly depend on the scenarios describing the uncertain parameters.
- The expected value never happens in reality. Only one of the scenarios will realize in future (not the expected value).
- In order to accurately describe the uncertain parameter, the decision maker needs large number of scenarios. The computation burden highly increases with the number of scenarios. For example in the analyzed example, there are 10 wind locations. In the IGDT approach, the number of wind related variables are 10 (wind sites)*24 (hours) =240. However in Stochastic formulation this number is multiplied by the number of scenarios. Additionally, the network variables (voltage angles) are 118 in IGDT approach but in stochastic technique this will
become $118 \times |\Omega_s|$, where $|\Omega_s|$ is the number of scenarios. The number of operating decisions of thermal units will increase from $(54 \times 24)$ in IGDT to $(54 \times 24 \times |\Omega_s|)$ in stochastic technique.

- If the on/off states variables (first stage decision variables) obtained by the stochastic approach is applied to the $R_c$ level of uncertainty at $\beta_c = 5\%$, then the operating cost would be the last column of Table III ($Cost(R_c)$) which is bigger than the value obtained by IGDT (i.e. $1176756.05$, in RA strategy).

- The CPU time highly increases with the increase of scenario numbers which is not an issue in IGDT framework.

- The stochastic technique cannot provide a confidence level for decision maker regarding the operating cost.

VI. CONCLUSION

This paper presents an Information Gap Decision Theory based network constrained UC to find the robust schedule of thermal generation units, demand response decisions and network parameters. The uncertainty of wind power generation is modeled using IGDT as an efficient uncertainty handling technique, especially in severe uncertainties.

The proposed approach is investigated on the IEEE 118-bus system to demonstrate its applicability. The conclusions drawn from this work are listed as follows:

1) The proposed IGDT strategy is exact and the obtained results are reliable for decision makers.
2) The computational burden of both RA and OS strategies are the same as base case (i.e. the case with no uncertainty). This means that the proposed technique is not computationally expensive compared to other uncertainty handling tools, and it could be utilized in large-scale systems.
3) The proposed IGDT based approach is applicable even if no probability density function is available for wind power generation.
4) DR flexibility has positive impacts in both RA and OS strategies, i.e. by increasing the DR flexibility in RA strategy the radius of wind power uncertainty increases eventually, while contrarily in the OS strategy it is decreased.
5) For the same amount of tolerable increase or desired decrease in the total cost, wind power generation uncertainty radius in OS is greater than its corresponding value in the RA strategy.
6) The factors that influence the robustness and opportunness degrees in the proposed model are thermal units characteristics, DR location and flexibility level, transmission network topology and wind farm location and capacity.
REFERENCES


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Fig. 4 Transmission network under study

Fig. 5 The robustness and opportuneness vs cost target for $\gamma_{b,t}^{\max/\min} = 10\%$ in RA and OS strategies

Fig. 6 The ratios of wind and thermal units power generations to the base case vs $\beta_c/\beta_o$ (RA/OS)

Fig. 7 The thermal unit schedules before and after wind realization in RA strategy ($\beta_c = 5\%$)

Fig. 8 The thermal unit schedules before and after wind realization in OS strategy ($\beta_o = 5\%$)

Fig. 9 The robustness and opportuneness vs cost target for different flexibility levels

Fig. 10 Monte Carlo scenario regions for testing the IGDT performance

Fig. 11 The histogram of RA cost for different wind regions ($\beta_c = 5\%$ and $R_c = 0.121$)

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Fig. 13 Wind scenarios considered for describing the wind generation at wind nodes (stochastic technique)

Fig. 14 The total cost ($) versus scenario number (stochastic technique)
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Fig. 2. The Input-output model of IGDT technique.
Fig. 3. The hourly variations of demand and forecasted wind power generation values.

Fig. 4. Transmission network under study.
Fig. 5. The robustness and opportuneness vs cost target for $\gamma_{b,t}^{max/min} = 10\%$ in RA and OS strategies.

Fig. 6. The ratios of wind and thermal units power generations to the base case vs $\beta_c/\beta_o$ (RA/OS).
Fig. 7. The thermal unit schedules before and after wind realization in RA strategy ($\beta_c = 5\%$)

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Fig. 13. Wind scenarios considered for describing the wind generation at wind nodes (stochastic technique)

Fig. 14. The total cost ($) versus scenario number (stochastic technique)
TABLE I
THE COMPARISON BETWEEN THE EXISTING METHODS AND THE PROPOSED METHOD

<table>
<thead>
<tr>
<th>Reference</th>
<th>Uncertain phenomenon</th>
<th>Uncertainty modeling</th>
<th>Constraints</th>
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<tr>
<td></td>
<td>Demand</td>
<td>Price</td>
<td>Wind</td>
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</tr>
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<td>[39]</td>
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<tr>
<td>[44]</td>
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</tr>
</tbody>
</table>
Proposed method | Yes | Yes   | Yes  | Yes           | Yes       | Yes   | Yes| Yes                  | Yes   | Yes|

TABLE II
THE LOCATION AND CAPACITY (IN MW) OF THE INSTALLED WIND FARMS

<table>
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<th>Connection node (b)</th>
<th>Wind capacity ($\Lambda_W^{b}$)</th>
<th>Connection node (b)</th>
<th>Wind capacity ($\Lambda_W^{b}$)</th>
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TABLE III
THE STOCHASTIC UC-DR AND IGDT PERFORMANCE PARAMETERS

<table>
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<th>[Ωs]</th>
<th>CPU-time (s)</th>
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<th>Cost($R_c$) ($)</th>
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