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Authors(s)	Pakrashi, Vikram, Basu, Biswajit, O'Connor, Alan
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A Statistical Measure for Wavelet Based Singularity Detection

Vikram Pakrashi[1], Biswajit Basu[2] and Alan O' Connor[3]

[1] Design Engineer, Roughan & O' Donovan Ltd, Sandyford, Dublin, Ireland. E-mail: pakrashv@tcd.ie

[2] Associate Professor, Department of Civil, Structural and Environmental Engineering, Trinity College Dublin, Ireland (corresponding author) E-mail: basub@tcd.ie

[3] Senior Lecturer, Department of Civil, Structural and Environmental Engineering, Trinity College Dublin, Ireland. E-mail: alan.oconnor@tcd.ie

Abstract

This paper presents a statistical measure for the identification of the presence, the location and the calibration of the strength of singularity in a signal or in any of its derivatives in the presence of measurement noise without the requirement of a baseline using a wavelet based detection technique. For this proposed wavelet based detection of singularities present in a signal, the problem of false alarm and its significant reduction by use of multiple measurements is presented. The importance of the proposed measure on baseline and non-baseline damage calibration has been discussed from the aspect of structural health monitoring. The findings in the paper can also be used for cross-checking of background noise level in an observed signal. The detection of the existence, location and extent of an open crack from the first fundamental modeshape of a simply supported beam is presented as an example problem.

Keywords: Singularity Detection, Wavelet Analysis, False Alarm, Signal to Noise Ratio, Open Crack

1. Introduction

The detection of the location and the strengths of singularities in a measured signal or in any of its derivatives contaminated by noise form a central and key aspect in a range of fields including structural health monitoring, damage detection and assessment techniques [1], aerospace engineering [2], detection of sensor failure [3], biomedical engineering [4] and finance [5]. The detection process usually consists of three distinct phases – (i) detection of the existence of the singularity, (ii) the detection of the location of the singularity and (iii) the estimation or calibration of the strength of the singularity. Of these, the first two phases of detection are more closely dependent on each other and can be often simultaneously detected while the third phase of detection turns out to be a very challenging problem due to the presence of noise and the consequent partial masking of the effects of singularity in the measurement data. The presence of singularity in a signal or in any of its derivatives affects the signal only in the neighbourhood of its presence while keeping the global nature of the signal nearly undisturbed. Techniques like wavelet analysis have become very popular in recent times for the detection of these singularities within a system arising out of phenomenological causes since the wavelet analysis of such signals produce a local extremum at the location of a singularity and the absolute value of the wavelet transform coefficient at the location of detection can be related to the strength of the detected singularity [6]. Apart from the aspect of this phenomenological singularity induced much localized perturbation within a signal or in any of its derivatives, the presence of measurement noise also gives rise to local and weak strength singularities at various locations of the signal. As a consequence, aspects of measurement noise induced masking, non-detection and possibilities of false alarm for the detection techniques are extremely topical.

Among various detection techniques, the wavelet based detection process, especially in structural health monitoring, has been observed to perform very effectively due to the flexibility and the choice of using various scales and basis functions for analysis [7, 8, 9]. Although the detection of the location and the presence of a singularity using wavelet analysis are not dependent on a pre-existing baseline, the calibration or estimation of the strength of the singularity are very much dependent on baselines which are either obtained from previous experiments or from numerical simulations. The statement also holds true for cumulant based detection techniques measuring the local deviation of a signal from Gaussianity. The problem of singularity detection in a signal or in any of its derivative is an important problem in the field of structural health monitoring with most existing literature dealing with the example of the detection of an open crack in a beam – like structure. The wavelet based detection of an open crack in the space domain is popular in this regard [10, 11, 12, 13]. Although wavelet based detection technique using a modeshape or a deflected shape can successfully detect the existence and the location of an open crack in a beam without the requirement of a pre – existing undamaged (or with a known damage) baseline model, the estimation or the calibration of the extent of the damage does require a baseline model to be established [14, 15, 16]. Additionally, the wavelet based detection is often masked by local extrema of high magnitude due to the presence of noise within the signal [10]. Thus, there exist the problems of non – detection, where no significant extremum is formed or the strength of the singularity is incorrectly represented. These two types of non – detection correspond to the errors associated with the identification of the location of the singularity and the estimate of the strength of the singularity respectively. There is also the associated problem of false alarm where a significantly high local extremum can be randomly formed at a location within the signal where singularity is not present. A detailed and very interesting summary of general wavelet based structural damage detection can be found in the literature review presented by Taha et al. [17]. The detection of the location and the presence of singularities without the requirement of a baseline model can also be realized using measures of local deviation from Gaussianity [18, 19]. However, these measures suffer from the same problems as described for wavelet based singularity detection. Also, these measures are inferior to wavelet based methods in terms of calibration of the strength of singularities and are less flexible since the wavelet transform is carried out on a number of scales. Both wavelet based and local deviation from Gaussianity based methods have been successfully applied on the numerical model of a plate with an open crack as well [20,21,22]. Recent experimental advances have made dense measurements within the space domain possible and the aforementioned wavelet based detection method has been validated [16, 23, 24, 25,26].

The objective thus lies in proposing a measure where the location and the strength of the singularity within a signal can be successfully found without a baseline model even in the presence of noise. From a structural health monitoring aspect, the problem is tantamount to the identification of the location and the extent of an open crack in an example beam from the modeshape or the deflected shape in the presence of possible measurement noise. The usual spatial responses like the modeshapes or the damaged static or dynamic deflected shapes usually contain a singularity in their derivative due to the presence of crack which locally perturbs the stress, strain and displacement fields and these perturbations sharply decay beyond the neighbourhood of the crack tip [27]. The use of multiple measurements or observations in the space domain has been exploited in this regard and a statistical quality control like measure has been proposed for the non – baseline detection of the damage extent. The measure is based on the deviation of the mean values of the significant extrema of the wavelet transform coefficients of the damaged modeshape at various locations of the beam from the average value of all such significant extrema along the length of the beam. The first modeshape of a damaged beam with an open crack has been simulated in this paper for the detection of damage. The choice of the first modeshape for simulation is justified by the fact that it is comparatively easier to obtain the first natural modeshape in real cases than the higher modes. The aspects of false alarm and robust detection for single and multiple measurements of the damaged modeshape in the presence of background noise have been investigated using a statistical comparison of the median values of the significant extrema of the wavelet transform coefficients of the damaged modeshape along the length of the beam.

The proposed measure identifies the location and the extent of damage with and without baseline, which is a definite advantage over the methods usually adapted. The method proposed can also provide an idea regarding the background noise of the measurements for a calibrated structure. The findings in this paper are general, not limited to the example presented and are potentially applicable on a wide range of fields as indicated in the references provided in this section.

2. Theoretical Background

2.1 Wavelet Based Singularity Detection

For a wavelet with no more than m number of vanishing moments, it can be shown [6] that for very small values of scales in the domain of interest, the continuous wavelet transform of a function f(x) in the square integrable function space can be related to the m^{th} derivative of the signal. For any wavelet basis function ψ(x), this relationship can be expressed as

$$\lim_{s\to 0} \frac{Wf(b,s)}{s^{m+1/2}} \infty \frac{d^m f(x)}{dx^m} \tag{1}$$

where W(.) is the continuous wavelet transform of f(x) and b and s are the translation and the scale parameters respectively. Hence it is possible for a wavelet to detect singularities in a signal or its derivatives through the incorporation of a proper choice of basis function. The measure of the local regularity in the neighbourhood of a point in a function can be related to the local Lipschitz exponent around that point [6]. A function f(x) in the square integrable space is pointwise Lipschitz $\kappa \geq 0$ at a point ν if there exists a K>0 and a polynomial p_ν of degree $\breve{m}$ such that

$$\forall x \in \mathbb{R}, \left|f(x) - p_\nu(x)\right| \leq K\left|x - \nu\right|^\kappa \tag{2}$$

The term κ provides the degree of singularity in the neighbourhood of the point x. If the function f(x) is uniformly Lipschitz $\kappa < \breve{n}$ over an interval $[\breve{a}, \breve{b}]$, then there exists an $\breve{A}$ >0 such that

$$\forall(b,s) \in [\breve{a}, \breve{b}] \times \mathbb{R}^+,\ |Wf(b,s)| \leq \breve{A} s^{\kappa + \frac{1}{2}} (1 + \left|\frac{b - \nu}{s}\right|^\kappa) \tag{3}$$

where $\mathbb{R}$ and $\mathbb{R}^+$ are the domains of real and positive real numbers respectively.

Thus, the magnitude of the wavelet coefficients around a point can be related to the local Lipschitz exponent, and hence to the degree of singularity present at that point.

2.2 Simply Supported Beam with an Open Crack

The first modeshape of a simply supported beam of length 'L' and depth 'h' with an open crack of depth 'c' at a distance 'a' from the left hand support has been considered. The popular rotational spring model [28] for an open crack has been chosen for the purpose of numerical simulations. The results however are not model dependent since the choice of the model ensures that the modeshape contains a singularity in its derivative due to the presence of damage and other damage models can also be used [27, 28, 29]. The rotational crack model considers the cracked beam to be an assembly of two sub-beams joined by a rotational spring at the location of the damage assuming the effects of damage to be localized in its immediate neighbourhood whereby the change of global modal properties are not significant. Continuities in displacement, moment and shear are present at the location of the crack while a discontinuity for slope is present at that location and is given in terms of the non dimensional crack section flexibility θ [28] dependent on crack depth ratio (δ=c/h) as

$$\Phi_R{}'(a) - \Phi_{\bar{L}}{}'(a) = \theta L \Phi_R{}''(a) \tag{4}$$

where Φ represents the mode shape and the subscripts R and $\bar{L}$ represent the right and the left hand side of the crack respectively. Each prime represents a differentiation with respect to the spatial variable x which is the distance from the left hand support of the beam. The term θ is expressed as a polynomial of δ as

$$\theta = 6\pi\delta^2 (h/L)(0.5033 - 0.9022\delta + 3.412\delta^2 - 3.181\delta^3 + 5.793\delta^4) \tag{5}$$

The modeshape derived from the damage model contains singularity in its derivative at the damage location.

3. Discussions on Numerical Investigations

3.1 Simulation of Data

An example problem is presented for a simply supported beam with an open crack where the length of the beam is 1 m, while the cross sectional area (A), the depth (h) and the moment of inertia (I) of the square beam being 0.0001 m^2, 0.01 m and 8.33×10^{-10} m^4 respectively. The Young's modulus (E) and the density of the beam (ρ) are assumed to be 190×10^9 N/m^2 and 7900 kg/m^3 respectively. The first fundamental modeshape, corrupted by noise (considered to be additive Gaussian white noise in this paper), is simulated for a number of times (100 times in this case) and each of the realization of the noisy modeshape is analyzed by Coif4 [30] wavelet basis function which has eight vanishing moments and is hence suitable for the detection of damage, if there be any. The modeshape data is premultiplied by a Hanning window of length equal to that of the modeshape data to reduce edge effects. The signal to noise ratio is kept approximately at 75 decibels. Applying window function to modeshape data as a preprocessing technique to significantly enhance the wavelet based singularity detection capability [11] has been observed before where a Hanning window was seen to be particularly useful. The edge effects relate to the very high valued coefficients near the edge of a non- windowed dataset. Since the points immediately beyond the support resemble a kind of discontinuity, a non-windowed data give rise to exceptionally high valued wavelet coefficients. This can mask the actual singularities when they exist within the zone of the high valued coefficients. The masking also takes place due to the order of magnitude of these edge coefficients that can render the actual singularity related extrema values unnoticed. The windowing smoothes out this edge effect by ascertaining a gradual transition at the ends. A plot of the damaged and the undamaged modeshape for a damage location 0.4m from the left hand support of the beam and a crack depth ratio (CDR) equal 0.35 is presented in Figure 1. Noise corruption is not shown. It is observed that even for a high damage, the change in the modeshape is extremely local and it is difficult to distinguish them from one another. For experimental damage scenario, the changes are usually higher as the theoretical models of open crack in a beam tend to represent the perturbation of strain in a very local fashion.

3.2 Discussions Related to the Comparison of Wavelet Calibration Medians

The existence of a significant wavelet coefficient extremum at a certain location for the analysed modeshape indicates the presence of damage at that location and the magnitude of the extremum forms the guiding factor for calibrating the extent of damage. This is not necessarily true when the measured signal is corrupted by noise since the noise itself contains many singularities. Thus there is a possibility of false alarm due to the presence of noise for an isolated measurement since there can be cases where a significant extremum forms at a location where there is no damage present. When multiple measurements of the damaged modeshape are possible, the concept of false alarm becomes less significant due to the fact that it is very difficult to obtain consistent and significant extrema values at a certain location of the signal after employing wavelet based damage detection since the source of variation of the location of such extrema is inherently random. As a result, it is expected that the probability this extremum value consistently occurring at one location or near one location due to noise effects is very small. Consequently the rate of false alarm rapidly decreases with the increase of the number of observations although for a single observation false alarm can exist. The presence of high background measurement noise might also mask an existing damage thus leading to a significant non – detectibility. This non – detection is different from false alarm since no significant extremum is formed. Thus, it might be so that the wavelet analysis fails to indicate a low strength singularity within the signal, thus resulting in high non – detection rate. Even then, for multiple observations, the possibility of false alarm can remain very small

Figure 2a illustrates the points discussed above. The boxplots of the significant maxima of the wavelet coefficients depicting the median, upper and lower quartiles and the extreme values are plotted for the simulated noisy modeshapes considering a non-overlapping ten point window sweeping across the length of the signal. The medians of the boxplot are qualitatively observed to be varying insignificantly although outliers do exist in each of the measurement clusters. These outliers, for isolated measurements form the basis of false alarm.

On the other hand, Figure 2b shows a similar boxplot under the same noise conditions for the beam with an open crack. The median value at the location of damage is seen to be significantly different than the neighbours, although for an isolated measurement there might be cases where the significant extremum is formed at some other location as well. A non-parametric Kruskal-Wallis test for the analysis of variance [31] to compare samples from multiple groups has been employed to compare the median values plotted in Figure 2b. Unlike a general analysis of variance method, the Kruskal-Wallis approach does not consider that the independently drawn samples all follow the same distribution. It was observed that on a 5% significance level, the null hypothesis that at least one group of measurements is significantly different from the rest is rejected for an undamaged case and accepted for a damaged case. For small number of measurements, when the undamaged background condition is available from measurements or from simulation, the significance level can be adjusted in terms of the variation of the median values for the undamaged case.

In practical situations, a number of parameters interconnected in a non – trivial way usually determine the choice of the number of points required for a successful detection. The chief contributors in this regard are the minimum size of the damage to be detected, the resolution of the measurement device, the sensitivity of the measurement device, the receiver operating characteristics (curves of the probability of detection versus the probability of false alarm) of the measurement technique, the signal to noise ratio and the location of damage. Consequently, the size of the damage that can be detected successfully depends on the location of the damage, the noise present in the measured signal and the resolution and the sensitivity of the measurement device. The number of points required to detect the damage decrease with the increase of damage size. Usually, a sufficiently dense representation of the measurement data is possible with devices like scanning laser vibrometer [16]. Exact cut-off numbers for data points should not be recommended for realistic situations. Rather, depending on the mechanical system, the minimum strength of singularity to be detected, the wavelet scale and the expected maximum level of noise within the signal, a conservative sampling interval should be used. Also, when a significantly higher number of observations are possible, the cut –off value of the number of points required is lower than its counterpart with low number of observations.

3.3 Discussions Related to the Comparison of Wavelet Calibration Means

An efficient damage detection and calibration method based on the means of the calibrated values of damage for multiple measurements of damaged modeshape and under noisy conditions is proposed next. The measure for the detection is defined as the deviation of the calibrated means of the damage at various groups of points along the length of the beam from the average of all the damage calibration values considered along the length of the beam. The grouping of the points is the same as Figure 1, i.e. a ten point non-overlapping window along the beam length. For a particular choice of a wavelet basis function and a scale, it can be known from numerical simulations whether the extremum formed at the location of damage present (if there be any) would be a maximum or a minimum. For the present case, the Coif4 wavelet basis function at scale 8 forms a maximum at the location of damage. Thus, any deviation on the negative side of the proposed damage detection measure can be safely ignored.

Figures 3a and 3b compare the proposed damage detection measure for undamaged and damaged cases respectively. The damaged condition is very easily identified. The presence and the location of damage does not need to be compared with an undamaged condition since the probability of a calibration average deviating more than twice of the standard deviation of the dataset is observed to be very small. Any significance test carried out along the lines of what has been shown in the previous section can distinguish between the undamaged and damaged conditions strongly as well. The approach closely resembles the idea of statistical quality control (SQC). Methods inspired by SQC have been applied to structural health monitoring in general successfully before [32]. Although some isolated measurements can possibly yield a significant extremum at a location when there is no damage, it is not consistent and thus the damaged condition can be picked up very easily and the idea of a false alarm in not required when sufficient measurements are available. For isolated measurements the concept of false alarms is still relevant.

Simulations were performed for the current problem on a large number of data (1000 realizations of additive Gaussian white noise for each damage location and extent) over a range of damage locations and extents and it was found that such false alarms are of the order 10^{-6} when considering that the proposed damage descriptor value at the location of damage (as in Figure 2) lie beyond five times the standard deviation of the entire series and of the order 10^{-3} when the limiting value is four times. The false alarm is thus directly related to how stringent the definition of damage descriptor is. It is also important to note here, that usually an extremely low false alarm rate also comes with a low detection rate for most detection methods, thus requiring the construction of receiver operating characteristic curves [33] to identify optimized measurement conditions. In fact, if the idea about the background noise corrupting the measurement is available even on an approximate basis, the proposed method can be used to calibrate the damage even without the need of a baseline. The distance of the damage descriptor in units of standard deviation of the pooled data series (referred to as One Sigma Level in Figure 2) would serve as an appropriate measure of damage extent calibration in those cases.

The same data, after the calibration of damage can be used for cross checking the average background noise level if the sensors are working correctly. This is a trivial by-product of the proposed method. The idea stems from the fact that a certain noise level would determine the one – sigma level bound of the data. Except for the local damaged regions, the rest of the length would always contribute to this one sigma level value dependent on the associated noise. If a baseline noise is measured or simulated beforehand to characterize the noise levels against the one sigma level wavelet coefficient values, then the approximate background noise can be quantified for an observation. When no baseline is present, the relative increase or the decrease of the corrupting noise can be found as a percentage change of the one sigma values. However, for pathological cases of simultaneous sensor failure and a major change in background noise level, or simultaneous sensor failure or dysfunction and the existence of a new damage location at the location of the failed or dysfunctional sensor, the proposed method cannot be used by itself. Numerical investigations, similar to what have been shown in this paper have been performed using different wavelet scales, different numbers of simulated noisy modeshape realizations, different number of data grouping along the length and have been applied to symmetric and asymmetric two-span beams with an open crack for various background noise levels. The findings of the paper were confirmed for all of these cases. The results are not spelled out in this paper to avoid repetition.

4. Conclusions

The problem of detecting the presence, location and the strength of singularity in a signal or in any of its derivatives in the presence of measurement noise was discussed in this paper. An example from the field of structural health monitoring was considered for illustration. Wavelet based damage detection and calibration method was employed on the first fundamental modeshape of the beam which consists of a damage induced singularity in its derivative. The problem of false alarm for isolated measurements and the significant reduction of such false alarm through a statistical comparison of median values of wavelet based damage calibration along the length of the beam were presented. A damage detection measure for multiple measurements based on the deviation of the wavelet based calibration means of damage at various groups of points along the length of the beam from the average of all the damage calibration values considered along the length of the beam has been proposed and shown to possess a definite potential to be employed for both baselined and non-baselined cases under certain conditions. In conjunction with other data, the proposed method was also observed to be important for acting as a double check for the approximate level of background noise. The findings hold good for any application involving the detection of singularity arising from some phenomenological cause.

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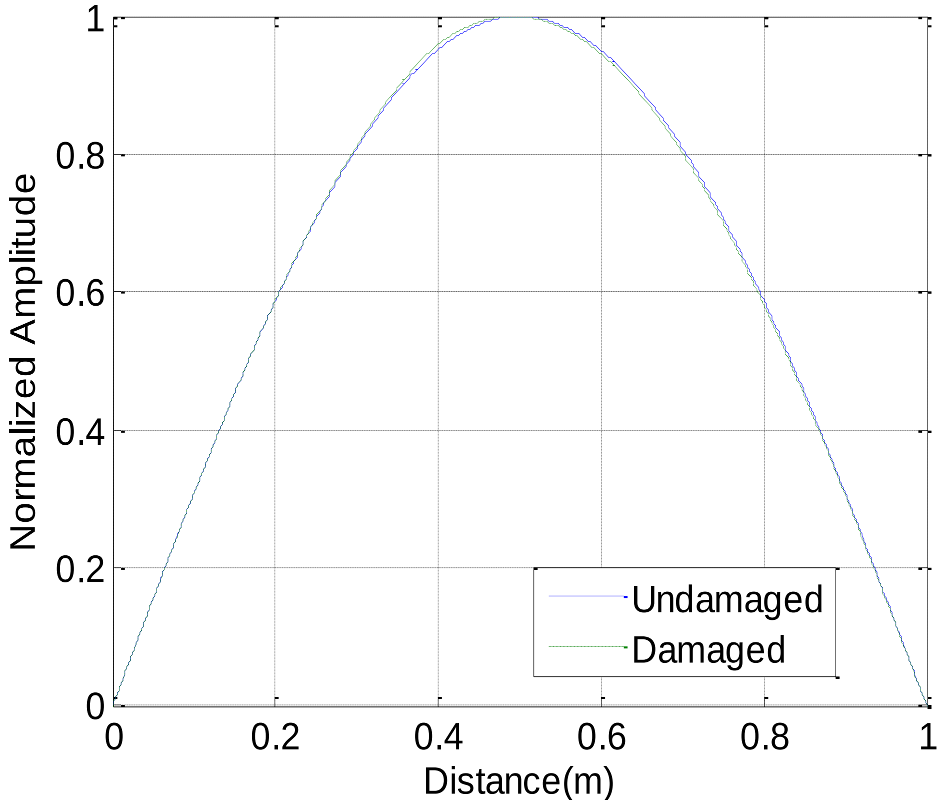
Figure 2a. Boxplot of wavelet based calibration values showing false alarm.

Figure 2b. Boxplot of wavelet based calibration values showing detected damage.

Figure 3a. Proposed detector values along the length for undamaged condition.

Figure 3b. Proposed detector values along the length for damaged conditions



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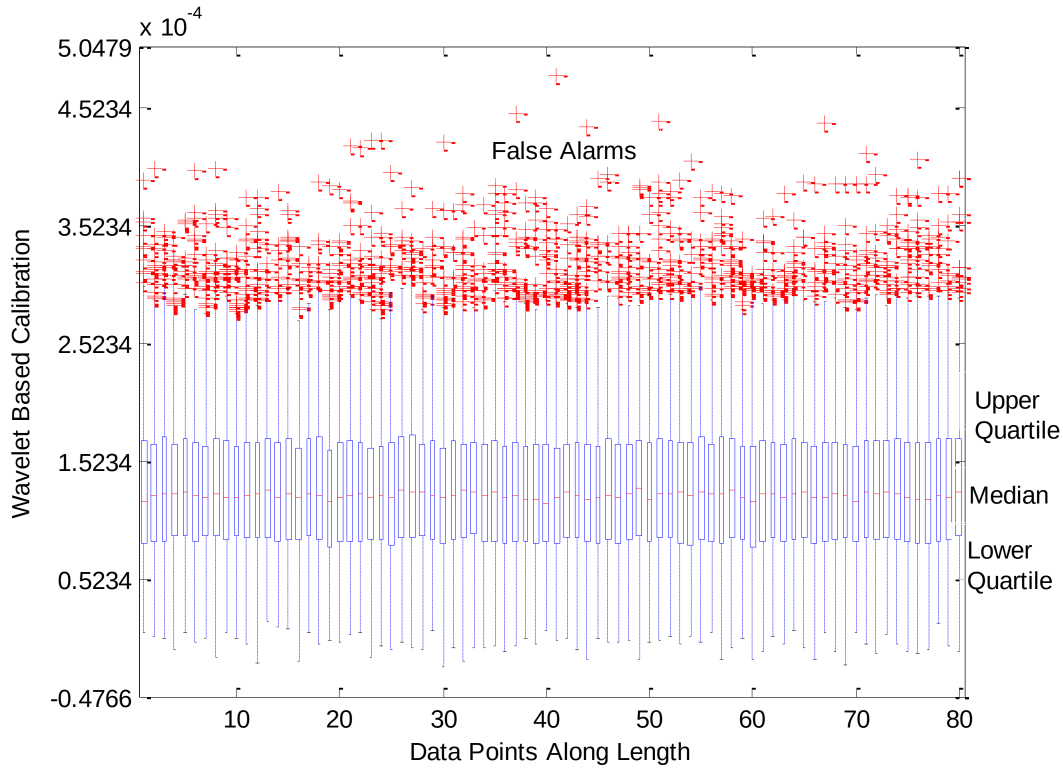

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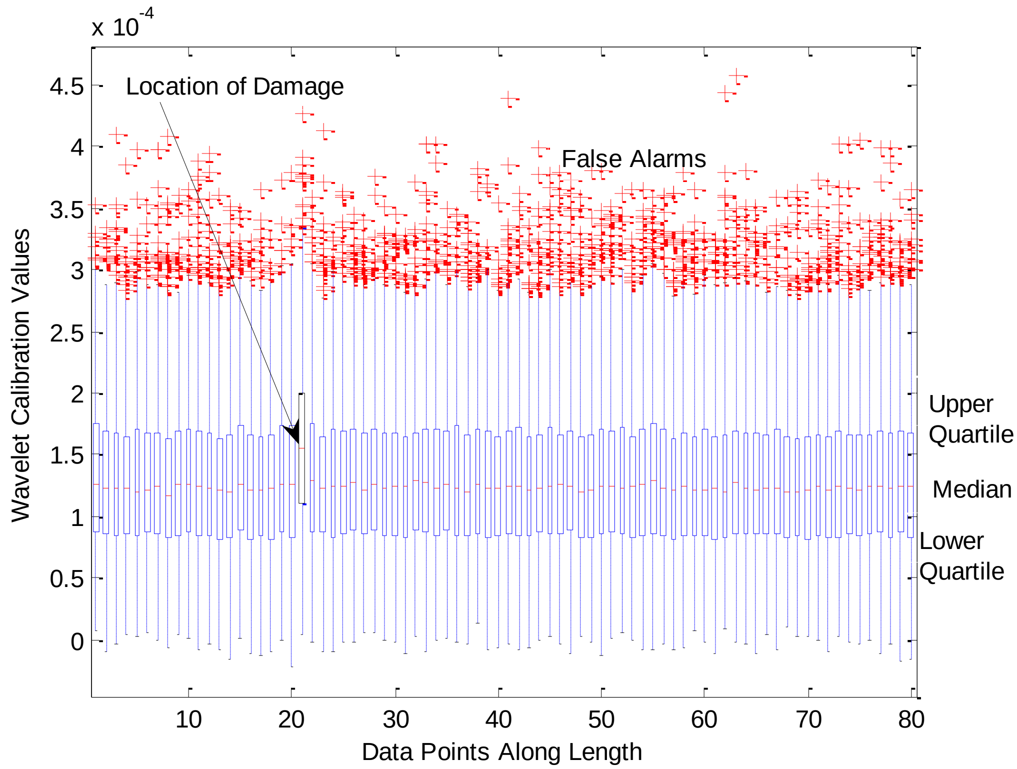

Figure 2b. Boxplot of wavelet based calibration values showing detected damage.

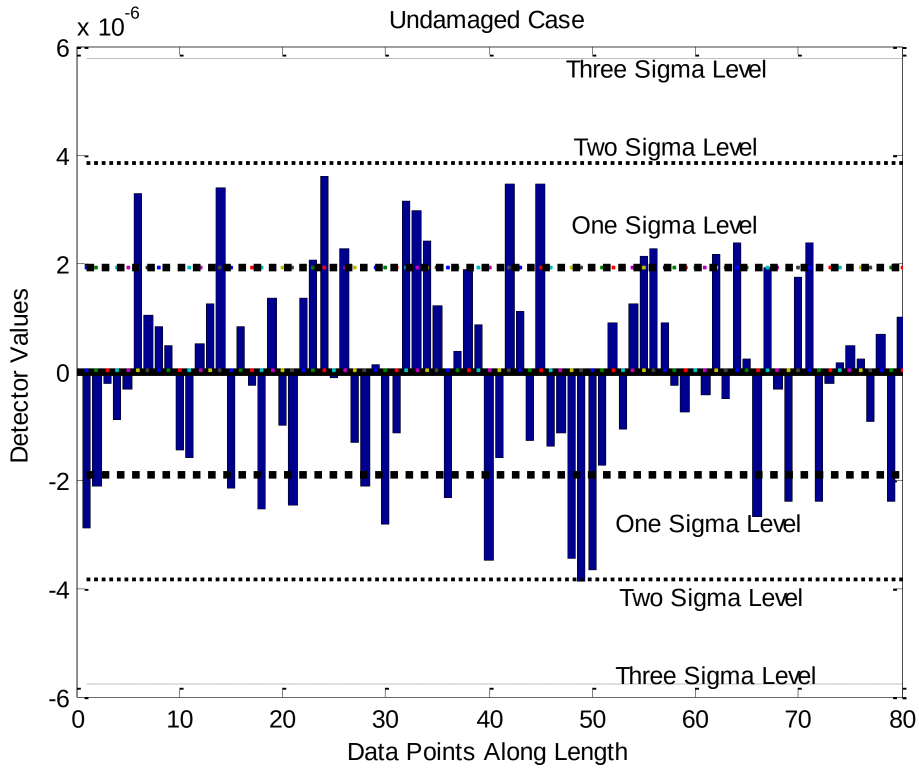

Figure 3a. Proposed detector values along the length for undamaged condition.

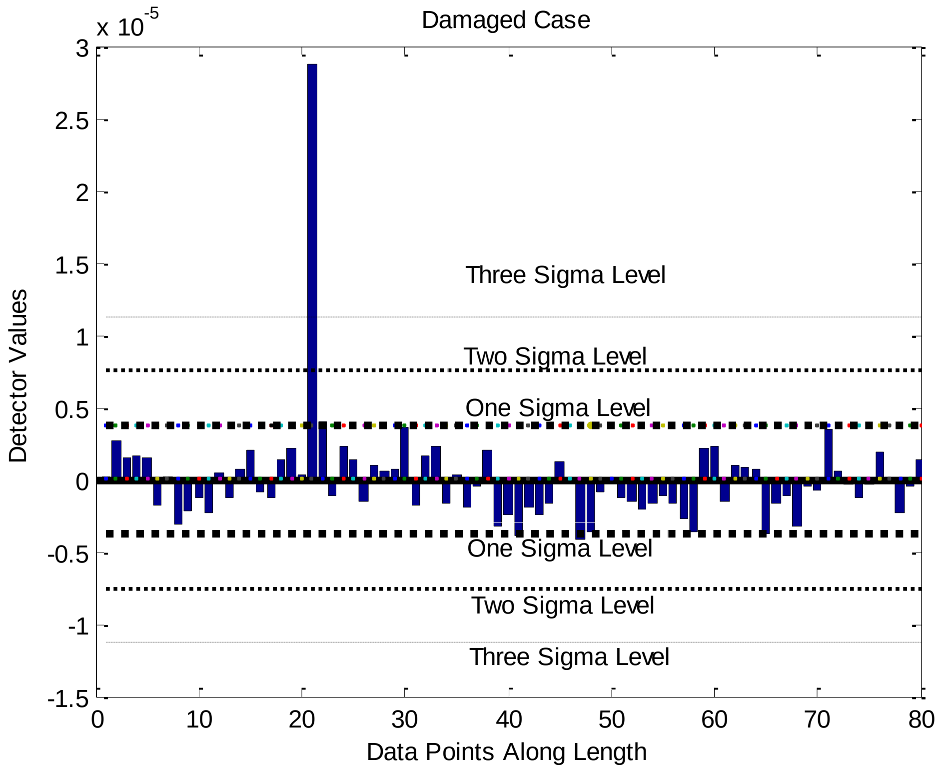

Figure 3b. Proposed detector values along the length for damaged conditions.