<table>
<thead>
<tr>
<th><strong>Title</strong></th>
<th>DOA Estimation for a Multi-Frequency Signal Using widely-spaced Sensors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Authors(s)</strong></td>
<td>Ballal, Tarig, Bleakley, Chris J.</td>
</tr>
<tr>
<td><strong>Publication date</strong></td>
<td>2010-08-27</td>
</tr>
<tr>
<td><strong>Conference details</strong></td>
<td>18th European Signal Processing Conference (EUSIPCO-2010), Aalborg, Denmark, 23 - 27 August, 2010</td>
</tr>
<tr>
<td><strong>Publisher</strong></td>
<td>IEEE</td>
</tr>
<tr>
<td><strong>Item record/more information</strong></td>
<td><a href="http://hdl.handle.net/10197/7035">http://hdl.handle.net/10197/7035</a></td>
</tr>
<tr>
<td><strong>Publisher's statement</strong></td>
<td>© © 2010 IEEE. Personal use of this material is permitted. Permission from IEEE must be obtained for all other uses, in any current or future media, including reprinting/republishing this material for advertising or promotional purposes, creating new collective works, for resale or redistribution to servers or lists, or reuse of any copyrighted component of this work in other works.</td>
</tr>
</tbody>
</table>

The UCD community has made this article openly available. Please share how this access benefits you. Your story matters! (@ucd_oa)

© Some rights reserved. For more information
DOA ESTIMATION FOR A MULTI-FREQUENCY SIGNAL USING WIDELY-SPACED SENSORS

Tarig Ballal and C. J. Bleakley

Complex & Adaptive Systems Laboratory, School of Computer Science & Informatics, University College Dublin
Belfield, Dublin 4, Ireland
phone: + (353) 1 716 2915, fax: + (353) 1 269 7262, email: {tarig.ballal, chris.bleakley}@ucd.ie

ABSTRACT

The estimation of sub-sample time-delay from the phase of the cross-power spectrum (CPS) of signals received by widely-spaced receivers requires unwrapped phase. Conventional phase unwrapping methods require a continuous CPS that starts at zero frequency or at a frequency with a known unwrapped phase. A novel phase unwrapping method is proposed herein that is capable of carrying out the task without these requirements. The proposed method is applied for direction-of-arrival (DOA) estimation of a bandpass signal, a case that conventional methods are unable to handle. Analytical performance and experimental results emphasize the effectiveness of the proposed method.

1. INTRODUCTION

A popular approach for DOA estimation is based on estimating the time-delay (or Time Difference Of Arrival (TDOA)) of the signal, as observed at a pair (or more) of spatially separated receivers, and then estimating the DOA by exploiting geometry (e.g., [1, 2, 3]). The most well known methods for time-delay estimation (TDE) are those based on the generalized cross-correlation (GCC) [4, 5]. In general, GCC based TDE consists of weighting the Cross-Power Spectrum (CPS) of the observed signals, and transforming the resulting spectrum to the time-domain. In the time-domain, the peak location is taken as the time-delay estimate, which is normally an integer number of samples.

In many cases, it is more convenient to estimate the time-delay directly from the phase of the CPS in the frequency-domain. The advantage of this is in obtaining sub-sample delay estimates without resorting to intra-sample interpolation [6]. Other advantages, including optimality, are discussed in [7, 8]. In spite of this, GCC based TDE has always been more popular than direct TDE from CPS phase. The main disadvantage of estimating time-delay directly from the CPS is the requirement that phase has to be unwrapped before it can be used for TDE [7, 6]. The occurrence of phase wrapping is common when the receivers are widely-spaced, i.e., the receiver separation exceeds \( \lambda_{\text{min}} / 2 \), half of the minimum wavelength of the impinging signal. In practice, wide spacing of receivers is required to enhance DOA resolution, reduce mutual coupling between receivers, or make the sensor placement physically realizable [9].

Phase Unwrapping is well known to be a difficult problem. Many phase unwrapping methods have been proposed in the literature (e.g., [2, 10, 11, 12, 13]). In general, these methods rely on the fact that the signal spectrum is continuous, and that it starts at zero frequency (i.e., the wideband signal case) or some wrapping-free low frequency that can be used as the starting point of the unwrapping process. The phase is unwrapped progressively starting from the lowest frequency, leaving the success of the whole process dependent on the success of the unwrapping at the low frequencies.

In some cases in practice, even the lowest frequency is subject to phase wrapping. An example of this is a bandpass signal, received by receivers whose separation exceeds \( \lambda_{\text{max}} / 2 \), half of the maximum wavelength [9]. Conventional phase unwrapping methods cannot be used in this case.

In this paper, we report on a new phase unwrapping method that unwraps phase at each frequency independently (not progressively). The new method, does not require continuous phase or phase that starts from zero, and can be applied even when the phase of the lowest available frequency is uncertain. The proposed method exploits knowledge of the upper limit of the phase-frequency line slope (the maximum time-delay) to unwrap the CPS phases using a frequency pairing approach. The proposed method is capable of handling the degenerate case of the bandpass signal discussed above, which is the example considered in this paper. The proposed method can be generalized to handle the DOA spatial aliasing problem for a multi-frequency signal using non CPS based methods. Analytical performance formuale and experiments with ultrasonic signals show the effectiveness of the proposed method.

This paper is organized as follows. Section 2 describes the generic approach for DOA estimation from CPS phase. In Section 3, the proposed phase unwrapping method for two frequencies is explained. Section 4 studies the effect of noise on the proposed phase unwrapping method. In Section 5, the proposed unwrapping method is applied to DOA estimation. Experimental results are presented and discussed in Section 6, and Section 7 concludes the paper.

2. DOA ESTIMATION FROM CPS PHASE

Consider two signals received by a pair of spatially separated sensors:

\[
x_1[t] = s[t] + r_1[t]
\]

(1)

\[
x_2[t] = s[t - \tau] + r_2[t]
\]

(2)

where \( s[t] \) (\( t \) represents discrete-time) is a transmitted multi-frequency signal; \( r_1[t] \) and \( r_2[t] \) are two random noise processes that are assumed to be uncorrelated with each other or with the transmitted signal; and \( \tau \) is the delay or TDOA of \( s[t] \) between the two receivers. The CPS of \( x_1[t] \) and \( x_2[t] \) can be estimated by dividing each of \( x_1[t] \) and \( x_2[t] \) into a number \( N \) (possibly overlapping) frames and estimating the complex CPS from these frames according to [14]

\[
\hat{G}_{x_1x_2}[\omega] = \frac{1}{N} \sum_{n=0}^{N-1} X_{n_1}[\omega]X_{n_2}^*[\omega]
\]

(3)

where \( \omega \) is the radian frequency which is assumed to be discrete; \( X_{n_1}[\omega] \) and \( X_{n_2}[\omega] \) are the discrete Fourier transforms (DFTs) of the \( n \)-th frames of \( x_1[t] \) and \( x_2[t] \) respectively, each frame is multiplied by an appropriate window function; and \( * \) denotes the complex conjugate operation. The CPS \( \hat{G}_{x_1x_2}[\omega] \) can be related to that of the transmitted signal by [6]

\[
\hat{G}_{x_1x_2}[\omega] \approx \hat{G}_s[\omega]e^{j\omega \tau + \epsilon}
\]

(4)

where \( \hat{G}_s \) is an estimate of the real power spectrum of \( s[t] \), and \( \epsilon \) is a phase error due to the effect of noise, finite data record, etc.
Now, assume that there are $M$ frequencies $\omega_m, m = 0, \ldots, M - 1$ in the passband. The phase at each frequency can be estimated as

$$\hat{\phi}_m = \arg \left( G_{12,m}[\omega_m] \right) = \phi_m + \varepsilon_m$$

(5)

where $\arg(.)$ denotes the angle of a complex quantity. Considering phase wrapping, the phases in Eq. (5) can be expressed as

$$\hat{\phi}_m = [\phi^p_m + \varepsilon^p_m] + 2\pi k_m$$

(6)

where $k_m \in \mathbb{Z}$ are phase wrapping parameters; $[\phi^p_m + \varepsilon^p_m] \in [-\pi, \pi]$ are noisy principal phase components with $\phi^p_m$ being the true principal phase and $\varepsilon^p_m$ representing the contributions of noise and estimation errors. Now, assume that there are $3$ phases, the following inequality must be satisfied:

$$|\phi_{[k_m]}| \leq 2\pi \left( \frac{\omega_b}{\omega_b - \omega_c} \right) > \omega_b d \frac{d}{c}, \forall q$$

(15)

where $k_m$ is a general integer variable whose true value ($k_m$) is being sought. Hence, the success/failure of the above condition can be written as

$$|\omega_b - \omega_c| < \pi d \frac{c}{2}$$

(17)

Eq. (17) is the end of the proof of Theorem 1. In the following, we show how the condition in (17) can be exploited to identify the true value of $\phi_0$. The same logic can be applied to $\phi^p$, or otherwise, the true value of $\phi^p$ can be directly determined from the true value of $\phi_0$ using (11). By inspecting Eq. (13) in light of the condition in Eq. (17), we obtain

$$|\phi_{[k]}| > \frac{d}{c} \left| \hat{\phi}_m \right| \frac{1}{\pi}.$$

(18)

It can be deduced from (18) that there are only three possible values for $k$ that one of them is anticipated to yield a valid phase (i.e., $|\phi_{[k]}| < \omega_b d / c$). Literally, the search has to be confined to the subset $\{1, 0, 1\}$. The true phase can simply be identified as the minimum of a triplet.

To recover the true noisy phase $\phi_0 \equiv \phi_{[k_m]}$ in a general case where only noisy versions of the principal phases are available (as in Eqs. (9) and (10)), and when (17) is satisfied, the following unwrapping algorithm can be used:

I. Set $\phi_{[k]} = \hat{\phi}_m$.

II. $\phi_0$ is the true value of $\phi_0$.

IV. $\phi^p = \phi_{[k_m]}$.

The objective of the last two steps is to absorb the effect of noise in the integer value representing $k_m$. The success/failure of the above algorithm will be discussed in the following section.

4. NOISE EFFECT

In this section, the effect of noise on the performance of the proposed phase unwrapping algorithm is considered. By considering
the noise terms in Eqs. (9) and (10), one can use the same procedure used to obtain (13), to obtain the noisy version of the same equation as

$$
\phi_u[k] = \left[ \frac{\omega_u}{\omega_u - \omega_o} \right] + [\theta_u^p - \phi_u^p + 2\pi k] + \epsilon_{uv}
$$

(19)

where \(\epsilon_{uv} = \omega_u (\epsilon_u^p - \epsilon_o^p) / (\omega_u - \omega_o)\) and represents the total effect of noise. For the purpose of analyzing the performance under noise, it will be assumed, without loss of generality, that \(\epsilon_u^p\) and \(\epsilon_o^p\) are two zero-mean Gaussian random variables with variances \(\sigma_u^2\) and \(\sigma_o^2\), respectively. Hence, \(\epsilon_{uv}\) will be a zero-mean Gaussian random variable with a variance given by

$$
\sigma_{uv}^2 = \left[ \frac{\omega_u}{\omega_u - \omega_o} \right] \left[ \sigma_u^2 + \sigma_o^2 \right]
$$

(20)

Noise can have the effect that the algorithm produces an erroneous (integer) values for \(k_{uv}\) and/or \(k_o\), a situation which we refer to as failure, while producing the correct integer is referred to as success. First consider step II of the proposed algorithm, where a selection between three phase values is made. Each of these three phase values corresponds to an integer; the integer corresponding to the correct selection is \(k_{uv}\), the other two integers corresponding to the false phases will be denoted as \(k_1\) and \(k_2\). The probability that step II will produce the correct answer—under noise—can therefore be written as

$$
P_s,m = P \left( |\phi_u[k_{uv}]| < |\phi_u[k_1]| \text{ and } |\phi_u[k_{uv}]| < |\phi_u[k_2]| \right)
$$

$$
= P \left( |\phi_u[k_{uv}]| + \epsilon_{uv} < |\phi_u[k_1]| + \epsilon_{uv} \right)
$$

$$
\quad \text{and} \quad P \left( |\phi_u[k_{uv}]| + \epsilon_{uv} < |\phi_u[k_2]| + \epsilon_{uv} \right)
$$

(21)

where \(\phi_u[k_{uv}], \phi_u[k_1]\) and \(\phi_u[k_2]\) are the noise-free candidate phases. The probability \(P_s,m\) depends on the actual values of these phases. A procedure for evaluating \(P_s,m\) from the cumulative distribution function (CDF) of the random variable \(\epsilon_{uv}\) is described in Appendix A.

Now, the probability of success of step III (in restoring the correct integer \(k_u\) under noise) given the success of step II can be expressed as the probability that \(\hat{k}_u = k_u\) when \(\phi_u = \phi_u[k_{uv}] + \epsilon_{uv}\). That can be stated as

$$
P_{s,r} = P \left\{ \left| \phi_u[k_{uv}] + \epsilon_{uv} - (\phi_u^p + \epsilon_u^p) \right| / 2\pi = k_u \right\}
$$

(22)

By substituting for \((\phi_u[k_{uv}] - \phi_u^p) / 2\pi = k_u\) and manipulating, Eq.(22) reduces to

$$
P_{s,r} = P \left\{ -0.5 \leq \frac{\epsilon_{uv} - \epsilon_u^p}{2\pi} < 0.5 \right\}
$$

$$
= P \left\{ -\pi \leq \epsilon_{uv} - \epsilon_u^p < \pi \right\}
$$

$$
= \Phi_{\epsilon_{uv} - \epsilon_u^p}(\pi) - \Phi_{\epsilon_{uv} - \epsilon_u^p}(-\pi)
$$

(23)

where \(\Phi_{\epsilon_{uv} - \epsilon_u^p}\) is the CDF of the random variable \(\epsilon_{uv} - \epsilon_u^p\), which has a Gaussian distribution with a zero mean and a variance equal to \(\sigma_{uv}^2 + \sigma_u^2\).

Now, the probability of success for the whole algorithm presented in Section 3 will be given by

$$
P_s = P_s,m P_{s,r}
$$

(24)

The probability \(P_s\) is directly a function of the variances \(\sigma_u^2\) and \(\sigma_{uv}^2\). In general, for large number of frames \((N)\), \(\epsilon_u^p\) and \(\epsilon_o^p\) are approximately Gaussian and \(\sigma_i^2, i = u, v\) can be approximated as \([6, 8]\)

$$
\sigma_i^2 \approx \frac{1 - |\gamma(\omega_i)|^2}{2N|\gamma(\omega_i)|^2}
$$

(25)

5. DOA ESTIMATION WITH PHASE UNWRAPPING

To estimate the DOA of a bandpass signal utilizing the CPS, each frequency in the passband is paired with all possible frequencies such that each pair satisfies (17). For each pair, the algorithm described in Section 3 is applied on the wrapped phases obtained from
6. EXPERIMENTAL RESULTS

The proposed approach for DOA estimation has been tested experimentally. The experiments were carried out in a normal office (see Fig. 2). Ultrasonic bandpass signals consisting of 20 equally-spaced frequencies in the range 35 - 49.5 kHz were used. The signals were constructed in a way that resembles a frequency hopping spread spectrum (FHSS) scheme [18], with a frequency duration of 3.2 ms. The sampling frequency was approximately 168 kHz. A frame size of 512 with 384 overlap between successive frames was used. A Fast Fourier Transform (FFT) was used to calculate the 512 length DFTs after windowing each frame by a Blackman window. The receivers were kept 2 cm apart (which is approximately 6 times \( \lambda_{\text{min}}/2 \) and 4 times \( \lambda_{\text{max}}/2 \), and approximately 170 cm from the transmitter. The receivers were maintained in the same location for all of the tests. Changing the angle of arrival of the signal was achieved by rotating the panel containing the two receivers. Four different angles were tested, nominally, 0°, 20°, 45° and 60°.

Fig. 3 shows examples of the unwrapped phases for each tested angle using the method described in Section 2.

Table 1 summarizes the results. The root mean square Error (RMSE) and the bias (estimated by subtracting the true angle from the mean) are used to evaluate the performance of the two proposed approaches. The results were obtained from 20 independent tests for each angle. Each test involved 3000 snapshots. The SNR was found to be approximately 30 dB. The column titled "All estimates" represents the results obtained directly from applying the proposed method. It can be seen that large error results in the cases of the largest tested angle—60°. As can be seen from Fig. 3 (d), more outliers appear for this angle compared to the smaller angles. Performance can be improved by excluding the outliers from estimation of the final DOA as demonstrated in the column titled "Ex. outliers". As it can be seen, some improvements have been obtained by applying such a two-stage approach in the larger angle cases.

To demonstrate performance at lower SNRs, artificial white Gaussian noise was added in simulation. Performance versus SNR is depicted in Fig. 4 for a DOA of 45°. The figure emphasizes the fact that outliers can dominate the performance at low SNRs, however, performance is seen to asymptotically improve as the SNR increases. The general trend in the figure is consistent with that in Fig. 1. Note that experimental results can be affected by other factors such as reverberation.

Table 1: Experimental Results: DOA errors for 4 different angles.

<table>
<thead>
<tr>
<th>DOA</th>
<th>All estimates</th>
<th>Ex. Outliers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bias</td>
<td>RMSE</td>
</tr>
<tr>
<td>0°</td>
<td>0.17°</td>
<td>6.76°</td>
</tr>
<tr>
<td>20°</td>
<td>-1.42°</td>
<td>1.42°</td>
</tr>
<tr>
<td>45°</td>
<td>-1.33°</td>
<td>1.51°</td>
</tr>
<tr>
<td>60°</td>
<td>-3.06°</td>
<td>4.30°</td>
</tr>
</tbody>
</table>

7. CONCLUSION

A novel method for linear phase unwrapping, applied to DOA estimation of a multi-frequency signal received at two widely-spaced receivers, is presented. The method does not depend on the existence of unwrapped phases at low frequencies. Experimental results with ultrasonic signals emphasize the feasibility of the approach.
APPENDICES

A. EVALUATION OF THE PROBABILITY \( P_{s,m} \)

First, let us start with the probability \( P(\{a + \epsilon\} < |a + \epsilon|) \), where \( \epsilon \) is a random variable with Gaussian distribution and CDF \( \Phi_{\epsilon}(x) \); and \( a \) and \( a_1 \) are real constants, \( a \neq a_1 \). This probability can be evaluated as

\[
P(\{a + \epsilon\} < |a + \epsilon|) = P(\epsilon > \zeta_1), \text{ for } a < a_1 \\
= P(\epsilon < \zeta_2), \text{ for } a > a_1
\]

where \( \zeta_1 = -a + a_1/2 \). Eq. (28) is also applicable for \( a_1 = a_2, etc. \)

Now, consider the probability \( P = P(\{a + \epsilon\} < |a + \epsilon|) \), \( a_1 = a_1 \neq a_2 \). Based on 28, four different cases can be recognized as follows:

**case 1: \( a < a_1 \) and \( a < a_2 \)**

\[
P = P(\epsilon > \zeta_1 \ \& \ \epsilon > \zeta_2)
\]

**case 2: \( a > a_1 \) and \( a > a_2 \)**

\[
P = P(\epsilon < \zeta_1 \ \& \ \epsilon < \zeta_2)
\]

**case 3: \( a < a_1 \) and \( a > a_2 \)**

\[
P = P(\epsilon > \zeta_1 \ \& \ \epsilon < \zeta_2)
\]

**case 4: \( a > a_1 \) and \( a > a_2 \)**

\[
P = P(\epsilon < \zeta_1 \ \& \ \epsilon > \zeta_2)
\]

where \( \zeta_2 = -a + a_2/2 \). Hence, for \( P = P_{s,m} \) given by (21), the above procedure can be used by setting \( \alpha_1 = \Phi_n[k_1], \ a_1 = \Phi_n[k_1], \ a_2 = \Phi_n[k_2] \) and \( \epsilon = \epsilon_{mn} \).

B. DERIVATION OF THE VARIANCE \( \sigma_i^2 \) AS A FUNCTION OF SNR

From the definition of \( |\gamma(\omega)|^2 \) in (26)

\[
1 - |\gamma(\omega)|^2 = \frac{1 - \frac{G_{m}[\alpha_1]}{G_{s}[\alpha_1]} - \frac{G_{m}[\alpha_2]}{G_{s}[\alpha_2]} + \gamma_{mn}(\omega)\gamma_{mn}^*(\omega)}{G_{s}[\alpha_1] + G_{m}[\alpha_2]} = \frac{G_{s}[\alpha_1] + G_{m}[\alpha_2] - G_{s}[\alpha_2]}{G_{s}[\alpha_1]} = \left\{ 1 + \frac{G_{m}[\alpha_2]}{G_{s}[\alpha_1]} \right\}^2 - 1
\]

However, for white noise

\[
\frac{G_{m}[\alpha_2]}{G_{s}[\alpha_1]} = \frac{P_{m}}{\alpha[\alpha_1]P_s} = \frac{P_0}{\alpha[\alpha_1]LP_s} = \{ \alpha[\omega]L \Lambda \}^{-1}
\]

where \( P_0 \) and \( P_s \) are, respectively, the total signal power and the total noise power; \( \alpha[\omega] = G_{s}[\omega]/P_s; L \) is the length of the DFT; and \( \Lambda \) is the linear SNR. Inserting (34) into (33), and inserting the latter back into (25), (27) is obtained.

ACKNOWLEDGEMENTS

The authors wish to thank Science Foundation Ireland (SFI) for supporting this work under grant number 06/RFP/CMS007, titled: “Robust High Precision Ultrasonic 3D Location for Ubiquitous Computing.”

REFERENCES