Clustering Nodes in a Directed Acyclic Graph By Identifying Corridors of Coherent Flow

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***Abstract*—This paper proposes a novel method for clustering nodes based on prevailing power flow conditions within a power grid. To this end, first, the network's active power flow state is modelled as a *directed acyclic graph*. This digraph explicitly represents where power is flowing and this can help in monitoring and analysing system vulnerabilities. The directed acyclic graph representation also allows easy identification of those buses that solely provide or absorb active power: these are pure *source* and *sink* nodes, respectively. An iterative path-finding procedure is applied to every node in the system, to enumerate the sources that is fed by, and the downstream sinks towards which it forwards power. The novel clustering algorithm is then applied, to group together those nodes which share the same set of reachable sources and sinks. This novel clustering methodology is proposed in the first instance as a tool to boost the situational awareness of control room operators by better summarising aggregate power flow dispositions in large grids. The proposed methodology is applied to two sample grids, and an analogy to river systems is articulated, applying such notions as *tributaries*, *distributaries* and the central *mainstream* to electrical networks.**

I. INTRODUCTION

CLUSTERING together related nodes can be helpful for analysing and monitoring operational issues within power networks [1]–[3]. In this paper, a clustering methodology is proposed that seeks to find the nodes which are coherently transmitting active power towards, or away from, specific zones within the grid. Detecting such groups of nodes, here termed *coherent corridors*, can improve the situational awareness in the control room as they represent the aggregate structure of power flow across the network. To apply the clustering algorithm, the prevailing power flow conditions within the grid are used to build its corresponding directed acyclic graph (DAG) so that the directions of active power flows within the grid set the branches directionality [4]. A DAG is a directed graph which has no cycles.

Investigations related to line outages and cascading failures in power systems may talk, somewhat ambiguously, about *'power flow corridors'* as one of the relevant factors [4], [5]. One way to monitor these flow corridors is to visualise the power flows in a meaningful way [6]. In this paper, a novel clustering method is proposed which enables the operators to see intuitive depictions of power flow dispositions, and also offers a more rigorous definition of the loose notion of a flow corridor.

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The DAG explicitly shows where the power is flowing and can help to analyse power flow vulnerabilities more clearly [4]. The DAG representation also allows easy identification of those buses that solely provide or absorb active power: these are pure *source* and *sink* nodes, respectively. In this paper, the Sugiyama layout [7] is used to portray the DAG, as this layout style intuitively shows how the active power is flowing, from the top to the bottom of the diagram. A Sugiyama layout is a layout where the vertices are placed in horizontal rows and the edges are directed downwards [7].

The nodes within a DAG can be analysed in terms of the sources which can reach them, and the sinks which they can reach. The nodes which are fed from the same sources, and which feed the same sinks, are termed a coherent corridor, as they feed power with uniform directionality from one functional segment of the grid to another. The novel clustering algorithm of this paper groups together those nodes that share the same set of reachable sources and sinks. It is shown that this reduction technique helps us to detect important cut-sets more straightforwardly and so may help to improve situational awareness.

In this paper, two further novel concepts are presented for the DAG representation, being a *tributary* and a *distributary* (following the fluvial nomenclature e.g [8]). A tributary refers to a coherent corridor of buses where all the constituent nodes are fed from just one identical source. A distributary refers to a coherent corridor where all the constituent buses feed exactly one identical sink. Using these concepts, the power flow state of a network can be conceptualised as tributaries successively merging to build more central coherent corridors, which in time are diminished by corridors splitting off to eventually feed various distributaries. The disposition of a particular node or edge within this hierarchy of conjoining power flow corridors may be insightful as a measure of component centrality or criticality.

The proposed methodology clearly identifies the sets of branches which connect together the different coherent corridors. These 'bridging' or 'flowgate' branches may be among the most important to monitor within the grid as they show where aggregate active power flows merge with or split from the rest of the system.

In the graph theory literature, *transitive reduction* is a well-known technique for reducing the number of edges in a DAG [9]. The transitive reduction of a DAG is another DAG with the same nodes and as few edges as possible, such that if there is a directed path from node i to node j within the original DAG, then there is also such a path in the reduced one [10]. In other words, this technique removes as many edges as possible to maintain the same reachability relationships within the DAG. The proposed node merging algorithm of the present paper seeks to build a reduced network that embodies the fundamental reachability relationships as the original DAG. In this sense, the proposed methodology could be seen as an taking inspiration from the transitive reduction approach.

The contributions of this paper are as follows. First, we visualise the instantaneous state of power flow within power grids using the Sugiyama layout of a DAG, to give an intuitive diagram where all flows proceed downward. Second, the aggregate power flow dispositions are investigated by a novel clustering method which identifies coherent corridors. To indicate one of the potential applications of the proposed methodology, it is suggested that the interconnections between the coherent corridors are of particular interest for maintaining situational awareness as aggregate power flow dispositions shift.

The rest of this paper is organised as follows. In section II, the suggested methodology of the visualisation and clustering is explained. In section III, the simulation results are presented. Finally, section IV concludes.

II. Methodology

In this section, a novel methodology to cluster and merge the nodes within a power grid is proposed.

A. Flow coherency within a network

Fig. 1 is a sample DAG representation of a small power grid: the prevailing active power flow directions set the branches' directionality. Two particular types of nodes in a DAG are relevant for this paper: source nodes and sink nodes. A source node is one in which there are no incoming edges, but only outgoing ones. A sink node is one in which there are no outgoing edges but which has incoming edges from other nodes. Considering fig. 1, it is clear which nodes are the sources $\{0, 1\}$ or the sinks $\{13, 14\}$, and it is easy to see how the active power is transmitted between these nodes, as a layered diagramming style is used, so flow runs from top to bottom. Note that source nodes must be generator connection points, and sinks loads. However, many generators will connect to *intermediate* nodes in the grid, where they feed power into an existing flow through the bus.

The novel clustering technique is proposed to identify corridors of coherent flow in such a grid. These coherent corridors are sets of nodes through which power flows with uniform directionality and with the same relationship to the sources and sinks in the network. If the network's function is conceptualised as transmitting power from source to sink nodes, then these coherent corridors are the groups of nodes which have the same functional role in this sense. One specific type of coherent corridor can be named a tributary, where the constituent nodes all share exactly one source node. The function of a tributary corridor is to deliver power into the wider grid. Likewise, a distributary is characterised by feeding just one sink: this type of coherent corridor siphons power out of the grid. The clustering method explicitly shows how tributaries progressively merge together to form more central network segments: a system's *mainstream* corridors may in fact receive power from *all* the sources, and forward it to *all* the sinks. A node's membership of these different types of coherent corridors could be viewed as a novel centrality measure [11].

Previous work in [4] defined the idea of a *coherent cut-set*, which is a set of edges whose removal would split the grid into two parts, with the novel restriction that all the traversing edges must have the same flow directionality. This particular type of cut-set was found to be especially critical in terms of system security. The present notion of a coherent corridor should extend this paradigm, as for instance it might be conjectured that the edges linking between corridors would likewise be especially critical.

B. Graph preparation

Before applying the novel clustering method, it is necessary to take some simple steps to prepare the grid. The initial step is to model the power grid as DAG. To this end, the prevailing active power flows, obtained from an operational snapshot or load flow calculation, are used to determine branch directions. As discussed in [4], the non-symmetric adjacency matrix, A, of the DAG is built up as follows:

$$\begin{cases} A_{nm} = 1 \quad iff \quad P_{n\to m} > 0 \\ A_{mn} = 1 \quad iff \quad P_{n\to m} < 0 \end{cases} \tag{1}$$

Where $P_{n\to m}$ is the signed active power flow along the branch connecting n and m. Considering the DC power flow assumptions, $P_{n\to m}$ can be obtained as follow:

$$P_{n\to m} = \frac{\delta_n - \delta_m}{X_{nm}} \tag{2}$$

Where, δ_n and δ_m are the voltage angles of buses n and m and X_{nm} is the reactance of the branch connecting them. As according to the DC-PF power flow assumptions, active power exclusively flows from buses of higher to lower voltage angles, the possibility of active power circulating in the grid is precluded. This means the network digraph built using equation (1) can be assumed to be acyclic i.e it is a DAG [4].

An additional, optional preparatory step is to remove the leaf nodes of the grid, so that only the nodes that are sources and sinks in a meshed sense are used within the reachability analysis. The leaf nodes in a graph are those with a degree of one, having a sole branch connecting them to the wider network. For instance, nodes $\{7, 11, 12\}$ in fig. 2 are the leaf nodes of the DAG. The removal of nodes that meet this condition does not affect the structural features of the overall meshed network: for instance, a leaf demand bus can be simply represented as an additional spot load at its connecting node. The removal of the leaf nodes is a recursive process, as the removal of a leaf node can sometimes create another leaf node.

After removal of the leaf nodes, we can proceed onto applying the novel clustering method to the remaining meshed nodes in the layout. The leaf nodes can be re-added to the final visual representation of the network if desired.

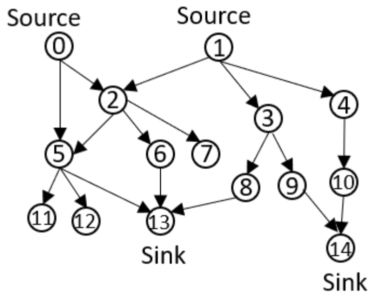

Fig. 1. A sample directed acyclic graph

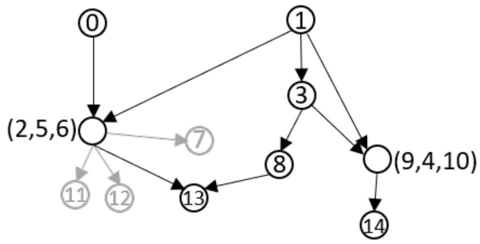

Fig. 2. The clustered graph, with pruned leaf nodes shown in grey

C. Clustering based on shared reachability of sources and sinks

This section describes the node clustering method that is the core novel contribution of this work. Essentially, the nodes are grouped together based on their common reachability from sources and to sinks, and are then merged to form *super-nodes*. For instance, in fig. 1, the sources nodes are identified as $\{0, 1\}$ while the sink nodes are $\{13, 14\}$. For nodes $4, 9$ and 10, the only reachable source is 1 and the only reachable sink is 14. Based on this, the nodes $\{4, 9, 10\}$ can be grouped together.

As applied in the context of a power grid DAG, this grouping of nodes describes the specific coherent corridors that sequentially transmit the active power from the sources to the sinks. Consequently, this novel algorithm depicts where aggregate coherent power flows conjoin and split, as though in a river system of hierarchical tributaries and distributaries.

For a DAG with N nodes, we notate the set of sources as G and the set of sinks as D and the set of remaining intermediate nodes as M. To be clear $N = G \cup M \cup D$. For notational clarity, the nodes of the DAG are assumed to be resorted in sequential order, starting with the sources G, then the intermediate nodes M and finally, the sinks D. This means that the intermediate nodes will be labelled from $|G| + 1$ to $|G| + |M|$.

To allow this sorting, the pure source and sinks nodes can be found using the directed adjacency matrix, A, presented in section II-B so that node i is a pure source iff:

$$\sum_{k=1}^{N} A_{ki} = 0 \tag{3}$$

Also, node i is a pure sink iff:

$$\sum_{k=1}^{N} A_{ik} = 0 \tag{4}$$

With the appropriately sorted DAG in hand, the *reachability* matrix [12] can then be populated using graph search methods [13]: $R_{ij} = 1$ if a directed path exists between i and j, otherwise $R_{ij} = 0$. By definition, the reachability matrix R will be non-symmetric for a DAG.

Due to the imposed node ordering, R can be partitioned into block matrices as follows to show the reachability relationships between the different node types:

$$R = \begin{array}{c c} & \begin{array}{ccc} G & M & D \end{array} \\ \begin{array}{c} G \\ M \\ D \end{array} & \begin{bmatrix} I_G & R_{GM} & R_{GD} \\ 0_{MG} & R_{MM} & R_{MD} \\ 0_{DG} & 0_{DM} & I_D \end{bmatrix} \end{array} \tag{5}$$

Where I_G and I_D are appropriately-sized identity matrices and 0 represents a zero matrix.

Each block matrix shows the corresponding reachability relations. For example, R_{GM} shows the reachability matrix between the sources G and the intermediate nodes M. Now, we define a composite clustering matrix, C, as follows:

$$C = [R_{GM}^T \;\; R_{MD}] \tag{6}$$

The dimensions of sub matrices R_{GM}^T and R_{MD}, extracted from R, are $(|M| \times |G|)$ and $(|M| \times |D|)$, respectively. Consequently, the dimension of matrix C is $(|M| \times (|G| + |D|))$. It should be noted that R_{GM}^T is the transpose of the reachability matrix between the sources and intermediate nodes, R_{GM}. The first $|G|$ columns of row i within matrix C show which sources can reach *to* the corresponding node within M i.e. M_i. Likewise, the last $|D|$ columns within row i identify the sinks that are reachable *from* the node M_i. Now, if rows i and j within C are the same, that means nodes M_i and M_j share the same set of reachable sources and sinks and therefore should be clustered together.

The steps taken by the algorithm to detect the clusters are as follows:

Input: A DAG, H
Output: Cluster assignment for each node in H

```
Extract the set of pure source nodes G using (3)
Extract the set of pure sink nodes D using (4)
Order the nodes [G, M, D]
Build the reachability matrix R
Partition R using (5)
Build the clustering matrix, C, using (6)
for i ≤ |M| do
    for j ≤ |M| do
        if C(i, :) = C(j, :) then
            Cluster nodes M_i and M_j
        end if
    end for
end for
```

D. Merging the clustered nodes and replacing them with super-nodes

Concerning graph reduction, the clusters detected by the suggested algorithm in Section II-C are merged together and replaced with a corresponding super-node. Each super-node will record the details of its constituent nodes. When creating a super-node all incoming edges to any constituent node in the cluster is treated as an incoming edge to the super-node, and vice-versa for outgoing edges. This preserves the reachability features of the original DAG. Only edges that are fully internal to a particular cluster can be discarded.

III. Results

A. Test platform

To evaluate the proposed methodology, two sample grids are considered: `nesta_case118_ieee` and `nesta_case145_ieee` from the repository at [14]. Creating the representative adjacency matrix, A is handled within MATPOWER [15]. The rest of the methodology is performed using a Jupyter Notebook with Python [16]. Underlying scripts and raw data are available at [17]. Also, the simulation results for `nesta_case73_ieee`, which are not discussed in this paper due to space constraints, can be found at [17].

B. Network clustering results

1) The results for `nesta_case118_ieee`*:* To visualise this network a Sugiyama layout is generated using the Networkx library [18]: this can be seen in fig. 3 (note that the original, rather than internally reordered, node labels are used in this figure, and elsewhere) This style of layout makes it easier to observe that the main source nodes are $\{12, 26, 66, 69, 80, 89\}$. Two intuitive corridors of nodes are annotated in fig. 3, which are subsequently clustered by the algorithm. These annotated coherent corridors show that, while the nodes involved in a cluster may range widely across the grid, the algorithm can still detect them accurately. The super-nodes which replace these two sample clusters are likewise annotated in fig. 4.

The Sugiyama representation provides an intuitive sense of the prevailing power flow state, which may aid situational awareness in a control room context. For example, the area annotated in fig. 3 as a distributary is splitting from the rest of the grid by an obvious cut-set, at the interface between node 100 on the sending side and nodes 103 and 104 receiving. This is a coherent cut-set as the power is being injected with uniform directionality *from* the grid *into* the distributary [4]. This visualisation approach helps us to identify instantaneous bottlenecks and operational vulnerabilities that emerge as power flow profiles change. The clustering methodology is proposed to more rigorously identify such situations.

The leaf nodes in the DAG are $\{10, 73, 87, 111, 112, 116, 117\}$. After the removal of these nodes, the clustering is performed. The nodes are merged into super-nodes, representing coherent corridors, according to their shared reachability with sources and sinks. Fig. 4 shows the network after applying the clustering algorithm and merging nodes into super-nodes. The source and sink nodes

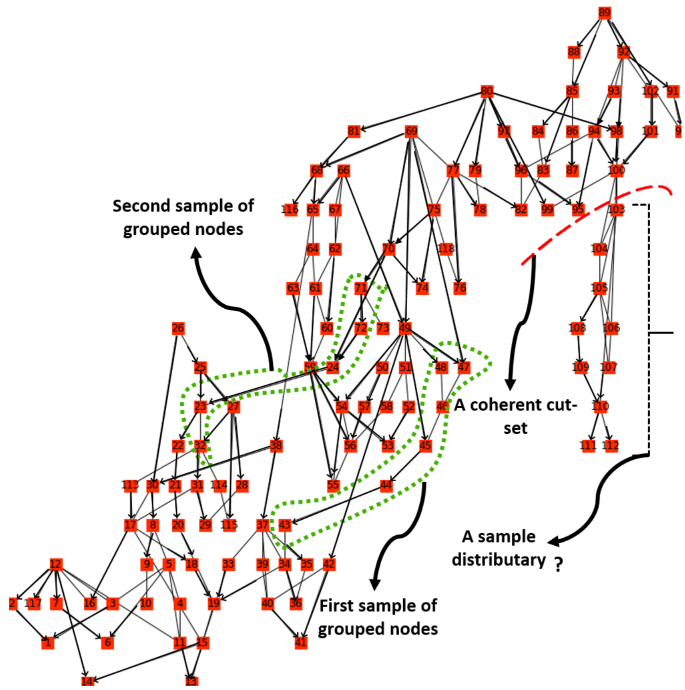

Fig. 3. Sugiyama layout of the DAG representing `nesta_case118_ieee`

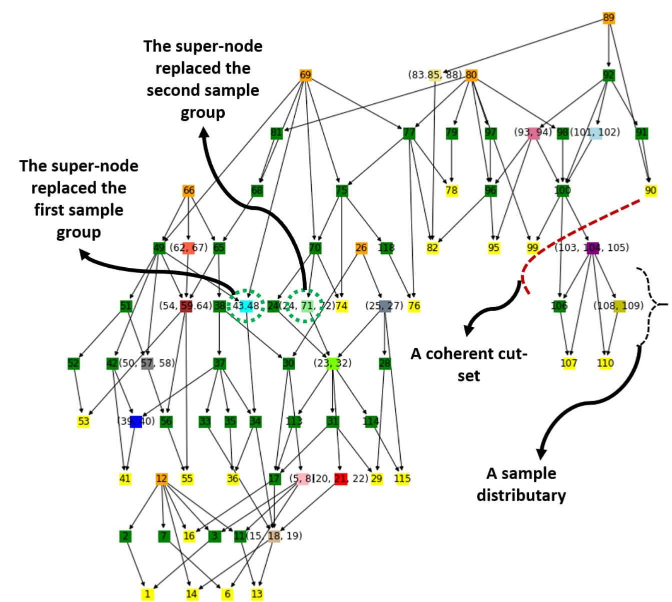

Fig. 4. Sugiyama layout of the DAG representing the clustered `nesta_case118_ieee`

are coloured orange and yellow, respectively, and unique intermediate nodes that are not part of any super-node are coloured green. Each super-node is given its own colours. To maintain readability of the super-node membership, longer clusters are represented with a period between its start and end node. e.g. $\{83.85, 88\}$ represents nodes $\{83, 84, 85, 88\}$.

TABLE I.
GRAPH AND LAYOUT CHARACTERISTICS FOR `nesta_case118_ieee`

Graph characteristics	**Original DAG**	**Clustered DAG**
Total nodes (inc. super-nodes)	118	78
Total edges	179	122
Average degree	0.65	0.63
Diameter	14	12
Layout characteristic		
Edge crossings	50	28

TABLE II.
GRAPH AND LAYOUT CHARACTERISTICS FOR `nesta_case145_ieee`

Graph characteristics	**Original DAG**	**Clustered DAG**
Total nodes (inc. super-nodes)	145	60
Total edges	420	93
Average degree	0.34	0.44
Diameter	13	10
Layout characteristic		
Edge crossings	192	44

It can be observed from fig. 4 that all the edges connected to and from the sub-nodes in the cluster are connected to and from the super-node thereby keeping the paths through the graph unchanged.

Considering figures 3 and 4, the distributary and coherent cut-sets determined in fig. 3 can be spotted more straightforwardly in fig. 4. This example indicates that the proposed clustering technique may improve situational awareness in the control room and offer new insights for network vulnerability assessment.

Table I records some particular graph metrics (taken from [19], [20]) for the grid before and after clustering the nodes. Note that there is a reduction of 33.89 % in the total nodes from the original graph. This means that the proposed clustering methodology brings the operators a smaller network which embodies the relationships between the different coherent corridors of flow within the original grid. This smaller network may prove easier to monitor and analyse.

Moreover, as this method is using the instantaneous power flow dispositions, when the operational conditions change, some (or maybe all) clusters will be affected. As the algorithm can be executed rapidly, this allows it to boost situational awareness by flagging when new large tributaries or distributaries have formed, or when important coherent corridors are linked by only a few lines.

2) The results for `nesta_case145_ieee`*:* Fig. 5 shows the Sugiyama layout for `nesta_case145_ieee`. As can be seen, the Sugiyama layout for this network is more disordered in comparison with it for `nesta_case118_ieee`, as this algorithm struggles to produce neat diagrams for larger networks (note the high number of diagram edge crossings recorded in table II). This again suggests how it can be useful to have a technique that can reduce the scale of a network while preserving the aggregate structure of the flow-exchange relationships. There are 35 leaf nodes removed, and fig. 6 shows the grid after applying the clustering algorithm. After applying

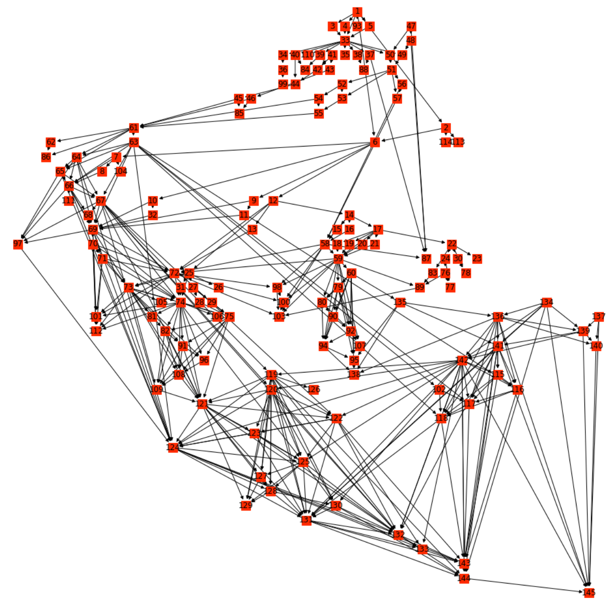

Fig. 5. Sugiyama layout of the DAG representing `nesta_case145_ieee`

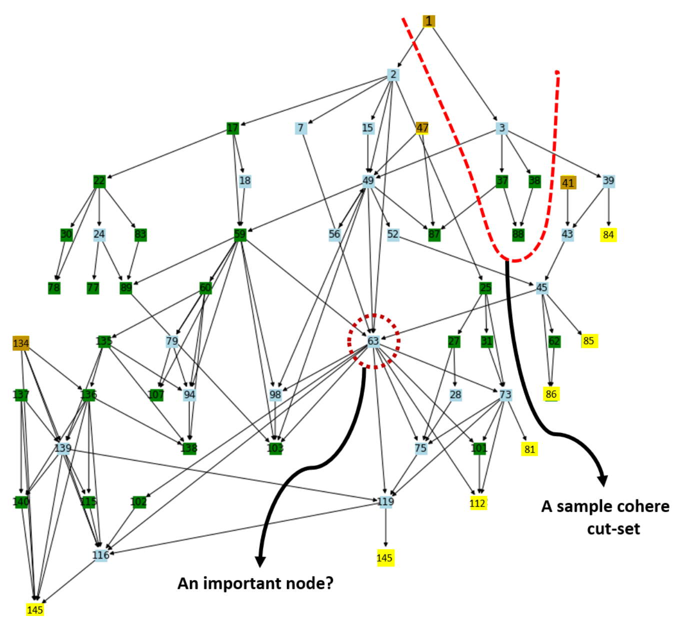

Fig. 6. Sugiyama layout of the DAG representing the clustered `nesta_case145_ieee`

the algorithm, 21 super-nodes were formed, and these are shown in light blue. Due to each cluster containing a larger group of nodes, each super-node is labelled by its lowest constituent node e.g. a super-node having $\{1, 4, 7, 8\}$ as its sub-nodes is labelled as 1.

The reduction of 59% in the total number of nodes and 78% in the total number of the branches shows how well the suggested methodology can produce a compact and more understandable network. Another important point is a 77% reduction in the edge crossing for the layout itself which greatly improves the diagram's quality.

The visual and intuitive importance of such a size reduction is highlighted by examining fig. 6. In this figure, the reduced grid visualisation brings to light an important super-node within the network annotated by the red dashed circle. The number of incoming and outgoing edges which are connected to this super-node suggests it as a very important nexus when considering with power flow vulnerabilities. This is relevant as concerning fig. 5, no one could visually detect such an important node within the grid. Moreover, the coherent cut-set, annotated by the red dashed line, which splits a tributary of the grid can be noticed easily using the reduced graph. While, concerning fig. 5, it is almost impossible to detect such an important cut-set and the associated tributary of the grid. Therefore, the proposed reduction technique enables us to detect such flowgates within even large grids in a more straightforward way. These considerations suggest how the novel clustering methodology could improve the situational awareness in the control room context and otherwise enhance the vulnerability assessment of power grids.

IV. Conclusion

In this paper, a novel technique to cluster nodes in a directed acyclic graph is proposed. The suggested method clusters the nodes based on their shared reachability relations with the sink nodes and source nodes in the network. The method groups together sets of nodes through which power flows sequentially and coherently. This method is proposed to aid the intuitive identification of bottlenecks within the grid, and to tangibly visualise the sets of line outages that could combine to cause islanding. The clustering technique can be applied to any directed acyclic graph, and may find applications beyond power engineering.

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