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A Modified Muskingum Routing Approach for Floodplain Flows: Theory and Practice

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Abstract

Hydrological or hydraulic flood routing methods can be used to predict the influences on a flood wave as it passes through a river reach. While hydraulic routing schemes solve the equation of continuity and the equation of momentum for river flows, the simpler data requirements of hydrological routing make it useful for preliminary estimates of the time and shape of successive positions of a flood wave. This paper presents a modified linear Muskingum methodology where the floodplain effects of flood peak attenuation and travel time are included in routing parameters. Developing the methodology involved routing hydrographs of different flood peak and duration through a channel model of a generalised river reach in which a range of geometrical and resistance parameters were varied. Comparisons of the upstream and simulated downstream hydrographs for each condition investigated, and the attenuation and travel time (i.e. standard Muskingum routing) of the f

wave to be estimated. Standard Muskingum routing was then used to de hydrographs for K values together with assumed storage weighting factors (x) ranging from 0 to 0.5. Flood peak attenuations were again determined throu upstream and routed downstream hydrographs and with these the attenuation, new Actual weighting factors corresponding to storag constants, were subsequently determined using these relationships and the attenuations determined from the 1-D model simulations. Using multi-variable regression analysis, the computed K and x were correlated to catchment and hydr properties and expressions for determining K and x from these properties w developed. The modified Muskingum routing method together with the regressed expressions for K and x was applied to a case study of the River Severn where good agreement between measured and routed hydrographs was observed.

Keywords: Overbank flow; Flood routing; Muskingum method; Hydraulic and hydrologic methods; floodplains; Modelling; Saint-Venant equations.

1 Introduction

Hydraulic or hydrological flood routing techniques are co and hydrologists to predict the temporal and spatial variations of a f reach (Choudhury et al., 2002). The methodologies that have been developed vary in their complexity with more analytically rigorous methods having in accommodate the dynamics and influences of floodplain beha flood wave in a natural channel. The Muskingum method of h approach. The popularity of the Muskingum method derives primar requirements. Knowledge of topographical catchment conditions are not r understand the propagation of dynamic flood waves [illegible] in a carried out using [illegible] (1977). The Muskingum approach represents hydrological flood routing techniques based on the equation of continuity and a relationship that describes [illegible]. At the opposite end of the spectrum, the spe[illegible] full-scale dynamic wave models based on the Saint Venant equations provide sophisticated hydraulic flood routing methods [illegible] (1985). Hydraulic routing uses the equation of continuity and momentum balance equation (the Saint Venants equation) involves the numerical solution using finite difference or characteristic me[illegible] Simplification of the momentum equation produces approximate solutions (e.g. mo[illegible] wave, convective diffusion) that are easier to calculate and r[illegible]

Advantages and disadvantages of using hydrological and hydraulic f[illegible] routing techniques. Although hydraulic routing techniques can more adequat[illegible] dynamics of unsteady flow in canals and rivers, these methods are more demanding i[illegible] information inputs and require data to accurately represent t[illegible] characteristics of the main channel and floodplain. Initial and boundary conditions are a[illegible] required. Conversely, the inputs and computational procedures for routing techniques are considered simple (Singh, 1988). These methods are useful wh[illegible] preliminary estimates of the time and shape of a flood wave a[illegible] are required, or where budgetary constraints may not facilita[illegible]

A modified non-linear Muskingum routing method that includes floodplain flows is presented in this paper. The method is based on the standard Muskingum[illegible] However, rather than determining the routing parameters through analysis of observed upstream and downstream hydrographs for given flows at sites, the pr[illegible] routing parameters from empirical relationships describing floodplain flood peak attenuation and flood wave travel time. The method therefore has similarities

Muskingum-Cunge method (Cunge, 1969) in that flood routing parameters are determined from geometrical and resistance properties of the channel, th calibration process. Development of the method involved a combination of one-dimensional hydraulic modelling and standard Muskingum flood routing. K the storage constant and storage weighting factor, x, for a range of catchment and hydrograph variate regression analysis was used to correlate K and x to these comp properties and expressions for K and x were developed. The modifie Muskingum routing method based on the new K and x expressions was applied to a case study of the River Severn where good agreement between measure hydrographs was observed. The method offers a simple and inexpensive m estimating the time and shape of a flood wave as it progresses along river channels of low to moderate sinuosity and in which backwater and iner small.

2 Muskingum Flood Routing

The Muskingum method (McCarthy, 1938) is a simple storage discharge relationship in river systems, extensively used in river engineering (Gill, 1979). The method performs best in river systems where inertia effects and backwater influence are small and where model parameters are chosen appropriately to represent the hydrau behaviour of the system (Chang, 1983). The original Muskingum used continuity and storage relationships expressed as:

Continuity
$$\frac{dS_t}{dt} = I_t - O_t \qquad (1)$$

Storage
$$S_t = K[xI_t + (1-x)O_t] \qquad (2)$$

where S_t, I_t and O_t are simultaneous amounts of storage, inflow and out given time t, K is a storage constant expressing the ratio between storage and river reach and x is a dimensionless weighting factor that varies between 0 an rivers. This weighting factor describes the relative importance o storage. The storage time constant, K, is equal to the flow travel time through the river reach (McCuen, 1998). If K and x are known, routing is performed using:

$$O_t = C_1 I_t + C_2 I_{t-1} + C_3 O_{t-1} \tag{3}$$

in which C_1, C_2 and C_3 are routing coefficients given by:

$$C_1 = \frac{0.5\Delta t - Kx}{(1-x)K + 0.5\Delta t} \tag{4}$$

$$C_2 = \frac{Kx + 0.5\Delta t}{(1-x)K + 0.5\Delta t} \tag{5}$$

$$C_2 = \frac{-0.5\Delta t + (1-x)K}{(1-x)K + 0.5\Delta t} \tag{6}$$

where parameters are as defined for Eq. 2 and Δt is the time step. I_{t-1} and O_{t-1} are the inflow and outflow at $t-\Delta t$. Once the coefficients (which sum to unity) are determined, Eq. 3 is used repeatedly to determine O_t at any time.

Values for K and x that describe the storage characteristics of a river are usually derived from observed upstream and downstream hydrographs records. These methods are well reviewed and broadly represented in five class (a) graphical method; (b) least squares method; (c) method of cumulants; (d) direct optimisation; and (e) method using Saint Venant equations (Singh and McCann, 1980). More recently, Yoon and Padmanabhan (1993) identified a further three methods for linear model parameter estimation. These are forward and backward optimisation, outliers filtering estimation method a quadratic programming algorithm.

Graphical methods are commonly applied. The most popular graphical approach used by McCarthy (1938) for the linear Muskingum model involves plotting $[xI_t + (1-x)Q_t]$, known as the weighted discharge, against accumulated storage f different assumed values of x. Different values of x in Eq. 2 produce a family of curves vary from being heavily looped to being reasonably linear. T the narrowest loop and can be fitted with a straight line is considered the best esti x. The inverse of the slope of this line gives the value of K. Although the graphical method is generally satisfactory (Chow, 1964; Linsley et al., 1975; Viessman et al., 1972 Wilson, 1990), it is time consuming. Furthermore, no objective selection criteria exists for choosing the appropriate x value; the method therefore requires a level of subjective interpretation to determine a value that is optimal (Gelegenis and Serrano, 2000; Yoon and Padmanabhan, 1993). Muskingum routing parameters have also been estimated using least squares scheme based on minimi sum of squares of the deviations between observed storage for given inflow and outflow hydrographs (Gill, 1978; Birkhead and Esen, 2007). The underlying principle of the graphical and least squares methods are the same and both methods should produce similar parameter values. The method of moments and the method of cumulants are based on relating the first and sec moments or cumulants of the instantaneous unit hydrograph (IUH) of the Muskingum reach to the Muskingum routing parameters K and x (Dooge, 1973). The method of direct optimisation is based on minimising the difference between observe hydrographs to determine directly the routing coefficients of the Muskingum model without explicitly estimating K and x (Gelegenis and Serrano, 2000).

More recent advances in computer technologies have allowed t Muskingum routing method to be linked with hydrodynamic software pac analysis of surface water drainage (see for example the HEC-HMS and TOPMODEL hydrological models). These are based on the Saint-Venant equations that a[re] derived from the principles of conservation of mass and mom[entum in] their 1-dimensional form:

$$\frac{\partial y}{\partial t} + y\frac{\partial V}{\partial x} + V\frac{\partial y}{\partial x} = 0 \qquad (7)$$

$$\underbrace{\frac{\partial V}{\partial t}}_{(I)} + \underbrace{V\frac{\partial V}{\partial x}}_{(II)} + \underbrace{g\frac{\partial y}{\partial x}}_{(III)} + \underbrace{g\left(S_f - S_0\right)}_{(IV)} = 0 \qquad (8)$$

where y is the flow depth, V is the flow velocity, g is the gravity acceleration, S_0 is the river bed slope, S_f is the slope of the energy line, x is the longitudinal distance and t is time. The equations are simultaneous, quasi-linear, first-order partial differential equ[ations] of the hyperbolic type and no general analytical solution. The first term (I) in Eq. 8 is the local inertia term, the second (II) is the convective inertia term, the third (III) is the pressure differential term and (IV) accounts for friction and bed slopes. Numerical methods for solving Saint-Venant equations are broadly classified in two categories: (a) approximate methods; (b) complete numerical methods. Approximate methods are based on the equations of continuity and a simplified equation of momentum.

Kinematic and diffusive wave models can be constructed from Eq. 7 and Eq. 8 assuming that the contribution of some terms is negligible (compared to ... Moussa and Bocquillon, 1996). Models that neglect inertial terms (I) and (II) are known as diffusion wave models (Cunge, 1969; Chang and Barry, 1997; Moussa and Bocquillon, 2009), and models that neglect both inertial (I, II) and pressure (III) terms are kinematic models (Smith, 1979). The full Saint-Venant equations have also been solved for routing applications using numerical techniques in channels with regular (cylindri...

cross-sectional geometries without the need for simplifying assumptions (Amein an 1970; Dooge et al., 1982; Wang et al., 2006).

Cunge (1969) included the effects of geometrical and resistanc reach in the original Muskingum method to develop the Muskingum-Cunge (M-C) model. Cunge showed that the Muskingum formula for solving flood routing a finite-difference approximation of the linearised diffusion wave being derived from the St-Venant equations by neglecting the inertial terms. In the M-C flood routing procedure, the necessity of calibration of the Muskingum parameters is not required and the routing parameters K and x are computed from hydraulic of the reach:

$$K = \frac{\Delta x}{c} \tag{9}$$

and

$$x = \frac{1}{2}\left(1 - \frac{Q}{B S_0 c \Delta x}\right) \tag{10}$$

where parameters are as described above and c is the flood wave celerity, Δx is the longitudinal distance in the channel, Q is the discharge, B is the average width of the channel.

The time step, Δt, used in M-C routing procedure is appropriately chos define the shape of the inflow hydrograph. Δt and Δx dependency ex routing procedures and the choice of the reach increment Δx is such that it is not significantly smaller than the distance travelled by the flood in Δt. This Δx interval chosen in the M-C routing method is therefore based on Δt, c, Q, the channel top width B and longitudinal channel slope S_0. The flood wave celerity, c, is obtained from the slope of the discharge-area curve for a given discharge Q. Details of the M-C method are discussed in Volume III of the Flood Studies Report (NERC, 1975). Tang et al. (1999a) investigated the M-Cp method for flood routing, usin hypothetical flood hydrographs in a prismatic channel with s indicated that M-Ct method suffers from a loss of outflow volume which depends on be slope and floodplain roughness. Furthermore, it was observed that ar and trailing edge oscillation occur in the rising and reces respectively. These oscillations can become more significant as the roughness of the increases, but gradually disappear with decreasing bed slope

The standard linear Muskingum routing method assumes K and x constant and these are determined by analysis of measured inflow and outflow hydrographs. The method therefore, does not accommodate changes in these pa accurately the routing of storm sequences in the river reac (Kundzewicz and Strupczewski, 1982). More recent developments however do allow for parameter variability with changing characteristi for example, Perumal, 1992; Guang-Te and Singh, 1992; Al-Humoud and Esen, 2006). Perumal (1992) developed a multilinear Muskingum flood routing method based or distribution scheme. The physically based Musking-um meth model in this method and the parameters are varied at eac prescribed flow zones in the inflow hydrograph. Guang-Te and Singh (1992) propose versions of the linear Muskingum method with variable para reach travel time (which depends on the storage, channel c obtained from a simplification of the St Venant equations. Al-Humoud and Esen (200 proposed two approximate methods for the estimation of lin parameters. The first method requires the computation of the hydrographs' point of intersection, and the computation of within the reach. The second method requires the computation of t

hydrographs at two specific points.

Although the current paper is based on the linear Muskingum, the relation between $[xI_t+(1-x)O_t]$ and S_t is usually nonlinear and Gill (1978) proposed the following storage relationship for the nonlinear Muskingum model:

$$S_t = K[xI_t+(1-x)O_t]^m \tag{1}$$

where m is an exponent that defines the nonlinear relation between accumulated storage and weighted flow. This exponent cannot be directly determined from inflow and outflow hydrographs and therefore, alternative parameter estimation methods have been presented (for example Gill, 1978; Tung, 1985; Yoon and Padmanabhan, 1993; Mohan, 1997; Kim et al., 2001; Geem, 2006; Chu, 2009). Gill (1978) proposed a routing scheme for the nonlinear model based on the segmented curve method where coefficients are determined by a least squares method. However, the technique is somewhat arbitrary process of selecting three points on the segmented curves for simultaneously solving the continuity (Eq. 1) and storage (Eq. 10) equations of the nonlinear method (Tung, 1985). Tung (1985) proposed procedures using the Hooke-Jeeves (HJ) pattern search technique in combination with simple linear regression (LR), the conjugate gradient (CG), and the Davidon-Fletcher-Powell (DF techniques and used the state variable technique to perform routing. These approaches were compared with Gill's methodology which showed that HJ+CG and HJ+DFP techniques produced better estimations of the routing parameters. Yoon and Padmanabhan (1993) proposed a least-squares regression technique which directly fits the nonli procedure iteratively and with initial assumption of the parameters using the Marqu algorithm (Marquardt, 1963). In addition, this method also analyzes the data and computes reasonably accurate values of the parameters to be estimated.

expedite the optimisation process. Mohan (1997) suggested a calibration tec determining K, x and m based on a genetic algorithm that avoids the need to make in assumptions. Kim et al. (2001) proposed a harmony search algorithm for estimation of the same parameters and observed that it outperformed the heuristic and mathematical algorithms, evolutionary programming and nonlinear programming. Geem (2006) presented Broyden-Fletcher-Goldfarb-Shanno (BFGS) technique for parameter estimation in nonlinear Muskingum model. The BFGS algorithm is a branch of Newton method based mathematical technique that searches for an optimised solution of the unconstrained nonlinear equation. Chu (2009) combined Fuzzy Inference System (FIS) implemented in an adaptive network with the Muskingum model to estimate the outflow hydrograph. However, the calibration procedure for finding the co values of the three parameters, K, x and m to determine this outflow is complicated (Kim et al., 2001).

3 Method

A multi-stage process (Fig. 1) that includes 1-dimensional HEC-RAS modelling of a generalised river reach, standard Muskingum regression analysis is adopted for developing expressions for storage constant (K) and weighting factors (x) in the modified Muskingum method is presented here. The process begins with implemen generalised HEC-RAS model using input flow hydrographs and geome determine travel times and relative attenuations which are ther to develop equations for estimating Muskingum model parame explain the process in detail.

Fig. 1

3.1 HEC-RAS Model of General River Reach

The HEC-RAS model of the generalised river reach was executed for an extensive range of geometrical and hydraulic properties. HEC-RAS (Hydrologic Engineering Centre River Analysis System) is a one-dimensional link and node river model developed by the US Army Corps of Engineers. It solves the dynamic Saint-Venant equations using an implicit, finite difference method. The inflow hydrographs of varying peak and shape, hydrographs of varying duration were key inputs to the model.

3.1.1 Hydrographs of Varying Peak Flow

Hydrographs for a range of peak flows were developed using a methodology (Fig. 2) and associated software from Work-Package 3.1 of the Irish Flood Studies U programme (O Connor and Goswami, 2010). This utilises a historical record of flow data

Fig. 2

Any gauged location where a record of good quality data was available w therefore be suitable and a long continuous flow record was chosen. Hydrograph development involved the following:

(1) The annual exceedence maximum flood events for the selected site were identified from record data (as shown for a site in Fig. 2 (a)).

(2) Annual exceedence flood hydrographs were isolated from th the complex segments on each side of the single peak hydrograph, leaving th component (Fig. 2 (b)).

(3) The isolated flood hydrographs were then normalised to have a peak value of uni

dividing all its flow ordinates by the peak flow.

(4) This unit peak is assumed to represent the 100 percentile flow and hydrograph widths were determined at percentiles of 98, 95, 90 &. 10 and 5.

(5) Widths corresponding to these flow percentiles were avera exceedence series and the rising limb of the hydrograph was approximated by a modified form of the Gamma curve with these widths. The s of the full unit hydrograph was obtained by an exponential recession drawn from the point of inflection of the Gamma curve (Fig. 2(c)).

(6) The required hydrographs were generated by scaling up the derived unit hydrograph with peak flows of different return periods (2, 5, 25, 50, 100, 500 ar A base flow was added to the particular return period ordinates (Fig. 2(d)).

Fig. 3

Annual maximum flows from Mosserleish catchment were found to follow a General Extreme Value (GEV) Type I distribution (NERC, 1975) and, based on this distribution, quantiles for the hydrographs in Fig. 2(d) vary from 91.41 m^3/s for the 2-year event to 153.90 m^3/s for the 100-year flood.

3.1.2 Hydrographs of Varying Duration

The hydrographs developed using this methodology have the same base length therefore flood volume is directly proportional to flood peak. Therefore, the relationship between flood volume and flood peak is not fully defined. Fl processes and to fully account for the random nature of their included if flood volume is to be accurately related to peak. Although attempts to develop relationships between volume and peak of direct runoff for ca

have been made (see for example Rogers, 1980, Mimikou, 1983, Singh and A

no validated relationship exists for Irish catchments and an attempt

relationship in the FSU was inconclusive (O Connor and Gosw

In the absence of a validated method to include the effects of the increased volume that would usually be associated with floods of longer duration, a simple appro duration is included independently of flood peak. The approach involved developing a triangular hydrograph of the same volume as the 1000-year hydrograph in Figure 2 (Panel D) and linking this volume to the hydrograph characteristics by:

$$Volume = \frac{1}{2} \times T_B \times Q_P \qquad (12)$$

where T_B is the hydrograph base width and Q_P is the flood peak (153 m^3/s). The duration of the 1000-year hydrograph corresponding to this is 265 hours approximately. This duration (T_B) was linked to the time to peak, T_P, by the Flood Studies Report (NERC, 1975) relationship:

$$T_B = 2.52\, T_P \qquad (13)$$

By further scaling the 1000-year hydrograph, the approach facilitated the second hydrograph of different duration (Fig. 2(e)).

3.1.3 Geometrical and Resistance Properties of Model in HEC-RAS

The basic HEC-RAS model geometry (Fig. 3) included basic bankfull (B_{bf}) and floodplain (B_{fp}) widths of 25 m and a bankfull depth (h_{bf}) of 2.5 m. This bankfull depth produced a bankfull flow for the hydrograph of the median flow having 2-year return period and ensured that floodplains in the generalised model flooded.

The main channel side slopes and floodplain boundaries were inclined at ^{0} giving trapezoidal geometries in both the inbank and overbank zones. The hydraulic resistance of the main channel (n_{mc}) and floodplain (n_{fp}) was expressed in terms of Manning's assigned values of 0.03 and 0.25 respectively. The high value of n_{fp} was chosen to ensure that measurable attenuations were obtained. The base model length (L) was 50 km and longitudinal slope of the floodplain (S_{fp}) was 1 m/km. In total, 65 variations of these basic properties were represented in the simulations, by A to H and in which one property was varied at a time (Table A1). Case A the effect of channel length (L), Case B investigated the effect of longitudinal floodplain slope (S_{fp}), Case C varied the floodplain resistance (n_{fp}), Case D the floodplain width (b_{fp}), Case E, the transverse floodplain slope (α) and Case F, main channel hydraulic resistance (n_{mc}). The influence of flood peak (Q_P) and flood duration (T_B) was explored by routing two sets of hydrographs (Fig. 2) through Panel D and F the generalised model in the Case G and Case H simulations. For each of these simulations, the effect of changes in each property on flood attenuation was examined by comparing input and output hydrographs.

3.2 Standard Muskingum Routing Model

The travel time of the peak of the flood wave determined from input and output hydrographs in HEC-RAS model was assumed to be equal to the constant K in the standard Muskingum routing method (Fig. 1, Panel A). Estimation of corresponding weighting factor x was then involved. For each combination of K with assumed values of x (increasing incrementally from 0 to 0.5 where low x high attenuation and vice versa), standard Muskingum flood routing of input hydrographs was repeatedly performed using Eq. 3, together with Eq. a series of outflow hydrographs. Peak outflows were determined with the peak inflows of the hydrographs (Fig. 1 Panel B) allowed a series of relative attenu to be determined, from:

$$\%\ \text{Relative attenuation} = \frac{Q_{P1} - Q_{P2}}{Q_{P1}} \times 100 \qquad (14)$$

where Q_{P1} and Q_{P2} are the peak inflow and outflow of the hydrographs (Fig. 1 Panel A). Linear relationships between these relative attenuations from assumed weighting factors (x) were developed. These relationships were used to determine the actual weighting factor for the relative attenuation calculated by comparison of the inflow and outflow hydrographs from HEC-RAS simulation (Fig. 1 Panel B). This produced a weighting factor for each of the 65 simulations from which storage constants were directly determined, covering the full range of catchment properties that were assessed for the different inflow hydrographs

3.3 Regression Analysis

Using multivariate regression analysis, the computed values of storage constant and weighting factors were correlated to catchment and hydro expressions for determining K and x in terms of these properties were developed

4 Results

Estimated values of K and x from both the HEC-RAS hydraulic modelling and the standard Muskingum method are shown in Table A1 and the variation of the resulting parameters with catchment and hydrograph properties are shown in Fig. 4 and Fig. 5 respectively. Variations in both peak inflow (Q_{P1}) and channel slope (S) were shown to have only a small influence on storage and weighting factors and were excluded from the analysis.

Fig. 4

Fig. 5

Results confirm that increasing floodplain length (Case A) and wid noted by Wolff and Burges (1994), increases the capacity of the overbank and delay the propagation of a flood wave along the channel. (Case B) is also important. Steep catchments have the capacity to convey floo than are more mildly graded and the increased conveyance results in reduced travel time (Fig. 4(b)) and attenuation (Fig. 5(b)). These trends are consistent with findings of Wolff and Burges (1994), Tang et al. (1999b) and Ghavasieh et al. (2006) where large attenuations with sharp reductions in the variability of the cumulative distribution were observed in low gradient catchments. Tang et al. (1999b) reported that the Muskingum Cun method suffers a certain amount of volume loss dependent on the longitudinal slop channel. Increasing the lateral slope (Case C) also results in a geometry in w overbank flow is continually redirected back towards the mai with steep lateral slopes will convey an increased proportion of the flood v channel. Furthermore, floodplain resistance in the generalis that in the main channel. The reduced attenuations and higher wave speed reduced travel time (Fig. 4(e)) in geometries with increasing transverse slopes exacerbated given that a diminishing proportion of flow is being influenced by the high floodplain roughness values. Increases in main channel (Case F) and floodplain resistance values (Case C) produce increased attenuation and travel times.

The full influence of floodplains on flood wave attenuatio

influenced by the flow peak magnitude (Q). Consider the resulting overbank depth. At low return periods (typically less that 2 years), flows will not significantly i will not be affected by the associated additional attenuation. With the Fig. 4(g) indicates that the wave speed is higher than for the bank-full event. At flows that produce low overbank depths (return period less than 5 years), floodplain influences increase attenuation and travel time. Case H simulations assessed flood dura attenuation and flood wave travel time. When combined, peak (Q_p) and duration (T_B) (define the flood volume. Floods with hydrographs with sharp peaks and short durations experience significantly higher attenuation than those w that are characterised by high volume and a long rising limb of the hydrograph w occupy floodplain storage that is available and once occupie available for the remainder of the flood. The attenuation pro cases is thus limited. In contrast, hydrographs with low, dispersed rising limbs fo the flood volume to contribute to storage and therefore relatively high downstream attenuations.

5 Development of Muskingum Parameters

The influences of channel geometry, resistance and hydrograph (K and X) proper Fig. 4 and Fig. 5 were included in multi-variate regression analysis to gener for these parameters. The floodplain width (B_{fp}) and the bankfull width (B_{bf}) expressed as a single parameter (B_{fp}/B_{bf}) that is consistent with other bank flow research (for example Knight and Shiono, 1996). The expressions

$$K = 0.794 \frac{L\, \eta_{fp}^{0.24}\, \eta_{mc}^{0.42} \left(\frac{b_{fp}}{B_{bf}}\right)^{0.60} T_B^{0.07}}{S_{fp}^{0.53} (\alpha)^{0.09} \left(\frac{Q_P}{2b_{fp}+B_{bf}}\right)^{0.06}} \tag{15}$$

$$x = 0.035 \frac{L^{0.03}\, S_{fp}^{0.16}\, T_B^{0.39}}{\left(\frac{b_{fp}}{B_{bf}}\right)^{0.05} (\eta_{mc})^{0.06}\, \eta_{fp}^{0.008}} \tag{16}$$

Application of Eq15 requires that $\alpha > 0$ and therefore horizontal floodpl

represented by a near-zero value of α. Similarly, the equation assesses flood

therefore peak flows should exceed bankfull discharge capacities in a It

should be noted that Eq15 and Eq16 are based solely on the influences of t

parameters on relative attenuations and delays of flood peaks dete by the HEC-RAS

modelling of the variations in the generalised river reach. The values of p

are therefore based on the simulated data only and as with reg

parameters that may intuitively be considered to be importan

fore in the analysis. The negative influence of floodplain roughness (η_{fp}) in Eq16 is a case in

point. It would be expected that significant floodplain roughnes

large storage and yield low storage weighting HEC-RAS modelling. However,

increasing floodplain resistance increased the proportion of flow being conveyed i

channel for all flows investigated, with the result that simula

performance of these is shown in Fig6 where the routing parameters, K and x, are

plotted on line against those calculated using Eq15 and Eq16

Fig6

Fig. 6 indicates that Eq. 15 and Eq. 16 reproduce reasonably well the simu for most of the geometrical, resistance and hydrograph However, some limitations exist. Simulated weighting factors (x) are shown to vary most significantly w duration (T_B). The poor fit may be a result of the assumption of independen between the flood peak (Q_P) and the flood duration (T_B) that was made when including duration as a parameter in the regression model equation, the low values of q_{fp} of Fig. 6 indicates the low influence of the routing parameter determined using the equations

6 Illustration of Modified Muskingum Flood Routing Method

The routing method presented was applied to the River Suir, Co. Tipperary, Ireland. The River Suir is typical of most Irish rivers in its slow main channel floodplain sinuosity. A 16.8 km reach between the New Bridge gauge (Station 16008) and Caher Park (Station 16009) gauging stations (Fig. 8) was tested. In addition to a third station at Killardry (Station 16007), where the flow of the River Aherlow (that joins the Suir between New Bridge and Caher) is measured, are characterised by good quality, digitised 15-minute flow records from Oct 1954 to 200 less significant tributaries join the river between these stations but are The catchment areas to the New Bridge gauging station and Caher Park are 1160 km^2 and 1602 km km^2 respectively and the area to Killardry on the Aherlow River km^2. The flood history of the river in this area also indicates that significan

Fig. 7

Illustrating the involved application of the modified Muskingum method using Eq15 and Eq16 to route a selection of hydrographs measured at New Bridge through the River Suir reach and then comparing those at Caher Park obtained from both measured data and from a HEC-RAS model of the river. The HEC-RAS model was developed from 35 recently obtained cross-sections between New Bridge and Caher Park that de the main channel geometry and floodplain to a width of appr channel banks of the Suir and Aherlowe rivers. This data was augmented by LIDA further define the floodplain topography to widths of approxi main channel. Longitudinal distances between measured cross approximately 400 m and this resolution in the model was inc interpolation. The lower reach of the River Aherlowe was inc 85% and 70% of the Suir and Aherlowe catchments respectively pasture and this land use dominates the floodplain. A Manning value of 0.05 for grassland pasture with areas of brush described the hydrauli a coefficient of 0.04 that is typical for a reasonably straight, clean channel at full st some obstructions and marginal vegetation defined the main 1956; Chow, 1959; Hollinrake and Millington, 1994).

The majority of natural hydrographs are complex and are multiple peaks that reflect both the temporal variability of th heterogeneity of the catchment. Although it is theoretically possible to res hydrograph into a series of simpler hydrographs, the method employed in this study used isolated storm events extracted from long flow records. Isolating events was a laborious task and was assisted by FSU hydrograph processing facilitate the identification of simple hydrographs at the three gauging stations for specific storm events. These events related to periods in Decem[ber] ... 5, August 1986/ September 1994 and October/November 2004 (Fig. 8).

Fig. 8

Measured outflow hydrographs at Caher Park can potentially be influenced by a number of tributary inflows (Fig. 7) for which no flow data is available. The impact on the peak and timing of hydrographs in the main River Suir is uncertain, therefore to ensure that measured outflow graphs at Caher Park were not unduly influenced by these tributaries and that measured data are suitable for testing the modified method, measured hydrographs at New Bridge and Killardry (Fig. 8) were routed through the HEC-RAS model of the river system (that excludes the tributary network for which no data is available) and compared to observed hydrographs at Caher Park. The agreement between hydrographs routed in the HEC-RAS model and those measured at Caher Park for the three events (Fig. 9), indicates that the contribution of the other than that from the Aherlow River, is not significant.

Fig. 9

Parameters in Eq 15 and Eq 16 apply only to a single reach and canno[t] extrapolated to a river system that includes a tributary network. For validat[ion] modified method, the Caher Park hydrographs for the three storm events that exclude the contribution from the Aherlow River were computed by routing the observed hydrographs at Killardry through a HEC-RAS model of the river fro[m] this location to Caher Park and subtracting these from the measured Caher Park hydrograph

Park. The resulting hydrographs are termed adjusted hydrographs. Although this process is somewhat artificial and backwater effects from interactio floodplain of the main channel and tributary are not included, these effects are likely to be local and in the context of a 16.8 km reach, the a acceptable. Comparison of adjusted hydrographs with those determined by measured hydrographs at New Bridge using the modified Musk performance of the approach to be illustrated.

The testing of the Muskingum routing using Eq 15 and Eq 16 was based on assigni appropriate values to parameters that describe the geometry channel and floodplains together with the characteristics of t Geometrical properties of the main channel and floodplain were determined from surve and where necessary, averaged. The floodplain slope (S_{fp}) and reach length (L) are FSU catchment descriptors for which numerical values are readily avail channel and floodplain Manning's resistances were estimated to be 0.0 and flood peaks and durations were determined from measured hydrographs at Ne Bridge. This data is summarised in Table 1 for the 1954/55, 1980 and 2004 flood eve For the initial testing of the method, floodplain widths (W_{fp}) in Table 1 were computed by averaging (for 400 m intervals over the 16.8 km reach) the main channel widths predicte the HEC-RAS model when routing the measured New Bridge hydrographs to Caher Park.

Table 1

Caher Park hydrographs using the modified Muskingum m (referred to as Muskingum with) these average widths are shown with adjusted in Fig. 10

Caher Park hydrograph. The Caher Park hydrographs from the HEC-RAS model (referred to as HEC-RAS) are also shown for comparative purposes.

Fig. 01

Although strong correlations between the Muskingum and HEC-RAS routed hydrographs are evident in Fig. 01, the usefulness of the proposed method as presented is limited in that the floodplain widths are derived from hydraulic m

predictive capacity of the method, floodplain attenuation indicators (FAIs) th

developed for the Irish Flood Studies Update (FSU) were util

that define the active river floodplain in Irish catchments for the 10-year (Q_{10}), the 100-year (Q_{100}) and the 1000-year (Q_{1000}) floods from normal depth modelling at FSU no

(approximate intervals of 500 m) on the main river network. The approach is based on the assumption that the Q_{med} flood, with a return period of 2 years, is equivalent to the bankfull flow in rivers. Given that bankfull recurrence intervals in ma

order of 1-3 years (see for example, Richards, 1982; Pauquet, 1997; and Castro and Jackson, 2001), this simple assumption is reasonable. The median flood is determi

using an FSU relationship (Swiecimski and Murphy, 2010) for ungauged catchments give

$$Q_{med} = 1.237 \times 10^{-5} AREA^{0.937} BFI^{-0.922} SAAR^{1.306} FARL^{2.217} DRAIND^{0.341} S1085^{0.185} (1 + ARTDRAIN2)^{0.408} \quad (17)$$

where AREA (km^2) is the catchment area of the river to the outlet p

S1085 (m/km) is the average slope of the river between 10% and 85

outlet, SAAR (mm) is the annual average rainfall in the catchment, FARL is a flood attenuation factor for reservoirs and lakes, DRAIND (km^{-1}) is the baseflo simple index that relates the length of the main stream (km) to the area of the hydrolog… the gauged catchment (km^2) and ARTDRAIN is an index of the arterial drainage extent defined as the percentage area of the catchment that is benefiting lands within the total catchment ar…

Simple multiplication of Q_{med} by appropriate growth curve factors defi… magnitudes of Q_{10}, Q_{100} and Q_{1000}. Floodplain flows were determined by subtracti… Q_{med} values from these flood quantiles and corresponding floodpla… iteratively at all nodes using the Manning equation and the geometry at that no… resistance coefficient that was consistent with the channel. Incorporating these depths into a Digital Terrain Model (DTM) facilitated the production of flood po… Q_{10}, Q_{100} and Q_{1000} for the Irish river network.

The return periods for the 16005 flood of 29/04/1986 on the River Suir between 5 and 10 yrs. The Q_{10} FAI was therefore the most relevant polygon from whic… estimate floodplain widths and an average value of 100 m was determined from the flo… extent at all nodes between Cahir and New Park. Hydrographs developed from modified Muskingum approach using this floodplain width of 100 … are shown in Fig. 11 with those generated from hydraulic routing and those develope…

Fig. 11

6.1 Discussion of Results

Visual comparison of the hydrographs in Fig. 10 and Fig. 11, although somewhat subjective, provide a quick and simple means of assessing the per… modified Muskingum routing method presented in this paper. Unsurprisingly, Cahir and New Park hydrographs in Fig. 10, correlate closely with the Muskingum method in which floodplain widths were extracted from HEC-RAS routing results. However, the given that floodplain widths are based on output from a hyd limited use. More meaningful assessments of the outflow hydrographs shown in Fig. 11. Here, good agreement is again observed between hydrographs from the modified Muskingum method in which floodplain widths ar determined from the FAI catchment and HEC-RAS output hydrographs. The goodness-of-fit between the Muskingum and HEC-RAS hydrographs was qualitatively and less subjectively assessed using the goodness-of-fit criteria recommended by Jewitt and Schulze (1999). Goodness-of-fit statistical tests measure the deviation of computed output from an observed output and different tests are applied to assess di components. Root mean square error (RMSE) determines the magnitude of error in the computed hydrographs and was estimated using the relationship by Schu given by:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(Q_{comp(t)} - Q_{obs(t)}\right)^2}{n}} \quad \text{for } i = 1, 2, 3, \ldots n \qquad (18)$$

where $Q_{comp(t)}$ and $Q_{obs(t)}$ are the computed and observed discharge at different time steps. Given that outflow is important in hydrological event modelling, percentage errors in computed and observed peak flow rates, peak timi determined using the following equations (Green and Stephenson, 19

$$E_{peak} = \frac{Q_{p-comp} - Q_{p-obs}}{Q_{p-obs}} \times 10 \qquad (19)$$

$$E_{time} = \frac{t_{p-comp} - t_{p-obs}}{t_{p-obs}} \times 10 \qquad (20)$$

$$E_{volume} = \frac{V_{comp} - V_{obs}}{V_{obs}} \times 10 \qquad (21)$$

where E_{peak}, E_{time} and E_{volume} are percentage errors in peak flow, timing and volumes respectively. $Q_{p\text{-}comp}$ and $Q_{p\text{-}obs}$ are computed and observed peak flows, $t_{p\text{-}comp}$ and $t_{p\text{-}obs}$ are computed and observed time to peak flow, and V_{comp} and V_{obs} are computed and observed hydrograph volumes.

Even though the RMSE, E_{peak}, E_{time} and E_{volume} statistics may assess model performance effectively, the shape of computed and observed hydrog not be accounted for fully. To overcome this, Nash and Sutcliffe (1970) proposed a dimensionless coefficient of model efficiency (E), given as:

$$E = \frac{F_o^2 - F^2}{F_o^2} \qquad (2)$$

in which $F^2 = \sum_{i=1}^{n} \left[Q_{obs(i)} - Q_{comp(i)}\right]^2$ and $F_o^2 = \sum_{i=1}^{n} \left[Q_{obs(i)} - Q_m\right]^2$.

The coefficient of efficiency in Eq. (2) provides a well accepted measure of fit computed and observed hydrographs, its value increasing toward unity as the simulated hydrograph progressively improves. Values of E that exceed 0.8 (Green and Stephenson, 1986) are considered to reflect a good correlation between the adjusted and the Muskingum and HEC-RAS hydrographs. In this study results of these statistical te the Muskingum and HEC-RAS hydrographs at Chainpur in Fig. 11 are compared to adjusted hydrographs developed from observed data are shown in Table 2.

Table 2

Table 2 shows that the modified Muskingum method produces outflow hydrog compare favourably with the adjusted hydrographs and are comparable to those developed through HEC-RAS modelling. The differences that exist between both the Muskingum HEC-RAS hydrographs and the adjusted hydrographs likely result from the si assumptions inherent in both methods. In reality, main channel and floodplain mom exchange in the River Suir may increase flood wave attenuation and decre However, the modified routing parameters were developed from analysis of ti data from 1-D modelling of a general channel in which the energy losses from t interaction are not included. Furthermore, the influences of geometrical, hydrograph properties in this analysis were assessed as independent Similarly, main channel and floodplain interactions are unaccounted for in HEC-RAS hydrographs developed from 1-D modelling of the River Suir study reach. these interactions, however, appear the relationships in Eqs 15 and 16 are considered to provide reasonable estimates of K and x for the presented method. The satisfactory performance of the modified method further implies that the storag Muskingum routing methods is a substitute for the momentum approach in typical Irish rivers therefore it is reasonable to relate the parameters to channel and flow characteristics (Perumal, 1992).

It should be noted however that the limitations of the app hydrological or Muskingum routing methods and therefore, the resu should be confirmed if applied to river reaches where backwa significant, where floodplain sinuosity is excessively high or momentum exchange between main channel and floodplain zones ar method however, does provide a simple and inexpensive meth estimates of the time and shape of a flood hydrograph at a river reach.

7 Conclusions

A modified linear Muskingum routing method for floodplains is suitable for presented in this paper. Muskingum flood routing methods are based on storage discharge relationships in river systems and can satisfactorily reproduce flow in river systems where inertia effects and backwater influences are s routing parameters, x and K, that describe the storage characteristics of usually derived analytically from observed upstream and downstream hydrographs extrac from historical flow records. The proposed method uses explicit relationships for K and x described in terms of standard geometrical and resistance properties of floodplains together with properties of the inflow hydrograph. The relationships we based on regression analysis of computational data generated by one-dimensional modelling of a generalised river reach. The expressions are:

$$K = 0.794 \frac{L\, n_{fp}^{0.24}\, n_{mc}^{0.42} \left(\frac{b_{fp}}{B_{bf}}\right)^{0.60} T_B^{0.07}}{S_{fp}^{0.53}\, a^{0.09} \left(\frac{Q_P}{(2b_{fp}+B_{bf})}\right)^{0.06}} \qquad x = 0.035 \frac{L^{0.03}\, S_{fp}^{0.16}\, T_B^{0.39}}{\left(\frac{b_{fp}}{B_{bf}}\right)^{0.05} \left(n_{mc}\right)^{0.06} n_{fp}^{0.008}}$$

Application of the method requires that $a > 0$ and therefore floodplains are represented by a near-zero area. Furthermore, the modified method assumes floodplain effects and therefore flows must be sufficient to produce overbank c Application of the method in a reach of the River Suir, Co. Tipperary, Irelan outflow hydrographs that compared favourably to the measured flow level records.

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Appendix

Table A1

List of Figures

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