Balanced Growth Revisited:
A Two-Sector Model of Economic Growth

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Abstract

The one-sector Solow-Ramsey model is the most popular model of long-run economic growth. This paper argues that a two-sector approach, which distinguishes the durable goods sector from the rest of the economy, provides a far better picture of the long-run behavior of the U.S. economy. Real durable goods output has consistently grown faster than the rest of the economy. Because most investment spending is on durable goods, the one-sector model's hypothesis of balanced growth, so that the real aggregates for consumption, investment, output, and the capital stock all grow at the same rate in the long run, is rejected by postwar U.S. data. In addition, to model these aggregates as currently constructed in the U.S. National Accounts, a two-sector approach is required.

Since the 1950s, the Solow-Ramsey model of economic growth, which models all output in the economy in terms of a single aggregate production function, has been the canonical model of how the macroeconomy evolves in the long run. The model has also featured prominently in the analysis of economic fluctuations: Business cycles are commonly characterized as correlated deviations from the model's long-run "balanced growth" path, which features the real aggregates for consumption, investment, output, and the capital stock, all growing together at an identical rate determined by the pace of technological progress. While multisector models, in which disaggregated sectors have different production functions, have been used to shed light on certain aspects of growth and business cycles, the one-sector growth model remains the workhorse for explaining the long-run evolution of the economy.[1]

The purpose of this paper is to make a simple point: Despite its central role in economics textbooks and in business cycle research, the traditional one-sector model of economic growth actually provides a poor description of the long-run behavior of the U.S. economy. A simple alternative two-sector model, which distinguishes the behavior of the *durable goods sector* from the rest of the economy, explains a number of crucial long-run properties of U.S. macroeconomic data that are inconsistent with the one-sector growth model, and is far better suited for modelling these data as currently constructed. As such, this two-sector approach provides a better "baseline" model for macroeconomic analysis.

Two principal arguments are presented in favor of this position.

1. The U.S. evidence firmly rejects the one-sector model's prediction of balanced growth, but is consistent with a two-sector approach.

The real output of the durable goods sector has consistently grown faster than the rest of the economy. Because most investment spending is on durable goods while the bulk of consumption outlays are on nondurables and services, real investment has tended to increase relative to real consumption. While a number of other factors masked this pattern for much of the past 40 years, real investment has grown

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[1]For example, Mankiw (2000) and Romer (1996) are two well-known undergraduate and graduate texts that place a central emphasis on the one-sector growth model. Long and Plosser (1983) is a well-known model of how multi-sectoral linkages can affect the business cycle. Hornstein and Praschnik (1997) and Huffman and Wynne (1999) are two more recent contributions in this area.

significantly faster than real consumption every year since 1991, and statistical tests now reject the one-sector model's prediction of long-run balanced growth.

The special behavior of real durable goods output has been the result of a large and ongoing decline in its relative price: The share of durable goods in *nominal* output has been stable, and the hypothesis that the logs of nominal consumption, investment, and output are cointegrated is still accepted. These patterns are easily modelled using a two-sector approach in which technological progress in the production of durable goods exceeds that in the rest of the economy.

2. The current published measure of U.S. real GDP can only be accurately modelled with a multi-sector approach.

In the one-sector model, all output has the same price. In this case, aggregate real output, as measured by weighting quantities according to a fixed set of base-year prices, should be independent of the choice of base year and should grow at a steady rate in the long-run. In reality, because the durable goods sector has faster growth in real output and a declining relative price, the growth rate of a fixed-weight measure of real GDP will depend on the choice of base year: The further back we choose the base year, the higher the current growth rate will be.

This dependence of real GDP growth on an arbitrary choice of base year is unsatisfactory. For this reason, the U.S. National Income and Product Accounts abandoned the fixed-weight approach in 1996, switching to a "chain index" approach, which uses continually updated relative price weights. These new measures of growth do not depend on an arbitrary choice of base year, but the change in methodology implies that a multi-sector approach, incorporating relative price movements, is necessary for modelling the long-run behavior of the major U.S. real aggregate series as currently constructed.

The rest of the paper is organized as follows. Section 1 outlines the empirical evidence that rejects the one-sector model but supports a two-sector approach. Section 2 presents the two-sector model, and Section 3 uses it to fit aggregate U.S. data constructed according to the current chain-aggregation procedures. Finally, Section 4 discusses the model's implications for a range of issues in empirical macroeconomics, including calibration of business cycle models, estimation of aggregate investment regressions, and the relationship between aggregates for TFP and output growth.

1 Evidence

1.1 A Look at the Data

The standard neoclassical growth model starts with an aggregate resource constraint of the form $C_t + I_t = Y_t = A_t L_t^{1-\beta} K_t^{\beta}$. Because all goods are produced using the same technology, no-arbitrage requires that a decentralized market equilibrium features consumption and investment goods having the same price. Thus, in the standard expression of the resource constraint, the variables C, I, and Y have all been deflated by an index for this common price to express them in quantity or real terms.

The model usually assumes that a representative consumer maximizes the present discounted value of utility from consumption, subject to a law of motion for capital and a process for aggregate technology. If the technology process is of the form $\log A_t = a + \log A_{t-1} + \epsilon_t$, where ϵ_t is a stationary zero-mean series, then it is well known that the model's solution features C, I, K, and Y all growing at an average rate of $\frac{a}{1-\beta}$ in the long-run. This means that ratios of any of these variables will be stationary stochastic processes.

This hypothesis of balanced growth and stationary "great ratios" has often been held as a crucial stylized fact in macroeconomics, with evidence in favor including the well-known contribution of Kaldor (1957). More recently, King, Plosser, Stock, and Watson (1991), using data through 1988, presented evidence that the real series for investment and consumption (and consequently output) share a common stochastic trend, implying stationarity of the ratio of real investment to real consumption. Figure 1 shows, however, that including data through 2000:Q2 firmly undermines the evidence for the balanced growth hypothesis. Since 1991, investment has risen dramatically in real terms relative to consumption, growing an average of 8.4 percent per year over the period 1992-99, 4.7 percentage points per year faster than consumption. Indeed, applying a simple cyclical adjustment by comparing peaks, there is some evidence that the ratio of real investment to real consumption has exhibited an upward trend since the late 1950s, with almost every subsequent cyclical peak setting a new high.

It might be suspected that the strength of investment since 1991 has been the result of some special elements in the current expansion such as its unusual length,

the surge in equity valuations, or perhaps the "crowding in" associated with the reduction of Federal budget deficits. If this were the case, then we would expect to see the ratio of real investment to real consumption declining to its long-run average with the next recession. However, while the factors just cited have likely played some role in the remarkable strength in investment, a closer examination reveals that something more fundamental, and less likely to be reversed, has also been at work.

A standard disaggregation of the data shows that the remarkable increase in real investment in the 1990s was entirely due to outlays on producers' durable equipment, which grew 9.1 percent per year; real spending on structures grew only 2.2 percent per year. Moreover, Figure 2a shows that this pattern of growth in real equipment spending outpacing growth in structures investment is a long-term one, dating back to about 1960. Importantly, this figure also documents that the same pattern is evident within the consumption bundle: Real consumer spending on durable goods has consistently grown faster than real consumption of nondurables and services. These facts are summarized jointly by Figure 2b, which shows that, over the past 40 years, total real output of durable goods has consistently grown faster than total real business output (defined as GDP excluding the output of government and nonprofit institutions).

The faster growth in the U.S. economy's real output of durable goods reflects an ongoing decline in their relative price. Remarkably, as can be seen from Figure 3a, the share of durable goods in *nominal* business output has exhibited no trend at all over the past 50 years. Similarly, the increase in real fixed investment relative to real consumption has been a function of declining prices for durable goods and the fact the such goods are a more important component of investment than consumption: Figure 3b shows that, once expressed in nominal terms, the ratio of fixed investment to consumption has also been trendless throughout the postwar period.[2]

Figure 4 illustrates the relevant relative price trends. Relative prices for durable goods, of both consumer and producer kind, have trended down since the late 1950s. The decline for equipment did not show through to the deflator for fixed investment until about 1980 because the relative price of structures, which show little trend

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[2]Over 1960-1999, durable goods accounted for 13 percent of consumption expenditures and 47 percent of investment expenditures.

over the sample as a whole, rose between 1960 and 1980 and fell thereafter.

Viewed from this perspective, the post-1991 increase in real investment relative to real consumption does not appear as a particularly cyclical phenomenon. Instead, it reflects long-running trends. Over the long run, nominal spending on investment and consumption have tended to grow at the same rate. But the higher share of durable goods in investment, and the declining relative price of these goods, together imply that real investment tends to grow faster than real consumption. Two other factors—the increase in the relative price of structures from the early 1960s until 1980, and then the decline in the nominal share of investment in the 1980s—somewhat obscured this trend until the 1990s. However, over the entire postwar period, these two variables have been stationary. Unless those patterns change going forward, the ratio of real investment to real consumption should continue to trend up.

1.2   Tests for Stationarity

Formal statistical tests confirm the intuition suggested by these graphs, that the real ratios are non-stationary and that the nominal ratios are stationary:

- An Augmented Dickey-Fuller (ADF) unit root test for the ratio of real fixed investment to real consumption gives a t-statistic of -0.78, meaning we cannot come close to rejecting the hypothesis that this series has a unit root.[3] The t-statistic for the corresponding nominal ratio is -2.99, which rejects the unit root hypothesis at the 5 percent level.[4]

- The ADF t-statistic for the ratio of real durables output to real business output is -0.05; the corresponding nominal ratio has an t-statistic of -4.03, rejecting the unit root hypothesis at the 1 percent level.

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[3]This test is based on a regression of the variable on its lagged level, an intercept, and lagged first-differences. The number of lags used (two) was chosen according to the general-to-specific procedure suggested by Campbell and Perron (1991), but the results were not sensitive to this choice.

[4]In fact, because this regression reveals a statistically significant intercept, the t-statistic in this case tends towards a Gaussian asymptotic distribution, implying rejection of the unit root hypothesis at the 1 percent level. See Hamilton (1994), pages 497 and 529.

These test regressions did not include a time trend, but adding one does not change the conclusion of non-stationarity of real ratios and stationarity of nominal ratios. While these tests are ambiguous about the presence of a unit root in the real ratios, they show that these ratios contain statistically significant positive time trends; the regressions for the nominal ratios reject both the inclusion of the time trend and the hypothesis of a unit root.

1.3 Some Data-Quality Issues

Before moving on to describe a model to fit the patterns just documented, a discussion of the data is appropriate. All series used here are from the U.S. National Income and Products Accounts (NIPA). For each of the categories presented, the NIPAs construct the nominal series and price deflators separately, with the real series then defined implicitly. While the nominal series are generally considered to be of high quality, questions have often been raised about the accuracy of the price deflators, and consequently the real series. This raises the question of whether the relative movements in prices and real output documented here are simply figments of mis-measurement.

On this issue, it is useful to note that much of the evidence on price mis-measurement in the official series, in particular the work of Robert Gordon (1990), has suggested price inflation for durable goods is overstated because valuable quality improvements are often ignored. This implies that the gap between the growth rate of real durable goods output and that of the rest of the economy may be even bigger than the NIPA data indicate. But countervailing evidence also exists. Durable goods are one of the areas where some of the adjustments for quality suggested by Gordon have been adopted by the NIPAs, most notably for computers. In contrast, much of the more recent evidence on price mis-measurement, including some of the work of the Boskin Commission, has focused on non-manufacturing sectors, such as financial services and construction, where there are few adjustments for quality.[5] In the absence of convincing evidence pointing one way or the other, the approach taken in this paper will be to model the NIPA data as published, while noting the need for further investigation of the quality of official price indexes.

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[5]For example, measured productivity growth in finance and construction has been extremely weak and may be underestimated, suggesting that price inflation in these industries may be overstated. See Corrado and Slifman (1999) and Gullickson and Harper (1999).

2 The Two-Sector Model

We have documented three important stylized facts about the evolution of the U.S. economy over the past 40 years:

- The real output of the durable goods sector has grown significantly faster than the real output of the rest of the economy.

- Because durable goods are a more important component of investment than consumption, real investment has grown faster than real consumption.

- These trends reflect relative price movements: The share of durable goods in nominal output has been stable, as have the shares of nominal consumption and investment.

Clearly, these patterns are inconsistent with the one-sector growth model: If we are to explain the trend in the relative price of durable goods, we need a two-sector approach that distinguishes these goods separately from other output. This section develops, and empirically calibrates, a simple two-sector model that fits the facts just described by allowing for a faster pace of technological progress in the sector producing durable goods.[6]

2.1 Technology and Preferences

The model economy has two sectors. Sector 1 produces durable equipment used by both consumers and producers, while Sector 2 produces for consumption in the form of nondurables and services and for investment in the form of structures. The notation to describe this is as follows: Sector i supplies C_i units of its consumption good to households, I_{ij} units of its capital good for purchase by sector j, and keeps I_{ii} units of its capital good for itself. The production technologies in the two sectors are identical apart from the fact that technological progress advances at a different pace in each sector:

$$Y_1 = C_1 + I_{11} + I_{12} = A_1 L_1^{1-\beta_1-\beta_2} K_{11}^{\beta_1} K_{21}^{\beta_2} \tag{1}$$

[6]There is plenty of available evidence that points towards a faster rate of technological progress in the durable goods sector. For example, the detailed Multifactor Productivity (MFP) calculations published by the Bureau of Labor Statistics reveal durable manufacturing as the sector of the economy with the fastest MFP growth. See http://stats.bls.gov/mprhome.htm.

$$Y_2 = C_2 + I_{21} + I_{22} = A_2 L_2^{1-\beta_1-\beta_2} K_{12}^{\beta_1} K_{22}^{\beta_2} \tag{2}$$

$$\Delta \log A_{it} = a_i + \epsilon_{it} \tag{3}$$

Equipment and structures depreciate at different rates, so capital of type i used in production in sector j accumulates according to

$$\Delta K_{ij,t} = I_{ij,t} - \delta_i K_{ij,t-1} \tag{4}$$

To keep things as simple as possible, we will not explicitly model the labor-leisure allocation decision, instead assuming that labor supply is fixed and normalized to one

$$L_1 + L_2 = 1 \tag{5}$$

and that households maximize the expected present discounted value of utility

$$E_t \left[\sum_{k=t}^{\infty} \left(\frac{1}{1+\rho} \right)^{k-t} (\alpha_1 \log D_{1t} + \alpha_2 \log C_{2t}) \right]$$

Here, D is the stock of consumer durables, which evolves according to

$$\Delta D_t = C_{1t} - \delta_1 D_{t-1} \tag{6}$$

2.2 Steady-State Growth Rates

We will focus on the steady-state equilibrium growth path implied by a deterministic version of the model in which the technology variables A_1 and A_2 grow at constant, but different rates; in a full stochastic solution, all deviations from this path will be stationary. In this steady-state equilibrium, the real output of sector i grows at rate g_i. We will also show below that the fraction of each sector's output sold to households, to Sector 1, and to Sector 2, are all constant, and that L_1 and L_2 are both fixed.

Along a steady-state growth path, if investment in a capital good grows at rate g then the capital stock for that good will also grow at rate g. Taking log-differences of (1) and (2), this implies that the steady-state growth rates satisfy

$$g_1 = a_1 + \beta_1 g_1 + \beta_2 g_2 \tag{7}$$

$$g_2 = a_2 + \beta_1 g_1 + \beta_2 g_2 \tag{8}$$

These equations solve to give

$$g_1 = \frac{(1 - \beta_2)\, a_1 + \beta_2 a_2}{1 - \beta_1 - \beta_2} \tag{9}$$

$$g_2 = \frac{\beta_1 a_1 + (1 - \beta_1)\, a_2}{1 - \beta_1 - \beta_2} \tag{10}$$

so the growth rate of real output in sector 1 exceeds that in sector 2 by $a_1 - a_2$.

2.3 Competitive Equilibrium

We could solve for the model's competitive equilibrium allocation by formulating a central planning problem, but our focus on fitting the facts about nominal and real ratios makes it useful to solve for the prices that generate a decentralized equilibrium. In doing so I assume the following structure. Firms in both sectors are perfectly competitive, so they take prices as given and make zero profits. Households can trade off consumption today for consumption tomorrow by earning interest income on savings. This takes the form of passing savings to arbitraging intermediaries who purchase capital goods, rent them out to firms, and then pass the return on these transactions back to households.

Relative Prices: Because relative prices, rather than the absolute price level, are what matter for the model's equilibrium, we will set sector 2's price equal to one in all periods, and denote sector 1's price by p_t. Given a wage rate, w, and rental rates for capital, c_1 and c_2, the cost function for firms in Sector i can be expressed as

$$TC_i\left(w, c_1, c_2, Y_i, A_i\right) = \frac{Y_i}{A_i} \left(\frac{w}{1 - \beta_1 - \beta_2}\right)^{1 - \beta_1 - \beta_2} \left(\frac{c_1}{\beta_1}\right)^{\beta_1} \left(\frac{c_2}{\beta_2}\right)^{\beta_2}$$

Prices equal marginal cost, so the relative price of sector 1's output is

$$p = \frac{A_2}{A_1} \tag{11}$$

Combined with (9) and (10), this tells us that the ratio of sector 1's nominal output to sector 2's nominal output is constant along the steady-state growth path: The decline in the relative price of durable goods exactly offsets the faster increase in quantity. We can also show that the fraction of each sector's output devoted to consumption and investment goods is constant. Together, these results imply that the ratios of all nominal series are constant along the steady-state growth path.

Clearly, this is a direct consequence of our restrictive choice of log-linear preferences and technology, but it fits with the evidence just presented.

So, our model is capable of fitting the stylized facts about the stability of nominal ratios, and differential growth rates for the real output of the durable goods sector and the rest of the economy. To calibrate the model empirically to match the behavior of *aggregate* real NIPA series, we will also need to derive the steady-state values of the nominal ratios of consumption, capital, investment, and output of type 1 relative to their type-2 counterparts, as functions of the model's fundamental parameters.

Consumption: Denominating household income and financial wealth in terms of the price of sector 2's output, the dynamic programming problem for the representative household is

$$V_t\left(Z_t, D_{t-1}\right) = \operatorname*{Max}_{C_{1t}, C_{2t}} \left[\alpha_1 \log D_t + \alpha_2 \log C_{2t} + \frac{1}{1+\rho} E_t V_{t+1}\left(Z_{t+1}, D_t\right)\right]$$

subject to the accumulation equation for the stock of durables and the condition governing the evolution of financial wealth:

$$Z_{t+1} = \left(1 + r_{t+1}\right)\left(Z_t + w_t - p_t C_{1t} - C_{2t}\right)$$

Here w_t is labor income and r is the real interest rate defined relative to the price of sector 2's output.

The first-order condition for type-2 consumption and the envelope condition for financial wealth combine to give a standard Euler equation:

$$\frac{1}{C_{2t}} = \frac{1}{C_{2,t+1}} \frac{1}{1+\rho} E_t\left[1 + r_{t+1}\right] \tag{12}$$

So, using a log-linear approximation, the steady-state real interest rate is

$$r = g_2 + \rho \tag{13}$$

Some additional algebra (detailed in an appendix) reveals the following equation

$$\frac{p\left(g_1 + \delta_1 + \rho\right) D}{C_2} = \frac{\alpha_1}{\alpha_2} \tag{14}$$

which tells us that the ratio of the rental value of the stock of durables to nominal consumption of nondurables and services is determined by the ratio of preference

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parameters $\frac{\alpha_1}{\alpha_2}$. Also, as noted earlier, the steady-state solution involves the stock of durables growing at rate g_1, so that $C_1 = (g_1 + \delta_1) D$ by equation (6). Substituting this expression for D into (14) implies the following condition for the ratio of nominal consumption expenditures for the two sectors:

$$\frac{pC_1}{C_2} = \frac{\alpha_1}{\alpha_2} \left(\frac{g_1 + \delta_1}{g_1 + \delta_1 + \rho} \right) \tag{15}$$

Investment: Capital goods are purchased by arbitraging intermediaries who rent them out to firms at Jorgensonian rental rates:

$$
\begin{aligned}
c_1 &= p \left(r + \delta_1 - \frac{\Delta p}{p} \right) = p (g_1 + \delta_1 + \rho) \\
c_2 &= r + \delta_2 = g_2 + \delta_2 + \rho
\end{aligned}
$$

The profit functions for the two sectors are

$$\pi_1 = p A_1 L_1^{1-\beta_1-\beta_2} K_{11}^{\beta_1} K_{21}^{\beta_2} - w L_1 - c_1 K_{11} - c_2 K_{21}$$

$$\pi_2 = A_2 L_2^{1-\beta_1-\beta_2} K_{12}^{\beta_1} K_{22}^{\beta_2} - w L_2 - c_1 K_{12} - c_2 K_{22}$$

The first-order conditions for inputs can be expressed as

$$
L_1 = \frac{p (1 - \beta_1 - \beta_2) Y_1}{w} \qquad K_{11} = \frac{\beta_1 Y_1}{g_1 + \delta_1 + \rho} \qquad K_{21} = \frac{p \beta_2 Y_1}{g_2 + \delta_2 + \rho} \tag{16}
$$

$$
L_2 = \frac{(1 - \beta_1 - \beta_2) Y_2}{w} \qquad K_{12} = \frac{1}{p} \frac{\beta_1 Y_2}{g_1 + \delta_1 + \rho} \qquad K_{22} = \frac{\beta_2 Y_2}{g_2 + \delta_2 + \rho} \tag{17}
$$

These conditions imply that, for each type of input, the ratio of the factor used in sector 1 relative to that used in sector 2 equals the ratio of nominal outputs, which is constant. One implication is that both L_1 and L_2 are fixed in steady-state, as assumed earlier.

From these equations, we can also derive the steady-state ratios for nominal investment in capital of type 1 relative to nominal investment in capital of type 2, as well as the corresponding ratio for the capital stocks. Letting

$$\mu_i = \frac{\beta_i (g_i + \delta_i)}{g_i + \delta_i + \rho} \tag{18}$$

these ratios are

$$\frac{p (I_{11} + I_{12})}{I_{21} + I_{22}} = \frac{\mu_1}{\mu_2} \tag{19}$$

$$\frac{p (K_{11} + K_{12})}{K_{21} + K_{22}} = \frac{\beta_1}{\beta_2} \frac{g_2 + \delta_2 + \rho}{g_1 + \delta_1 + \rho} \tag{20}$$

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where equation (19) uses the fact that $I_{ij} = (g_i + \delta_i) K_{ij}$ in steady-state.

Output: Combining the resource constraints, (1) and (2), with the first-order conditions for capital accumulation, (16) and (17), and the fact that $I_{ij} = (g_i + \delta_i) K_{ij}$ in steady-state, we obtain the following expressions for real output in the two sectors:

$$
\begin{aligned}
Y_1 &= C_1 + \mu_1 Y_1 + \frac{\mu_1}{p} Y_2 \\
Y_2 &= C_2 + p\mu_2 Y_1 + \mu_2 Y_2
\end{aligned}
$$

Combined with (15) these formulae can be re-arranged to obtain the share of consumption goods in the output in each sector:

$$
\begin{aligned}
\frac{C_1}{Y_1} &= \frac{1 - \mu_1 - \mu_2}{1 - \mu_2 + \frac{\alpha_2 \beta_1}{\alpha_1}} \\
\frac{C_2}{Y_2} &= \frac{1 - \mu_1 - \mu_2}{1 - \mu_1 + \frac{\alpha_1 \beta_2}{\alpha_2}}
\end{aligned}
$$

Note these equations, together with the constancy of the ratios of capital of type i used in sector 1 to capital of type i used in sector 2, implied by (16) and (17), confirm our earlier assumption (made when deriving the steady-state growth rates) that the fraction of each sector's output sold to households, to Sector 1, and to Sector 2, are all constant.

Finally, we can combine these expressions with equation (15) to obtain the ratio of nominal output in sector 1 to nominal output in sector 2:

$$
\frac{pY_1}{Y_2} = \frac{1 - \mu_2 + \frac{\alpha_2 \beta_1}{\alpha_1}}{1 - \mu_1 + \frac{\alpha_1 \beta_2}{\alpha_2}} \frac{pC_1}{C_2} = \left(\frac{1 - \mu_2 + \frac{\alpha_2 \beta_1}{\alpha_1}}{1 - \mu_1 + \frac{\alpha_1 \beta_2}{\alpha_2}} \right) \frac{\mu_1}{\beta_1} \frac{\alpha_1}{\alpha_2} = \theta \tag{21}
$$

2.4 Calibration

To use the model for empirical applications, we need values for the following eight parameters: a_1, a_2, β_1, β_2, δ_1, δ_2, ρ, and α_1/α_2 (only the ratio of these two parameters matters). The depreciation rates are fixed a priori at $\delta_1 = 0.13$ and $\delta_2 = 0.03$, on the basis of typical values used to construct the NIPA capital stocks for durable equipment and structures.[7] The values of the other parameters are set so that the

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[7]See Katz and Herman (1997) for a description of these depreciation rates.

model's steady-state growth path features important variables matching their average values over the period 1957-99, where this sample was chosen to match the start of the pattern of declining relative prices for durable goods:

- A ratio of nominal consumption of durables to nominal consumption of non-durables and services of 0.15.

- A ratio of nominal output of durables to other nominal business sector output of 0.27.

- A growth rate of real durables output per hour worked of 3.9 percent.

- A growth rate of real other business sector output per hour worked of 1.8 percent.[8]

- An average share of income earned by capital of 0.31.

- A rate of return on capital of 6.5 percent, as suggested by King and Rebelo (2000).

In terms of the model's parameters these conditions imply the following six equations:

$$\frac{\alpha_1}{\alpha_2} \left(\frac{\frac{(1-\beta_2)a_1+\beta_2 a_2}{1-\beta_1-\beta_2} + \delta_1}{\frac{(1-\beta_2)a_1+\beta_2 a_2}{1-\beta_1-\beta_2} + \delta_1 + \rho} \right) = 0.15 \tag{22}$$

$$\theta = 0.27 \tag{23}$$

$$\frac{(1-\beta_2)\,a_1 + \beta_2 a_2}{1 - \beta_1 - \beta_2} = 0.039 \tag{24}$$

$$\frac{\beta_1 a_1 + (1-\beta_1)\,a_2}{1 - \beta_1 - \beta_2} = 0.018 \tag{25}$$

$$\beta_1 + \beta_2 = 0.31 \tag{26}$$

$$\frac{\beta_1 a_1 + (1-\beta_1)\,a_2}{1 - \beta_1 - \beta_2} + \rho = 0.065 \tag{27}$$

These solve to give parameter values $a_1 = 0.03$, $a_2 = 0.009$, $\beta_1 = 0.145$, $\beta_2 = 0.165$, $\rho = 0.047$, and $\frac{\alpha_1}{\alpha_2} = 0.19$.

We will now use these parameter values to discuss some empirical applications of the model.

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[8] All per-hour calculations in this paper refer to total business sector hours.

3 Fitting Aggregate Data

We have described how each of our sectors grows over time, how they price their output, and how they split this output between consumption and investment goods. In this section, we consider the combined behavior of the two sectors: How do *aggregate* real variables behave?

3.1 Unbalanced, Unsteady Growth?

Because the two sectors grow at steady but different rates, we have called our solution a steady-state growth path. However, it is clearly not a *balanced* growth path in the traditional sense. Indeed, according to the usual theoretical definition of aggregate real output as the sum of the real output in the two sectors, aggregate growth also appears to be highly *unsteady*. The growth rate will tend to increase each period with sector 1 tending to become "almost all" of real output. Similarly, we should expect to see the growth rates for the aggregates for consumption and investment also increasing over time, as the durable goods components become ever larger relative to the rest.

To illustrate this pattern, consider what we usually mean when we say that aggregate real output is the sum of the real output of the two sectors. If our economy produces four apples and three oranges, we don't mean that real GDP is seven! Rather, we start with a set of prices from a base year and use these prices to weight the quantities. Recalling our assumption that the price of output in sector 2 always equals one, we can construct such a measure by choosing a base year, b, from which to use a price for sector 1's output:

$$
\begin{aligned}
Y_t^{fw} &= p_b Y_{1t} + Y_{2t} \\
&= \left(p_0 e^{(g_2 - g_1)b} \right) \left(Y_{10} e^{g_1 t} \right) + Y_{20} e^{g_2 t} \\
&= p_0 Y_{10} e^{g_2 b} e^{g_1 (t - b)} + Y_{20} e^{g_2 t}
\end{aligned}
\tag{28}
$$

The growth rate of this so-called "fixed-weight" measure of real output will tend to increase every period, asymptoting towards g_1. And the time path of this acceleration will depend on the choice of base year: The further back in time we choose b, the faster Y_t^{fw} will grow. Intuitively, this occurs because the fixed-weight series has the interpretation "how much period t's output would have cost had all prices

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remained at their year-b level" and the percentage change in this measure must depend on the choice of base year, b. Because the fastest growing parts of the output bundle (durable goods) were relatively more expensive in 1960 than in 1990, the cost of the this bundle in 1960 prices must grow faster than the cost in 1990 prices.[9]

This problem of unsteady and base-year-dependent growth is not just a prediction of our model; it is an important pattern to which national income accountants have had to pay a lot of attention. In the past, the Department of Commerce, which publishes the U.S. NIPA data, dealt with this problem by moving the base year forward every five years. This re-basing meant that current-period measures of real GDP growth could always be interpreted in terms of a recent set of relative price weights. However, this approach also had problems. For example, while 1996 price weights may be useful for interpreting 1999's growth rate, they are hardly a relevant set of weights for interpreting 1950's growth rate, given the very different relative price structure prevailing then. So, re-basing improves recent measures of growth at the expense of worsening the measures for earlier periods. In addition, periodic re-basing leads to a pattern of predictable downward revisions to recent estimates of real GDP growth.

3.2 Steady-State Growth: Chain-Aggregates

Because of the problems with fixed-weight measures, the U.S. Commerce Department abandoned this approach in 1996. Instead, it now uses a so-called chain index method to construct all real aggregates, including real GDP. Rather than using a fixed set of price weights, chain indexes continually update the relative prices used to calculate the growth rate of the aggregate. Since the growth rate for each period is calculated using relative price weights prevailing close to that period, chain-weight measures of real GDP growth do not suffer from the interpretational problems of fixed-weight series, and do not need to re-calculated every few years. The "base year" for chain-aggregates is simply the year chosen to equate the real and nominal series, with the level of the series obtained by "chaining" forward and backward

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[9]This pattern, known as substitution bias, is the quantity version of the well known problem with fixed-weight price indexes: For example, much of the recent research on the bias in the Consumer Price Index has focused on how the index's fixed expenditure weights tend to overstate price inflation by understating the role of new goods with declining relative prices. See Boskin et al (1998).

from there using the index; the choice of base year does not affect the growth rate for any period.[10]

The specific index used by the NIPAs is the "ideal chain index" pioneered by Irving Fisher (1922). Technically, the Fisher index is constructed by taking a geometric average of the gross growth rates of two separate fixed-weight indexes, one a Paasche index (using period t prices as weights) and the other a Laspeyres index (using period $t-1$ prices as weights). In practice, however, the Fisher approach is well approximated by a Divisia index, which weights the growth rate of each category by its current share in the corresponding nominal aggregate. The stability of nominal shares, documented earlier, implies that unlike fixed-weight series, chain-aggregates do not place ever-higher weights on the faster growing components, and will grow at a steady rate over the long run, even if one of the component series continually has a higher growth rate than the other.

The switch to a chain-aggregation methodology has important implications for the analysis of U.S. macroeconomic data. Table 1 illustrates that there are very important differences between these series and their fixed-weight counterparts. It shows the growth rates of the chain-weight and 1992-based fixed-weight aggregates for GDP, investment, and consumption over the period 1992-98.[11]

For every year after 1992, each of the fixed-weight series grows faster than the corresponding chained series, with these differences increasing over time in a nonlinear fashion. This pattern is most apparent for investment because durable equipment is a larger component of that series, so the relative price shift between equipment and structures is more important. For 1998, six years after the base year, the chain-aggregate for investment grows 11.4 percent while the 1992-based fixed-

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[10]Chain-aggregated levels need to interpreted carefully. For example, because the level of chain-aggregated real GDP is not the arithmetic sum of real consumption, real investment, and so on, one cannot interpret the ratio of real investment to real GDP as "the share of investment in real GDP" because this ratio is not a share: The sum of these ratios for all categories of GDP does not equal one. See Whelan (2000) for more discussion of the properties of chain-aggregated data.

[11]The fixed-weight figures shown in this table are unpublished estimates obtained from the Department of Commerce's STAT-USA website. Earlier estimates, going through 1997, were published as Table 8.27 of Department of Commerce (1998). In this table, I have used 1992-based data, which pre-date the 1999 comprehensive revision to the NIPAs, to better illustrate how the chain- and fixed-weight series differ as we move away from the base year. However, all other data used in the paper are current as of September, 2000 and use a base year of 1996.

weight series grows 22.5 percent. The difference between fixed- and chain-weight measures of real GDP, as we move away from the base year for the fixed-weight calculation, are also quite notable. For 1997 and 1998, the fixed-weight measure of GDP grows 5.2 percent and 6.6 percent, while the corresponding chain series grows at a steady 3.9 percent pace.

These figures provide an important lesson for those looking to match theoretical models with the published macroeconomic data. The current measure of real GDP behaves very differently from measures based on summing the underlying component real series. As a result, we cannot expect theoretical models that use a concept of real output based on summing real consumption and real investment to replicate the properties of the published data.

In contrast, our two-sector model is well-suited to replicating the behavior of the NIPA chain-aggregates. From (15), (19), (20), and (21), we can derive the share of durable goods in nominal consumption, investment, capital stock, and output as functions of our model's parameters. Using the Divisia approximation, our model predicts the steady-state growth rates for the chain-aggregates for output, consumption, investment, and the capital stock (all on a per hour basis) should be as follows:

$$g_Y = \frac{\theta}{1+\theta}g_1 + \frac{1}{1+\theta}g_2 \tag{29}$$

$$g_C = \frac{\alpha_1\left(\frac{g_1+\delta_1}{g_1+\delta_1+\rho}\right)}{\alpha_1\left(\frac{g_1+\delta_1}{g_1+\delta_1+\rho}\right)+\alpha_2}g_1 + \frac{\alpha_2}{\alpha_1\left(\frac{g_1+\delta_1}{g_1+\delta_1+\rho}\right)+\alpha_2}g_2 \tag{30}$$

$$g_I = \frac{\frac{\beta_1(g_1+\delta_1)}{g_1+\delta_1+\rho}}{\frac{\beta_1}{g_1+\delta_1+\rho}+\frac{\beta_2}{g_2+\delta_2+\rho}}g_1 + \frac{\frac{\beta_2(g_2+\delta_2)}{g_2+\delta_2+\rho}}{\frac{\beta_1}{g_1+\delta_1+\rho}+\frac{\beta_2}{g_2+\delta_2+\rho}}g_2 \tag{31}$$

$$g_K = \frac{\frac{\beta_1}{g_1+\delta_1+\rho}}{\frac{\beta_1}{g_1+\delta_1+\rho}+\frac{\beta_2}{g_2+\delta_2+\rho}}g_1 + \frac{\frac{\beta_2}{g_2+\delta_2+\rho}}{\frac{\beta_1}{g_1+\delta_1+\rho}+\frac{\beta_2}{g_2+\delta_2+\rho}}g_2 \tag{32}$$

We have calibrated the model to match the average share of durable goods in nominal consumption and output, and also the average real growth rates of our two sectors. Thus, essentially by design, the model's predicted steady-state values for growth in chain-aggregated output, consumption, and investment match their empirical averages closely (see Table 2). The model's steady-state growth

rate for output per hour exactly matches the published value of 2.3 percent, and the predicted values for consumption and investment are one-tenth different from the empirical averages of 2.2 percent and 3.1 percent respectively. The steady-state growth rate for the chain-aggregated capital stock (per hour) is 2.3 percent, a good deal higher than the empirical average value of 1.7 percent. Given that the composition of the capital stock adjusts slowly, empirical averages for this variable may be less likely to correspond to a long-run growth path. Using a slightly longer sample starting in 1948, this average value increases to 2.0 percent, closer to our predicted steady-state value.

Another result worth noting here is that our calibrated steady-state values for g_K and g_Y are both 2.3 percent. This prediction of a constant steady-state capital-output ratio may seem familiar from the Solow-Ramsey model but, in this case, it is simply due to a coincidence of coefficients taking the right values rather than being a fundamental feature of the model's steady-state growth path.

4   Implications for Empirical Macroeconomics

We have seen that a two-sector model fits some important facts about the long-run evolution of the U.S. economy that are inconsistent with a one-sector approach, and that it can be used to model the current chain-aggregated measures of real output, consumption, investment, and capital stock. In this section, we explore some of the model's other implications for empirical macroeconomics. In particular, we consider the potential pitfalls in assuming the current U.S. macroeconomic real aggregates are fixed-weight series generated by a one-sector growth model.

4.1   Calibration of Business Cycle Models

While general equilibrium business cycle analysis has now moved well beyond merely adding stochastic technology shocks to the basic neoclassical growth model, with modern treatments incorporating variable utilization, more complex preferences, and other elements, it remains the case that the long-run growth paths for most models in this literature conform to the balanced growth assumption of the Ramsey model.[12] Thus, the preference and technology parameters for these models tend to

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[12]See King and Rebelo (2000) for an excellent treatment of current developments in this area.

be calibrated by ensuring that, over the long-run, the models feature the traditional "great ratios," like the real consumption-output ratio or the real capital-output ratio, conforming to their sample averages.[13] However, our model predicts that, in the long run, the real aggregates for output, consumption, investment, and the capital stock, as currently measured, can each tend to grow at different rates, reflecting their different mixes of durable goods and other output. As a result, these ratios are unlikely to be stationary, a prediction confirmed by the statistical tests presented earlier. An important implication of this is that average values of these ratios taken from a particular sample will have little meaning, so there is little reason to calibrate model parameters to match these empirical averages.

4.2 Aggregate Investment Regressions

Most macroeconomic regression equations for investment are derived from the assumption that there is one type of capital good, which depreciates at a constant rate. In this case, the ratio of aggregate real investment to the lagged aggregate real capital stock, $\frac{I_t}{K_{t-1}}$, summarizes the growth rate of the capital stock. For this reason, the aggregate $\frac{I_t}{K_{t-1}}$ ratio has been the most commonly used dependent variable in macroeconomic investment regressions.[14] Our model suggests that, if applied to current NIPA data, such regressions would be mis-specified. The problem, of course, is that not all types of capital are identical. In particular, our model captures two crucial differences between equipment and structures: The stock of equipment grows faster than the stock of structures, and equipment depreciates significantly faster.

From equations (18), (19) and (20), our model predicts that, because $g_1 + \delta_1$ is greater than $g_2 + \delta_2$, equipment should make up a higher share of total nominal investment than of the total nominal capital stock. This pattern is confirmed by the data: Over 1957-99, equipment accounted for 47 percent of nominal investment, and 19 percent of the nominal capital stock. Because it places a higher weight on the fast-growing asset (equipment), the growth rate of chain-aggregated investment will be higher than the growth rate of the chain-aggregated capital stock. Thus, the aggregate series for $\frac{I_t}{K_{t-1}}$ as currently measured will tend to grow without bound,

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[13]Cooley and Prescott (1995) is a useful guide to how these models are calibrated.

[14]See, for example, Blanchard, Rhee, and Summers (1994), Hayashi (1982), and Oliner, Rudebusch, and Sichel (1995). A notable counter-example that treats multiple capital goods explicitly is Hayashi and Inoue (1991).

making it a very poor proxy for g_K(which is constant along the steady-state growth path).

For this reason, investment regressions using this dependent variable - whether of the traditional form in which the explanatory variables are the growth rate of output and the real cost of capital, or the alternative form using Tobin's Q - will be mis-specified.[15] In contrast, one can easily re-arrange the first-order conditions for profit maximization to obtain separate specifications for equipment and structures, in which $\frac{I_t}{K_{t-1}}$ is a function of chain-aggregated output growth and the growth rate of the asset-specific real cost of capital.

4.3 Technological Progress and Growth

Solow's (1956, 1957) pioneering work on growth not only highlighted the crucial role played by technological progress in determining the economy's steady-state growth rate, but also showed how technological progress could be measured. In the one-sector model, the long-run growth rate of the economy is $\frac{a}{1-\beta}$, where a is the growth rate of aggregate technological progress—also known as Total Factor Productivity (TFP)—and β is capital's share of income: The scaling factor $\frac{1}{1-\beta}$ measured the extent to which the induced effect on capital accumulation adds to the direct effect on production of technological progress. Empirically, TFP growth is calculated by subtracting a weighted average of the growth of inputs from aggregate output growth, where the weights correspond to the share of nominal income generated by that input.

Our two-sector model suggests a more complex relationship between aggregate output growth and TFP growth. Some simple algebra shows that, using chain-aggregated real output growth, our model predicts steady-state TFP growth is a weighted average of the growth rate of technological progress in each sector, where the weights are the sector's share in nominal output:

$$\frac{\dot{TFP}}{TFP} = \frac{\theta}{1+\theta}a_1 + \frac{1}{1+\theta}a_2 \tag{33}$$

The one-sector model predicts that g_Y should average $\frac{1}{1-\beta_1-\beta_2}\frac{\dot{TFP}}{TFP}$, but this prediction does not hold in the two-sector case. Using the calibrated parameters, the

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[15]See Tevlin and Whelan (2000) for a further discussion of this issue.

steady-state value for $\frac{\dot{TFP}}{TFP}$ is 1.4 percent. Scaling this up by $\frac{1}{1-\beta_1-\beta_2}$, we get 2.1 percent, while the steady-state growth rate for output is 2.3 percent.

Where does the one-sector model's prediction go wrong? Inserting equations (9) and (10) into the formula for g_Y, we get

$$
\begin{aligned}
g_Y &= \frac{\theta}{1+\theta}\left[\frac{(1-\beta_2)\,a_1+\beta_2 a_2}{1-\beta_1-\beta_2}\right] + \frac{1}{1+\theta}\left[\frac{\beta_1 a_1+(1-\beta_1)\,a_2}{1-\beta_1-\beta_2}\right] \\
&= \frac{1}{1-\beta_1-\beta_2}\left[\frac{\beta_1+(1-\beta_2)\,\theta}{1+\theta}a_1 + \frac{\beta_2\theta+(1-\beta_1)}{1+\theta}a_2\right]
\end{aligned}
\tag{34}
$$

This equation gives a more complex picture of the relationship between technological progress and aggregate output growth: With two types of technological progress, there are two separate direct effects on output, and with two capital goods used in two sectors, there are four separate induced accumulation effects. So, while the durable goods sector is a relatively small part of total nominal output (θ is relatively small), equipment is a relatively important input into the production process (β_1 is nearly as big as β_2). For this reason, the faster pace of technological progress in the production of durables has more effect on g_Y than on $\frac{\dot{TFP}}{TFP}$. Indeed, in contrast to the one-sector rule-of-thumb in which an extra percentage point of technological progress translates into an extra 1.5 ($\approx \frac{1}{1-\beta_1-\beta_2}$) percentage points of aggregate output growth, plugging our parameter values into (33) and (34), we find that an extra percentage point of a_1-type technological progress has twice as large an effect on output growth as on TFP growth, while an extra point of a_2-type technological progress has only 1.3 times the effect on output growth as on TFP growth.

More generally, our parameter values imply that 58 percent of the long-run growth in business sector output per hour (as currently measured) can be attributed to technological progress in the production of durable goods (a_1), with 35 percentage points of this coming from the direct effect on the production of durables, and the other 23 points representing the induced effect of this technological progress on the production of other goods and services.[16] In contrast, of the 42 percent of

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[16] A similar calculation has been presented by Greenwood, Hercowitz, and Krusell (1997). They also concluded that 58 percent of growth was due to "investment-specific" technological progress, which could be identified with technological progress in the production of producers' durable equipment. However, they used a different measure of aggregate real output, obtained by deflating aggregate nominal output by the price of consumer nondurables and services. This measure of real output will tend to grow at rate g_2, the growth rate of the sector producing nondurables, services,

long-run growth due to technological progress in the production of nondurables, services, and structures (a_2), only 2 percentage points represents induced effects on the production of durable goods.

5   Conclusion

This paper has argued that, despite its central role in economics textbooks and in business cycle research, the one-sector model of economic growth provides a poor description of the long-run evolution of the U.S. economy. In particular, the model's central prediction, that the real aggregates for consumption, investment, output, and the capital stock, should all grow at the same rate over the long run, is firmly rejected by postwar U.S. data.

The reason for the failure of the balanced growth hypothesis is the model's inability to distinguish the behavior of the durable goods sector from that of the rest of the economy. The real output of the durable goods sector has consistently grown faster than the rest of the economy, and most investment spending is on durable goods while most consumption spending is on nondurables and services. As a result, real investment has tended to grow faster than real consumption, a pattern that has been particularly evident since 1991. These patterns are easily modelled using the simple two-sector approach of this paper.

The special behavior of the durable goods sector has also had important implications for the measurement of real GDP and the interpretation of data from the National Income and Product Accounts. Because of the large relative price shifts associated with the faster pace of technological progress in the durable goods sector, the major U.S. real aggregates are now constructed using a chain-index methodology, which relies on continually updated relative price weights. An important conclusion of this paper is that treating these data as if they are fixed-weight aggregates generated by a one-sector model can often result in incorrect conclusions.

and structures. Thus, a comparable calculation is not our figure of 58 percent, but rather the contribution of a_1 to g_2, which is 36 percent according to our parameter values. The difference between these estimates is primarily due to the use of a different series for the relative price of durable goods. Greenwood, Hercowitz, and Krusell used Gordon's alternative price deflator for durable equipment but used the NIPA deflator for consumer nondurables and services. As noted above, such a measure likely overstates the relative price decline for durable goods by ignoring price mismeasurement for nondurables and services.

In contrast, the two-sector approach developed in this paper can be easily used to model the current data.

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A   Derivation of Equation (14)

The first-order conditions for consumption expenditures are

$$\frac{\alpha_1}{D_t} + \frac{1}{1+\rho} E_t \left[\frac{\partial V_{t+1}}{\partial D_t} \right] = \frac{1}{1+\rho} E_t \left[p_t \left(1 + r_{t+1}\right) \frac{\partial V_{t+1}}{\partial Z_{t+1}} \right] \tag{35}$$

$$\frac{\alpha_2}{C_{2t}} = \frac{1}{1+\rho} E_t \left[\left(1 + r_{t+1}\right) \frac{\partial V_{t+1}}{\partial Z_{t+1}} \right] \tag{36}$$

while the envelope conditions for the value function are

$$\frac{\partial V_t}{\partial D_{t-1}} = \left(1 - \delta_1\right) \frac{\alpha_1}{D_t} + \frac{1 - \delta_1}{1+\rho} E_t \left[\frac{\partial V_{t+1}}{\partial D_t} \right] \tag{37}$$

$$\frac{\partial V_t}{\partial Z_t} = \frac{1}{1+\rho} E_t \left[\left(1 + r_{t+1}\right) \frac{\partial V_{t+1}}{\partial Z_{t+1}} \right] \tag{38}$$

From (35) and (36), we know that

$$\frac{\alpha_1}{D_t} + \frac{1}{1+\rho} E_t \left[\frac{\partial V_{t+1}}{\partial D_t} \right] = \frac{p_t \alpha_2}{C_{2t}} \tag{39}$$

Thus, (37) can be re-expressed as

$$\frac{\partial V_t}{\partial D_{t-1}} = \left(1 - \delta_1\right) \left(\frac{p_t \alpha_2}{C_{2t}} \right)$$

Shifting this equation forward one period and taking the expectation we get

$$E_t \left[\frac{\partial V_{t+1}}{\partial D_t} \right] = \left(1 - \delta_1\right) E_t \left(\frac{p_{t+1} \alpha_2}{C_{2,t+1}} \right)$$

Inserting this into (39) and re-arranging we get

$$\frac{\alpha_1}{D_t} = \frac{p_t \alpha_2}{C_{2t}} - \frac{1 - \delta_1}{1+\rho} E_t \left(\frac{p_{t+1} \alpha_2}{C_{2,t+1}} \right)$$

which becomes

$$\frac{\alpha_1}{\alpha_2} \frac{C_{2t}}{p_t D_t} = 1 - \frac{1 - \delta_1}{1+\rho} E_t \left(\frac{C_{2t}}{C_{2,t+1}} \frac{p_{t+1}}{p_t} \right)$$

Using our formulae for steady-state growth rates of consumption and relative prices, this becomes

$$\frac{\alpha_1}{\alpha_2} \frac{C_{2t}}{p_t D_t} = 1 - \frac{\left(1 - \delta_1\right)\left(1 + g_2 - g_1\right)}{\left(1+\rho\right)\left(1 + g_2\right)} \approx g_1 + \delta_1 + \rho$$

as required.

	GDP		Consumption		Investment	
	Chain-	Fixed-	Chain-	Fixed-	Chain-	Fixed-
1992	2.7	2.7	2.8	2.7	5.7	5.5
1993	2.3	2.4	2.9	3.0	7.6	7.7
1994	3.5	3.6	3.3	3.4	8.6	9.0
1995	2.3	2.7	2.7	2.9	5.5	7.2
1996	3.4	4.1	3.2	3.6	8.8	11.2
1997	3.9	5.2	3.4	4.1	8.3	12.7
1998	3.9	6.6	4.9	6.1	11.4	22.5

Table 1: 1992-Dollar Chain- and Fixed-Weight Growth Rates for Major Aggregates
Source: Department of Commerce STAT-USA Website (www.stat-usa.gov)

	Steady-State Values	Average, 1957-99
Output Per Hour	2.3	2.3
Consumption Per Hour	2.1	2.2
Investment Per Hour	3.0	3.1
Capital Stock Per Hour	2.3	1.7

Table 2: Growth Rates for Chain-Aggregates
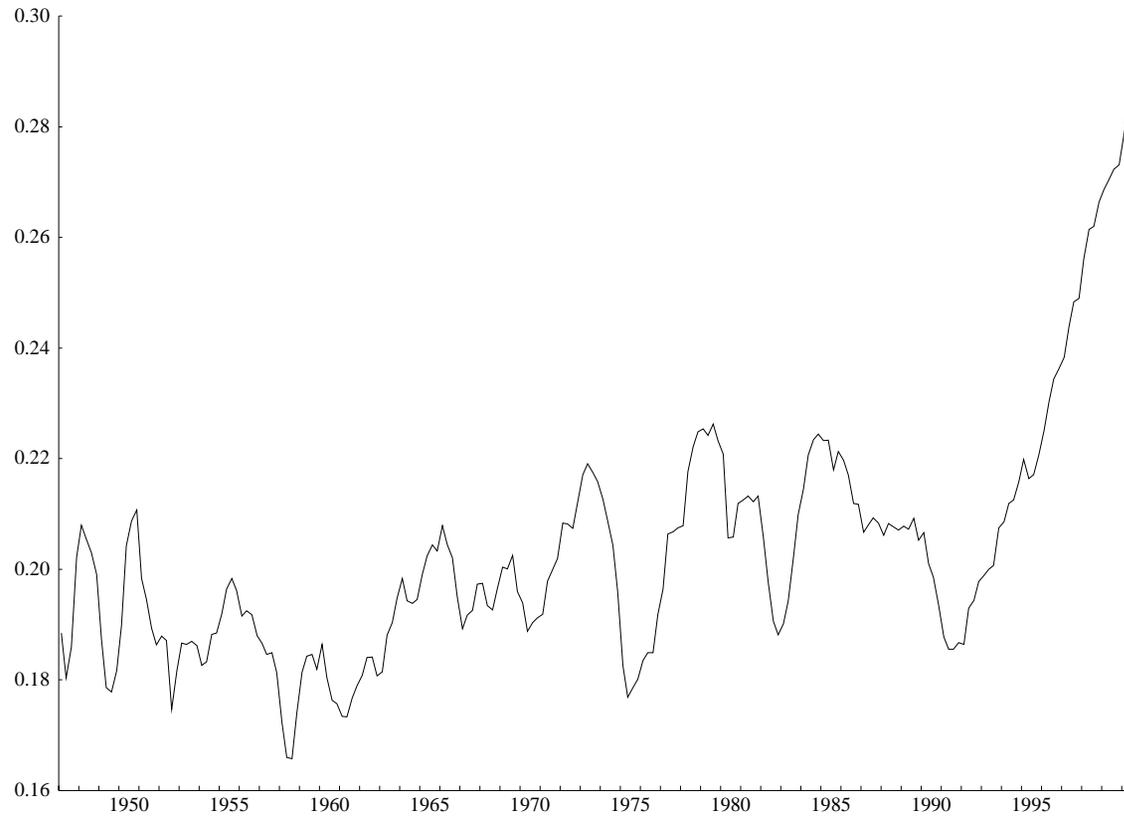
Figure 1
Ratio of Real Private Fixed Investment to Real Consumption

Figure 2a
Ratios of Major Components of Investment and Consumption

Real Equipment Investment / Real Structures Investment →
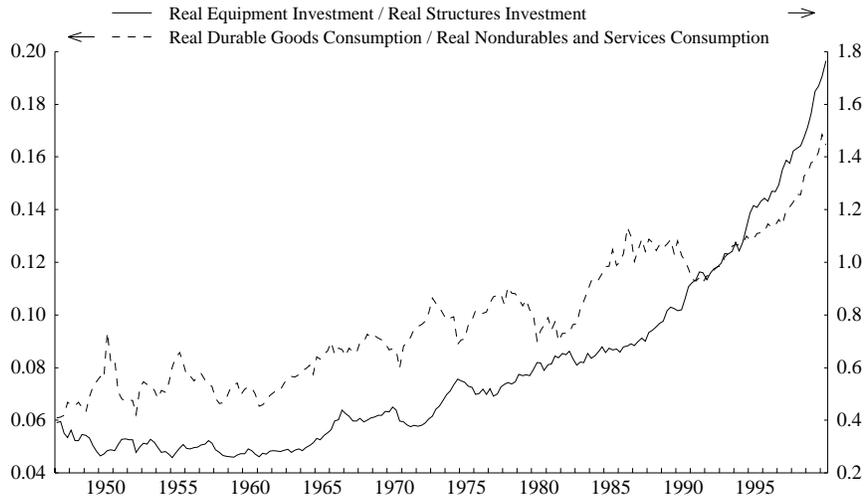← Real Durable Goods Consumption / Real Nondurables and Services Consumption

Figure 2b
Ratio of Total Real Durables Output to Real Business Output

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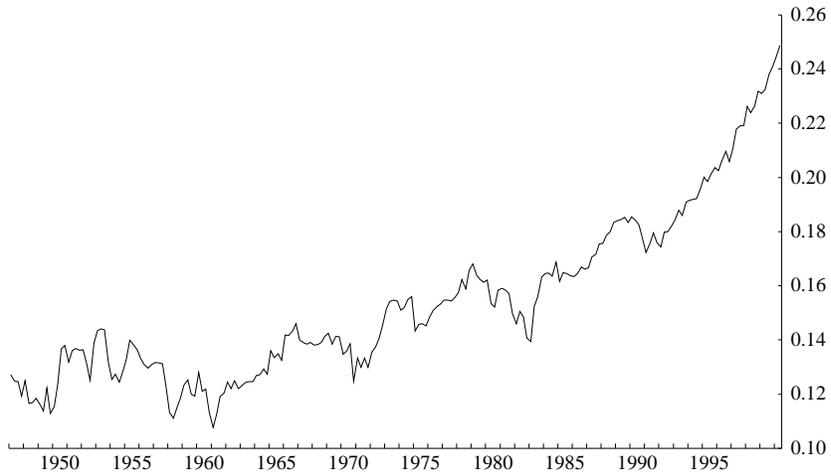
Figure 3a
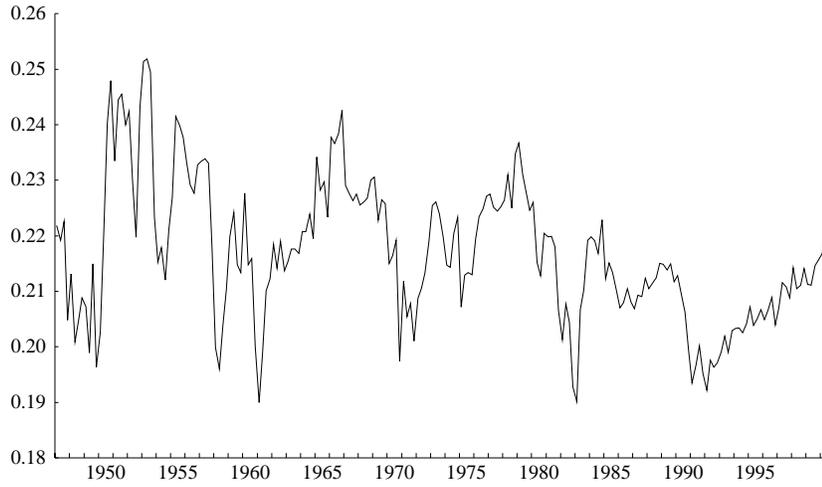**Ratio of Total Nominal Durables Output to Nominal Business Output**

Figure 3b
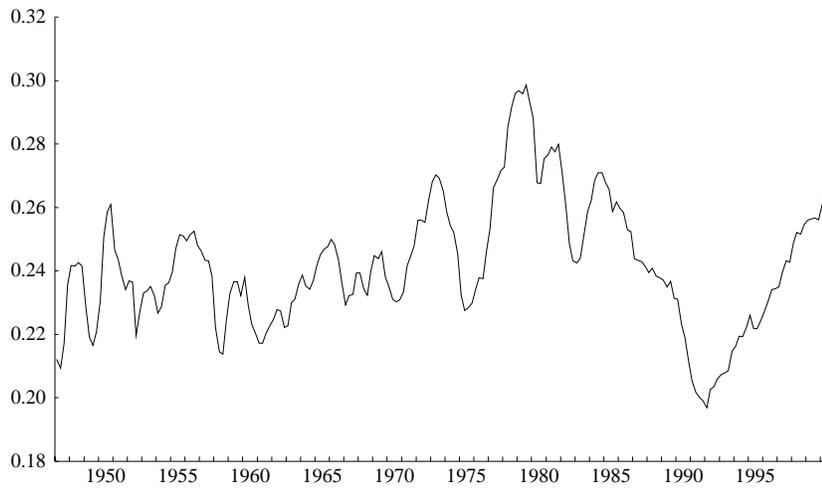**Ratio of Nominal Private Fixed Investment to Nominal Consumption**

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Figure 4
Relative Prices of Durables and Fixed Investment
(In Logs, All Prices Relative to Price of Consumer Nondurables and Services)

All Durable Goods

Producers' Durable Equipment

Fixed Investment

Structures