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Rotor Speed-free Estimation of the Frequency of the Center of Inertia

Federico Milano, IEEE Fellow

Abstract—This letter proposes a formula to estimate the frequency of the center of inertia based exclusively on measures of bus frequencies, obtained, for example, from phasor measurement units; the network admittance matrix; and two parameters of synchronous generators, namely, the inertia constant and the internal reactance. The proposed formula can be utilized on-line and requires a highly reduced set of measures of bus frequencies. The letter discusses the theoretical background of the proposed expression and tests it with a 1,479-bus model of the all-island Irish transmission system.

Index Terms—Frequency estimation, center of inertia, frequency divider, phasor measurement unit.

I. INTRODUCTION

The Center of Inertia (COI) is a well-known concept utilized in transient stability analysis of power systems [1]–[3]. The frequency of the COI is the common synchronous frequency to which generators tend in stead-state conditions. In simulations, referring machine speed deviations to the frequency at some relevant (e.g., pilot) bus of the system.

In practical applications, the frequency of the COI cannot be estimated because its calculation requires the availability of the measures of all synchronous machine rotor speeds. System operators do not estimate the frequency of the COI on-line but, rather, measure the frequency at some relevant (e.g., pilot) bus of the system. The behavior of the frequency of the pilot bus, however, does not represent the average frequency of the system as it follows the dynamics of the closest synchronous generators.

In recent years, the ability of measuring the frequency at buses has been dramatically boosted by the development of Phasor Measurement Unit (PMU) devices [5]. Actually, any device equipped with a phase-lock loop, such as power electronic converters of distributed energy resources, can accurately estimate the frequency of the voltage phasor at the point of connection [6]. Then there is a generalized trend to connect every device to the internet, which is, in turn, one of the most innovative aspects of the smart grid.

It can be thus safely assumed that bus frequency measures will be ubiquitous and easy to collect in the near future. How to meaningfully use such huge amount of information, however, is still not fully clear. This letter proposes an application for PMUs and discusses how to estimate through a simple yet accurate formula the frequency of the COI using exclusively bus frequency measures.

II. THEORY

Let us consider an interconnected ac grid with n buses and m synchronous machines. The starting equations are the definition of the angular speed of the COI and the frequency divider (FD) formula presented in [7]. The expression of the speed of the COI is:

\[ \omega_{COI} = h^T \omega_G, \]  

where \( \omega_G \) is a \( m \times 1 \) vector of synchronous machine rotor speeds and \( h \) is a \( m \times 1 \) vector of normalized inertia constants, i.e., the i-th element of \( h \) is:

\[ h_i = H_i / H_T, \]

where \( H_i \) is the inertia of the i-th machine and \( H_T = \sum_{j=1}^{m} H_j \).

The FD formula presented in [7] is as follows:

\[ \omega_B - \omega_G = D(\omega_G - \omega_{m,1}), \]

where

\[ D = -(B_{BB} + B_{BS})^{-1} B_{BG}, \]

where \( \omega_B \) is a \( n \times 1 \) vector of the frequencies at system buses; \( B_{BB} \) is the \( n \times n \) network susceptance matrix, i.e., the imaginary part of the standard network admittance matrix; \( B_{BG} \) is the \( n \times m \) matrix obtained using the stator and step-up transformer impedances of the synchronous machines; and \( B_{BS} \) is a \( n \times n \) diagonal matrix that accounts for the internal susceptances of the synchronous machines at generator buses. Equation (3) can be utilized also with external networks assuming that one can measure the frequency at the point of connections and knows the equivalent Tevenin impedances and inertias of these networks. The complete set of hypothesis and the detailed mathematical derivation of the FD formula are given in [7].

The purpose of this letter is to define an expression where \( \omega_{COI} \) is determined through a linear combination of as few elements as possible of the vector of frequency buses \( \omega_B \). This result is obtained through algebraic manipulations of equations (1) and (3), as follows. Observe that (1) can be rewritten as:

\[ \omega_{COI} = h^T (\omega_G - \omega_{m,1}), \]

in fact, (1) derives directly from (2) which implies that \( h^T \omega_{m,1} = \sum_{i=1}^{m} h_i = 1 \). Then, using the definition of the matrix \( D \) given in (4), (3) can be rewritten as:

\[ (B_{BB} + B_{BS})(\omega_B - \omega_{m,1}) = -B_{BG}(\omega_G - \omega_{m,1}), \]

where \( B_{BG} \) has \( m \) linearly independent columns if each power plant is modeled with a single equivalent machine. It is thus possible to define the Moore-Penrose pseudo-inverse of \( B_{BG} \) as:

\[ B_{BG}^{-1} = (B_{BG}^T B_{BG})^{-1} B_{BG}^T \]

Matrix \( B_{BG}^{-1} \) is the left inverse of \( B_{BG} \), i.e., \( B_{BG}^T B_{BG} = I_m \). The pseudo-inverse \( B_{BG}^{-1} \) provides the least-square solution of (6), where \( \omega_G = \omega_{m,1} \) is the vector of unknowns. Such a solution is unique because \( B_{BG} \) has rank \( m \). Multiplying (6) by \( B_{BG}^{-1} \) and equating (6) to (5) lead to:

\[ \omega_{COI} - 1 = -h^T B_{BG} (B_{BB} + B_{BS}) (\omega_B - \omega_{m,1}) \]

where \( \xi^T = -h^T B_{BG} (B_{BB} + B_{BS}) \) is the sought vector of weights that allows calculating the frequency of the COI from the frequency of the buses \( \omega_B \).

The following remarks are relevant.

- Equation (8) only involves sparse matrix-vector products. Moreover, the product \( (B_{BB}^T B_{BG})^{-1} \) is a diagonal matrix, whose elements are the inverse of the square of the internal reactances of the synchronous machines.

- The elements of \( \xi \) can be viewed as the weights of the measurements \( \omega_B \) for the evaluation of \( \omega_{COI} \). Note also that \( \xi^T \omega_{m,1} \approx 1 \), which derives from the definition of \( h \) and the properties of the rows of matrix \( D \), as discussed in [8].

- A property of \( \xi \) is that a large number of its elements is actually null or \( \ll 1 \). This fact has a relevant practical consequence: only a reduced number of measures of bus frequencies are needed to estimate \( \omega_{COI} \).
The proposed expression to estimate the frequency of the COI is:

$$\omega_{\text{COI}} = \xi^T \omega_B + \alpha$$  \hspace{1cm} (9)

where $\alpha = 1 - \xi^T \mathbf{1}_{n_1}$ is an offset, with $|\alpha| \ll 1$. This equation is valid in time. $\xi$ and, hence, $\alpha$ are piece-wise constant and need to be recomputed only when a topological change occurs, e.g., a line outage, or a synchronous machine is connected to or disconnected from the grid.

The following section provides a numerical appraisal of the formula (9) based on a large real-world power system.

III. CASE STUDY

The Irish Transmission system grid, which has been made available by EirGrid, the Irish TSO, to researchers in the author’s research group, consists of 1,479 buses, 1,851 transmission lines and transformers, 245 loads, 22 conventional synchronous power plants modeled with 6th order synchronous machine models with AVR's and turbine governors, 6 PSSs, and 176 wind power plants, of which 142 are DFIGs and 34 CSWTs. This model provides a dynamic representation of the Irish electrical grid which is topologically accurate and approximates the dynamics of the actual Irish grid. All results shown in this section are obtained using Dome [9].

Expression (9) is computed using a vector of coefficients $\xi$ where the elements $|\xi_i| < 10^{-3} = \varepsilon$ are set to 0. This threshold leads to a vector with 42 non-null elements, which is the 2.8% of the total number of buses of the system. That is: only 42 bus frequencies out of 1,479 needs to be measured, e.g., by means of PMUs devices, to provide an accurate estimation of $\omega_{\text{COI}}$. The accuracy of the estimation provided by (9) is illustrated in Fig. 1, which shows the frequency response following a three-phase fault that occurs at $t = 1$ s and is cleared after 50 ms.

Observe that it would not be accurate to simply use the frequencies $\omega_{B,G}$ measured at the terminal buses where synchronous machines are connected, i.e.,

$$\omega_{\text{COI}} = \mathbf{h}^T \omega_{B,G}.$$  \hspace{1cm} (10)

Figure 2 shows the dynamic evolution of $\omega_{\text{COI}}$ for the same contingency shown in Fig. 1. Clearly, (10) is not able to properly estimate the frequency of the COI in the first seconds after the clearing of the fault. The rationale behind this result, which might appear surprising, is that the frequencies at the terminal buses of the synchronous machines are not exactly equal to the rotor speeds because of the internal reactances of the machines themselves and the topology of the network. In other words, the term $\mathbf{B}_{BG}(\mathbf{B}_{BH}+\mathbf{B}_{HB})$ in (8) is a non-trivial vector and cannot be neglected without introducing a significant estimation error.

While using only $\mathbf{h}$ leads to inconsistent results, one may wonder whether it is possible to obtain accurate results with a reduced number of measures by increasing the threshold $\varepsilon$.

Fig. 1. Three-phase fault for the all-island Irish system – Trajectories of the actual frequency of the COI and an estimated value of $\omega_{\text{COI}}$ obtained using (9) and $\varepsilon = 0.001$.

Fig. 2. Three-phase fault for the all-island Irish system – Trajectories of the actual frequency of the COI and an approximated value ($\hat{\omega}_{\text{COI}}$) obtained using (10).

Observe that it would not be accurate to simply use the frequencies $\omega_{B,G}$ measured at the terminal buses where synchronous machines are connected, i.e.,

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REFERENCES