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<tr>
<td><strong>Authors(s)</strong></td>
<td>McKenna, Killian, Keane, Andrew</td>
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<td><strong>Publication date</strong></td>
<td>2015-06-10</td>
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<tr>
<td><strong>Publication information</strong></td>
<td>McKenna, Killian, and Andrew Keane. “Residential Load Modeling of Price Based Demand Response for Network Impact Studies” 7, no. 5 (June 10, 2015).</td>
</tr>
<tr>
<td><strong>Publisher</strong></td>
<td>IEEE</td>
</tr>
<tr>
<td><strong>Item record/more information</strong></td>
<td><a href="http://hdl.handle.net/10197/7431">http://hdl.handle.net/10197/7431</a></td>
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<tr>
<td><strong>Publisher's version (DOI)</strong></td>
<td>10.1109/TSG.2015.2437451</td>
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Residential Load Modeling of Price Based Demand Response for Network Impact Studies

Kilian McKenna, Student Member, IEEE, Andrew Keane, Senior Member, IEEE

Abstract—This paper presents a comprehensive low-voltage residential load model of price based demand response. High-resolution load models are developed by combing Monte Carlo Markov Chain bottom-up demand models, hot water demand models, discrete state space representation of thermal appliances and composite time-variant electrical load models. Price based demand response is then modeled through control algorithms for thermostatically controlled loads, optimal scheduling of wet appliances and price elasticity matrices for representing the inherent elastic response of the consumer. The developed model is used in a case study to examine the potential distribution network impacts of the introduction of dynamic pricing schemes. The effects of cold load pick-up, rebound peaks, decrease in electrical and demand diversity and impacts on loading and voltage are presented.

Index Terms—Demand response, distribution networks, dynamic pricing, load modeling, residential load sector, smart grids.

I. INTRODUCTION

IMPERATIVE to the success of demand response (DR) schemes is to ensure that their implementation does not adversely affect the operation and control of the networks to which they are connected. Residential DR has been identified as a load sector with high potential, particularly when subjected to dynamic pricing schemes such as time of use pricing (TOUP), day ahead pricing (DAP), real time pricing (RTP) and critical peak pricing (CPP) [1]. These pricing schemes have been identified as providing the potential high-level system benefits that have come to typify demand side management. These include reduced peak demand, forgoing investment in new peaking capacity and network upgrades, facilitating the integration of renewable generation, providing regulating capacity and lowering the required reserve margin and hence reducing inefficiencies in capacity payment markets [2]. These high level system objectives need to be consolidated with the constraints of low-voltage (LV) distribution networks, such as thermal loading and voltage deviation. There is uncertainty around the potential impacts of such pricing schemes on the load diversity metrics for which the planning of traditional distribution networks were based.

Bottom-up demand models allow high resolution simulation of residential energy consumption, incorporating occupancy and consumer behavior characteristics. The models require extensive demographic, socioeconomic, lifestyle and appliance operation data [3]–[5]. The US Department of Energy has developed a residential distribution simulation environment, GridLAB-D, [6], which has been used in the literature, notably in [7] where an equivalent thermal parameter model and time-variant load model of air-conditioning units was coupled with the software. In [8], a sequential Monte Carlo (MC) simulation platform for residential networks was developed with a multi-phase network and a load/generation behavior model. Models of physical based appliances for demand response (DR) studies, such as heating and cooling loads, have also been used [9].

Time-Use Survey (TUS) data has been widely used in the literature for developing consumer occupancy and behavior models [10]–[12]. These data are used to create transition probability matrices for modeling occupancy using a Markov Chain Monte Carlo (MCMC) approach, and also for creating activity probability profiles for modeling behaviors using MC simulations. Lighting models have been developed that include dependence on irradiance levels allowing seasonal variations to be captured [13], [14]. Further development in this area has incorporated these energy demand models with electrical load models to create high-resolution time-variant steady-state load models [15].

There is wide potential for price-based DR in the residential sector, this response comes primarily in two forms; inherent elastic response of the consumer and automatic DR controllers. Price elasticity matrices (PEM), traditionally used under the assumption that demand is continuous with uniform response across all hours, [16], have been used in the literature to assess elastic DR. Time-varying elasticity values of hour resolution have been used to examine the effects of residential DR on distribution networks [17], and statistical demand-price elasticity models have been used to assess the optimal real time price [18].

There has also been much research into developing control algorithms and residential energy optimization frameworks. In [19] an optimal residential energy consumption scheduling framework is proposed with price prediction to manage the trade-off between minimizing the electricity payment and waiting time. The potential of spot price based control algorithms for residential load, in particular scheduling, [20], and thermal storage potentials have been investigated [21]. Discrete time difference equations have been used in the literature to assess...
the potential control of thermostatically controlled appliances (TCAs) for the purposes of providing regulation service, [22], appliance commitment for load scheduling, [23], dynamic controllers for real-time demand response, [24], and generalized control strategies for TCAs in a competitive electricity market [25]. The effect of these different control algorithms on the creation of rebound peaks, and potential solutions using randomized scheduling, maximum household power signals, and concurrent use of multiple pricing schemes to increase diversity have also been investigated [26].

There is uncertainty and concern around how many of the proposed algorithms will work in practice, in particular when the complex interactions between consumer behavior, appliance characteristics, electrical load dependency and the distribution network are taken into account [27]. There is a need to consolidate the detailed bottom-up load modeling research with the proposed control algorithms and pricing structure to see the overall effect on consumer welfare and network operation. Consumer demand is discrete in both magnitude and duration, and can be highly volatile at a sub-hourly time frame. Hence, it is imperative that such studies are conducted at high temporal resolution to capture these effects.

To that end this paper proposes a TUS based bottom-up load modeling platform for price based residential elastic and automatic demand response. The model extends the methodology for capturing the elastic response of consumers by the authors in [28], by addition of thermal modeling of TCAs with hot water demand models, time-variant electrical load models, static and dynamic control algorithms for thermal appliances, optimal scheduling of wet appliances and distribution network load flow and analysis.

The methodology for the load model is presented in Section II with elastic and automatic residential demand response presented in Sections III and IV respectively. Model implementation is presented in Section V and Section VI presents details of the case study. Section VII presents the results and discussion, examining the network impacts and effects on consumer revenue. Finally, Section VIII presents the conclusions of this paper.

II. RESIDENTIAL LOAD MODEL

The residential load model developed in this paper is comprised of modeling consumer occupancy, activity, appliance electrical and thermal operation and consequently the electrical demand.

A. Consumer Activity Modeling

Activity profiles are used for up to six different activities, for up to four occupants, for both weekday and weekend profiles using activity profiles and occupancy transition probability matrices from [29]. Using transition probability matrices from TUS data, occupancy profiles can be generated using MCMC methods [11]. The activity profiles are linked to appliance use, with the sharing of appliances captured both through using occupancy and activity profiles as a function of the number of occupants present resulting in a non-linear increase in probability.

B. Appliance Use

Appliances are assigned to each household based on ownership statistics at initialization. Data is required for each appliance detailing cycle power, duration, standby power and cycles per year, among other parameters [28].

1) General Appliance Model: The general appliance model for bottom-up models is used for the consumer use of cooking, information communication technology (ICT) and consumer electronics (CE) appliances, and wet appliances [13]. For this model the switch-on probability, \( P_a \), of any appliance, \( a \), for any time step, \( t \), is dependent on a number of factors. These are the binary variable \( O(t) \) dependent on the presence of an active occupant, the calibration scalar, \( C_a \), for each appliance which is used to calibrate the number of switch-on events based on appliance data. Finally, the activity probability itself, \( A(t,n) \), which is dependent both on the number of occupants, \( n \), and time. Using these variables the probability of a switch-on event for appliances with an associated activity are determined by (1) and those solely dependent on occupancy are determined by (2), see Section V for further details.

\[
P_a(t) = (O(t) \times C_a \times A(t,n)) \tag{1}
\]

\[
P_a(t) = (O(t) \times C_a) \tag{2}
\]

2) Lighting: The lighting model assigns each household one of 500 possible lighting configurations based on UK statistics on bulb penetration, type and installed wattage, [30], all of which are necessary for the electrical load model. The switch-on probability, \( P_b \), of a bulb, \( b \), can be determined by five factors, see (3), [13]. These are the occupancy binary variable, \( O(t) \), relative-use weighting of the bulb, \( W_b \), the effective occupancy, \( \text{Eff}(t,n) \), a lighting calibration scalar, \( CL \), and a binary irradiance variable, \( \text{Irr}(t) \), based on a household irradiance threshold, an adjustment is made such that 20% of switch-on events are independent of irradiance levels to take account of the significant lighting load used during hours of low irradiance [31]. The use of night lights, which are both independent of occupancy and irradiance levels are modeled separately, with mean start times, durations and penetrations.

\[
P_b(t) = (O(t) \times CL \times \text{Irr}(t) \times W_b \times \text{Eff}(t,n)) \tag{3}
\]

3) Domestic Water Heating: High resolution hot water demand profiles can be created by using washing activity profiles coupled with calibration scalars that represent the frequency of each washing event such as hand washing, showers and baths [32]. The probability of a washing event occurring, \( P_w \), is then a function of occupancy, the number of occupants, \( n \), the probability of a washing activity taking place, \( A(t) \), and the washing event calibration scalar, \( C_{\text{wash−act}} \) (4). Then each household is given a set of discharge rates and duration for each washing event.

\[
P_w(t) = (O(t) \times n(t) \times C_{\text{wash−act}} \times A(t)) \tag{4}
\]

The hot water demand profiles are used as an input into dual-element electric water heater model which can be represented through discrete thermal dynamic equations [22]. For
the domestic hot water (DHW) model, the energy balance equations for the two layers of the tank are described in (5) and (6) respectively where \( c \) is the specific heat capacity of water, \( \rho \) is the density of water, the temperatures and liter capacity of the lower and upper sections are \( T_1, T_2, L_1 \) and \( L_2 \) respectively, \( p \) is the electrical power input with the subscripts denoting the element \((p_{1c}, p_{2c})\), the heat radiation \((p_{a1}, p_{a2})\), the heat transfer from each section \((p_{21})\), the hot water heat demand \((p_{hw})\) and the binary variables controlling the electrical input \((Y_1, Y_2)\). The heat radiation from each section \((i = 1 \text{ or } 2)\) is given in (7), where \( \tau \) is the time constant and \( T_{amb} \) is the ambient temperature.

\[
cp L_1 \frac{dT_1}{dt} = Y_1(t) p_{1c} - p_{a1} - p_{21} \tag{5}
\]
\[
cp L_2 \frac{dT_2}{dt} = Y_2(t) p_{2c} - p_{a2} + p_{21} - p_{hw} \tag{6}
\]
\[
p_{ai} = cm L_i (T_i - T_{amb}) / \tau \tag{7}
\]

The time constant, (8), is calculated by finding the thermal capacitance, \( C_{th} \), and the thermal resistance, \( R_{th} \), of the electric water heater, see (9) and (10) respectively. The thermal conductivity of the insulating material, \( \kappa \), its thickness, \( x \), and the total area, \( A \), are needed to calculate the thermal resistance. For the thermal capacitance, the mass, \( m \), and specific heat capacity, \( c_p \), are needed.

\[
\tau = C_{th} R_{th} \tag{8}
\]
\[
R_{th} = \frac{x}{\kappa A} \tag{9}
\]
\[
C_{th} = mc_p \tag{10}
\]

The power consumption of the electric water heater is controlled thermostatically with a dead-band, \( \delta_a \), around the temperature set point, \( T_{set,a} \). The state of the binary variables, \( Y_1 \) and \( Y_2 \), the hysteretic control of the electrical power input for the two heating elements where 1 is on and 0 is off, are described by (11) and (12) respectively.

\[
Y_1(t+1) = \begin{cases} 
0, & T_1(t) > T_{set,a} + \delta/2 \\
1, & T_1(t) < T_{set,a} - \delta/2 \text{ and } T_2 \leq T_1 \\
Y_1(t), & \text{otherwise} 
\end{cases} \tag{11}
\]
\[
Y_2(t+1) = \begin{cases} 
0, & T_2(t) > T_{set,a} + \delta/2 \\
1, & T_2(t) < T_{set,a} - \delta/2 \text{ and } T_2 > T_1 \\
Y_2(t), & \text{otherwise} 
\end{cases} \tag{12}
\]

4) Other Thermostatically Controlled Appliances: Other TCAs are modeled using a discrete time difference equation representation, (13), which is commonly used in the literature [22], [23]. Here the time step is represented by \( h \), the coefficient of performance is \( COP_a \), equal to 1 for non-heat pumps, for each appliance \( a \), the electrical power input is \( P_{elec,a} \) and \( \theta_a \) is the temperature gain (14). This methodology also uses the thermal capacitance and the thermal resistance of the appliance, requiring data on the thermal mass, insulation material and area of the device.

\[
T_a(t+1) = e^{-h/\tau} T_a(t) + (1 - e^{-h/\tau}) (T_{amb}(t) - Y_a(t) \theta_a) \tag{13}
\]
\[
\theta_a = COP_a P_{elec,a} R_{th,a} \tag{14}
\]

The control strategy for these thermostatically-controlled appliances is similar to that of DHW heaters. Equation (15) is presented for refrigeration appliances, for heating devices the same equations are used but with the important difference of a change in sign of the term \( \theta_a \) in (14) and the status of the binary variables in (15) are reversed.

\[
Y_a(t+1) = \begin{cases} 
0, & T_a(t) < T_{set,a} - \delta/2 \\
1, & T_a(t) \geq T_{set,a} + \delta/2 \\
Y_a(t), & \text{otherwise} 
\end{cases} \tag{15}
\]

C. Electrical Load Model

In order to use the demand model in power system simulations a time-variant steady state load model is implemented using a composite polynomial load model with appliances represented in terms of constant impedance, constant current and constant power loads for active power, \( P \), for each appliance, \( \alpha \), \( Z_{Pa}, I_{Pa} \) and \( P_{Pa} \), and for reactive power \( Z_{Qa}, I_{Qa} \) and \( P_{Qa} \), respectively. Using displacement power factors for each appliance, \( PF_a \), allows the loads to be represented by both their active, \( P_a \), and reactive, \( Q_a \), power demand (20, 21) and aggregate household demands (18, 19). For each house, the aggregate effect of each appliances on the total load model \( P_{Ph}, I_{Ph} \) and \( P_{Ph} \) parameters can be calculated (16, 17) for each time step, as the polynomial household components change depending on what appliances are being operated.

\[
\begin{bmatrix}
Z_{Ph}(t) \\
I_{Ph}(t) \\
P_{Ph}(t)
\end{bmatrix} = \sum_{a=1}^{A} P_a(t) \begin{bmatrix}
Z_{Pa} \\
I_{Pa} \\
P_{Pa}
\end{bmatrix} \tag{16}
\]
\[
\begin{bmatrix}
Z_{Qh}(t) \\
I_{Qh}(t) \\
P_{Qh}(t)
\end{bmatrix} = \sum_{a=1}^{A} Q_a(t) \begin{bmatrix}
Z_{Qa} \\
I_{Qa} \\
P_{Qa}
\end{bmatrix} \tag{17}
\]

\[
P_h(t) = \sum_{a=1}^{A} P_a(t) \tag{18}
\]
\[
Q_h(t) = \sum_{a=1}^{A} Q_a(t) \tan^{-1}(PF_a) \tag{19}
\]

\[
P_h(t) = P_{h,0}(t) \left[ Z_{Ph}(t) \left( \frac{V(t)}{V_0} \right)^2 + I_{Ph}(t) \left( \frac{V(t)}{V_0} \right) + P_{Ph}(t) \right] \tag{20}
\]
\[
Q_h(t) = Q_{h,0}(t) \left[ Z_{Qh}(t) \left( \frac{V(t)}{V_0} \right)^2 + I_{Qh}(t) \left( \frac{V(t)}{V_0} \right) + P_{Qh}(t) \right] \tag{21}
\]

Each appliance is categorized according to library of 12 archetypal appliances from research conducted which presented a time-variant steady state bottom-up load model [15], with some appliances, such as wet appliances, behaving as different load types depending on which stage of its cycle
it is operating at. The composite load model for each house is updated at each time-step, resulting in time-variant voltage dependencies.

III. ELASTIC DEMAND RESPONSE

To represent the elastic DR of consumers the effects of consumer occupancy, natural temporal activity preferences, the inter and intra-temporal effects, stochastic nature due to lack of complete visibility of consumer utility and lastly the discrete nature of both the level and duration of demand need to be taken into account. The authors developed a methodology for capturing these elastic consumer effects in [28], which is briefly presented in this section.

A. Price Elasticity Matrices

The methodology takes into account the discrete nature of the consumer by altering the probability of consumption, \( P \), and using MC techniques to test switch-on probabilities. Over a large number of simulations this method is equivalent to the classical elasticity function which alters demand as a continuous variable; it is observed that the classical elasticity approach is only an approximation of a large number of discrete changes in demand. This approach results in elasticity defined as relating a change in price, \( \Delta p \), to a change in the probability of a switch-on event, \( \Delta P \), given the elasticity \( \varepsilon \), with respect to an equilibrium point, of the reference price, \( p_0 \), and reference probability, \( P_0 \). To take into account of the intra and inter-temporal effects of electricity price on demand, the model uses both self-elasticity, \( \varepsilon_{ii} \), and cross-elasticity, \( \varepsilon_{ij} \), for different time intervals, \( (i,j) \). A matrix of self and cross elasticity coefficients can be constructed to form a price elasticity matrix (22). The elasticity coefficients can be used to represent lossless and lossy cases of demand response.

\[
\begin{pmatrix}
\Delta P_i \\
\Delta P_j
\end{pmatrix}
= 
\begin{pmatrix}
\varepsilon_{ii} & \varepsilon_{ij} \\
\varepsilon_{ji} & \varepsilon_{jj}
\end{pmatrix}
\begin{pmatrix}
P_i \\
P_j
\end{pmatrix}
\tag{22}
\]

The window over which consumers are willing to shift their demand is represented by \( W \), and it is over this value for which consumers react to changes in price. Rational consumers schedule their demand taking into account the price difference over the duration, \( D \), of the appliance, and hence the probability of a switch-on event is altered to be a reaction to price differentials over the duration of appliance operation (23).

\[
\Delta P_i = \sum_{j=i-W}^{i+W} P_j \varepsilon_{ij} \left( \frac{\sum_{d=j}^{j+D-1} \Delta p_d}{D} \right) - p_0
\tag{23}
\]

Occupancy dynamically affects not only the window over which a consumer responds but also the level of the response. Cross-elasticity coefficients must be weighted according to occupancy over the duration of the window. To represent these effects, the cross-elasticity coefficients are weighted by the occupancy factor, \( a_j \), and all coefficients are multiplied by the binary variable for occupancy, \( O_j \) (24, 25). The developed methodology is shown graphically in Fig. 1, showing the change in normalized change in probability of a switch-on event of appliance, \( P_a \), in equations (1) and (2).

\[
a_j = \left\{ \begin{array}{ll}
\frac{2W}{\sum_{i-j=1-W} O_j} & j \neq i \\
1 & j = i
\end{array} \right.
\tag{24}
\]

\[
\Delta P_i = \sum_{j=i-W}^{i+W} O_j a_j P_j \varepsilon_{ij} \left( \frac{\sum_{d=j}^{j+D-1} \Delta p_d}{D} \right) - p_0
\tag{25}
\]

Fig. 1. Elasticity method: (a) TOUP signal and reference price (b) Change in normalized probability with different device duration (c) Occupancy profile (d) Effect of occupancy on change in normalized probability with device duration of 60 minutes.

IV. AUTOMATIC DEMAND RESPONSE

There are two categories of control modeled, the first is optimal scheduling of wet appliances, and the second is price-based control of the set-point of TCAs. Residential appliance automation and control can be facilitated by in-home smart technologies and home-area communication networks for the internet of things, such as ZigBee [33].

A. Optimal Scheduling

Wet appliances can be optimally scheduled as the presence of the consumer is not required after start has been initiated. To model this, a simple cost minimization is implemented based on the known energy cycle of the appliance, \( E_d \), and the prices, \( P_d \), over a given horizon, \( D \) (26). Consumers initiate the start of the appliance, \( \alpha_{a,n} \), and their requested end time, \( \beta_{a,n} \), which is modeled by a logarithmic distribution, \( \gamma_{a,n} \), with, \( X \), a random number of uniform distribution between 0 and 1, with a mean wait time, \( T_{m,a} \), from the initial end time, \( \mu_{a,n} \), should the appliance have initiated on start (27). The end time is also modeled with the constraint that it must be less than the next start time of the appliance, \( \alpha_{a,n+1} \), and it must occur
in the next period of occupancy greater than or equal to the variable \( \gamma_{a,n} \) which is denoted \( \zeta_{a,n} \).

\[
\min \left( \sum_{d=1}^{D+1} E_d P_{t+d} \right) \text{ for } t = [\alpha_{a,n}, \ldots, \beta_{a,n} - D] \quad (26)
\]

\[
\gamma_{a,n} = -T_{m,a} \log(X) + \mu_{a,n} \quad (27)
\]

\[
\beta_{a,n} = \begin{cases} 
\alpha_{a,n} + 1 & \gamma_{a,n} \geq \alpha_{a,n} + 1 \\
\zeta_{a,n} - 1 & \gamma_{a,n} \leq \zeta_{a,n} \\
\gamma_{a,n} & \text{otherwise}
\end{cases} \quad (28)
\]

B. Thermostatic Controller

TCAs have the potential to change their energy consumption by altering their thermostatic set-point to either increase or decrease energy consumption [22]–[24]. There are two control strategies used in this paper for thermal appliances, the first is a static controller implemented for TOUP, and the second is a dynamic controller implemented for pricing schemes such as DAP.

1) Static Demand Response Peak Controller: Static controllers are best implemented for TOUP, where a set number of tariff rates of fixed duration are repeated day on day. The controller has prior knowledge of the peak tariff rate, due to its fixed time for entire seasons, and tries to maximize pre-peak period thermal inertia to ensure maximum ride through, hence lowering in-peak energy consumption. For refrigeration appliances the controller achieves this by altering the thermostatic set-point of the controller, \( T_{\text{ctrl},a} \), to a lower set-point before the peak, \( T_{\text{pre},a} \), where \( t_{\text{start}} \) and \( t_{\text{store}} \) denote the start-time of the peak period and the time before the peak for which the thermostatic set point is altered. During the peak, the set-point of the controller is set to a higher temperature, \( T_{\text{peak},a} \), to reduce energy consumption during the peak. The controller then stays at this setting until the end of the peak period, \( t_{\text{end}} \).

\[
T_{\text{ctrl},a}(t) = \begin{cases} 
T_{\text{pre},a}, & t < t_{\text{start}} - t_{\text{store}} \\
T_{\text{peak},a}, & t_{\text{start}} \leq t < t_{\text{end}} \\
T_{\text{set},a}, & \text{otherwise}
\end{cases} \quad (29)
\]

2) Dynamic Demand Response Elastic Controller: For non-periodic pricing signals, such as CPP, DAP or RTP, dynamic controllers are better suited to adapting energy consumption to the price signal. The controller implemented in this paper adjusts the thermostatic set-point based on a maximum and mean price parameters set by the consumer, \( P_{\text{max},a} \) and \( P_{\text{avg},a} \) respectively. For these parameters, the consumer gives a corresponding maximum and mean temperature set-point, \( T_{\text{max},a} \) and \( T_{\text{avg},a} \), given these parameters a linear relationship between price and temperature can be established with the consumer lastly setting a minimum temperature, \( T_{\text{min},a} \), based on the established linear response, which determines the full range of response for the controller (30, 31). The controller responds to price linearly across this range, and remains at the maximum or minimum temperature set-point should the price exceed the price boundaries (32, 33).

\[
\alpha_{\text{DDREC},a} = \frac{T_{\text{max},a} - T_{\text{set},a}}{P_{\text{max},a} - P_{\text{avg},a}} \quad (30)
\]

\[
\beta_{\text{DDREC},a} = T_{\text{max},a} - P_{\text{max},a} \alpha_{\text{DDREC},a} \quad (31)
\]

\[
T_{\text{elas},a}(t) = \alpha_{\text{DDREC},a} P(t) + \beta_{\text{DDREC},a} \quad (32)
\]

\[
T_{\text{ctrl},a}(t) = \begin{cases} 
T_{\text{max},a}, & P(t) \geq P_{\text{max},a} \\
T_{\text{min},a}, & P(t) \leq \frac{1}{\alpha_{\text{DDREC},a}} T_{\text{min},a} \\
T_{\text{elas},a}(t), & \text{otherwise}
\end{cases} \quad (33)
\]

The linear controller is adjusted such that the temperature set-point is a stepped approximation of the linear slope, see Fig. 2. This reflects that standard TCAs do not have linear control over temperature settings; for this paper the appliances are assumed to operate in discrete steps of 0.5°C. The above formulation is for refrigeration appliances, where higher temperature set point are used in periods of high prices to reduce energy consumption with the reverse strategy is used for heating appliances, such as DHW heaters.

V. MODEL IMPLEMENTATION

A. Input Data

Whilst there are obvious advantages to bottom-up load modeling techniques, a disadvantage is the extensive data requirements needed. However, this data is becoming more readily available with increased communications, smart grid and load monitoring trials. For the activity profiles and occupancy transition matrices, these can be extracted from national time-use survey data. Appliance ownership and operation can be extracted from a combination of national statistical data and detailed load monitoring surveys. Thermal appliance resistance and capacitance can be calculated using a bottom up approach, using commonly available information on mass, typically water, and insulation material. Electrical load monitoring is becoming increasingly prevalent in the literature, and is frequently being conducted on a per-appliance data; it is these
data that can be used for the time-variant load model. For this paper Republic of Ireland data is primarily used, and when not available, UK data is used as the two countries have similar temperate, economic and social characteristics [28].

B. Model Platform

The model was implemented in MATLAB, [34], and a steady-state three-phase unbalanced load flow for a test distribution network run in Power Factory [35]. A flow-chart of the load model is given in Fig. 3, the time step for the model is of one minute, and appliance category and associated activities are shown in Table I. The random variables used to test the switch-on probability of any appliance are created in MATLAB using a uniform distribution between 0 and 1. The random number generator is seeded for each simulation such that they can be directly comparable, i.e. going from a flat-tariff to a DAP simulation the test random variables are the same. Some notable areas of difficulty can be dealing with both the memory requirements and time to run these simulations. These problems can both be minimized through parallelisation for increased speed and implementation of sparse matrices for cumbersome data arrays. MATLAB is used to simulate the load model demand, producing active, reactive and polynomial electrical load model data with automated communication to Power Factory via .csv files for power flow analysis.

VI. CASE STUDY

A case study for a typical Irish suburban residential distribution network is presented, examining multiple scenarios of different levels of DR under the three different pricing schemes, with the flat rate tariff used as a base case for comparison. Energy regulators are currently investigating mandating the use of these dynamic pricing signals and smart metering technologies [27]. The simulation is run for a Winter week, as this typically represents the highest loading conditions for the network as heating and lighting loads are at their most prevalent.

A. Test Network

The test network is a LV suburban residential distribution feeder in Dublin, Ireland. The LV substation serves a total of 85 nodes, 11 of which are three-phase with the remaining 74 being single-phase customer nodes, Fig. 4. The network includes a 400 kVA, 10/0.4 kV step-down transformer supplying the customer nodes through 1.2 km of 3-phase copper mains

Fig. 3. Overall model implementation and basic flow chart describing key inputs (i/p) and outputs (o/p) of each module for residential appliances (Appl.).

Table I

<table>
<thead>
<tr>
<th>ACTIVITIES</th>
<th>THERMAL</th>
<th>WET</th>
<th>DHW</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Freezer, Fridge-freezer,</td>
<td>Refrigerator, Storage heaters</td>
<td>(3) Dish Washer, Washing Machine</td>
<td>(8) Immersion Heater</td>
</tr>
<tr>
<td>(2) Space Heating</td>
<td>(2) Active</td>
<td>(4) House Cleaning</td>
<td>(5) Icing</td>
</tr>
<tr>
<td>(3) Cooking</td>
<td>(3) Cooking</td>
<td></td>
<td>(5) Icing</td>
</tr>
<tr>
<td>(4) House Cleaning</td>
<td>(4) House Cleaning</td>
<td></td>
<td>(5) Icing</td>
</tr>
<tr>
<td>(5) Icing</td>
<td>(5) Icing</td>
<td></td>
<td>(5) Icing</td>
</tr>
<tr>
<td>(6) Cleaning</td>
<td>(6) Cleaning</td>
<td></td>
<td>(5) Icing</td>
</tr>
<tr>
<td>(9) Laundry</td>
<td>(9) Laundry</td>
<td></td>
<td>(5) Icing</td>
</tr>
<tr>
<td>(10) Windows</td>
<td>(10) Windows</td>
<td></td>
<td>(5) Icing</td>
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</table>

For N Houses

MATLAB

<table>
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<th>n=1</th>
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<th>O/P</th>
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<td>I/P</td>
<td>O/P</td>
<td>I/P</td>
<td>O/P</td>
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<td>I/P</td>
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A case study for a typical Irish suburban residential distribution network is presented, examining multiple scenarios of different levels of DR under the three different pricing schemes, with the flat rate tariff used as a base case for comparison. Energy regulators are currently investigating mandating the use of these dynamic pricing signals and smart metering technologies [27]. The simulation is run for a Winter week, as this typically represents the highest loading conditions for the network as heating and lighting loads are at their most prevalent.

A. Test Network

The test network is a LV suburban residential distribution feeder in Dublin, Ireland. The LV substation serves a total of 85 nodes, 11 of which are three-phase with the remaining 74 being single-phase customer nodes, Fig. 4. The network includes a 400 kVA, 10/0.4 kV step-down transformer supplying the customer nodes through 1.2 km of 3-phase copper mains
cables and 980 m of single-phase copper service cables. The Irish distribution network is operated at a nominal voltage of 230/400 V with a tolerance of +/- 10%, giving a minimum and maximum allowable line to ground voltage of 207 V and 253 V respectively as specified by Electricity Supply Board Networks [36]. The mains cable has a three-phase maximum current rating of 424 A and the maximum import capacity for average domestic households is 12 kVA. To capture the voltage drop and profile at the LV side of the distribution transformer accurately two three-phase lumped loads are modeled, at the 0.4 kV and 10 kV side of the distribution transformer, with 74 and 1628 customers respectively. This configuration allows for greater accuracy in the medium voltage (MV) line flows between the LV network and the primary substation, which has tap changing abilities and regulates its MV side voltage to 1.045 per unit.

B. Pricing Schemes

The TOUP signal used is tariff D from the Consumer Behavior Trial conducted by the Irish Commission for Energy Regulation (CER), that tariff was chosen as it has the biggest peak to day tariff price differential [37]. The reference price, \( P_0 \), was taken to be 18.5 cents/kWh price and is also the price for the flat rate tariff [37]. For the DAP price signal, the paper uses Winter data for the corresponding period for the ex-ante system marginal price DAP price, that is released 24 hours in advance by the Irish Single Electricity Market Operator [38]. The price signal is of half hourly resolution, and is altered to include the distribution use of system charge that would be experienced by residential customers.

C. Consumer Characteristics

For the simulations in this paper elasticity coefficients for self and cross elasticity of -0.3 and 0.001667 are chosen based on a review of values of elasticity of electricity demand and a shifting window, \( W \), of 90 minutes representing a lossless simulation [39]. This results in the total period in which a consumer will consider substituting their demand to be a 3 hour moving horizon, with customers willing to consider shifting their demand forward or back an hour and a half from their current time step. Global irradiance data used for the lighting model was data of 15 minute resolution from a Dublin weather station close to the LV network, with the data corresponding to the Winter week used for the DAP signal. Water heating is a significant load proportion and 76% of households in Ireland have electric immersion heaters [40], [41]. From available data only 10% of households use immersion heaters in the Winter months, due to winter use of central heating systems, and 67% of these use timers which mainly operate on a morning and evening cycle corresponding to periods of hot water demand [40].

VII. Results And Discussion

The developed methodology allows a high-resolution detailed analysis to be conducted on the network impacts of price based demand response for the residential sector, capturing increased volatility in the demand profile due to step changes in price.

A. Model Validation

The developed model was validated against data for the Irish residential sector, validating both the load profile and energy consumption per appliance sector [41]. Fig. 5 shows the model load profile against Irish data for the residential sector for a typical Winter weekday [42]. Comparing this data against the model gives a Pearson correlation coefficient of 0.97 and a mean absolute percentage error of 14.45% and a root-mean square error of 7.6%. The annual energy consumption data for each sector, [41], such as heating and cooling, was compared against Irish data for the residential sector and all sectors compared favorably within +/- 5% apart from the circulation pumps and fans sector which was not modeled and accounts for approximately 4% of average annual residential energy consumption.

B. Price Demand Response

The three different elements of price response modeled in this paper each present different characteristic effects on demand and electrical diversity. Coupled with the elastic response (ER) different penetration levels of automatic demand response (ADR) are examined, with the analysis here presenting adoption rates of the control strategies presented in Section IV in steps of 20%.
Fig. 6. (a) TOUP Signal and (b) Demand response composition for elastic consumers for the entire network with 50% penetration of automatic demand response compared against the base case scenario.

Fig. 7. (a) DAP Signal and (b) Demand response composition for elastic consumers for the entire network with 50% penetration of automatic demand response compared against the base case scenario.

Fig. 8. Change in (a) Active $(P)$ and (b) Reactive $(Q)$ power flows at the MV side of the distribution transformer under each scenario for TOUP signal.

Fig. 9. Change in (a) Active $(P)$ and (b) Reactive $(Q)$ power flows at the MV side of the distribution transformer under each scenario for DAP signal.

1) Elastic Demand: Through the comprehensive modeling techniques developed in this paper it is shown that there is low potential to shift elastic demand to off peak hours, as there is traditionally lower levels of occupancy and low activity preferences for these hours. The duration of appliances heavily effects the magnitude and duration of pre and post-peak price rebounds. The pre-peak rebound tends to be more diversified whilst the post peak rebound tends to have a large coincidental operation of appliances as the duration of demand is no longer a major determining factor in switch-on probability.

2) Thermostatic Controls: Dynamic pricing schemes coupled with thermostatic control introduces cold load pick-up effects traditionally seen in network restoration, with water heating primarily affecting active power demand and refrigeration appliances also significantly affecting the reactive power demand profile. As refrigeration appliances constitute the main reactive power demand in LV networks, particularly when wet appliances have been optimally scheduled to be off peak, any control actions within this sector causes large scale reactive power fluctuations, see Fig. 8 and Fig. 9. The pre and post-peak rebounds cause significant voltage dips due to the increase in active power demand.

3) Optimal Scheduling: Wet appliances are optimally scheduled subject to the consumer constraints in Section IV. Typically they are scheduled to off-peak hours, but cascaded operation create new spikes in demand, as can be seen in Fig. 6 and Fig. 7. Furthermore, these appliances have similar electrical characteristics, typically operating a resistive element followed by motor operation, leading to reduced electrical diversification when the coincidence of operation is increased.
C. Network Impacts

The introduction of dynamic pricing schemes introduce the effects of rebound peaks, synchronizing of appliances and cold load pick-up effects that have impacts on network loading, voltage profile and voltage unbalance.

1) Line Loading: Incidents of high loading are increased under dynamic pricing with pricing effects affecting normal loading distribution, see Fig. 10. This is particularly due to the operation of thermostatically controlled devices reacting to peak pricing signals. Although loading increases there is low risk of causing any major problems as LV lines are typically conservatively rated.

Fig. 10. Logarithmic plot of the total percentage duration curve of voltage and loading conditions under a TOUP for (a) Phase C Loading on Line 1-2 (b) Phase C Voltage at Pillar 9.

2) Voltage Deviation: The increased volatility of active power and reactive power demand results in a redistribution in the voltage profile, see Fig. 10. This is prevalent around peak tariff periods, causing significant short-term voltage dips. The reactive power demand profile is primarily affected due to the thermostatic price response control of refrigeration appliances which typically have low power factors. There is a marginal increase in voltage unbalance under dynamic pricing schemes, as although load diversity decreases, thermal loading on each phase becomes more a function of the appliance composition on that line, as their operation becomes synchronized.

3) Load Diversity: Fundamentally the introduction of dynamic pricing reduces diversity of demand, increasing coincidental response and promoting the same characteristics of response among consumers and automatic appliances. Table II shows the effect of the different scenarios on maximum demand (MD), after-diversity maximum demand (ADMD) and the load factor (LF) for the entire network of customers in the case study. It can be seen that maximum demand increases in all scenarios, and up to 30% in the full roll out of automatic appliances under time-of-use pricing.

D. Welfare Analysis

The consumer savings are less than 10% in all cases, see Table III for the weekly energy expenditure in all cases with customers broken down between those with electric domestic hot water (DHW) heating and those without. The greatest savings on weekly expenditure are seen to be 8% with elastic consumer response and full installation of price controllers for wet appliances and TCAs.

In the individual automatic response categories, wet appliances provide the greatest value in response, with savings of over 25% seen on the budget for that sector. Refrigeration appliance response to prices saves in the region of 12%, and water heating controls saving in the region of 10%.

VIII. CONCLUSION

Quantifying the impact that dynamic pricing schemes will have on consumer demand, and consequently distribution network operation, is a difficult and important challenge. Consumer trials are difficult to implement, and rigorous comparisons between demand under different scenarios is compounded by the lack of complete knowledge of consumer utility and environmental factors. This paper presents a comprehensive high-resolution model for simulating both the elastic and automatic price responsiveness of demand. The developed modeling methodology enables the network impacts of dynamic pricing strategies to be analyzed and it is shown that the dynamic pricing signals in this paper increase maximum demand by decreasing both diversity of demand and the electrical load. This erosion of load diversity is of concern as it is this assumed diversity of demand that distribution networks were originally designed. This calls into question the motivation for the wide-scale introduction of dynamic pricing schemes, however, the effects seen on LV networks need to be consolidated with the potential benefits, or otherwise, of such schemes to the high-voltage transmission network.