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# DYNAMIC AMPLIFICATION FACTORS FOR BRIDGES WITH VARIOUS BOUNDARY CONDITIONS

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## Abstract

It is important that the assessment of bridges be as accurate as possible. Dynamic amplification is one area in which the codes are sometimes overly conservative. Bridge dynamics is well researched but the majority of this research has concentrated on simply supported bridges. This paper examines the effect that introducing support rotational restraint has on bridge dynamic amplification factors. Finite element models are used with rotational springs at the bridge supports. This paper examines the effect of vehicle velocity on the dynamic amplification for the different end restraints.

**Keywords:** (Boundary Conditions, Dynamic amplification, Speed)

## 1. Introduction

Bridge design is often excessively conservative. Conservatism is relatively inexpensive in new construction but can lead to unnecessary repair or replacement of existing bridges, with consequent waste of maintenance funds and non renewable resources. Standards and codes of practice make significant allowances for dynamics. For example, the Eurocode (admittedly for the design of new bridges) allows for a 20-70% increase in stress (Dawe 2003)

Dynamic amplification takes into account the phenomenon that the stress produced at a given point in a bridge when a vehicle is moving across it, differ from that generated, had the vehicle been stationary. The change in stress is a consequence of the complex relationship between a moving vehicle, the road profile and the bridge underneath. This relationship is dependent on many parameters including the natural frequencies of the bridge, the road surface and, the vehicle(s) causing the loading. There is much literature on these factors and their effect on bridge dynamics. This paper concentrates on the effect of speed and boundary conditions.

The term used to measure the dynamics in this paper is dynamic amplification factor (DAF), defined by Zhang et al. (2001) as,

$$DAF = \frac{E_{tot}}{E_{stat}} \quad (1)$$

where  $E_{stat}$  is the maximum static load effect induced at a point in the bridge by a stationary vehicle and  $E_{tot}$  is the maximum total (static plus dynamic) load effect induced by the same vehicle when it is moving.

### 1.1 Boundary Conditions

Most of the literature on bridge dynamics has been carried out on bridges with simple supports or certainly on bridges with one set of support conditions (Nowak and Hong 1991; Brady et al. 2006). Salawu and Williams (1995) acknowledge that significant

assumptions are usually made with regard to the boundary conditions. Even for bridges incorporating spherical or pot bearings, some resistance to rotation is frequently evident in site measurements (Doebbling et al. 1997).

Although most of the literature uses one set of boundary conditions, generally the simply supported case, there are some that investigate changes in the support conditions. Fryba (1971) is the most notable of these sources and the equations he developed are used to verify the finite element model used in this paper. Akin & Massood (1989) present a method of determining beam response to a moving mass with varying support conditions. They look at displacement at a certain point for different beams while varying the boundary conditions.

This paper examines the change in frequency as a result of a change in boundary conditions. Law et al. (1995) suggest that the fundamental frequency of a T-beam slab bridge deck is not sensitive to changes in boundary conditions. This is in contrast to Mertlich et al. (2007) who report that reducing the end rotational restraint had a significant impact on the natural frequencies. Brownjohn et al. (2003) report a considerable increase in frequency for a bridge that underwent rotational restraint and the installation of a guardrail. They go on to say that it is the increased rotational resistance that has had the greater influence on the bridge frequency.

## **1.2 Rotational Springs**

Rotational springs are used in this paper to model rotational resistance at the supports. This approach is used in many scenarios such as to analyse the effect of increasing support stiffness after bridge upgrading works (Brownjohn et al. 2003) or, similarly to this paper, to use the adjustable stiffness of the springs to model a range of end conditions (Wu and Law 2004). In many cases rotational springs are used to fit finite element models to the experimental data (Doebbling et al. 1997; Zhang and Aktan 1997; Sanayei et al. 1999; Feng et al. 2004). All of these authors acknowledge that the boundary conditions affect the dynamic characteristics. Salawu (1997) notes that dynamic characteristics may respond to changes in boundary supports.

## **2. Fryba Beam model**

This paper represents the vehicle moving across the bridge as a simple point force traversing a beam. This is a simplification and can have a significant influence on the results (Akin & Massood 1989) but using such models facilitates a focus on the influence of specific parameters (Brady 2004). The bridge is 25m in length ( $l$ ), has a span to depth ratio of 1/20 with square cross section meaning the second moment of area ( $I$ ) is  $1.38\text{m}^4$ . The Youngs modulus ( $E$ ) is  $3.6 \times 10^{10}\text{N/m}^2$  and density ( $\mu$ ) is  $2446\text{kg/m}^3$ . The stiffness ( $k$ ) of the rotational springs that model the boundary conditions changes. Fryba (1971) provides a closed form solution to the Bernoulli-Euler equation for a beam. He first presents a solution for simply supported boundary conditions but goes on to give a general equation for displacement at a point  $x$  and time  $t$  that can be adapted depending on the boundary conditions:

$$\begin{aligned}
v(x, t) = \sum_{j=1}^{\infty} \frac{P}{V_j} v_{(j)}(x) & \left\{ \frac{1}{\omega_{(j)}^2 + \omega^2} \left[ \left( \sin \omega t - \frac{\omega}{\omega_{(j)}} \sin \omega_{(j)} t \right) (-\cos \lambda_j + A_j \sin \lambda_j) + \right. \right. \\
& \left. \left( \cos \omega t - \cos \omega_{(j)} t \right) (\sin \lambda_j + A_j \cos \lambda_j) \right] + \\
\frac{1}{\omega_{(j)}^2 + \omega^2} & \left[ \left( \sinh \omega t - \frac{\omega}{\omega_{(j)}} \sin \omega_{(j)} t \right) (-B_j \cosh \lambda_j - C_j \sinh \lambda_j) + (\cosh \omega t - \right. \\
\cos \omega_{(j)} t) & \left. (B_j \sinh \lambda_j + C_j \cosh \lambda_j) \right] \left. \right\} \quad (2)
\end{aligned}$$

where the values  $A_j, B_j, C_j$  and  $\lambda_j$  are constants dependent on the boundary conditions,  $P$  is force,  $\omega$  is the load circular frequency,  $\omega_{(j)}$  is the natural circular frequency of the beam, and where:

$$v_{(j)}(x) = \sin \frac{\lambda_j x}{l} + A_j \cos \frac{\lambda_j x}{l} + B_j \sinh \frac{\lambda_j x}{l} + C_j \cosh \frac{\lambda_j x}{l} \quad (3)$$

and

$$\begin{aligned}
V_j = \frac{ul}{2} & \left\{ 1 + A_j^2 + B_j^2 + C_j^2 + \frac{1}{\lambda_j} \left[ 2C_j - 2A_j B_j - B_j C_j - \frac{1}{2} (1 - A_j^2) \sin 2\lambda_j + \right. \right. \\
2A_j \sin^2 \lambda_j & + (B_j^2 + C_j^2) \sinh \lambda_j \cosh \lambda_j + 2(B_j + A_j C_j) \cosh \lambda_j \sin \lambda_j + \\
2(-B_j + A_j C_j) & \sinh \lambda_j \cos \lambda_j + 2(C_j + A_j B_j) \sinh \lambda_j \sin \lambda_j + 2(-C_j + \\
A_j B_j) \cosh \lambda_j \cos \lambda_j & + B_j C_j \cosh 2\lambda_j \left. \right\} \quad (4)
\end{aligned}$$

Using the relation,  $M(x, t) = -EI \frac{\partial^2 v(x, t)}{\partial x^2}$  (5), an equation can be derived for moment which accounts for boundary conditions.

### 3. Finite Element Model

In order to implement the finite element method, both the stiffness and mass matrices must be defined. For the elements that do not have rotational springs attached, the stiffness matrix,  $k_e$ , is defined by Rockey et al. (1983) and is given below. Variations of the element stiffness matrix must be defined to allow for a rotational spring at the left ( $k_{eS-L}$ ) or the right ( $k_{eS-R}$ ) end of the beam.

$$k_{eS-L} = \begin{bmatrix} A & H & -A & Q \\ B & G & -B & D \\ -A & -H & A & -Q \\ C & O & -C & R \end{bmatrix} \quad (6) \quad k_e = \frac{EI}{l_e^3} \begin{bmatrix} 12 & 6l_e & -12l_e & 6l_e \\ 6l_e & 4l_e^2 & -6l_e & 2l_e^2 \\ -12 & -6l_e & 12 & -6l_e \\ 6l_e & 2l_e^2 & -6l_e & 4l_e^2 \end{bmatrix} \quad (7)$$

$$k_{eS-R} = \begin{bmatrix} A & Q & -A & H \\ C & R & -C & O \\ -A & -Q & A & -H \\ B & D & -B & G \end{bmatrix} \quad (8)$$

where,

$$A = \frac{3EI}{l_e^3} + \frac{3\pi}{2l_e} \quad (9) \quad B = \frac{3}{2l_e} / \left( \frac{l_e}{4EI} + \frac{2}{k} \right) \quad (10) \quad C = Al_e - B \quad (11) \quad D = \frac{1}{\frac{l_e}{2EI} + \frac{4}{k}} \quad (12)$$

$$R = \frac{3EI}{l_e} + \frac{D}{2} \quad (13) \quad Q = \frac{D + R}{l_e} \quad (14) \quad G = \frac{1}{\frac{l_e}{4EI} + \frac{1}{k}} \quad (15) \quad H = \frac{3G}{2l_e} \quad (16) \quad O = \frac{G}{2} \quad (17)$$

Where  $l_e$  is the length of each element and  $k$  is the stiffness of the spring. The mass matrix, also defined by Rockey et al. (1983), is the same for all elements. Moment can be calculated after finding the strain ( $\varepsilon(x)$ ) using:

$$\varepsilon(x) = \frac{-y}{l_e^2} [(6l_e - 12(x_e))u_i + (4l_e^2 - 6l_e(x_e))\theta_i + (-6l_e + 12(x_e))u_{i+1} + (2l_e^2 - 6l_e x_e)\theta_{i+1}] \quad (18)$$

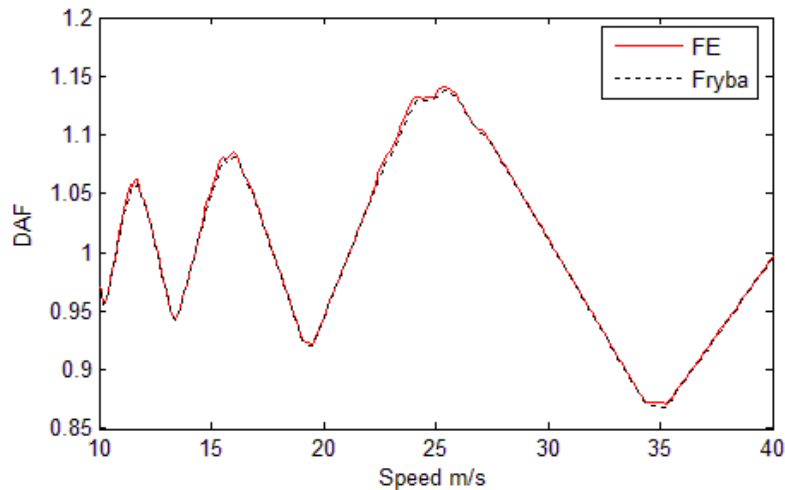
and,

$$M = \frac{EI}{y} \varepsilon(x) \quad (19)$$

Where  $x_e$  is the distance from the left hand edge of the element that the strain is measured,  $u_i$  is the displacement degree of freedom at node  $i$  and  $\theta_i$  is the rotational degree of freedom.

#### 4. Finite Element Model agreement with Fryba Beam model

As seen in Figure 1 the Finite Element (FE) and the Fryba (1971) beam model are in good agreement. The level of matching is dictated by the number of elements used in the FE model and the number of mode shapes considered when using the Fryba method. It can be seen that there are a series of progressively larger peaks in dynamic amplification with speed. Whether dynamics increases or decreases with speed depends on where typical highway speeds fall on this graph and the critical speeds are directly related to bridge first natural frequency.



**Figure 1** - Variation of Dynamic Amplification (DAF) with Speed: Comparison of Finite Element and Fryba models; Simply Supported End Conditions

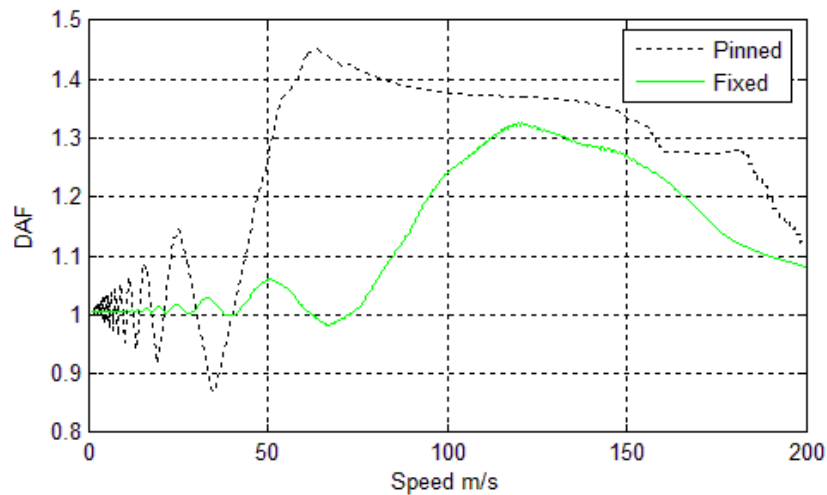
#### 5. Pinned versus Fixed

Figure 2 shows the variation of DAF with speed for both pinned and fixed boundary conditions. One of the first things of note is the difference in magnitude of DAF between the pinned and fixed bridges. It can be seen that the peaks for the pinned end conditions are considerably higher than for the fixed. The highest values are 1.45 and

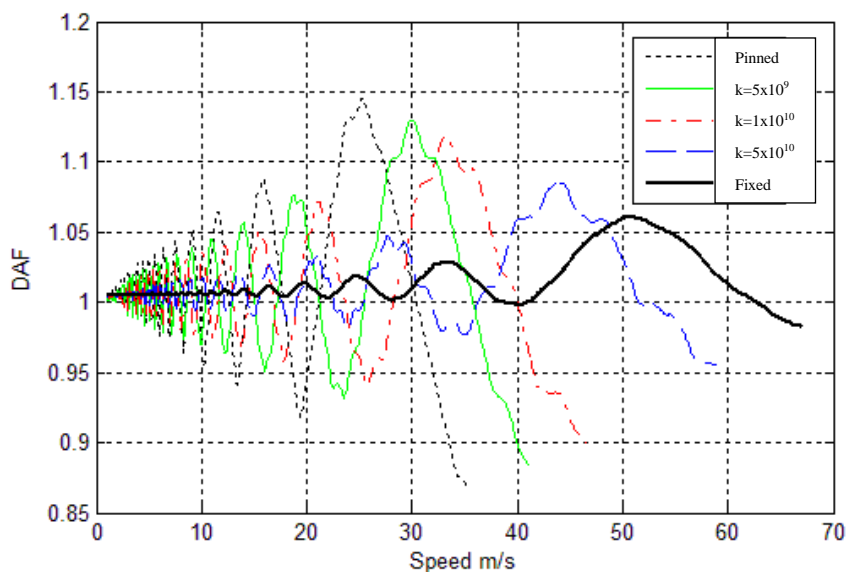
1.33 for the pinned and fixed bridges respectively. These values occur at 230 km/h and 431 km/h which are clearly not realistic speeds for road traffic (critical speeds will be different for bridges with different fundamental frequencies). The next peaks of 1.15 and 1.06 happen at 91 km/hr and 183 km/hr which illustrate that not only does the fixed beam have lower magnitude DAF but that these values are more difficult to reach for normal traffic as they occur at higher speeds.

Considering beams that have a stiffness in between pinned and fixed, it can be seen that the peak dynamic amplification factor decreases with an increase in stiffness and that higher speeds are needed to reach the peaks. This is shown by the trend to the right in Figure 3 for the final peak shown for each bridge. This indicates that changing the resistance to rotation has an effect on the frequency of the bridge; something which is investigated further in the next paragraphs.

Figure 4 shows the peak DAF's and corresponding critical speeds for each of the five boundary conditions used in Figure 3. The straight lines show that the peak DAF's decrease as the critical speed increases in an approximately linear way. The slopes of the lines joining the peaks increase with each peak, but only slightly.



**Figure 2** - Variation of Dynamic Amplification with Speed: Fixed vs Pinned End Conditions

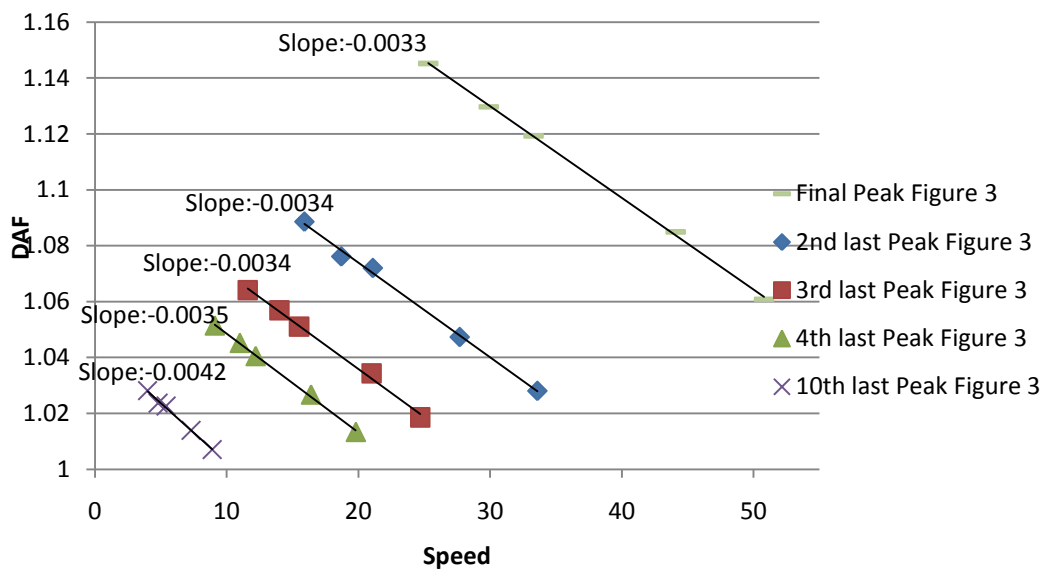


**Figure 3** - Variation of Dynamic Amplification with Speed: Various End Condition Rotational Stiffnesses

Brady (2004) reports that for a given bridge, when the moving force reaches a certain velocity, it coincides with a critical bridge frequency and a peak DAF results. It is therefore important to look at what effect the increase in stiffness has on the bridge frequency. This is done by examining certain points in Figure 3 and creating the bending moment versus time responses for the relevant velocities. From these, the bridge frequency can be calculated. The resulting frequencies are presented in Table 1:

**Table 1 - Change in Frequency with End Rotational Stiffness**

Boundary Stiffness	Pinned	$5 \times 10^9$	$1 \times 10^{10}$	$5 \times 10^{10}$	Fixed
Frequency (Hz)	3.54	4.19	4.68	6.25	7.55

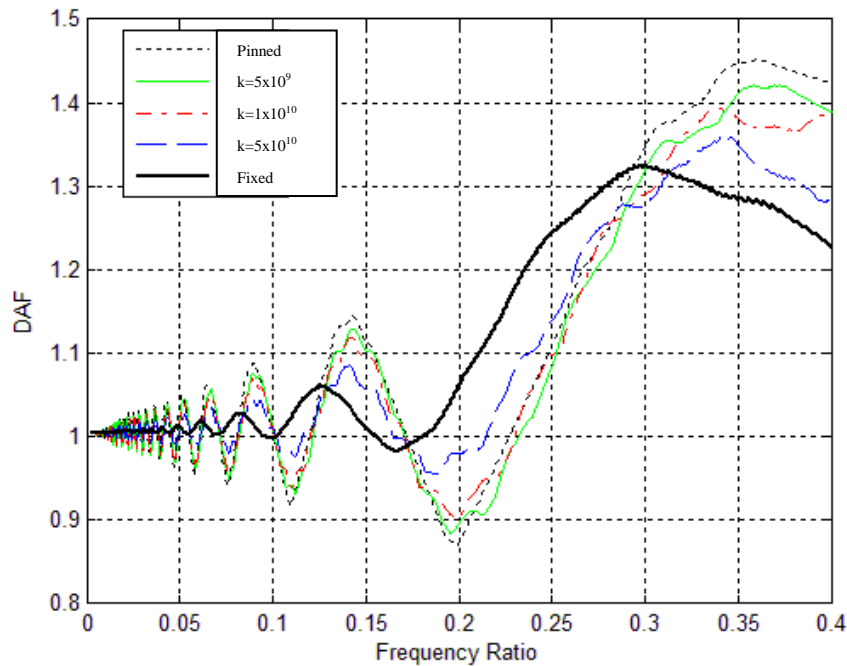


**Figure 4 - Peak values of Figure 3**

It can clearly be seen that increasing the resistance to rotation at the ends of the bridge increases the first natural frequency. This is consistent with Figure 3; the shift to the right illustrates that a higher speed is needed to produce the equivalent peak in each end stiffness. These peaks occur at certain frequency ratios, defined as the ratio load circular frequency to first beam circular frequency.

Accordingly, a higher speed for the same peak indicates that that bridge has a higher first natural frequency. Figure 5 is similar to Figure 3 but, in this case, the horizontal axis is rescaled by dividing by bridge fundamental frequency. This aligns the peaks to a considerable extent, removing the influence of changing frequency.

It is clear from Figure 5 that the peaks do not occur at the exact same frequency ratio. This behaviour is consistent with what is shown in Figure 4. The slope of the lines linking the peaks are not exactly the same. This implies that if the graphs were normalised so that the final peaks of the five bridges were directly under each other, then the other peaks would not line up accordingly.



**Figure 5 - Frequency Ratio vs Dynamic Amplification**

## 6. Conclusion

The majority of the literature investigating dynamic amplification has looked at bridges with simple supports. By modelling rotational springs at the supports it is possible to model different levels of rotational restraint and hence to show the effect of the different boundary conditions on dynamic amplification. It is shown in this paper that by introducing rotational restraint the magnitude of dynamic amplification factor (DAF) reduces and occurs at higher vehicle speeds.

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