<table>
<thead>
<tr>
<th><strong>Title</strong></th>
<th>The dynamics of Smithian growth</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Authors(s)</strong></td>
<td>Kelly, Morgan</td>
</tr>
<tr>
<td><strong>Publication date</strong></td>
<td>1997-08</td>
</tr>
<tr>
<td><strong>Publication information</strong></td>
<td>Kelly, Morgan. “The Dynamics of Smithian Growth” 112, no. 3 (August, 1997).</td>
</tr>
<tr>
<td><strong>Publisher</strong></td>
<td>MIT Press Journals</td>
</tr>
<tr>
<td><strong>Item record/more information</strong></td>
<td><a href="http://hdl.handle.net/10197/521">http://hdl.handle.net/10197/521</a></td>
</tr>
<tr>
<td><strong>Publisher’s statement</strong></td>
<td>All rights reserved</td>
</tr>
<tr>
<td><strong>Publisher’s version (DOI)</strong></td>
<td>10.1162/003355397555398</td>
</tr>
</tbody>
</table>
THE DYNAMICS OF SMITHIAN GROWTH

MORGAN KELLY

This paper analyzes the evolution of an economy where growth is driven by increased specialization caused by the geographical expansion of markets. It proves that such Smithian growth exhibits generic threshold behavior. Below a critical density of transport linkages, the economy is split into isolated local markets with limited specialization. Above the critical density, these markets begin to fuse into a large, economywide market causing growth to accelerate. This allows an explicit test of the consensus among historians of Sung dynasty China that the economic revolution during that period was a result of commercialization caused by the creation of a national waterway network.

I. INTRODUCTION

Despite its long pedigree, Smithian progress remains very much a poor relative among theories of economic growth. There is no dispute that when markets expand, increased specialization will cause output to rise. The relative neglect of Smithian growth arises, rather, from the perception that the resulting growth will be very gradual. To explain rapid bursts of growth, economists turn instead to the standard neoclassical sources of innovation: learning by doing, and private capital accumulation.

This paper challenges the idea of Smithian progress as a gradual affair. It analyzes an economy where geographical markets expand gradually through time. This growth of market linkages between sites leads to what Stigler [1951] called "vertical disintegration" in production. A firm at one site can specialize in the production of some subset of goods, and trade with other specialized producers that are located in the same connected market.

The central result of this paper is that a gradual expansion of market linkages does not lead to continuous steady growth, as intuition might suggest. Instead, it is proved that Smithian growth exhibits generic threshold behavior. Below a critical density of market linkages, the economy is split into isolated, local markets that limit the scope for specialization. When the critical density is reached, these small local markets start to coalesce into a large market that spans the economy. The resulting increase in

* Earlier versions of this paper were presented at SITE, the SEIC, University of Virginia, and the Econometric Society. For helpful comments, I would like to thank William Brock, Paul David, Steven Durlauf, Emily Hill, Kevin O'Rourke, and, particularly, Cormac O Grada, Andrei Shleifer, and two anonymous referees. They gave valuable critiques of the submitted draft. All errors are mine.

© 1997 by the President and Fellows of Harvard College and the Massachusetts Institute of Technology

The Quarterly Journal of Economics. August 1997
division of labor as local markets fuse together causes a sudden acceleration of growth. In other words, Smithian growth leads to a takeoff. Below the critical density, adding linkages causes small, local markets to emerge; above the critical density, adding linkages causes these local markets to coalesce. As these markets coalesce, labor is reallocated from the inefficient subsistence production to the more efficient commercial technology, allowing output to expand.

As more linkages are added, all sites become incorporated into this large market, and the potential for growth through increased specialization becomes exhausted. If the sources of further growth—capital accumulation, innovation, and learning by doing—are blocked, the acceleration that occurs when the Smithian threshold is reached will be followed by stagnation.

This threshold property is shown first for a parametric model in Sections II and III, and is then proved to hold in general in Section IV.

The prediction that Smithian progress implies a takeoff in growth allows a formal evaluation of the role of market expansion in the economic transformation of the Sung dynasty China. Between the ninth and twelfth centuries, China changed from a simple subsistence economy to a level of development not surpassed elsewhere until the late eighteenth century. The consensus among historians of the period is that this economic revolution was the result of a commercialization of Chinese society caused by the creation of a national waterway network. The cheap transportation offered by this network—created by the government for fiscal and strategic purposes—allowed regions to switch from subsistence to large-scale commercial activities, causing output to rise and creating incentives for technological innovation.

If true, this commercialization hypothesis predicts a sudden acceleration of growth around the time that the national waterway network was completed, an event that occurred at the end of the tenth century. To test this, an annual series of real money growth is constructed for the period 805 AD to 1075 AD. This gives a proxy for the growth rate of real output. The series does indeed show a distinct and statistically significant acceleration around 1000 AD: it is not possible to reject the commercialization hypothesis.

The existing literature on Smithian growth has been less concerned with the dynamics of growth than with the factors that
determine the degree of specialization in an economy. Early developments of Adam Smith’s proposition “that division of labor is limited by the extent to the market” are Young [1928] and Stigler [1951]. More recent formal analyses include Rosen [1978], Locay [1990], Yang and Borland [1991], Becker and Murphy [1992], Takayama [1992], Ades and Glaeser [1994], and Goodfriend and McDermott [1995]. An extensive survey is provided by Yang and Ng [1994].

II. SMITHIAN GROWTH

We start by analyzing a simple parametric model of Smithian growth. The model used is a variant of standard textbook Schumpeterian growth models [Aghion and Howitt 1992; Grossman and Helpman 1991, Chapter 3]. In Schumpeterian models, firms invest to improve the quality of their product and gain monopoly profits. In the model here, by contrast, they invest to expand their geographical markets, and so reduce costs through greater specialization. The model abstracts from all other sources of growth: private capital accumulation, technological progress, and learning by doing. Market expansion will be the only source of growth considered here.

II.1. Geography and Markets

The economy is illustrated in Figure I. It comprises $N$ sites that branch over a large plain. These sites can be thought of as villages located in a river basin, with one village situated at every confluence of two tributary streams. Each site starts out in autarky, using a basic technology to produce all goods for its own subsistence. As time passes, sites may form transport linkages with other sites, allowing them to trade goods and factors with each other. A set of connected sites is called a market. The defining feature of these linkages between sites is that they allow large volumes of goods to be transported at low cost. This permits a specialized producer at one site to produce a good, and to supply it to all other sites in the same market more cheaply than they can produce it themselves using the subsistence technology.

In this model each site can connect with its three nearest neighbors to the south, northeast, and northwest. These potential linkages are shown in Figure I as light lines, while actual linkages are depicted as dark lines. All transport costs are incurred when a linkage is being constructed: once a linkage has been
Subset of the Economy, with Four Connected Markets
formed, goods and factors can be moved costlessly between connected sites. In terms of the river analogy, linkage formation corresponds to deepening channels and removing obstacles to make the stream navigable; and also to the construction of docks, warehouses, and boats, and the hiring of their crews.

II.2. Preferences

At each site consumers have preferences \( U = \int_{-\infty}^{\infty} e^{-\rho t} \log D(t) \, dt \), where \( \rho \) is the discount rate and \( D \) is the consumption good. This consumption good is produced from \( N \) intermediate goods \( x \), using the CES technology \( D = (\sum_{i=1}^{N} x_i^\alpha)^{1/\alpha} \), where \( 0 < \alpha < 1 \). Equivalently, \( D \) may be viewed as a basket of final goods.

Demand for each intermediate good is \( x_j = E p_j / \sum_{i} p_i^{1-\varepsilon} \), where \( E \) is total expenditure and \( \varepsilon = 1/(1 - \alpha) \) is elasticity of substitution. The Euler equation corresponding to the consumers’ maximization problem is \( \dot{E}/E = r(t) - \rho \), where \( r \) is the interest rate. Using the standard normalization, \( E(t) = N \) implies that \( r(t) = \rho \).

II.3. Technology and Linkage Formation

At every site in the economy each one of the \( N \) intermediate goods \( x \) may be produced by a constant returns technology, where one unit of labor is transformed into one unit of the good. However, there is also a specialized producer at each site \( j \in N \) who can produce a single good \( x_j \) more efficiently than subsistence producers.\(^1\)

As geographical markets expand, each site can specialize more in the production of one good, and purchase other goods from specialized producers at other sites. This rise in regional specialization in the production of intermediate goods corresponds to Young’s [1928] and Stigler’s [1951] idea of Smithian progress as vertical disintegration in production, where each firm undertakes fewer and fewer steps of the production process itself, and instead buys its inputs from one set of producers and sells its output to other producers.

The specialized producer at site \( j \) requires only \( \gamma < 1 \) units of labor to produce each unit of \( x_j \), and has a fixed cost of \( F \) units of labor. It is assumed that \( \gamma/\alpha \geq 1 \) so that, as in Murphy, Shleifer, and Vishny [1989], competition with the competitive fringe forces

---

1. Because goods are symmetric, the mechanism by which specialized production is allocated among sites does not matter here.
the specialized producer to set its profit-maximizing price equal to the wage rate, \( p = w \). Given that the price of each good is identical and aggregate expenditure is \( N \), it follows that nominal expenditure on each good \( j \) in a connected market \( E_j = 1 \). This just means that the price level falls as markets and output grow.

Each specialized producer may invest in forming trading linkages with its two nearest neighbors to the northeast and northwest. Investing in linkage formation lowers the productivity of labor engaged in production: if the firm invests at rate \( i \) in linkage formation, it needs \( \gamma + f(i) \) units of labor to produce each unit of output where \( f(0) = 0, f' > 0, \) and \( f'' > 0 \).

The specialized producer earns a trading profit \( \pi = px - \{\gamma + f(i)\}wx - wF \). Given \( w = p = 1/x \), profit per firm equals
\[
\pi = \{1 - \gamma - f(i)\} - pF. \tag{1}
\]

Once formed, a linkage lasts forever. If a firm invests in linkage formation at intensity \( i \) for a time of length \( dt \), it succeeds with probability \( i dt + o(dt) \). The Poisson success rate of linkage investment implies that a firm will invest all its resources in completing one linkage before it attempts to form another. It will be shown below that, if a firm can form two linkages, it will be indifferent as to which it forms first.

**II.4. Labor Market**

Prices are determined by labor market clearing. There is an identical endowment of labor \( L \) at every site so, in a connected market of \( n \) sites, there are \( nL \) units of labor available for production and linkage formation. Of the \( N \) types of good, \( (N - n) \) are produced by the traditional constant returns technology, and \( n \) are produced by specialized firms.\(^2\) Labor demand is determined by linkage formation as well as production: among the \( n \) specialized producers, \( n_1 \) will be investing in linkage formation.

Recalling that output of each good \( x_n = 1/p_n \) (where the subscript refers to the number of sites in the connected market), the labor market clearing condition may be written as
\[
nL = \frac{1}{p_n} \left( (N - n) + (n - n_1)\gamma + \sum_{j \neq n_1} \{\gamma + f(i)} \right) + nF. \tag{2}
\]

\(^2\) Unlike Locay [1990] and Goodfriend and McDermott [1995], there is an instantaneous movement of resources from the traditional to the modern sector as the market expands.
This gives the price and wage level,

\[ p_n = \frac{N - (1 - \gamma)n + \sum_{j \leq n} f(i_j)}{n(L - F)}. \]  

Prices fall as market size \( n \) rises, reflecting the rise in output resulting from increased division of labor. It is assumed that \( L \) and \( \rho \) are sufficiently large relative to \( N, F, \) and \( \gamma \) that specialized firms always earn positive profits and produce

\[ L(1 - \gamma) > F[ N + f(f^{-1}(NF/\rho(L - F)))]. \]  

**II.5. Capital Markets**

The cost to a firm of attempting to form a linkage is the increased number of workers it needs. At price level \( p_n \) it produces \( x_n = 1/p_n \) goods using \( \{\gamma + f(i)\}/p_n \) variable workers who earn a wage equal to the price level \( p_n \). Investing in linkage formation at rate \( i \) for an interval \( dt \) therefore increases the firm’s wage bill by

\[ f(i)dt. \]

This (nominal) cost is independent of market size \( n \). As in Becker and Murphy [1992], the real cost of linkage formation rises as markets grow and prices fall.

The benefit of market expansion is the rise in profits resulting from increased specialization. A linkage that joins markets \( n \) and \( m \) into market \( n + m \) has instantaneous payoff rate,

\[ \sum_{j \leq n + m} \pi_j - \left( \sum_{k \leq n} \pi_k + \sum_{l \leq m} \pi_l \right). \]

The successful formation of a linkage affects aggregate profits in two ways. First, prices change, affecting profits through the last term of (1). Second, the change in market size may cause investment in linkage formation by other firms to change, affecting the first bracketed term in (1), and the last term in the numerator of (3). These price and spillover effects will be discussed in turn.

*Price effects.* When a linkage forms, it affects price in two ways. First, because market size \( n \) is larger, labor is allocated more efficiently; causing output to rise and price (3) to fall. The present value of the increase in instantaneous profits resulting from this source is, from (1), (3), and (6),
\begin{equation}
    v = \frac{1}{\rho} \frac{NF}{(L - F)}.
\end{equation}

Again, this is independent of the size of the markets being linked.

The second way that price changes when a linkage occurs is when a firm ceases to invest because there are no more potential linkages that it can form. This reduces demand for labor and causes the price level to fall. However, this second price effect cannot be captured by the firm: while a firm can bar access to the linkage to firms that refuse to pay, it has no credible means of returning price to the level it was at when the firm was hiring workers to form the linkage.

Spillover effects. When a linkage forms, it may affect the rate of investment of other firms by facing them with a larger market. This spillover effect is absent here. To see this, note first that from (5) and (7) there exists one symmetric Nash equilibrium where the costs and benefits of linkage formation are independent of market size. Because the payoff to a firm of increasing its rate of linkage formation declines as other firms increase their linkage formation and drive up wages, by Proposition 1 of Cooper and John [1988] this equilibrium is unique.

II.6. Linkage Formation

To summarize, the expected payoff from investing at rate \(i\) for time \(dt\) is \(iv\ dt\), where \(v\) is given by (7). Equating the expected marginal value of investment with its marginal cost, from (5) and (7) firms invest at rate

\begin{equation}
    i = f^* - [NF/\rho(L - F)].
\end{equation}

The purpose of linkage formation is to economize on labor by allowing more production to be undertaken by specialized producers. As a result, the scarcer is labor \(L\) relative to fixed input requirements \(F\) and the number \(N\) of goods produced in the economy, the greater will be the incentive to invest in expanding the market. This is in contrast to Schumpeterian models [Aghion and Howitt 1992; Grossman and Helpman 1991, Chapter 3] where abundant labor allows more resources to be devoted to investment.

The effects on investment of government taxation and expropriation are easily added. If profits are taxed at rate \(\tau\), the term in brackets in (8) is multiplied by \((1 - \tau)\). If firms face expropriation by the state at exponential rate \(\delta\), the term \(\rho\) in (8) is replaced by \((\rho + \delta)\).
II.7. Linkage Probabilities

Let \( P_k(t) \) denote the probability that any site has formed \( k \) transport linkages by time \( t \). Given that firms' investment in linkage formation is a Poisson process with rate \( i \),

\[
P_k(t) = \begin{cases} \frac{ite^{-u}}{1 - e^{-u}(1 + it)} & k = 1 \\ & k = 2. \end{cases}
\]

As time passes and investment in linkage formation occurs at a constant rate, connected markets grow causing specialization and output to rise.

III. TAKEOFF AND STAGNATION

Given that the probability (9) that any site is linked to its neighbors increases gradually through time, this section considers the resulting growth of specialization and output. Intuition suggests that Smithian progress associated with the gradual growth in the number of transport linkages in the economy should take the form of continuous, steady growth. This intuition will be shown to be erroneous: Smithian growth leads to a sudden takeoff when a critical density of market linkages is reached. The economy will move quickly from small, isolated markets that permit limited specialization, to a large market with considerable division of labor.

**DEFINITION 1.** An economy is said to possess a large market if there exist two sites arbitrarily far apart that are joined by a continuous path of linkages, where distance between sites is measured by the minimum number of linkages needed to connect them together.

The dynamics of Smithian growth in an economy where markets form according to (9) is given by the following result. Normalize the investment rate \( i \) to unity.

**PROPOSITION 1.** For an economy with \( N \) sites and linkage probabilities (9), there exists a critical value \( t_c \) of \( t \) such that

\[(2 + t_c)e^{t_c} = 1.\]

For \( t \leq t_c \), the probability that a large market exists tends to zero as \( N \) becomes large. For \( t > t_c \), the probability that a large market exists tends to one as \( N \) becomes large.
Proof. Each site can be thought of as the originating point of a branching process with generating function $G(s) = ste^{-t} + s^2(1 - (1 + t)e^{-t})$. Its expected number of linkages is therefore

$$G'(1) = 2 - (2 + t)e^{-t},$$

If this is less than or equal to one, the branching process dies out. If greater than one, the process originating at each point survives indefinitely with positive probability. For $N$ large, applying the Kolmogorov 0–1 law over all points, the linkage process survives to link arbitrarily distant points with probability 1.

Smithian growth exhibits critical behavior. Given linkage formation normalized to rate one, there exists a critical time $t_\gamma$, where $t_\gamma \approx 1.15$. At this time, although half the potential linkages have formed, the economy is split into local markets that permit only limited division of labor. Once the critical time is reached, adding further linkages causes isolated local markets to begin to fuse together into a large, economywide market which permits a substantial increase in specialization and the efficiency of production. Briefly, below the critical density, adding extra linkages causes small isolated markets to appear; above it, adding new linkages causes these markets to fuse together. Each site now produces few goods for its own subsistence and is able instead to reallocate labor to its specialized activity, trading with specialized producers at other sites in the same connected market. This reallocation of labor from the subsistence technology to the increasing returns technology causes growth to accelerate when the threshold is reached.

The threshold effect is illustrated in Figure II which simulates output growth in an economy with $N = 2^{10} - 1 = 1023$ sites. It is assumed that the labor endowment at each site $L = 2N$, each specialized producer has fixed labor input $F = 0.2$, and variable labor requirement $\gamma = 0.5$. Labor used in linkage formation is ignored for simplicity.

Investment in linkage formation at a rate normalized to unity begins at $t = 0$. The initial growth of the economy is slow until the critical density of linkages is reached around $t = 1.15$. Growth accelerates noticeably as local markets fuse together. This accelerated growth continues until around $t = 5$ by which time nearly all sites have been incorporated into one single mar-

See, for example, Grimmett and Stirzaker [1992, pp 153, 290]
ket. At this stage the possibilities of growth through increased specialization have been exhausted. In the absence of other sources of growth such as capital accumulation, innovation, or learning by doing the economy will stagnate.

**III.1. Costly Transportation**

How does this result change if transport cost is not constant but rises with distance? Suppose that the probability that a site is linked is still given by (9) but linkages are now built exogenously by government. Below the threshold density of linkages, the average number of sites in a connected market is, from (10),

\[
1/[(2 + t)e^t - 1].
\]

If transport costs are sufficiently high that the largest feasible market for a specialized producer is reached at \( t < t_c \), then the accelerated growth of markets after \( t_c \) will have little effect on output.

As time passes, transport networks with successively lower cost are built, and vehicle technology improves, allowing the feasible market for a specialized producer to expand. Each improvement will have little impact on economic activity until transportation cost is sufficiently low that maximum feasible
market size is reached after $t_*$. When this occurs, expansion of transport networks will lead to a general takeoff. For instance, in the economy graphed in Figure II, the largest connected market rises from around 20 sites at $t = 1$ to over 100 at $t = 2$. If the transport technology limits specialized producers to markets smaller than 10, then market expansion will have little impact, whereas if the feasible market size is 200, then a general takeoff will occur as markets expand. In general, goods cost different amounts to transport so that sectors with low transport cost relative to value will undergo a commercial revolution with primitive transportation networks, whereas sectors with higher transport costs must wait for a lower cost network before a Smithian takeoff occurs.

III.2. Catastrophes

While the impact of commercialization is usually assumed to be entirely positive, the expansion of geographical markets can increase an economy’s exposure to occasional, catastrophic shocks. The increase in prosperity caused by trade can make the economy more attractive to foreign conquerors, while the greater movement of people associated with trade facilitates the spread of epidemics.

In the model here, these shocks correspond to the destruction of trading linkages. The impact of the shock will be more or less serious, therefore, depending on whether the economy remains above the critical density of linkages, and, if it is driven below, how far it falls. Finally, the shock can lower the rate $i$ at which replacement linkages form.

IV. A General Result

Having demonstrated a threshold effect in Smithian growth for a parametric example, it will now be shown that this result holds in general. This section will make no assumptions about preferences, technology, or the geographical structure of endowments and markets. In particular, the rate at which new linkages appear can vary across sites, reflecting different natural endowments or the existing pattern of linkages with other sites. All that

---

4. I am grateful to an anonymous referee for this point.

5. In eighteenth century France, small villages, which were isolated from commercial activity, were virtually unaffected by the recurrent plague epidemics that visited large urban centers [Cohen 1989, p 53].
the takeoff result requires is that there be a large number of sites which can connect together, that geographical markets grow as time passes, and that this expansion of geographical markets cause output to grow.

IV.1. Geography and Trade

The economy comprises a set of sites that become linked together into a single market as time passes.

Assumption 1. An economy is a set of sites $Y$ with cardinality $N$. $X = \{1, 2, \ldots, K\}$ denotes the set of possible transport linkages between these sites. If all $K$ potential linkages exist, all sites in $Y$ belong to a single connected market.

At any given time an economy has $k(K)$ linkages in existence.

Assumption 2. The number of linkages in existence $k$ grows through time.

IV.2. Smithian Growth

Smithian progress occurs in this economy. This requires two things. First, the economy must possess an increasing returns technology, allowing production to be undertaken by more efficient specialized producers as markets expand. Second, the linkages that emerge must allow goods to be transported at sufficiently low cost for this increasing returns technology to be exploited. If transport costs are too high, existing patterns of production will not change as new linkages appear.

Assumption 3. Aggregate output is strictly increasing in $k$.

IV.3. Threshold Effects

We are concerned with the emergence of a large, connected, economywide market that permits substantial division of labor. An economy with such a market is said to have property $M$. Formally,

Definition 2. A subset of linkages $A \subseteq X$ is defined to have property $M$ if some of its elements are part of a set of linkages connecting $s$ sites in $Y$, where $s$ is of order $O(N)$.

Elements of $X$ with property $M$ are identified as belonging to set $M$. Consider $X^k$, the set of subsets of $X$ with $k$ elements, where $k$ is monotonically increasing in $K$. For $M \subseteq F_X$ (the $\sigma$-algebra corresponding to $X$), denote the fraction of markets of size $k$ that
have property property $M$ as $\text{Pr}_k(M) = |M \cap X^k|/|X^k| = |M \cap X^k|/\binom{k}{k}$.

**Definition 3.** $k^*(K)$ is said to be a threshold function for property $M$ if, when $\lim_{K \uparrow \infty} k/k^* = 0$, $\lim_{K \uparrow \infty} \text{Pr}_k(M) = 0$; and when $\lim_{K \uparrow \infty} k/k^* = \infty$, $\lim_{K \uparrow \infty} \text{Pr}_k(M) = 1$.

**Theorem 1.** There exists a threshold function $k^*(K)$ for property $M$.

**Proof.** If a subset of linkages, $A \subseteq M$, and $A \subseteq B$, then $B \subseteq M$; and neither the empty set $\emptyset$, nor $\emptyset$ equal $M$. $M$ therefore satisfies the conditions to be a nontrivial, monotonic property of set $X$. By Theorem 4 of Bollabas and Thomason [1986] there exists a $k^*(K)$ such that for any monotone increasing function $\omega(K) \geq 1$, if $k \leq k^*/\omega$, $\text{Pr}_k(M) \leq 1 - 2^{-1/\omega}$; and if $k \geq (k^* + 1)/\omega$, $\text{Pr}_k(M) \geq 1 - 2^{-\omega}$. Taking limits as $K$ becomes large, the result follows. □

For almost all patterns of linkage formation in a large economy, as the density of trading linkages exceeds a critical value, a large connected market appears. The existence of a growth threshold is a generic property of Smithian progress.

**V. The Transformation of Sung China**

Between the eight and twelfth centuries, China changed from a simple subsistence economy to a level of commercial and industrial development not matched elsewhere until the end of the eighteenth century. Historians specializing in this period attribute this transformation to the development of a network of waterways permitting regions to specialize in one activity and to trade with other regions. Theorem 1 permits a formal test of this hypothesis. If China's growth in this period was driven by Smithian progress, then we should be able to find evidence for a sudden

---


THE DYNAMICS OF SMITHIAN GROWTH

acceleration in growth around the time that an economywide transport network appeared.

This section will attempt to see whether such a takeoff occurred. It does three things. First, it rehearses the qualitative evidence on the role of transport networks in the development of China in this period. Second, it applies Theorem 1 to test the null hypothesis that China’s growth is an example of Smithian progress by looking for a sudden acceleration in growth, followed possibly by stagnation once the limits of specialization had been reached. Finally, it considers the alternative hypothesis that China’s growth in this period was a result of unusually low government exactions compared with later periods of slow growth.

V.1. Waterways and the Economic Transformation of Sung China

The economy of the Tang dynasty in 750 AD differed little from that of the Han dynasty 850 years before [Hartwell 1963, p. 3]. Both were largely subsistence economies, with little trade other than in luxuries and salt. Use of money outside large administrative centers was unknown, and most taxes were paid in kind. Manufacturing was on a tiny scale, serving the needs of the aristocracy and Buddhist clergy.

By the third quarter of the eleventh century, many manufacturing enterprises operated on a large scale in mining, metallurgy, and shipbuilding, employing hundreds or sometimes thousands of workers, and serving large geographical markets along a national waterway network. The use of money was universal, with a variety of negotiable instruments used to facilitate large transactions over long distances. Many innovations occurred for which the period is famous: the use of coal in smelting iron, printing, the adoption of early-ripening strains of rice, pound locks, a Bessemer process for producing steel, a water-powered spinning wheel, deep drilling wells, and gunpowder to name a few. Hartwell [1963, p. 54] concludes that “in terms of absolute output, rate of growth, scale of business enterprise, and adoption of new technology, the Northern Sung had reached a level of commercial and industrial development surpassed by no society before the last decades of the eighteenth century.”

For historians of the Tang and Sung, the decisive change in this period is the shift in China’s population southwards, from the wheat-millet region around the Yellow River and North China Plain, to the rice-producing Yangtze basin. In 609 only 25 percent
of China's population lived in Southern China, compared with 21 percent in 2 AD. However, as peasants moved south to exploit newly opened territories and to escape the depredations of steppe nomads, South China's share of population rose to 46 percent in 750, 65 percent in 1080, and 71 percent in 1200 [Hartwell 1982, Table 1].

This southward shift in China's demographic center of gravity created two problems for the state. First, it had to exert political control over these regions. Second, it was faced with the difficult logistical task of transporting large quantities of tax grain from the south to feed the imperial capital and armies that were concentrated in the north.

To solve both problems, the Chinese state began to construct a national waterway network [Hartwell 1971, p. 307]. Early canals were used mostly to transport soldiers and bureaucrats. However, as the peasants of South China began to master the techniques of wet rice cultivation, waterways came increasingly to be used for transporting tax grain. Construction of new waterways and the improvement of existing ones through deepening, removing rapids, and adding locks, continued steadily. While important areas were still isolated in 900 AD, by the end of the tenth century all economically important regions of China had been linked into a single, 30,000 mile long national network of canals and navigable rivers [Chi 1936, p. 132].

Although built for political and fiscal reasons, this waterway network transformed the Chinese economy. The ability to transport goods cheaply over long distances enabled regions to switch from subsistence to commercial activities. Regions were able to concentrate on commercial agricultural and, sometimes, industrial activities and to import grain for subsistence from specialized producing areas [Shiba 1970a, Chapter 2]. The way that waterways transformed a subsistence economy into a commercial one was noted by contemporary observers (see the remarks of the twelfth century writer Yeh Shih in Ma [1971, p. 16]), and also by Adam Smith [1776]. In the chapter "That the Division of Labor is Limited by the Extent of the Market," he notes how China, and other ancient empires such as Egypt, "seem all to have derived their great opulence from this inland navigation" [1776, p. 25].

Commercialization transformed urban life. Trade spread from the tightly regulated urban markets of the T'ang period

8 Hill [1988] discusses the influence of China on Smith's thought
[Twitchett 1966, 1968] to enmesh the entire country in a complex hierarchy of articulated markets, ranging from giant specialized markets in the large cities, to periodic markets in small villages [Ma 1971, Chapter 4; Shiba 1970a, Chapter 4; Shiba 1975].

Market expansion not only caused output to grow by permitting increased specialization; it also increased the incentives for innovation. This is shown by the most important innovations in agriculture and industry in this period: the adoption of early-ripening strains of rice; and the substitution of coke for charcoal in smelting iron. The adoption of early-ripening strains of rice in the eleventh century and the intensive development of strains suited to local conditions would not have been worthwhile in a subsistence economy without the means of transporting the increased surplus to markets [Ho 1956; Shiba 1970b]. Similarly, most of the quadrupling of iron output during the eleventh century was accounted for by a few very large ironworks in northeastern China, each employing thousands of workers and operating dozens of furnaces continuously. The large scale of iron working, initially using charcoal for fuel, caused the rapid deforestation of northeastern China [Hartwell 1963, p. 29], and provided both the inducement and the resources to develop bituminous coke as an alternative fuel source.

V.2. Takeoff and Stagnation

While this qualitative evidence does indicate that waterways played some role in the transformation of China between the mid-ninth and early twelfth centuries, growth over a period of 250 years hardly qualifies as the sort of sudden takeoff that Theorem 1 predicts. Given that China's waterway network came to span the economy sometime around 1000 AD, one should expect an acceleration in growth some time before this if agents reacted immediately to the expanded potential market. To the extent that there was some delay in agent's realizing the gains to increased specialization or developing the organization and technology to specialize and trade over longer distances, this acceleration should take place later.

To see whether a Smithian takeoff did occur sometime around 1000 AD requires quantitative data on China's real output during this period. Although no direct data on output are available, we can use data on real money supply as a proxy for real output growth.

Recall from the Quantity Equation that the growth rate of
real money supply is equal to the growth rate of real output, if velocity is constant. There are two rival theories of how velocity changes in the course of development: Wicksell's view that velocity rises as transactions opportunities increase; and Friedman's view that rising real income reduces the need to economize on real balances (cf. Bordo and Jonung [1987, Chapters 2–3]). Although the Wicksell hypothesis of rising velocity through increased commercial activity seems more plausible for Sung China, because we are concerned with structural breaks any error in the estimated growth rate due to changing velocity will be of secondary importance here.

A price level series was constructed by interpolating data on rice prices from Hartwell [1967, footnote 8]. Data on annual increments to three components of money supply were interpolated and summed to produce an estimate of the annual increment to nominal money supply. These components are coinage issues [Gernet 1982, Table 14], precious metal issues [Hartwell 1963, p. 6], and paper money issues [Hartwell 1963, p. 218]. Data on one important component of money supply—negotiable instruments—are not available. This probably means a serious underestimate of money supply growth during the eleventh century.

When interpolating the data, the presence of a structural break caused a cubic spline to misbehave predictably. Assuming a constant growth rate between observations, linear interpolation was therefore applied to the log of each data series.

This allowed the construction of an annual series of increments to real money supply. The series begins in 805 AD, which is the earliest year for which data for all series are available, and ends in 1075 AD. After this date, large, and inflationary, issues of paper money were made by the collapsing Northern Sung state.9

To estimate the growth rate of real money supply from these annual increments, we must assume some value for the initial real money supply. This was done by assuming that the money supply increment in 805 AD represented a real growth of 0.64 percent. This figure was chosen to equal the revised Crafts-Harley [1992, Table 4] estimate of British growth in the period 1760–1780. Assuming a wide range of other values did not affect the qualitative pattern of growth in the data.

The log of the real money supply series from 805 to 1075 is

---

9 This episode of early inflation is described in a fascinating paper by Liu [1983]
plotted in Figure III. It is immediately apparent that there is a marked acceleration in the growth rate around 1000 AD. Recall that this date corresponds to the completion of a waterway network linking all the more important parts of China.

The monetary evidence for a takeoff of the Chinese economy during the eleventh century is supported by data on the output and price of iron. Hartwell [1966] estimates that total iron output quadrupled between 998 and 1078, causing its price (in terms of grain) to fall by more than two-thirds. Both the per capita output and price of iron are comparable with Britain’s in the early eighteenth century.

**Stagnation.** After this revolution, China’s development stalled. Aggregate output certainly continued to grow: population quadrupled between the twelfth and the late nineteenth centuries, and was fed (more or less) by increasing the area under cultivation and by adopting new world crops. However, the intensive growth of the Sung period ended. There are two sources of quantitative evidence for this assertion: real wages, and output of metallurgical industries.

Data on real wages of different occupations from 50 BC until 1818 have been assembled by Chao [1986, Table 9.2]. They indicate that real wages peaked around 1100, and afterwards declined continuously, reflecting the faster growth of population than output.

Given the spectacular progress in metallurgy during the eleventh century, its subsequent regression is particularly interesting. The iron and steel industries of northeast China went into decline as a series of natural and man-made disasters destroyed the transportation network in the region [Hartwell 1968, 1969]. Although the network was rebuilt in the fifteenth century, the iron industry had disappeared by then. Comparing metallurgical production in 1919 with that of the Sung, Hartwell [1963, p. 35] finds that iron output was lower in per capita terms, while non-ferrous output had fallen even in absolute terms.

While the exact timing of this stagnation is uncertain, it does appear that the Sung economy was in difficulty well before the massive disruption of the Mongol invasions and rule. Evidence of increasingly severe rural subsistence crises and commercial recessions leads Hartwell [1982] and Smith [1991, p. 313] to conclude that living standards began to fall during the twelfth century.

The reasons why China failed to grow further once the limits
of specialization had been reached are fascinating but are beyond the scope of this paper. Mokyr [1990, Ch. 9] provides an extensive survey of the older literature. Smith [1991, pp. 313–18] gives an excellent discussion of recent research on this topic, which emphasizes the rise to power of a conservative, localist gentry at the expense of an interventionist, centralized bureaucratic state.

V.3. Government and Growth

The idea that government retarded Chinese economic development is a familiar theme of comparative economic histories [Jones 1988, Chapter 4; Landes 1969, pp. 15–21] and general his-
tories of China [Fairbank 1992, Chapter 8]. It is argued that high levels of taxation, the low status of mercantile activity, poorly developed commercial law, and high probabilities of state expropriation all discouraged private capital accumulation, and diverted wealth into land ownership and securing government posts for family members. At the same time, government spent its revenue uselessly instead of using it to improve the capital infrastructure.

That government probably retarded the growth of China after the eleventh century is not at issue here. However, the proposition that high taxes and low infrastructural spending necessarily imply low growth is problematic because, for Sung China, its contrapositive is false. The economic revolution of the eleventh century occurred at a time when government taxation and expropriation were much higher than at any later time in Chinese history.

Feuerwerker [1984] estimates that government spending in the Sung accounted for approximately 13 percent of national income, compared with 5 percent under the later Ming and Qing dynasties.\(^{10}\) This spending was funded not only by taxation, but by state monopolies (which accounted for one-third of state revenues in 1077 [Smith 1991, p. 340]); direct expropriation through state organized guilds [Ma 1971, pp. 82–91]; and outright confiscation of unusually profitable activities such as tea growing [Smith 1991] and salt drilling [Glahn 1987, Chapter 3]. This need to generate revenue is explained by the precarious military position of the Sung which faced a succession of well-organized steppe states, forcing it to spend over 80 percent of revenue on its army by the end of the eleventh century.

While taxation and military spending were uniquely onerous, the rate of public infrastructural expenditure under the Sung does not seem to have been higher than in later periods. Looking at hydrological projects—building canals and deepening rivers for transportation and irrigation purposes—Chi (cited by Needham [1971 p. 282]) estimated that the average number begun annually under the Sung was considerably lower than under later dynasties.

The economic transformation of Northern Sung China shows that low rates of government activity are not always necessary for growth. Nor are they sufficient. By the late fourteenth cen-

\(^{10}\) Hartwell, cited by Golas [1986], derives a similar figure of 9–12 percent, while Golas [1986] reaches the less plausible estimate of 24 percent.
tury, Ming Dynasty China had very much lower government expenditure than under the Sung, and a better transport infrastructure. Despite this, the intensive growth of the Sung did not recur. This reinforces the view that China crossed a once-off growth threshold in the eleventh century.

VI. CONCLUSIONS AND EXTENSIONS

This paper analyzed the evolution of an economy where growth is driven by increased specialization caused by the geographical expansion of markets. It showed that gradual formation of new transport linkages does not imply gradual growth, as intuition might suggest. Instead Smithian progress was proved to exhibit threshold behavior. Below a critical density of transport linkages, the economy is split into small markets with limited scope for division of labor. Once the critical density is reached, these small markets begin to fuse together into a large, economywide market. The resulting increase in specialization causes an acceleration in the growth rate. Once all sites have been incorporated into one market, the possibility of further growth through increased division of labor is exhausted.

This result supports the consensus among historians of Sung China that the economic transformation between the ninth and eleventh centuries occurred in response to the creation of a national waterway network. Section V showed that the economic takeoff around 1000 AD coincided with the completion of a national waterway network that had been growing steadily since the seventh century. Moreover, this economic transformation occurred in a period of heavy taxation and state expropriations, but lasted only a short period and was followed by stagnation rather than by an industrial revolution.

Two extensions are possible. First, while the paper focused on transport costs, the analysis extends straightforwardly to many other transactions costs emphasized by North [1991] and others. Second, it is possible to consider transport revolutions other than Sung China. In doing this, it is useful to distinguish, as in Section III, between development of low cost transport net-

11 Market expansion can have other effects that are not considered here, such as evolution of more sophisticated political structures and development of more cosmopolitan outlooks, which will also affect growth.
works, such as canals and rail, that permit specialization in all sectors; and the development of more primitive networks that only affect a few sectors producing light, high value goods.

The European "Commercial revolution of the thirteenth century" fits into the latter category. Roover and others have shown how improving security and communications created by the development of strong states caused the pattern of trade in Western Europe to shift in the late twelfth century from small, heavily armed caravans of itinerant merchants to organized networks where large firms based in Italy traded with their agents in large cities, sending goods—textiles most importantly—by specialized carrier and making use of sophisticated credit and insurance instruments [Roover 1965; Spufford 1988, Chapter 11]. By contrast, the weaker states and lower population density of Eastern Europe kept the organization of Hanseatic trade at a rudimentary level. Outside Europe, a huge variety of trade networks are described by Curtin [1984], but shortage of source material restricts his descriptions to individual points of time, and does not allow the evolution of trade to be analyzed.

For the development of low cost networks outside China, one must look to the eighteenth and nineteenth centuries, when the impact of commercialization is harder to disentangle from that of the concomitant technological progress. In eighteenth century Britain, the rapid emergence of integrated national turnpike and canal networks is well illustrated by the maps produced in Langford [1992, Figures 6 and 7]. In 1740 turnpikes existed only near major towns, but by 1770 these local linkages had fused into a dense national network. Similarly, in 1760 inland navigation was confined to large rivers, while by 1789 every major town was linked into a national network of canals and navigable rivers whose role in growth is discussed by Mantoux [1988, Sections 2.7–2.8] and others. For the United States the pre-Fogel consensus assigned railways the central role in economic development in the last two-thirds of the nineteenth century [Jenks 1944, Section 4; Rostow 1971, p. 55]. For Tsarist Russia, Metzer [1974] shows how the development of branch railway lines in the period 1890–1910 transformed the geographical pattern of specialization in grain production. What distinguishes China's experience of Smithian progress from these cases is that other sources of growth did not emerge in China once the limits of specialization had been reached; whereas Western growth in the nineteenth
century was able to continue, and accelerate, through a combination of capital accumulation, innovation, and learning by doing.

University College Dublin

REFERENCES


Chi, Ch'ao Ting, *Key Economic Areas in Chinese History* (London: George Allen and Unwin, 1936)


THE DYNAMICS OF SMITHIAN GROWTH


---, “A Cycle of Economic Change in Imperial China: Coal and Iron in Northeast China,” *Journal of the Economic and Social History of the Orient,* X (1967), 102–59


Ma, Laurence J. C., *Commercial Development and Urban Change in Sung China* (960–1279) (Ann Arbor, MI: Department of Geography, University of Michigan, 1971)


Tamura, Robert, "Regional Economies and Market Integration," Working paper, University of Iowa, 1992


——, "Merchant, Trade and Government in Late T’ang," *Asia Major* (n.s.), XIV (1968), 63–95


Yang, Xiaokai, and Siang Ng, "Specialization and Division of Labor: a Survey," working paper, Monash University, Department of Economics, 1994