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Towards the Use of Sink Ancestors and Source Descendants as a Clustering Method in a Directed Acyclic Graph

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***Abstract*—Clustering techniques play an important role in analysing complex networks such as electrical power systems grids. They can help in identifying congestion bottlenecks and other planning and operation activities. This paper presents a clustering method for partitioning a power network by grouping the nodes and edges based on their reachability to or from the various sources and sinks in the system. These clusters can be discussed using familiar terminology from river networks, such as tributaries and distributaries. A goal is to concretely visualise how nodes and edges are related, and their operational dependencies within the network. The clustering results may give power system operators new insights into the system's structure, enabling situational awareness, fault/vulnerability analysis, and planning. The proposed methodology is applied to two sample grids to demonstrate the types of clusters it can identify.**

I. Introduction

The existing power system grid confronts various challenges due to its increasing complexity, rapid expansion, and departure from the traditional power systems structure. For identifying innate patterns in datasets, *clustering* has a wide range of applications in several disciplines, including electrical power systems networks, social network studies, market analysis, biological network studies, computer science, to mention only a few [1]. Splitting or grouping the large, interconnected power system into small groups or clusters can simplify analysis and control problems. It potentially can also increases operator situational awareness by facilitating an intuitive mental model of the prevailing grid conditions.

The present paper focuses on clustering in the Directed Acyclic Graph (DAG) that represents the instantaneous state of a power system. Such a DAG representation can help in identifying sets of lines that collectively carry an aggregate power transactions [2], [3]. A key idea is the idea of a *source node*: this is a bus that has no incoming power flows on any of its lines. Likewise, a *sink node* only receives power through its incoming lines. Notably, not all generator buses will be sources, nor all loads sinks. In fact, most nodes within a grid will be *intermediate nodes*, which, notwithstanding their own load or generation, receive some quantum of power from the wider grid and also forward power onward to downstream nodes.

This paper revisits and modestly extends the DAG clustering technique first presented in [4]. In the present treatment, the language of ancestor and descendant sets is used to describe the algorithm's operation, and novel results are presented for clustering based on these sets, seperately and jointly.

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Within a river network, water flows from high to low altitudes, and different tributary branches sequentially merge together to form a *mainstem*. With the present work, we are aiming to further articulate a similar intuitive model for discussing the instantaneous state of a power system. To this end, the instantaneous state of power flow within the power system network is depicted using a layered graph drawing [5], [6] of the corresponding DAG. These graph drawing algorithms position each node within a two-dimensional canvas such that all oriented edges are aligned, and therefore all power flows are from the top towards the bottom of the diagram. Rather than depicting the geographical positions of the grid's buses, such a diagram explicitly shows the bulk power transactions arising from the prevailing generator dispatch and system loading.

Building on this perspective, the presented technique clusters together the nodes and edges in a power system network based on a shared fingerprint of sources that feed them and/or sinks that they feed towards. Each cluster collectively carriers a bulk power transactions from one functional area of the network to another. Related precursor work in [7] proposed Sankey diagrams as another way to depict and discuss such aggregate power flow transactions.

The authors in [4] described such clusters of nodes as *"coherent corridors"*. It is hoped that defining such clusters can enhance the power system operators' situational awareness in the control room. This paper hopes to modestly build on the work in [4], by presenting its principles in a new way and to showcase a broader set of clustering results.

By identifying all sources and associated sinks, buses, and edges, this clustering technique may help facilitate the assessment of overloaded lines, faults, and vulnerable sections of the grid, enabling appropriate actions to avoid blackouts.

In electrical power systems, different researchers have analysed and applied various clustering techniques to solve various problems, such as K-means, partitioning, hierarchical, density based, model based, and grid based [8], [9]. For instance, decentralised power system control strategies based on clustering for distributed generation are presented in [10], [11], these control techniques balance the supply and demand as well as the cluster frequency inside the clusters while providing a hierarchical and multi-level control framework. With secondary level voltage regulation at lower levels, this clustering offers decentralised system control. In a very large, interconnected power system, optimal power flow can be a laborious problem to solve. The methodologies for distributed optimizations solutions that use clustering are discussed in [12], [13]; these techniques make use of adjacent bus data from the physical system and scheduled load data from the Lagrangian-solved economic load scheduling.

Clustering methods based on graph theory are discussed in [14]–[16]. By considering several similarity metrics, these methods respectively employ multi-constraint and multi-objective

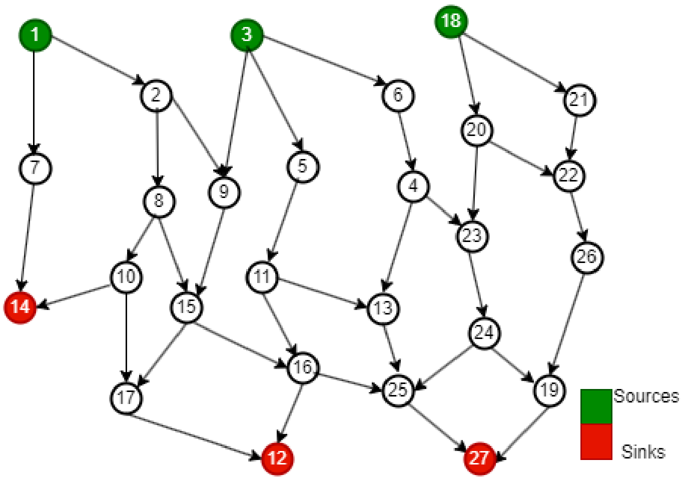

Fig. 1. A notional DAG containing three pure sources and three pure sinks.

graph partitioning theory, an unnormalised spectral clustering algorithm, and a connected graph constrained Knapsack problem. The clusters are utilised for a variety of power system applications, including the modelling of electrical networks, decision-making regarding switching operations, and evaluation of the power system under blackout conditions. Clustering is sometimes employed as the first step in the real-time power system control process because it may make the process of offering solutions to problems with intelligent/future power systems grid easier. However there are few network clustering algorithms specifically focusing on directed graphs, which are directed networks which can encode the instantaneous state of a power grid.

The remainder of this article is structured as follows. The process for clustering the nodes and edges of a power systems network, the modified Horton-Strahler stream order numbering, and an intuitive process of power system visualisation are discussed in section II. The simulation results are shown in section III. Finally, Section IV brings this study to a conclusion.

II. Methodology

In this section, the proposed clustering technique, the modified Horton-Strahler stream order numbering applicable to the electrical power systems, and the proposed way to visualised the electrical power systems are discussed.

A. The Representation of Power Systems as DAG

A couple of simple measures must be taken to prepare the electrical power system network before applying the clustering algorithm. The power network must first be modelled as a DAG. A directed graph without cycles is known as a DAG [17]. The instantaneous power flow circumstances within the power system are used to generate an appropriate DAG to perform the clustering method, setting the directionality of the branches according to the active power flow directions within the network, as depicted in Fig.1.

To represent the power systems network as DAG, let $G = (V, E)$ be a directed acyclic graph, where V is the set of nodes (buses) and E is the set of directed edges (branches i.e powerlines or transformers). The cardinality $|V|$ is the total number of nodes in the network, each labelled with a unique index v, where $1 \leq v \leq |V|$. An $edge(s, r) \in E$, indicates that there is a oriented connection from a sending node s to a receiving node r. The DAG representing the instantaneous state of the power grid is constructed with edges oriented with the prevailing direction of flow of active power in each branch. The non-symmetric adjacency matrix, A, of the DAG can be built up by assigning a 1 element on the appropriate side of the diagonal for each pair of connected nodes s and r:

$$A_{(s,r)} = 1 \text{ iff } P_{(s\to r)} > 0 \tag{1}$$

$$A_{(r,s)} = 1 \text{ iff } P_{(s\to r)} < 0 \tag{2}$$

Where $P_{(s\to r)}$ is the signed active power flow on the branch connecting node s and node r. $P_{(s\to r)}$ can be calculated using the following assumptions on the DC power flow as depicted in equations 1 & 2 or alternatively could be empirically metered or calculated with more accurate AC assumptions as expressed in equation 3.

$$P_{(s\to r)} = \frac{\delta_s - \delta_r}{X_{sr}} \tag{3}$$

Where, δ_s and δ_r denotes the voltage angles of nodes s and r and the reactance of the branch (edge) connecting the nodes is denoted by X_{sr}.

B. Defining Descendant and Ancestor Subgraphs

The term *reachability* in graph theory describes whether a path exists from one node to another node or edge [18]. That is to say, for a given set of vertices in a graph, p is said to be reachable from q if and only if some path leads from p to q [19]. For two nodes p and q to be connected, there must be a path between them consisting of only adjacent nodes, with path beginning at p and ending at q [18], [20]. Denote with $B \subset V$ the sub-set of pure source nodes within the network and with $F \subset V$ the sink nodes within the DAG.

1) Descendant Reachable Subgraphs: Let D_v denote the set of *descendants* reachable from a particular node $v \in V$. The descendant set is the subgraph of nodes and edges which are reachable downstream from that particular node. In this work, the downstream subgraph reachable from each pure source node is of particular interest: for this reason a superscripted asterisk decorator * is used to denote the descendant set in such cases: $D^*_{v\in B}$.

Reachable subgraphs can be extracted from networks with various algorithms, for instance using a *depth first search* [21]. In the present example, such a search would start from a pure source node, and would explore as far as possible along each edge in the network and record all the nodes visited, backtracking when no further onward path is available.

2) Ancestor Subgraphs: The sets of nodes and edges that can reach *to* a particular node in a network are also of interest. Let A_v denote the set of *ancestors* that can reach to a particular node $v \in V$. This ancestor set is the subgraph of nodes and edges which are directly upstream from a node and can reach to it. Once again, in the special case that we are considering the ancestor set for a pure sink node, we use the decorated notation $A^*_{v\in F}$.

The depth first search can likewise extract the set of nodes and edges that can reach to a specified node e.g by building a reversed digraph with flipped edge orientations and searching as before.

C. Clustering Based on Strict Intersections between Descendant and Ancestor Sets

A particular node may be fed by one or many source nodes: that is, it may be appear in several source descendant sets D^*. Likewise, a node may forward power to one or several pure sinks: it can be an element of numerous sink ancestor sets A^*. The proposed clustering technique groups nodes together that have a matching *fingerprint* in terms of the sources that feed them, or the sinks that they themselves feed, or both of these characteristics. Stated another way, this grouping procedure clusters nodes based on the segment they would occupy in the Venn diagram showing intersections between all sets D^*, or all sets A^*, or indeed [D^*, A^*]

D. Illustrative Example

The notional diagram in Fig. 1 shows a small DAG. This network contains three pure sources node (nodes 1, 3, & 18) and three pure sinks (12, 14, and 27). It can be observed that each pure source nodes (1,3, & 18) has a corresponding descendant set D_1^*, D_3^*, & D_{18}^*, respectively. Likewise, the pure sinks are fed from upstream subgraphs denoted as ancestor sets A_{12}^*, A_{14}^*, and A_{27}^*.

The Venn diagram in Fig. 2 enumerates the node membership of each descendant set D^* and shows how these sets overlap. There are six non-empty clusters evident here. This approach to clustering based on intersections between D^* sets is here termed *source reachability clustering*.

Likewise, the Venn diagram in Fig. 3 provides the node membership of each ancestor set A^* and again shows how these sets overlap. Grouping nodes together based on the segment they occupy in such a Venn diagram is here termed *sink reachability clustering*.

There are are seven non-null segments to occupy in a three way Venn diagram such as these: this number is given by $2^N - 1$. It is also possible to look at the various conjunctions of all six sets A_{12}^*, A_{14}^*, A_{27}^*, D_1^*, D_3^*, & D_{18}^*, although a six-way Venn diagram is awkward to draw in two dimensions. Such a grouping approach here is termed *source/sink reachability clustering*. Here, a particular node or edge could appear in any of 63 different clusters (= $2^6 - 1$).

This approach to clustering is motivated by identifying sets of nodes and edges within a particular power grid that simultaneously share a common purpose: either receiving power from the same sources, or feeding power towards the same sinks. Such commonality is termed a *"corridor of coherent flow"* in [4]. This may offer a new way to monitor aggregate or parallel flows in a section of a network.

E. The Modified Horton-Strahler Stream Order Numbering

This section introduces a modified version of the Horton-Strahler stream order numbering. The Horton-Strahler concept, named after the scientists Horton and Strahler who originally proposed it [22], establishes a statistical relationship between the river segments of different "orders" within a drainage basin, spanning from small orginating streams which sequentially merge to form a large river mainstem. Stream order numbers quantify where in this branching hierarchy a particular river segment falls.

In our research, we have adapted this concept to a power system network, drawing an analogy between the power system network and a river network. To do, we first group the nodes and edges in the network into clusters based on source, sink, and

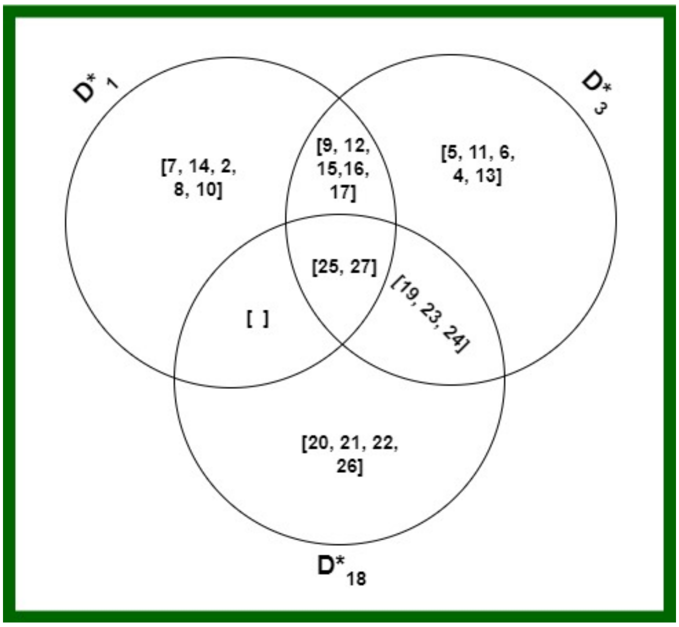

Fig. 2. A Venn diagram showing the nodes in the sources' descendant sets in the notional DAG. Grouping nodes based on the segment they would occupy in such a Venn diagram is the principle behind the proposed *source reachability clustering*

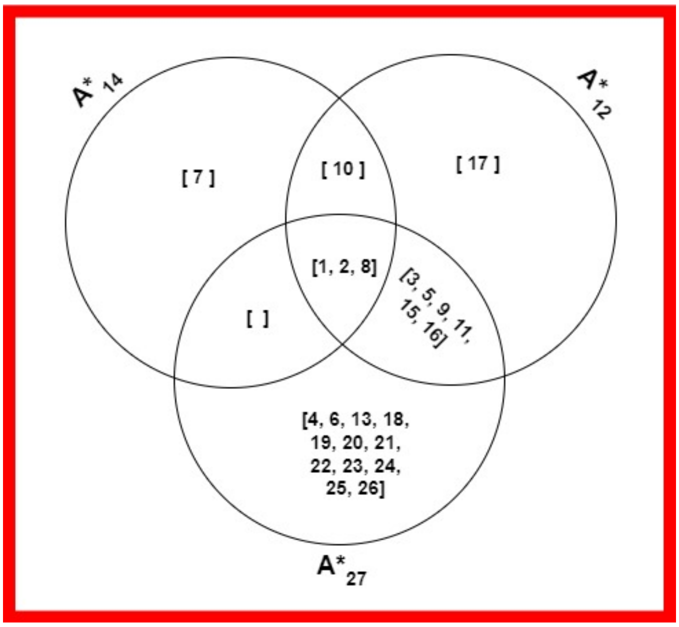

Fig. 3. A Venn diagram showing the nodes in each sink's ancestor set in the notional DAG. Grouping nodes based on the segment they would occupy in such a Venn diagram is the principle behind the proposed *sink reachability clustering*

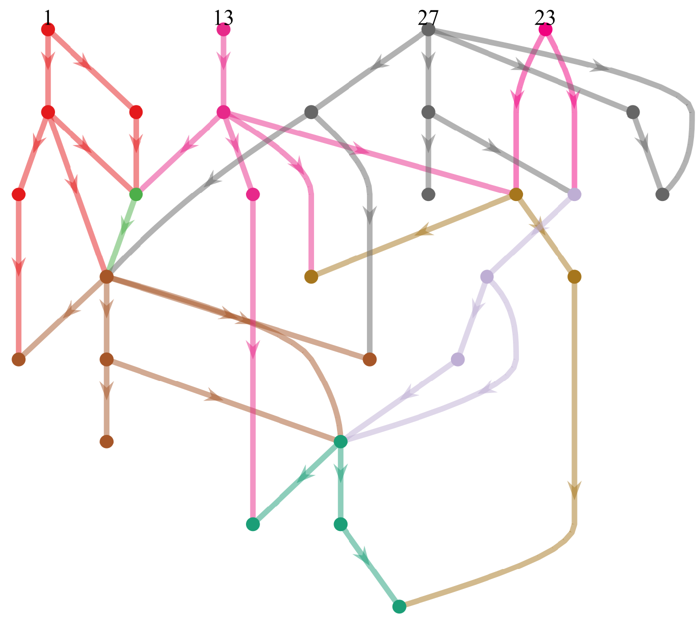

Fig. 4. A layered graph drawing of the `case30` network, such that power flows downwards from the pure sources at the top. Nodes and edges are coloured according to the *source reachability* cluster they occupy

TABLE I.
TOTAL LOAD DEMAND AND POWER GENERATED FOR EACH CLUSTER BASED ON SOURCE REACHABILITY FOR `case118`

Cluster	Source Fingerprint	Total Load Demand (MW)	Total Generation (MW)
1	[10]	433	535
2	[10,26]	304	7
3	[26]	239	534
4	[10,26,66]	173	0
5	[10,26,66,69]	121	0
6	[66,69]	693	271
7	[66]	499	1098
8	[66,69,80]	184	0
9	[69]	0	516.4
10	[26,69]	84	0
11	[26,69,80,89]	216	0
12	[69,80,89]	132	0
13	[80]	184	477
14	[80,89]	168	0
15	[89]	684	899
16	[87,89]	21	0
17	[87]	0	4
18	[89,111]	107	0
19	[111]	0	36

source-sink reachability as discussed in the preceding sections. Each of these clusters loosely corresponds to a particular segment of a river (which runs between confluences)

In the case of clustering nodes based on the reachability of sources, the clusters containing a set of nodes accessible from a single source (tributaries) are assigned a first-order number. Subsequently, the algorithm identifies clusters emanating from suh tributaries; when two clusters with the same order number are linked to a new downstream cluster, the order number of the downstream cluster increments by 1. However, if the two incoming clusters have different order numbers, the downstream cluster assumes the order number of the higher upstream cluster and increments by one. This aspect differentiates our algorithm from the conventional Horton-Strahler concept. The propogation of stream orders numbers through the network can be performed by a recursive algorithm.

III. RESULTS

A. Test platform

To evaluate the proposed clustering and stream order numbering approaches of this paper, two sample grids are used: `case30` and `case118` from MATPOWER [23]. The creation of the representative adjacency matrix, A, and all other calcuations is also handled within MATPOWER. Note that both the `case30` and `case118` test systems are specified with a default generator dispatch and network loading: the DAGs presented here are depicting this particular snapshot of the network. When generator dispatch and load conditions would change, a new DAG would be formed, and so a new set of network clusters would emerge.

B. Clustering Examples

1) Clustering Results: Source Reachability: Figures 4 and 5 depict the `case30` and `case118` systems respectively. It can be observed that the nodes (buses) and the edges (branches) are grouped into respectively nine and nineteen different clusters based on their source reachability. The colours clearly demarcate the subgraphs that compose the different clusters. The style of the layout of the DAG was chosen to make it easier to observe the main source nodes, from fig. 4 and fig. 5, the source nodes are [1, 13, 23, 27] and [10, 26, 66, 69, 80, 87, 89, 111] respectively. For instance, in 4 the nodes and edges immediately downstream of source 1 are shown in bright red: these are the elements of D_1^* that do not appear in any other sets: $D_1^* - D_{13}^* - D_{23}^* - D_{27}^*$.

The proposed clustering techniques may be useful for the power system operators, as this could more clearly show pictorially the downstream portion of the grid that would be affected if a set of lines were outaged.

Table I shows the total power demand and the total generations in each of the clusters shown in Fig. 5. This is also extended to all the clustering techniques discussed in the previous sections, but due to lack of space, we only discussed clustering based on source reachability for IEEE `case118`. It can be observed from table I that some clusters have zero total generation and a non-zero value of the total load demand. This indicates that these clusters rely entirely on external power sources to meet their load demand, and so may be more vulnerable to disruptions in the external power supply which could result to power shortage or blackouts in these region. Such insights could be useful in power system planning and infrastructure considerations.

2) Clustering Results: Sink Reachability: It can be observed from Fig. 6, that the sink nodes are 24 in number as depicted in the figure. Each cluster of colours form a partition; a closer look at Fig. 6 shows that there are forty-five clusters based on sink reachability. Nodes in the same group are likely to have a similar power flow characteristics with same level of depe ndence, and nodes in a higher group or cluster depend on the nodes in a lower group (lower groups are groups that a directly connected to the sink nodes).

3) Clustering Results: Source/Sink Reachability: One of the same test systems from the preceding sections is used to evaluate the accuracy and efficiency of the proposed algorithm in clustering the power system network nodes and edges based on the source-sink reachability. The algorithm seeks to group the nodes based on the ability of a node or edge to reach both primary source(s) and final sink(s) in the power system

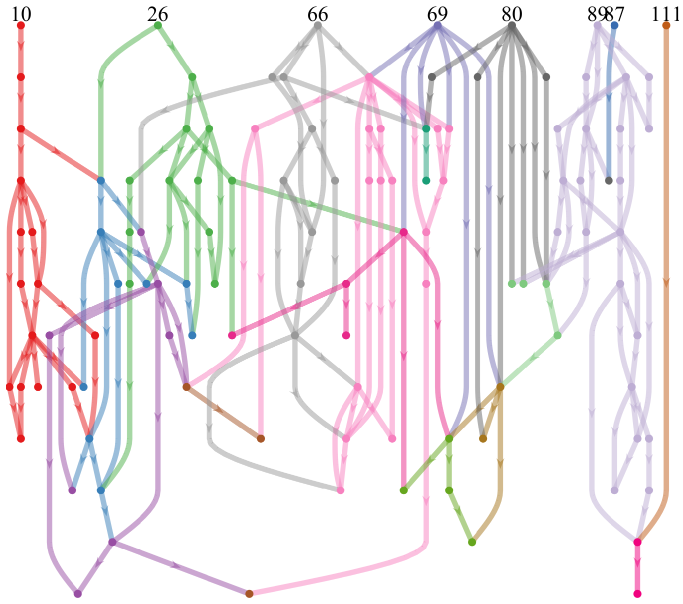

Fig. 5. A layered graph drawing of the `case118` network. Nodes and edges are coloured according to the *source reachability* cluster they have been grouped into

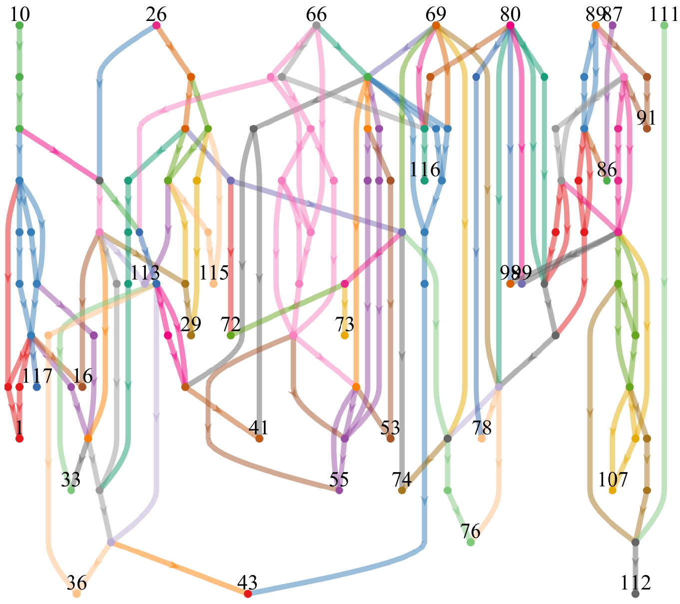

Fig. 7. A network diagram showing the clusters formed based on conjoint *source/sink reachability* for `case118`

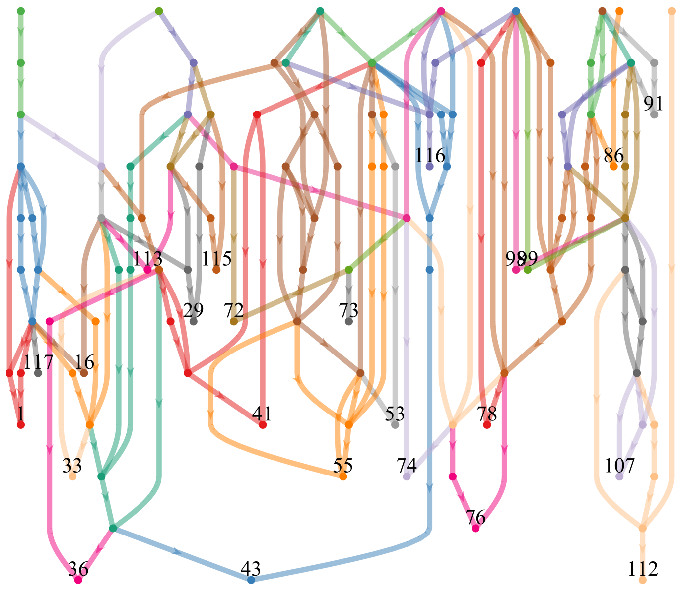

Fig. 6. A network diagram showing the clusters formed based on *sink reachability* for `case118`

network.

There are total of eighty clusters of nodes and edges in the `case118` network, as depicted in Fig. 7. By identifying these clusters of nodes with common sources and sinks, we found out that it provides a useful insight into the cohesion or flow of power, interaction, and behaviour of these nodes withing the network. This suggests that these nodes (buses) work together and receive the same amount of power from the generator feed common loads. This can also help the power system operators to curb failure propagation and ensure resilience by understanding the dependencies and interconnection between nodes with common sources and sinks. This will also enhance error-handling mechanisms.

C. Horton-Strahler Order Numbering Technique

The `case118` system is used to evaluate the proposed novel algorithm which seeks to apply the concept of Horton-Strahler stream order numbering schema. This has historically been used to classify and analyse the hierarchical structure of rivers network. The proposed algorithm assigns a numerical value to each cluster, which could be using the source reachability, sink reachability, or source-sink reachability, based on the cluster's position within the hierarchy of the power system network.

In Fig. 8, we demonstrated the effectiveness of this proposed algorithm. The modified Horton-Strahler order numbering is applied to the clusters which grouped nodes and edges based on the source reachability. It can be observed from Fig. 8 that we have four orders/levels in the networks which are depicted by the sequence of the colour, and thickness/width of the edges. The higher orders, the bigger or thicker the edge width. All the clusters that are tributaries, that is supplied by only one source, gets the order number of one, these clusters are shown with red colour in the figures and the distributaries emanating from these tributaries are assigned order numbers based on their position and relative importance in the network.

IV. Conclusions

In this paper, we proposed a novel algorithm to cluster the nodes and edges in any directed acyclic graph based on the descendant and ancestor subgraphs corresponding to each pure source and sink in the network. We further assigned order numbers based on this clustering, to enumerate where each cluster sits on the spectrum between a pure tributary fed by just one source, through to to the broad mainstem of the network.

The clustering technique in this paper can be applied to any directed acyclic graph, not just those representing the instantaneous state of a power grid. These findings could potentially be applied to task scheduling and workflow management, data

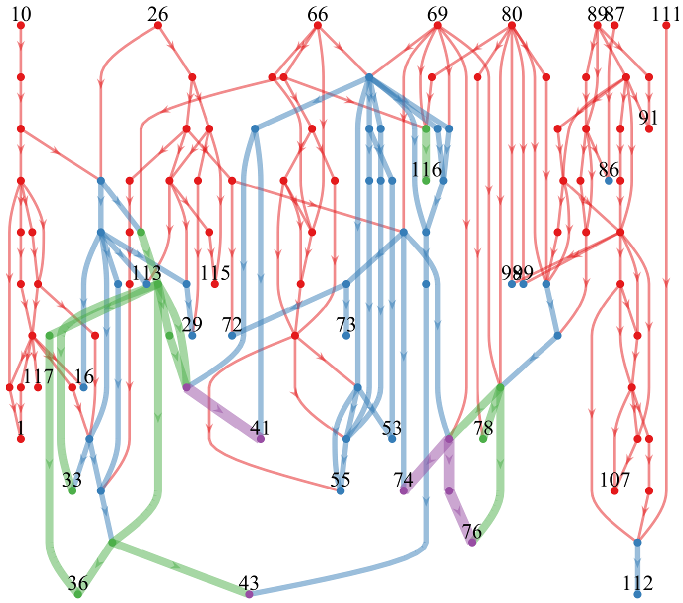

Fig. 8. Source reachability Horton-Strahler order numbering for `case118`. First order tributaries are shown in red, second order in blue, third in green and fourth in purple

processing and data analysis, compiler optimisation and data processing, graph algorithms and network analysis, parallel processing, and dependency analysis, to mention only a few.

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