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Non-Detection, False Alarm and Calibration Insensitivity in Kurtosis and Pseudofractal Based Singularity Detection

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Abstract- This work isolates cases of non-detection, false alarm and insensitivity for a general class of problems dealing with the detection and characterization of existence, location and extent of singularities embedded in signals or in their derivatives when employing kurtosis and pseudofractal based methods for the detection and characterization process. The non-detection, false alarm and insensitivity for these methods are illustrated on an example problem of damage identification and calibration in beams where the singularity to be identified lies in the derivative of the measured signal. The findings are general, not constrained to linear systems, and are potentially applicable to a wide range of fields including engineering system identification, fault detection, health monitoring of mechanical and civil structures, sensor failure, aerospace engineering and biomedical engineering.

CE Subject Database Headings: Numerical Analysis, Cracking, Identification, Calibration.

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I. INTRODUCTION

The importance of the detection of the presence, location and extent of singularity in a measured signal or in any of its derivatives (Robertson et al. 2003) has gained considerable interest in various fields of engineering (Dimarogonas 1996)), medicine (Addison 2005) and economics (Ide and Sornette 2002). Among the various methods that detect and characterize this essentially local phenomenon, the kurtosis (Hadjileontiadis et al. 2005) and pseudofractal based methods (Hadjileontiadis et al. 2005) have been successfully applied and experimentally validated in recent times. Although this identification process is similar to wavelet based singularity detection (Gentile and Messina 2003) in the sense that the computed detectors form an extremum at the location of the singularity and the absolute value of the extremum so formed can possibly be related to the degree of singularity at its location, no study has been performed addressing the issues of possible non-detection, false alarm and inconsistencies in the calibration of the extent of singularity which arise directly from the detection scheme. These non-detection, false alarm and inconsistencies are not related to measurement noise and thus are epistemic in nature. Assessment of the kurtosis and pseudofractal based techniques is thus considered topical and important in this regard. In this paper, we consider a structural health monitoring system comprised of a damaged beam with an open crack as an example problem. The objective is to find the existence, location and the severity of the damage through the identification and calibration of the damage induced singularity embedded in the first derivative of the measured response (modeshape, static or dynamic deflected shape) of the beam. Boundary condition dependent non-detection and consistent false alarm have been successfully isolated and identified for a kurtosis based singularity detection scheme, while situations of possible loss of relevance in the calibration of the degree of singularity related to pseudofractal based detection have been found.

II DETECTION TECHNIQUES – A BRIEF OVERVIEW OF KURTOSIS CRACK DETECTOR (KCD) AND PSEUDOFRACTAL CRACK DETECTOR (PFCD)

Kurtosis and pseudofractal based detections of singularity consider a cumulant based scheme incorporating a moving window. Since damage often introduces a singularity in a measured static or dynamic response in the spatial domain, kurtosis and pseudofractal based detectors can be helpful for damage detection. These detectors can identify damage by investigating the local deviation of the damaged response from a Gaussian signal. The Gaussian signal serves as a benchmark and a sudden, significant change of this local deviation from Gaussianity can be interpreted as damage at that location. The extent of such sudden deviation with respect to its neighbouring regions can be then possibly connected to the extent of damage at that region.

The kurtoses of an empirically chosen width of window within the signal are computed for the KCD scheme. The window slides along the signal at each point so that a local kurtosis value is computed at each location of the moving window. This single step sliding of the windows is what has been previously reported in literature as a nearly 99 percent overlap of the windows (Hadjileontiadis et al. 2005). The global mean of the local kurtosis values is computed and the absolute deviation of the local kurtosis values for each position of the sliding window from the mean acts as an indicator of damage. At the location of damage, the deviation of local kurtosis values (corresponding to the central position of each sliding window) from the local mean forms an extremum and identifies the location. The Kurtosis Crack Detector (KCD) is defined as

$$\mathrm{KCD} = \left|\beta_i - \overline{\beta}\right| \tag{1}$$

where β_i are the local kurtosis values at each position of the sliding window and $\overline{\beta}$ is the mean of these kurtosis values.

The pseudofractal dimension based crack detection scheme (Hadjileontiadis et al. 2005) is similar to the Kurtosis Crack Detector (KCD) and is referred to as Pseudofractal Crack Detector (PFCD) in this paper. A sliding window, similar to what has been described for the KCD scheme is considered for PFCD. The piecewise linear length of the portion of the signal (since only discrete values are obtained in reality) corresponding to a certain sampling step size is computed first. The same signal, sampled at a different rate employing a different step size is used again to find the new piecewise linear length. The location and the extent of the sliding window for both the cases are same. When a single signal is available, a different sampling step and the piecewise linear length within the windowed part of the signal can be computed by downsampling the signal. In the current paper, the step size is doubled by downsampling the signal by two. The PFCD crack detector is a measure defined somewhat similar to the way a fractal box counting measure is obtained for a signal. The PFCD based crack detection scheme is defined in this paper as the computed measure

$$\mathrm{PFCD} = \mathrm{abs}\left(\frac{\log\left(\frac{L_1}{L_2}\right)}{\log\left(\frac{S_2}{S_1}\right)}\right) \tag{2}$$

where $L_{(.)}$ are the respective lengths of the windowed part of the signal for each location of the sliding window computed by employing a step size of $S_{(.)}$. The subscripts of L and S in equation 2 represent the cases corresponding to two different step sizes used. Similar to KCD, the PFCD measure detects the damage by forming an extremum at its location. The sudden change in the signal or its derivative at the location of damage is magnified.

As has been discussed, KCD and PFCD are essentially a measure of the local deviation of a measured signal from Gaussianity. The measure of the local regularity in the neighbourhood of a point in a function can be related to the local Lipschitz exponent around that point (Mallat 2001). A function f(x) in the square integrable space is pointwise Lipschitz $\kappa \geq 0$ at a point ν if there exists a K>0 and a polynomial p_ν of degree $\breve{m}$ such that

$$\forall x \in \mathbb{R}, \left|f(x) - p_\nu(x)\right| \leq K\left|x-\nu\right|^\kappa \tag{3}$$

The term κ provides the degree of singularity in the neighbourhood of the point x. It is important to find how the absolute value of the local extremum formed by KCD or PFCD at the location of singularity is related to the strength of the singularity at that location.

III APPLICATION ON STRUCTURAL HEALTH MONITORING

A typical problem related to the identification of singularity in a signal or its derivative arises in the field of structural health monitoring where the presence, the location and the extent of an open crack in a beam is to be detected. The presence of an open crack in the beam introduces a singularity at the crack tip and brings about a sharp change in the displacement and the stress-strain fields in the neighbourhood of the location of the crack (Carneiro and Inman 2002). As a result, the first derivative of the typical spatial responses of the beam, like modeshapes or static and dynamic displaced shapes contain a damage induced singularity. The first modeshape (noise-free) of a simply supported beam of length ‘L’ and depth ‘h’ with an open crack of depth ‘c’ at a distance ‘a’ from the left hand support is considered as an example. The choice of the first modeshape is also important from the point of view that it is comparatively simpler to obtain from a real structure. Damage models of various complexities and detail for an open crack in a beam can be considered (Carneiro and Inman 2002, Narkis 1998, Bovsunovsky and Matveev 2000). However, the choice of a damage model essentially serves the purpose of simulating a damaged modeshape containing a singularity in its derivative, which in turn is the key to the detection process. Reports of laboratory based studies have validated the presence of singularity in the derivative of the first modeshape or deflected shape of a beam with an open crack (Okafor and Dutta 2000, Rucka and Wilde 2006, Pakrashi et al. 2007). In this paper, the cracked beam is modelled as an assembly of two sub-beams joined by a rotational spring at the location of the damage assuming the effects of damage to be localized in its immediate neighbourhood whereby the change of global modal properties are not significant. The free vibration equation for the beams on either side of the crack is given as

$$EI\frac{\partial^4 y}{\partial x^4}+\rho A\frac{\partial^2 y}{\partial t^2}=0 \qquad (4)$$

where E, I, A and ρ are the Young’s modulus, the moment of inertia, the cross sectional area and the density of the material of the beam on either side of the crack. The displacement of the beam from its static equilibrium position is y(x,t), at a distance of x from the left hand support along the length of the beam at an instant of time t. Continuities in displacement, moment and shear are present at the location of the crack while a discontinuity for slope is present at that location and is given in terms of the non dimensional crack section flexibility θ (Narkis 1998) dependent on crack depth ratio (δ=c/h) as

$$\Phi_R{}'(a) - \Phi_L{}'(a) = \theta L \Phi_R{}''(a) \tag{5}$$

where Φ represents the mode shape and the subscripts R and L represent the right and the left hand side of the crack respectively. The term θ is expressed as a polynomial of δ□ as

$$\theta = 6\pi\delta^2 (h / L)(0.5033 - 0.9022\delta + 3.412\delta^2 - 3.181\delta^3 + 5.793\delta^4) \tag{6}$$

The modeshape derived from the damage model contains singularity in its derivative at the damage location.

IV ISOLATION OF NON-DETECTION, FALSE ALARM AND CALIBRATION INSENSITIVITY

The damaged first modeshape is simulated for a square beam of length 1 m with the cross sectional area (A), depth (h) and the moment of inertia (I) being 0.0001 m^2, 0.01 m and $8.33\text{x}10^{-10}$ m^4 respectively. The Young's modulus (E) and the density of the beam (ρ) are assumed to be $190\text{x}10^9$ N/m^2 and 7900 kg/m^3 respectively. Figure 1 shows the successful detection of the location of an open crack situated at 0.4m from the left hand support using KCD and PFCD. No noise is considered. It is clearly observed that the KCD has a potential to fail to identify the presence and the location of damage if it is exactly or near to the centre of the beam. This aspect of non-detection is dependent on the boundary condition of the structural system and will not be observed for a cantilever (Hadjileontiadis et al. 2005) due to its monotonically increasing modeshape and thus has not been reported before. The modeshape of a simply supported beam contains a contraflexure at the midpoint and the local deviation from Gaussianity around that point is so significant that it consistently overwhelms the effect of any damage that might be present at or near to the point of contraflexure. On the other hand, it is interesting to note that even when no damage is present, the local extremum near the mid-point still exists and thus generates a false alarm. The number of locations where this non-detection and false alarm will be present is equal to the number of locations of significant contraflexure in the signal. Oscillations due to measurement noise can mask the location of damage for low signal to noise ratios (SNR) but will not generate consistent extremum location due to the inherent random nature of noise. Such false alarm is not present for the PFCD detection scheme as it targets the sudden jump in the derivatives. The change in contraflexure is not important in this case since a unique derivative exists at the location of contraflexure.

The calibration surface of damage extent (related to the degree of singularity at its location) employing the KCD detection method with a 10 point sliding window is shown in Figure 2a for a wide range of crack depth ratios and damage positions. The variation of the calibration is dependent on the number of points within the window. This is shown in figures 2b and 2c showing the absolute percentage deviation of the calibration values (from the 10 point window calibration values) for a 9 point and a 19 point sliding window respectively. The nature of the calibration however has been checked to be independent of the number of points in the sliding window.

The PFCD calibration is exceptionally sensitive on the number of points present on the window and especially on whether the number of points in the window is even or odd. This is due to the fact that the position of a downsampled signal within a given window is not unique. A number of observations are made from the PFCD calibration. Figures 3a and 3b show the PFCD calibration values employing a 19 point sliding window, but by computing the downsampled length (L_2) from the first and the second point of the original windowed signal (of length L_1) respectively. Although the calibration values are close enough, the nature of the calibration changes completely. On the other hand, the calibration values are highly dependent on whether the number of points in the sliding window is even or odd. Figures 3c and 3d repeat the calibration as described for Figures 3a and 3b, but for an 18 point sliding window. The dramatic variation of the calibration values is clearly observed. For windows of smaller widths, this change is comparatively less in terms of the nature and the magnitude of the calibration values. Figures 3e and 3f show graphs similar to Figures 3a and 3b for a 5 point sliding window. However, in this case, the calibration becomes exceptionally sensitive based on whether the last point of downsampled signal matches with the original signal or not. This is observed in Figure 3g

This non-consistency for PFCD method is not present or apparent at the damage location detection level. Figure 3 shows the comparison of PFCD based damage extent calibration between two cases where the first calibration is based on a moving window consisting of an even number of points while the second calibration considers the same window with odd number of points. While the calibration with a moving window with even number of points is consistent and sensitive, considering a window with odd number of points poses a serious problem in terms of sensitivity against increasing damage extent.

V CONCLUSIONS

Events of non-detection, false alarm and calibration insensitivity have been isolated for kurtosis and pseudofractal based identification of the presence, location and extent of singularity in a signal or its derivatives. The kurtosis based singularity detection technique is susceptible to non-detection and false alarm at the level of singularity location identification and is dependent on the signal geometry. The pseudofractal based calibration of the extent of singularity at its location is dependent on whether the number of points employed in the moving window for the computation pseudofractal measure is odd or even. For a window with odd number of points, the sensitivity of the pseudofractal measure against the degree of singularity is insignificant in comparison with a window with even number of points, although the successful detection of the location of singularity is unaffected. The findings have been illustrated using the problem of damage detection of a beam with an open crack.

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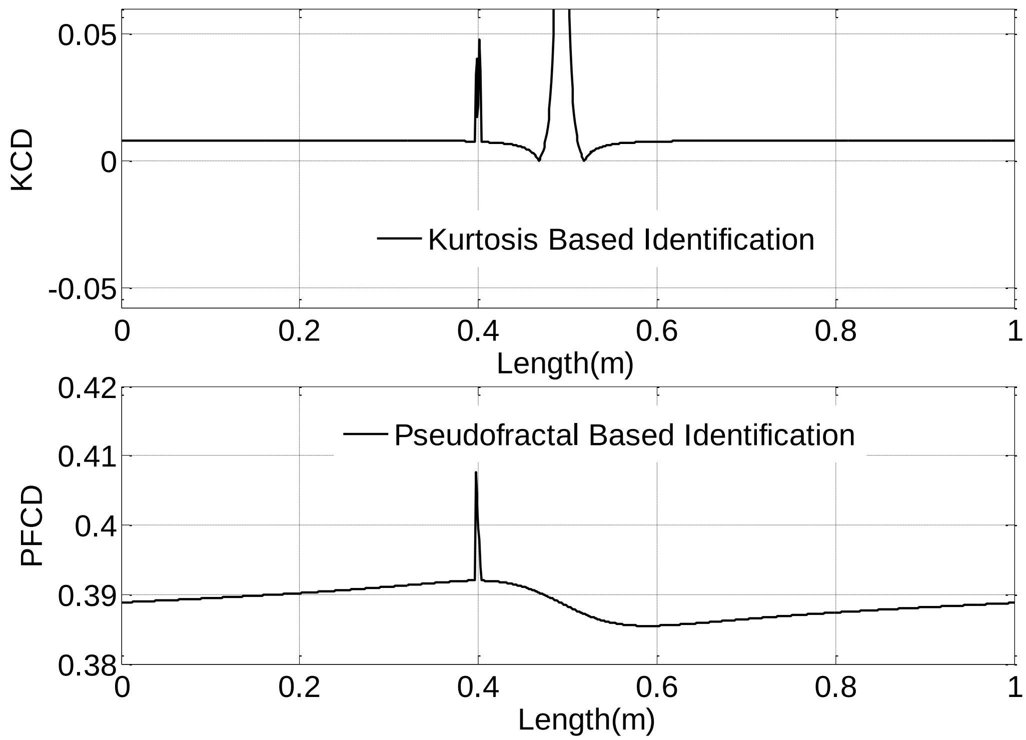

0.05
0
-0.05
KCD
Kurtosis Based Identification
0
0.2
0.4
0.6
0.8
1
Length(m)
0.42
0.41
0.4
0.39
0.38
PFCD
Pseudofractal Based Identification
0
0.2
0.4
0.6
0.8
1
Length(m)

Figure 1.

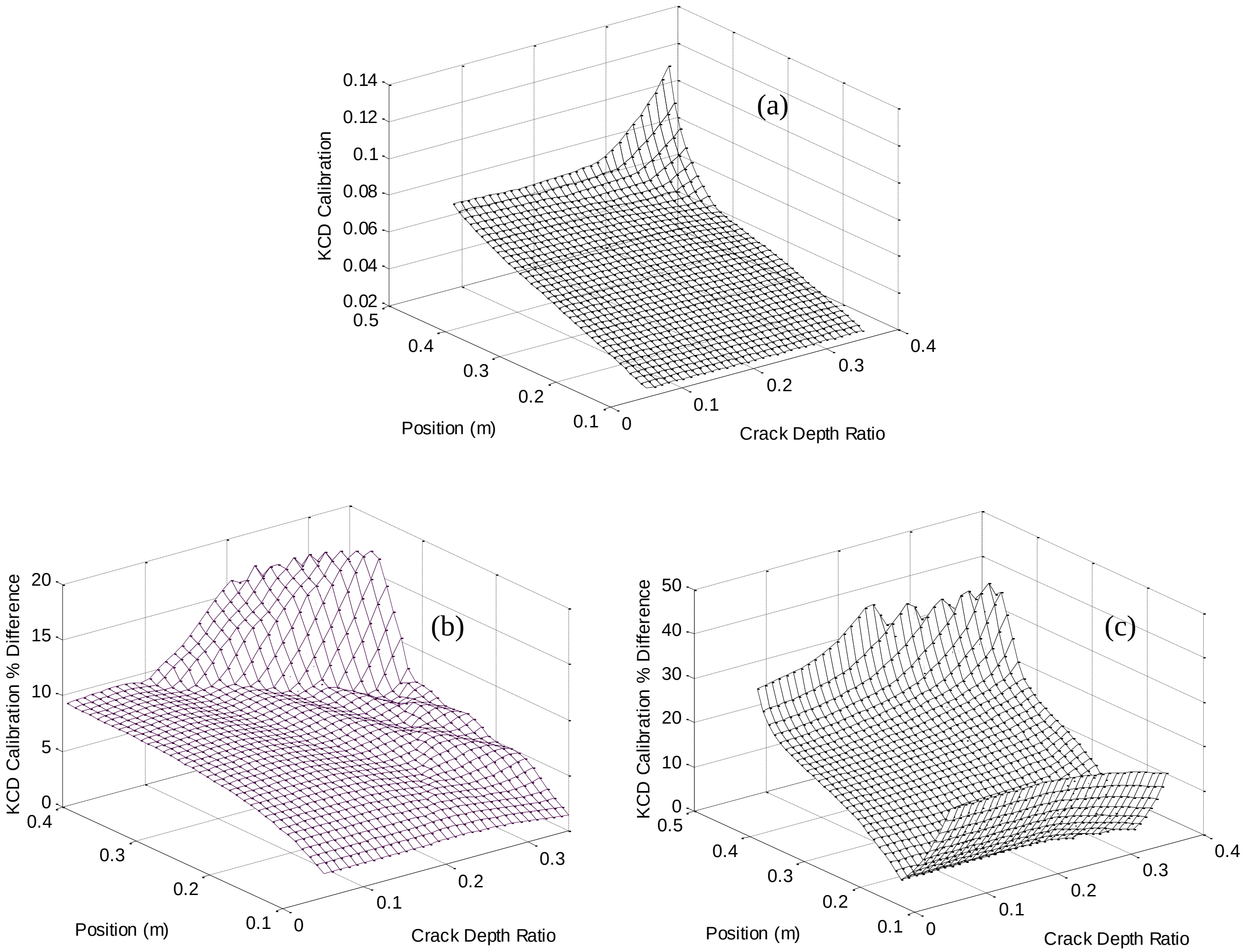
(a)
KCD Calibration
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0.5
0.4
0.3
0.2
0.1
0
Position (m)
0.1
0.2
0.3
0.4
Crack Depth Ratio
(b)
KCD Calibration % Difference
20
15
10
5
0
0.4
0.3
0.2
0.1
0
Position (m)
0.1
0.2
0.3
Crack Depth Ratio
(c)
KCD Calibration % Difference
50
40
30
20
10
0
0.5
0.4
0.3
0.2
0.1
0
Position (m)
0.1
0.2
0.3
0.4
Crack Depth Ratio

Figure 2.

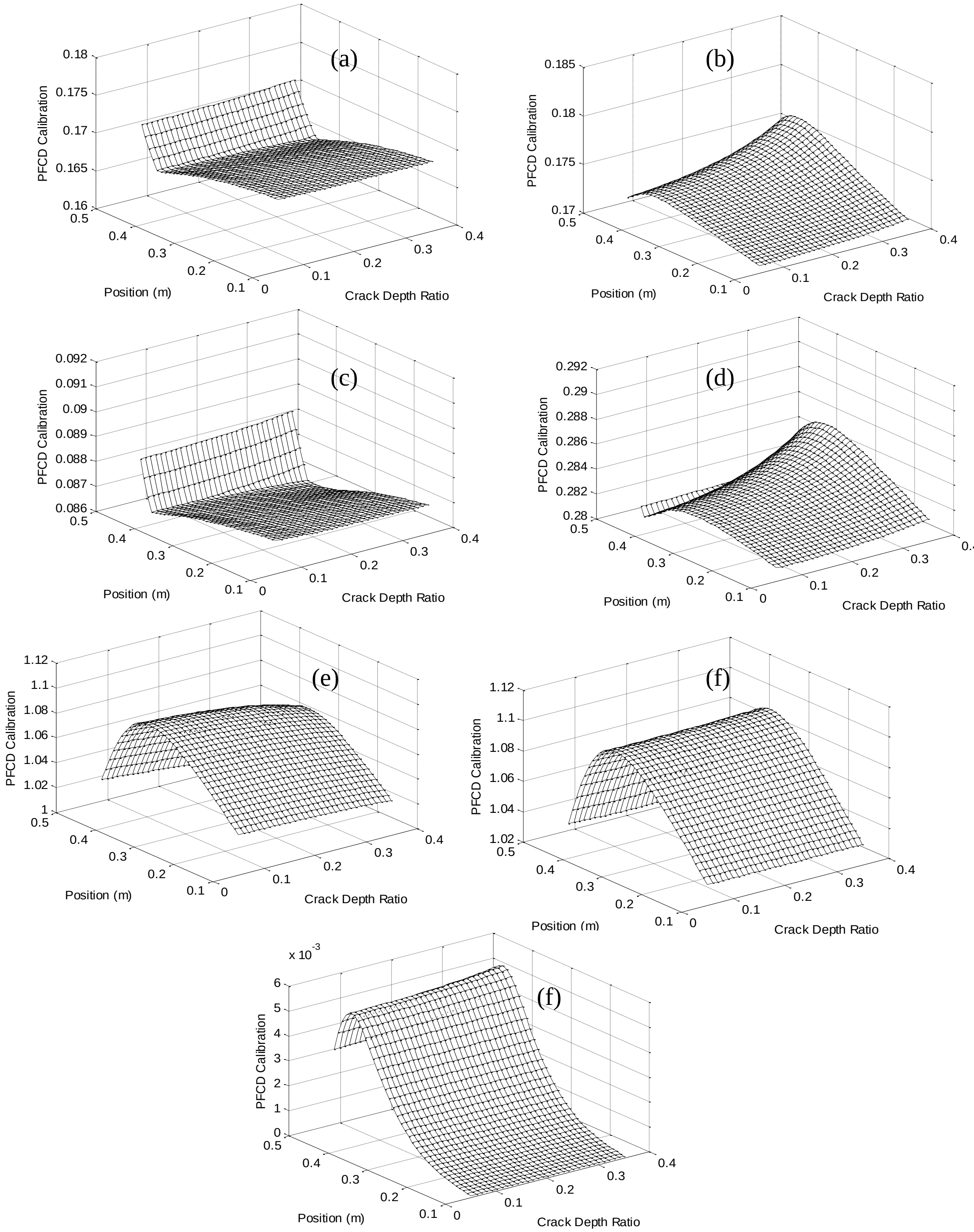
(a)
0.18
0.175
0.17
0.165
0.16
PFCD Calibration
0.5 0.4 0.3 0.2 0.1
Position (m)
0 0.1 0.2 0.3 0.4
Crack Depth Ratio
(b)
0.185
0.18
0.175
0.17
PFCD Calibration
0.5 0.4 0.3 0.2 0.1
Position (m)
0 0.1 0.2 0.3 0.4
Crack Depth Ratio
(c)
0.092
0.091
0.09
0.089
0.088
0.087
0.086
PFCD Calibration
0.5 0.4 0.3 0.2 0.1
Position (m)
0 0.1 0.2 0.3 0.4
Crack Depth Ratio
(d)
0.292
0.29
0.288
0.286
0.284
0.282
0.28
PFCD Calibration
0.5 0.4 0.3 0.2 0.1
Position (m)
0 0.1 0.2 0.3 0.4
Crack Depth Ratio
(e)
1.12
1.1
1.08
1.06
1.04
1.02
1
PFCD Calibration
0.5 0.4 0.3 0.2 0.1
Position (m)
0 0.1 0.2 0.3 0.4
Crack Depth Ratio
(f)
1.12
1.1
1.08
1.06
1.04
1.02
PFCD Calibration
0.5 0.4 0.3 0.2 0.1
Position (m)
0 0.1 0.2 0.3 0.4
Crack Depth Ratio
x 10-3
6
5
4
3
2
1
0
(f)
PFCD Calibration
0.5 0.4 0.3 0.2 0.1
Position (m)
0 0.1 0.2 0.3 0.4
Crack Depth Ratio

Figure 3.