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Forecasting with Instabilities: an Application to DSGE Models with Financial Frictions *

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12th October 2015

Abstract

This paper examines whether the presence of parameter instabilities in dynamic stochastic general equilibrium (DSGE) models affects their forecasting performance. We apply this analysis to medium-scale DSGE models with and without financial frictions for the US economy. Over the forecast period 2001-2013, the models augmented with financial frictions lead to an improvement in forecasts for inflation and the short term interest rate, while for GDP growth rate the performance depends on the horizon/period. We interpret this finding taking into account parameters instabilities. Fluctuation test shows that models with financial frictions outperform in forecasting inflation but not the GDP growth rate.

Keywords: Bayesian estimation, Forecasting, Financial frictions, Parameter instabilities

JEL code: C11, C13, C32, E37

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1 Introduction

Dynamic stochastic general equilibrium (DSGE) models have become extensively used both in central banking and in the academia. Thanks to advances in Bayesian estimation tools, DSGE models are used for policy implications and forecasting comparison (see Del Negro and Schorfheide, 2013, among others).

The forecasting evaluation of these macroeconomic models is subject to the estimation of the parameters of the model and, as documented by Giacomini and Rossi (2015), there is ample evidence of instabilities in parameters that might affect their forecasting performance. Giacomini and Rossi (2010) show that in the presence of structural instability of the parameters the forecasting performance of two alternative models may be itself time-varying. To overcome this issue, they propose two specific tests, the Fluctuation test and the One-time Reversal test, to analyze the evolution of the models’ relative performance considering the instabilities.

The DSGE empirical literature offers four approaches to deal with the issue of parameters instabilities. The first approach features both stochastic volatility and parameter drifting in the Taylor Rule, estimating a non-linear model as in Fernández-Villaverde et al. (2010), Caldara et al. (2012) and Bekiros and Paccagnini (2013). Second, Eo (2009), Bianchi (2013) and Foerster et al. (2014), among others, propose a Markov-switching DSGE set-up, modeling and estimating the regime change in some of the key parameters. Third, Inoue and Rossi (2011) propose a formal test to identify which parameters are stable over time. Their so-called ESS procedure (Estimate of Set of Stable parameters) identifies unstable parameters via maximum-likelihood estimation. Jerger and Röhé (2014) apply the ESS procedure developed by Inoue and Rossi (2011) to a New Keynesian DSGE model on French, German, Italian, and Spanish data, where parameters are estimated using maximum likelihood. They indeed find parameter instabilities. Finally, Castelnuovo (2012) and Hurtado (2014) present a rolling-window estimation in order to investigate possible instabilities in the structural parameters of a medium scale model. However, all these approaches detecting the role of instabilities do not perform a forecasting evaluation analysis.

The main focus of our paper is to evaluate empirically whether the presence of parameter
instabilities can affect the prediction performance in a DSGE framework without considering a nonlinear DSGE or a Markov-switching set up. We implement a forecasting comparison among medium-scale DSGE models with and without financial frictions for the US economy.

Financial factors have played a central role in the recent financial crisis by affecting the amount of credit available in the economy. Before the Great Recession, the DSGE models proposed by Bernanke et al. (1999) and Iacoviello (2005) consider financial frictions on the firms and households, respectively. Under these settings, borrowers can obtain funds directly from lenders without any active role for the banking sector. In the wake of the financial turmoil understanding the disruption in financial intermediation has become a priority. In the model by Gertler and Karadi (2011) an endogenous leverage constraint on banks effectively ties the provision of credit to the real economy. This mechanism creates a loop between financial intermediaries’ balance sheet, firms’ asset prices and GDP.

The forecasting literature has partly assessed the empirical relevance of DSGE models with financial frictions for the US economy. Del Negro and Schorfheide (2013) show that a Smets and Wouters (2007) (hereafter, SW) economy augmented by financial frictions à la Bernanke et al. (1999) can forecast output growth during the Great Recession better than the SW model, while the latter model generates more accurate forecasts in previous times. Villa (2015) compares the forecasting performance of two DSGE models, one featuring financial frictions as in Bernanke et al. (1999) and the other as in Gertler and Karadi (2011). She finds that no model dominates the other in terms of point forecasts. Kolasa and Rubaszek (2015) find that adding frictions in the housing sector proves very helpful during the financial turmoil, providing a forecasting performance better than both the frictionless benchmark and the alternative that incorporates financial frictions in the corporate sector.

Differently from them, we study the role of financial frictions on firms and on financial intermediaries in forecasting real and nominal variables, focusing on the role of instabilities in parameters. To this end, we compare the workhorse Smets and Wouters (2007) model with two models: a SW economy augmented by a banking sector as in Gertler and Karadi (2011) (hereafter, SWBF);\(^1\) and a SW economy augmented with financial frictions as Bernanke et al.

\(^{1}\)In comparison to other DSGE models with a banking sector, their model features an agency problem which poses a limit to the financial intermediaries ability to acquire assets, and hence, to lend to the private
(1999) (hereafter, BGG). Using Bayesian techniques, we recursively estimate the three models – SW, SWBF and BGG. The out-of-sample forecasting period is from 2001Q1 to 2013Q4, split into two subsamples: 2001Q1-2008Q4 and 2009Q1-2013Q4. The point forecast analysis, based on Mean Absolute Errors (MAE) and Root Mean Square Forecast Error (RMSFE), and the density forecast evaluation, based on the average of the log predictive density scores (LPDS) and the probability integral transform (PIT) histograms, shows that there is no clear evidence of an outperformed model in terms of forecasting accuracy. In the first sample – 2001Q1-2008Q4 – the model featuring financial frictions exhibit the best performance for output growth and inflation only in the longer horizon. In the second sample – 2009Q1-2013Q4 – the BGG model outperforms the SW model in forecasting inflation and the short term interest rates, but not output growth; the SWBF model, instead, dominates the SW model in forecasting inflation in the long horizon and interest rate in the short horizon.

In order to rationalize these results, we investigate possible instabilities in parameters. We do not aim at stressing the importance of endogeneous instabilities related to a modeling feature, but we detect a change in parameters over the estimation samples and we assume that the way the parameters can vary over time presents a feature of instability. For this reason, we employ a recursive estimation of the three DSGE models and we find a considerable degree of parameter variation. Hence, we compute the Fluctuation test as in Giacomini and Rossi (2010, 2015) to test the hypothesis that the financial DSGE models and the SW model have equal performance at every point in time over the sample. This implies that the forecasting performance of the models with financial frictions is not statistically different from that of the SW model for GDP growth rate, except in 2006. The accuracy of inflation forecasts of the former models, instead, is generally higher, but in 2009 and 2010 when none of the models clearly dominates the other. Hence, the empirical ranking among models changes over time.

The remainder of the paper is organized as follows. Section 2 briefly sketches the three models. The Bayesian estimation procedure is discussed in Section 3. Section 4 evaluates the forecasting accuracy. Section 5 concludes. An appendix complements the paper by providing technical details about the construction of the dataset, variance decomposition analysis and sector. In addition, it is fairly elegant and computationally fairly tractable (Cole, 2011).
robustness exercises of forecasting accuracy.

2 The fully-fledged DSGE models

This section briefly describes three linearized models: (i) the model by Smets and Wouters (2007) (SW); (ii) the model by Gertler and Karadi (2011) (SWBF); and (iii) the model by Bernanke et al. (1999) (BGG). The last two models are included in an otherwise setup of SW. The BGG model is nested in the SW model, whereas the SWBF model is not.

The economy is composed by households, labor unions, labor packers, financial intermediaries (in the SWBF model), a productive sector and a monetary authority. Households consume, accumulate government bonds and supply labor. A labor union differentiates labor and sets wages in a monopolistically competitive market. Competitive labor packers buy labor services from the union, package and sell them to intermediate goods firms. Output is produced in several steps, including a monopolistically competitive sector with producers facing price rigidities. The monetary authority sets the short-term interest rate according to a Taylor rule.

In the SWBF model, the presence of an agency problem limits the ability of financial intermediaries to obtain deposits from households. This, in turn, affects the leverage ratio of financial intermediaries. On the contrary, in the BGG model intermediate firms stipulate a financial contract to obtain funds from the lenders in presence of costly state verification problem. In this case the external finance premium is the result of asymmetric information because only the borrowers directly observe the rate of return on the investments. The higher the leverage ratio of a firm, the higher the default probability, and hence the higher the premium on the risk free interest rate.

**Households.** The economy is populated by a continuum of households of measure unity. In the SWBF model within each household there are two types of members: a share $f$ are workers while a share $1 - f$ are bankers. The first earn wages while the latter manage financial intermediaries. The maximization problem of households yields the following Euler
\[ c_t = c_1 c_{t-1} + (1 - c_1) E_t c_{t+1} + c_2 (l_t - E_t l_{t+1}) - c_3 \left( r_t - E_t \pi_{t+1} + e_t^b \right), \]  

(1)

where \( c_1 = \frac{h/\gamma}{1+h/\gamma} \), \( c_2 = \frac{(\sigma_c-1)(w_t l_t/c_t)}{\sigma_c(1+h/\gamma)} \) and \( c_3 = \frac{1-h/\gamma}{\sigma_c(1+h/\gamma)} \). The parameter \( h \) measures the degree of superficial external habits in consumption, \( \sigma_c \) the coefficient of relative risk aversion of households, \( \gamma \) is the steady state growth rate and \( e_t^b \) captures the risk premium shock following an AR(1) process, \( \rho_e \) is an autoregressive coefficient and \( \varepsilon_t^e \sim N(0, \sigma_e^2) \).

Current consumption, \( c_t \), is affected by past and future consumption, \( c_{t-1}, E_t c_{t+1} \), expected increasing growth hours \( (l_t - E_t l_{t+1}) \) and the real interest rate \( (r_t - E_t \pi_{t+1}) \).

**Labor market.** Wage setting is characterized by sticky wages, as shown by:

\[ w_t = w_1 w_{t-1} + (1 - w_1) (E_t w_{t+1} + E_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + e_t^w, \]  

(2)

where \( w_1 = \frac{1}{1+\beta \gamma - \sigma_e} \), \( w_2 = \frac{1+\beta \gamma - \sigma_e}{1+\beta \gamma - \sigma_e} \), \( w_3 = \frac{\xi w}{1+\beta \gamma - \sigma_e} \) and \( w_4 = \frac{(1+\xi w)(1-\sigma_e)}{(1+\beta \gamma - \sigma_e)\xi w(\phi_w-1)e^w+1} \).

The parameter \( \beta \) represents the households discount factor, \( \xi w \) indicates the Calvo probability of not adjusting nominal wages, \( \tau_w \) denotes the degree of wage indexation of non-adjusting unions, \( (\phi_w - 1) \) is the steady state labor market markup, and \( e^w \) is the curvature of the Kimball aggregator in the labor market. The wage markup disturbance, \( e_t^w = \rho_w e_{t-1}^w + \varepsilon_t^w - \mu_w \varepsilon_{t-1}^w \), is an exogenous shock to the wage markup following an ARMA(1,1) process, \( \rho_w \) is an autoregressive coefficient and \( \varepsilon_t^w \sim N(0, \sigma_w^2) \).

The wage mark-up is the difference between the real wages and the marginal rate of substitution between consumption and labor:

\[ \mu_w = w_t - \left[ \sigma l_t + \frac{1}{1-h} (c_t - hc_{t-1}) \right], \]  

(3)

where \( \sigma l_t \) is the elasticity of labor supply with respect to the real wage.

\(^2\)All variables are log-linearized around their steady state balanced growth path and starred variables represent steady state values.
Production sector. The production sector is characterized by different types of firms. A continuum of infinitely-lived intermediate firms of measure one produce an intermediate good using labor and capital under perfect competition. They use a standard Cobb-Douglas production function, according to:

$$y_t = \phi_p [\alpha (k_{t-1} + u_t) + (1 - \alpha) l_t] + e_t^a,$$  \hspace{1cm} (4)

where $u_t$ is capital utilization, $e_t^a$ is the transitory technology shock following an AR(1) process, $\rho_a$ is an autoregressive coefficient and $e_t^a \sim N(0, \sigma_a^2)$. The parameter $\phi_p$ represents one plus the share of fixed costs in production.

The optimal rate of utilization, $u_t$, depends on the marginal product of capital, $z_t^k$, as follows:

$$u_t = u_1 z_t^k,$$  \hspace{1cm} (5)

with $u_1 = \frac{(1-\psi)}{\psi}$, where $\psi$ represents the positive function of elasticity of the capital utilization adjustment cost.

A continuum of retail firms differentiate intermediate goods and set prices following a process à la Calvo (1983), in analogy to the labor market:

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 \pi_{t+1} - \pi_3 \mu_t^p + e_t^p,$$  \hspace{1cm} (6)

with $\pi_1 = \frac{\iota_p}{1 + \beta \gamma^{1-\sigma}e^{c_{tp}}}$, $\pi_2 = \frac{\beta \gamma^{1-\sigma}e^{c_{tp}}}{1 + \beta \gamma^{1-\sigma}e^{c_{tp}}}$, $\pi_3 = \frac{(1-\beta \gamma^{1-\sigma}e^{c_{tp}})(1-\xi_p)}{(1+\beta \gamma^{1-\sigma}e^{c_{tp}})\xi_p[(\phi_p-1)e^{p}+1]}$. $\iota_p$ represents the indexation parameter, $\xi_p$ the degree of price stickiness in goods market and $e^p$ is the curvature of Kimball aggregator in the goods market. The price markup disturbance follows an ARMA(1,1) process, $e_t^p = \rho_p e_{t-1}^p + e_t^p - \mu_p e_{t-1}^p$, $\rho_p$ is the AR(1) coefficient and $e_t^p \sim N(0, \sigma_p^2)$. The term $(\phi_p - 1)$ is the steady-state markup in the goods market.

The price markup, $\mu_p$, is equal to the difference between the marginal product of labor and the real wage:

$$\mu_p = \alpha (k_{t-1} + u_t - l_t) + e_t^a - w_t.$$  \hspace{1cm} (7)

Cost minimization by firms implies that the marginal product of capital is negatively
related to the capital-labor ratio and real wages:

\[ z_t^k = -(k_{t-1} + u_t - l_t) + w_t. \]  

(8)

A continuum of competitive capital goods producers repair depreciated capital and re-
build new productive capital. Capital producers purchase depreciated capital from the
intermediate-goods producers and combine it with investment goods to produce new cap-
ital. The newly produced capital is sold back to intermediate goods firms and any profits are
transferred to households. The investment is subject to adjustment costs as in Christiano
et al. (2005):

\[ i_t = i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \varepsilon_t^i, \]  

(9)

where \( i_1 = \frac{1}{(1 + \beta \gamma^{1-\sigma})} \) and \( i_2 = \frac{i_1}{\gamma^2 \varphi} \). The parameter \( \varphi \) is the elasticity of investment adjust-
ment costs and \( \varepsilon_t^i \) is an investment-specific technology shock following an AR(1) process with
\( \rho_e \) the AR(1) coefficient and \( \varepsilon_t^i \sim N(0, \sigma_e^2) \).

The arbitrage equation for the value of capital is given by:

\[ E_t r_{t+1}^k = q_1 E_t z_{t+1}^k + q_2 E_t q_{t+1} - q_t, \]  

(10)

where \( q_1 = \frac{z_b}{r_t} \), \( q_2 = \frac{(1-\delta)}{r_t^{1-\sigma}} = \beta \gamma^{1-\sigma_e} (1 - \delta) \). The parameter \( \delta \) represents the depreciation
rate and \( E_t r_{t+1}^k \) is the rental rate of capital.

The law of motion of installed capital evolves as follows:

\[ k_t = k_1 k_{t-1} + (1 - k_1) i_t + k_2 \varepsilon_t^i, \]  

(11)

where \( k_1 = \frac{(1-\delta)}{\gamma} \) and \( k_2 = [1 - \frac{(1-\delta)}{\gamma}](1 + \beta \gamma^{1-\sigma_e}) \gamma^2 \varphi \).

**Monetary authority.** The central bank sets the nominal interest rate according to a
Taylor rule of the form:

\[ r_t = \rho_r r_{t-1} + (1 - \rho_r) [\rho_y \pi_t^p + \rho_y (y_t - y_t^p)] + \rho_{\Delta y_t} [(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \varepsilon_t^r, \]  

(12)
where \( r_t \) is the gross nominal interest rate, \( y^p_t \) represents the level of output that would prevail under flexible prices and wages, while \( \rho_r, \rho_\pi, \rho_y \) and \( \rho_{\Delta y} \) are policy parameters referring to interest-rate smoothing, and the responsiveness of the nominal interest rate to inflation, to the output gap and to changes in the output gap, respectively. The term \( e^i_t \) represents an exogenous shock following an \( AR(1) \) process, \( \rho_i \) is an autoregressive coefficient and \( e^i_t \sim N(0, \sigma^2_i) \).

**Equilibrium.** Equilibrium conditions in labor and goods market require that the resource constraint is satisfied in every period:

\[
y_t = c_y c_t + i_y i_t + z_y z^k_t + e_g,
\]

where \( c_y \) is the steady state share of consumption, \( i_y \) the steady state share of investment, \( z_y = z^k_t k^*_t / y^*_t \) represents the steady-state rental rate of capital, and \( e_g \) exogenous government spending disturbance that follows an \( AR(1) \) process and it is also affected by the technology shock as in Smets and Wouters (2007), with \( \rho_g \) is the AR coefficient and \( e^g_t \sim N(0, \sigma^2_g) \).

The SW model features seven exogenous disturbances: total factor productivity, price mark-up, wage mark-up, investment-specific technology, risk premium, exogenous spending, and monetary policy shocks.

**Financial intermediaries (SWBF).** In the SWBF model, a continuum of mass-one banks owned by the households lend funds to non financial sector. The balance sheet of the risk-neutral financial institution features assets, \( s_t q_t - s_t \) is the quantity of financial claims on non-financial firms and \( q_t \) is the relative price of each claim – and net worth \( n_t \) as well as deposits on the liabilities side.

As each financial intermediary pays the risk free interest rate on deposits, \( r_t \), and receives on loans \( E_t r^{k_{t+1}}_t \), a credit spread \( r^{ep}_t \) arises:

\[
r^{ep}_t = E_t r^{k_{t+1}}_t - (r_t - E_t \pi_{t+1}^r) .
\]

(14) To limit the liability of financial intermediaries a moral-hazard costly enforcement problem
occurs so that they cannot borrow indefinitely from households. At the beginning of each period the banker can choose to divert the fraction $\phi$ of available funds from the project and transfer them back to their household. Depositors can force the intermediary into bankruptcy and recover the remaining fraction $1 - \phi$ of total assets. However, costly enforcement implies that it is too costly for the depositors to recover the diverted fraction of funds by the banker. To limit the expansion of bankers’ assets, a positive exit probability prevents bankers from accumulating sufficient net worth to finance equity investment internally. In each period $1 - \varpi$ bankers exit and transfer their earnings back to their corresponding households. Those bankers are replaced by an equal number of workers who are endowed by start-up funds, $\xi$, by their households. The amount of assets that financial intermediaries can acquire depends on the equity capital:

$$q_t + k_t = lev_t + n_t,$$

(15)

where the leverage, $lev_t$, is endogenously determined as

$$lev_t = \eta_t + \frac{v}{\phi - v} v_t.$$

(16)

Note that the leverage depends both on the gain of having net worth, $\eta_t$, and on the gain of expanding assets, $v_t$. The first is specified as follows:

$$\eta_t = \eta_1 (E_t \Lambda_{t+1} - \Lambda_t + z_t + E_t \eta_{t+1}),$$

(17)

where $\eta_1 = \frac{\omega \beta}{\pi_c} z_s$ and $\Lambda_t$ is the Lagrange multiplier associated to the utility maximization problem. The gross growth rate of net worth, $z_t$, is represented by

$$z_t = z_1 r_t^k + z_2 (r_{t-1} - \pi_t) + z_3 lev_{t-1},$$

(18)

where $z_1 = \frac{lev_t r_t^k}{z_s}$, $z_2 = r_s (1 - lev_s)$ and $z_3 = lev_s (r_s^k - r_s)$.

The gain of expanding assets, $v_t$, can be expressed as:

$$v_t = v_1 (E_t \Lambda_{t+1} - \Lambda_t + x_t + E_t v_{t+1}) + v_2 \left[ r_s^k r_t^k - r_s (r_{t-1} - \pi_t) \right] + v_3 (E_t \Lambda_{t+1} - \Lambda_t),$$

(19)
where \( v_1 = \frac{\omega \beta}{\gamma \sigma} z_* \), \( v_2 = \frac{(1-\omega)\beta}{\gamma \sigma} \), \( v_3 = \frac{(1-\omega)\beta}{\gamma \sigma} (r^k - r_*) \). The gross growth rate in assets, \( x_t \), is

\[
x_t = lev_t - lev_{t-1} + z_t.
\]  

Finally, total net worth, \( n_t \), is composed by the sum of the net worth of existing bankers, \( n^e_t \), and the net worth of new bankers, \( n^n_t \):

\[
n_t = n_1 n^e_t + n_2 n^n_t,
\]  

where \( n_1 = \frac{n^e}{n} \) and \( n_2 = \frac{n^n}{n} \). The law of motion of the net worth of existing bankers depends on the gross growth of net worth and on exogenous shock, \( \epsilon^n_t \), to the net worth of banks following an AR(1) process, \( \rho_n \) is an autoregressive coefficient and \( \epsilon^n_t \sim N(0, \sigma^2_n) \):

\[
n^e_t = n^e_{t-1} + z_t + \epsilon^n_t.
\]  

New bankers receive a “start-up” transfer from households, equal to a fraction \( \xi \) of total assets. Therefore, their net worth is:

\[
n^n_t = \xi lev_*(q_t + k_t).
\]  

**External borrowing (BGG model).** A fraction of capital acquisition is financed by net worth, \( n_t + 1 \), and the remaining by borrowing from lenders. Bernanke et al. (1999) assume that an agency problem makes external finance more expensive than internal funds. Lenders must pay a fixed auditing cost to observe an individual borrower’s return. The monitoring cost is a proportion of the realized gross payoff to the firm’s capital. The financial contract implies an external finance premium (EP), i.e. the difference between the cost of external and internal funds. Hence, in equilibrium, the marginal external financing cost must equate the external finance premium gross of the riskless real interest rate:

\[
E_t \left[ \hat{r}^{k}_{t+1} \right] = \hat{r}_t + \hat{EP}_t
\]  

11
The EP depends on the inverse of the firm’s leverage ratio:

\[ \hat{EP}_t = \kappa \left( \hat{q}_t + E_t [\hat{k}_{t+1}] - E_t [\hat{n}_{t+1}] \right) \] (25)

where \( \kappa \) measures the elasticity of the external finance premium with respect to the leverage position of intermediate firms. Net worth evolves as the difference between earning on assets and borrowing repayments,

\[
\frac{1}{\theta_r} E_t [\hat{n}_{t+1}] = \frac{k_s}{n_s} \hat{x}^k_t - \left( \frac{k_s}{n_s} - 1 \right) (r_{t-1} - \pi_t) - \kappa \left( \frac{k_s}{n_s} - 1 \right) \left( \hat{k}_t + \hat{q}_{t-1} \right) + \left( \frac{k_s}{n_s} - 1 \right) \kappa + 1 \hat{n}_t + e^n_t \] (26)

where \( \theta \) represents the survival rate of firms – to avoid the possibility of full self financing – and \( k_s/n_s \) is the steady state leverage ratio.

**SW model.** The standard SW economy does not feature capital producers and financial intermediaries. The price of capital, equation (10), is given by

\[
q_t = q_{1r} E_t q_{t+1} + (1 - q_{1r}) E_t \hat{x}^k_t - (r_t - \pi_{t+1}), \] (27)

where \( q_{1r} = \frac{1 - \delta}{\hat{x}^k_t + (1 - \delta)} \). The exogenous disturbance to net worth of financial intermediaries is clearly absent.

### 3 Estimation procedure

We estimate the model using Bayesian methods. The loglinearized model is solved by applying the algorithm proposed by Sims (2002). As in Bayesian practice, the likelihood function (evaluated by implementing the Kalman Filter) and the prior distribution of the parameters are combined to calculate the posterior distribution. The posterior Kernel is then simulated numerically using the Metropolis-Hasting algorithm with 150,000 replications for two chains. The three DSGE models are estimated for the US quarterly data over the period 1984Q1-
2000Q4 as in Schorfheide et al. (2010). To estimate the SW model we use the standard seven observables: GDP, investment, consumption, wages, hours of work, GDP deflator inflation and the federal funds rate. In the SWBF model we also include net worth of financial intermediaries as a financial observable since the model features a net worth shock. The additional financial observable in the BGG model is, instead, the credit spread similarly to Del Negro and Schorfheide (2013).

All the models are estimated with a number of shocks equal to observable variables to avoid the stochastic singularity. The following set of measurement equations shows the link between the observable variables in the dataset and the endogenous variables of the DSGE model:

\[
\begin{bmatrix}
\Delta Y_o^t \\
\Delta C_o^t \\
\Delta I_o^t \\
\Delta W_o^t \\
L_o^t \\
\pi_o^t \\
\sigma_{s.o}^t \\
\Delta N_o^t \\
EP_t^o
\end{bmatrix}
= \begin{bmatrix}
\hat{\gamma} \\
\hat{\gamma} \\
\hat{\gamma} \\
\hat{\gamma} \\
\hat{I} \\
\hat{\pi} \\
\hat{\pi^n} \\
\hat{\gamma^N} \\
\hat{EP}
\end{bmatrix}
+ \begin{bmatrix}
\bar{Y}_t - \bar{Y}_{t-1} \\
\bar{C}_t - \bar{C}_{t-1} \\
\bar{I}_t - \bar{I}_{t-1} \\
\bar{W}_t - \bar{W}_{t-1} \\
\hat{L}_t \\
\bar{\Pi}_t \\
\bar{R}_t^n \\
\bar{N}_t - \bar{N}_{t-1} \\
\bar{EP}_t
\end{bmatrix},
\tag{28}
\]

where \( \hat{\gamma} = 100(\gamma - 1) \) is the common quarterly trend growth rate of GDP, consumption, investment and wages; \( \bar{I} \) is the steady-state hours of work; \( \bar{\pi} \) is the steady-state quarterly inflation rate; and \( \bar{\pi^n} \) is the steady-state quarterly nominal interest rate; \( \hat{\gamma^N} = 100(\gamma^N - 1) \) is the quarterly trend growth rate of net worth of financial intermediaries in the SWBF model, as in Gelain and Ilbas (2014); and \( \hat{EP} \) is the steady-state quarterly credit spread.

Our general calibration and estimation strategy follows the standard procedure proposed by Smets and Wouters (2007) adapted to models augmented with financial frictions. In particular, we calibrate the parameters i) using a priori source of information and ii) to match some stylized facts over the period of consideration. The time period in the model

---

3Although observations on all variables are available at least from 1973Q2 onward, we concentrate on this period because it is characterized by the same monetary policy regime.

4Appendix A contains a detailed discussion of data sources, definitions and transformations, while Appendix C investigates the sensitivity of forecasting performance of the SWBF model to an alternative financial variable, the credit spread.

5A hat over a variable represents log-deviation from steady state.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital depreciation rate</td>
<td>δ</td>
</tr>
<tr>
<td>Kimball aggregator in the goods market</td>
<td>( \eta_p )</td>
</tr>
<tr>
<td>Kimball aggregator in the labor market</td>
<td>( \eta_w )</td>
</tr>
<tr>
<td>Gross mark-up in the labor market</td>
<td>( \lambda_w )</td>
</tr>
<tr>
<td>Government share of output</td>
<td>( \frac{G}{Y} )</td>
</tr>
<tr>
<td>Survival rate of financial intermediaries/firms (SWBF/BGG)</td>
<td>( \varpi )</td>
</tr>
<tr>
<td>Fraction of divertable assets (SWBF)</td>
<td>( \phi )</td>
</tr>
<tr>
<td>Fraction of assets given to new bankers (SWBF)</td>
<td>( \xi )</td>
</tr>
<tr>
<td>Firm’s leverage ratio (BGG)</td>
<td>( \frac{k_<em>}{n_</em>} )</td>
</tr>
</tbody>
</table>

Table 1: Calibration

corresponds to one quarter in the data. As shown in Table 1, the discount factor, \( \beta \), is set equal to 0.99, implying a quarterly steady state real interest rate of 1%. The depreciation rate of capital, \( \delta \), is set equal to 0.025. The Kimball aggregators in the goods and labor market are equal to 10, and the steady state gross wage mark-up is set to 1.5. The share of government to GDP is equal to 0.18. Similarly to Villa (2015), the financial parameters are calibrated as follows. In the SWBF model \( \varpi \), \( \phi \) and \( \chi \) are calibrated to target an average working life of bankers of almost a decade, a steady state spread of 150 basis points and a steady state leverage ratio of financial intermediaries equal to 4. In the BGG model, the expected working life of firms is almost a decade and the leverage ratio is set equal to 2 as in Bernanke et al. (1999).

The remaining parameters governing the dynamics of the model are estimated using Bayesian techniques. The locations of the prior mean correspond to those in Smets and Wouters (2007). Similarly to De Graeve (2008), we set a Uniform distribution between 0 and 0.3 for the parameter measuring the elasticity of external finance premium with respect to the leverage position of firms in the BGG model.

### 4 Evaluating forecast accuracy

The models are recursively estimated from 1984Q1 to 2000Q4. The pseudo out-of-sample forecasting estimation considers two periods: from 2001Q1 to 2008Q4 (with 32 forecast periods in the last recursive sample) and from 2009Q1 to 2013Q4 (with 20 forecast periods in the

---

\(^6\)Dynare toolbox for Matlab is used for the computations.
last recursive sample). A multi-steps forecasting analysis is implemented with forecasts for the horizon \( h \in (1, 2, 4, 6, 8, 12) \). We assess the predictability of the two models by evaluating the point and the density forecasts.\(^7\) The point forecasting accuracy is evaluated in terms of Mean Forecast Error (MFE) and Root Mean Squared Forecast Error (RMSFE). For the density evaluation, we report the average of the log predictive density scores (LPDS) and the probability integral transform (PIT) histograms.

### 4.1 Point Forecast Evaluation

Before presenting the statistics computed on the forecast errors, Figure 1 shows the one-quarter forecast series for output growth, investment growth, inflation, and federal funds rate (FFR) for the three models for the whole forecasting period, which starts in 2001Q1.\(^8\) The graphical demonstration of the forecasting performance is useful to evaluate which model exhibits a better forecasting performance in the recent years, similarly to Gürkaynak et al. (2013) and Marcellino and Rychalovska (2014). Output growth sharply falls at the end of 2008; investment growth and inflation reached the minimum value in 2009, while the nominal interest rate reached the zero lower bound in 2009. With exception of the FFR, the fall of the three other variables is followed by a recovery.

The forecasts of the three models are similar for inflation and for the FFR, while the BGG model yields a better prediction for output and for investment growth in the 2009 recession. Overall, three main results emerge from Figure 1: first, the SW and SWBF models are not able to predict the sharp contraction in output and investment occurred during the financial crisis. Second, all models produce good forecasts for the FFR and, to a minor extent, for inflation. Third, the figure suggests us that we can split the forecasting sample as follows:

2001Q1 - 2008Q4 and 2009Q1 - 2013Q4.\(^9\)

---

\(^7\)As described in Wolters (2015) and in Kolasa and Rubaszek (2015), for each parameter a large number of values are drawn from the parameter’s posterior distribution. We take each 20th draw from the final 150,000 parameter draws calculated by the Metropolis-Hastings algorithm, which produces 7,500 draws from the posterior distribution. For each of them, we draw seven shock trajectories to generate the predictions for the seven macrovariables of interest. The obtained 52,500 trajectories are draws from the predictive density and hence can be used to evaluate the density forecasts. The point forecasts are calculated as means of these draws (see Wolters, 2015, for technical details).

\(^8\)Since the sample ends in 2013Q4, we compute forecast errors on the basis of 52 observations for the one-quarter forecast.

\(^9\)For point and density forecasts analysis, we also split the sample before and after 2007/2008, considering
Table 2 reports the Mean Forecast Error (MFE) for the period 2001Q1-2008Q4 for seven macroeconomic variables. Note that positive MFE implies that on average the model underpredicts the historical values of the series. For all horizons, all models are not biased in forecasting consumption; instead they underpredict inflation, interest rates, wage and hours. The SWBF model overpredicts output growth, while the SW and the BGG models do not show a statistically significant bias. All models overpredict investment, although the MFEs are not statistically different from zero on shorter horizons for the BGG model and on longer horizons for the other two models.

Table 3 reports the MFE for the period 2009-2013. Results are remarkably similar to ones shown in Table 2, especially for output growth which is still underpredicted by the SWBF model. The bias is statistically significant during this period for hours in the three models and for investment in the SWBF model.

A possible explanation for the model-based forecasts for investment can be found in the assumption of a common trend growth rate to real GDP, consumption and investment which is not in line with the data, as noted also by Kolasa and Rubaszek (2015). In fact, investment exhibits in the data a growth rate which is about 40% lower than that of output the financial turmoil and crisis. The results are similar and robust to changing the ending/starting date of the two samples.
Table 2: Mean Forecast Error for the sample 2001Q1-2008Q4. Asterisks ***, ** and * denote, respectively, the 1%, 5% and 10% significance levels.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Output</th>
<th>Inflation</th>
<th>FFR</th>
<th>Wage</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SW 0.01</td>
<td>0.13** 1.12*** 0.46*** 0.25** -0.80** 0.37</td>
<td>SWBF -0.37** 0.27*** 1.11*** 0.32** -0.08 -0.92** 1.74***</td>
<td>BGG -0.02 0.19*** 1.03*** 0.51*** 0.21* -0.48 0.54</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>SW -0.02 0.15*** 0.98*** 0.31** 0.17 -0.93** 0.34</td>
<td>SWBF -0.29** 0.31*** 0.98*** 0.35*** -0.02 -0.81*** 1.52***</td>
<td>BGG -0.06 0.21*** 0.93*** 0.53*** 0.17 -0.66 0.56</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>SW -0.14 0.19*** 0.74*** 0.46*** 0.22 -1.12*** 0.21</td>
<td>SWBF -0.34*** 0.30*** 0.76*** 0.28** -0.06 -1.06*** 1.12*</td>
<td>BGG -0.10 0.24*** 0.75*** 0.52*** 0.15 -0.70* 0.55</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>SW 0.00 0.21*** 0.58*** 0.35** 0.17 -0.67 0.33</td>
<td>SWBF -0.43*** 0.27*** 0.60*** 0.17** -0.15 -1.10*** 0.81</td>
<td>BGG -0.16 0.25*** 0.64*** 0.44*** 0.05 -0.80* 0.49</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>SW -0.17 0.29*** 0.46*** 0.23* 0.00 -0.61** 0.20</td>
<td>SWBF -0.37*** 0.24*** 0.46** 0.22** -0.15 -0.46 0.89</td>
<td>BGG -0.19 0.26*** 0.54*** 0.34*** 0.00 -0.67* 0.46</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>SW -0.07 0.27*** 0.32** 0.14 -0.10 -0.30 0.46</td>
<td>SWBF -0.25* 0.24*** 0.30 0.05 -0.17 -0.31 0.78</td>
<td>BGG -0.14 0.27*** 0.44** 0.17 -0.03 -0.43 0.58</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

and consumption over the whole sample. Moreover, during the forecasting period 2001-2013, the growth rate of output is 0.17%, that of consumption is 0.22%, while the growth rate of investment is -0.27%. As far as inflation is concerned, forecast values are lower than the actual ones. The average inflation rate is 0.62% in the period 1984-2000 and it becomes 0.50% in the period 2001-2013. The possible reason for the bias can be found in the recursive estimates of the Calvo parameter: Figures 3, 5 and 7 show an increasing trend, which is particularly evident in the latter forecasting period. A lower probability of adjusting prices implies an inflation rate which might be too low compared to its actual value and this can affect inflation forecasts. The FFR is on average below its actual value because the model-based forecast might suggest a nominal interest rate below its lower bound and this becomes particularly relevant during the 2009-2013 period.

We continue our forecasting analysis by comparing the second moments of the forecast errors. Table 4 and Table 5 show the ratio of the RMSFE of the SWBF model relative to the SW model for the forecasting periods 2001-2008 and 2009-2013, respectively, whereas Table
Table 3: Mean Forecast Error for the sample 2009Q1-2013Q4. Asterisks ***, ** and * denote, respectively, the 1%, 5% and 10% significance levels.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Output</th>
<th>Inflation</th>
<th>FFR</th>
<th>Wage</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SW</td>
<td>0.18</td>
<td>0.37***</td>
<td>1.90***</td>
<td>0.22</td>
<td>0.32***</td>
<td>-0.32</td>
</tr>
<tr>
<td></td>
<td>SWBF</td>
<td>0.02</td>
<td>0.42***</td>
<td>1.87***</td>
<td>0.30*</td>
<td>0.41*</td>
<td>-1.42**</td>
</tr>
<tr>
<td></td>
<td>BGG</td>
<td>0.46**</td>
<td>0.37***</td>
<td>1.75***</td>
<td>0.56***</td>
<td>0.72***</td>
<td>0.44</td>
</tr>
<tr>
<td>2</td>
<td>SW</td>
<td>0.07</td>
<td>0.40***</td>
<td>1.84***</td>
<td>0.33*</td>
<td>0.29***</td>
<td>-0.62</td>
</tr>
<tr>
<td></td>
<td>SWBF</td>
<td>-0.05</td>
<td>0.43***</td>
<td>1.69***</td>
<td>0.34**</td>
<td>0.43**</td>
<td>-1.64**</td>
</tr>
<tr>
<td></td>
<td>BGG</td>
<td>0.39**</td>
<td>0.38***</td>
<td>1.65***</td>
<td>0.56***</td>
<td>0.67***</td>
<td>0.18</td>
</tr>
<tr>
<td>4</td>
<td>SW</td>
<td>-0.09</td>
<td>0.45***</td>
<td>1.63***</td>
<td>0.33***</td>
<td>0.30**</td>
<td>-1.15</td>
</tr>
<tr>
<td></td>
<td>SWBF</td>
<td>-0.34**</td>
<td>0.37***</td>
<td>1.44***</td>
<td>0.22</td>
<td>0.41**</td>
<td>-3.05***</td>
</tr>
<tr>
<td></td>
<td>BGG</td>
<td>0.31*</td>
<td>0.38***</td>
<td>1.45***</td>
<td>0.59***</td>
<td>0.58***</td>
<td>-0.08</td>
</tr>
<tr>
<td>6</td>
<td>SW</td>
<td>-0.09</td>
<td>0.39***</td>
<td>1.51***</td>
<td>0.39**</td>
<td>0.08</td>
<td>-1.69***</td>
</tr>
<tr>
<td></td>
<td>SWBF</td>
<td>-0.51***</td>
<td>0.35***</td>
<td>1.18***</td>
<td>0.17</td>
<td>0.20</td>
<td>-2.94***</td>
</tr>
<tr>
<td></td>
<td>BGG</td>
<td>0.20</td>
<td>0.40***</td>
<td>1.33***</td>
<td>0.65***</td>
<td>0.41***</td>
<td>-0.38</td>
</tr>
<tr>
<td>8</td>
<td>SW</td>
<td>-0.12</td>
<td>0.45***</td>
<td>1.37***</td>
<td>0.39**</td>
<td>0.09</td>
<td>-1.56***</td>
</tr>
<tr>
<td></td>
<td>SWBF</td>
<td>-0.63**</td>
<td>0.32***</td>
<td>0.97***</td>
<td>0.30**</td>
<td>-0.02</td>
<td>-3.02***</td>
</tr>
<tr>
<td></td>
<td>BGG</td>
<td>-0.02</td>
<td>0.42***</td>
<td>1.24***</td>
<td>0.57***</td>
<td>0.25*</td>
<td>-0.80*</td>
</tr>
<tr>
<td>12</td>
<td>SW</td>
<td>-0.36*</td>
<td>0.44***</td>
<td>1.12***</td>
<td>0.08</td>
<td>-0.05</td>
<td>-1.48***</td>
</tr>
<tr>
<td></td>
<td>SWBF</td>
<td>-0.56**</td>
<td>0.30***</td>
<td>0.60***</td>
<td>0.14</td>
<td>-0.38**</td>
<td>-2.33***</td>
</tr>
<tr>
<td></td>
<td>BGG</td>
<td>-0.18</td>
<td>0.45***</td>
<td>1.03***</td>
<td>0.46**</td>
<td>-0.01</td>
<td>-1.36***</td>
</tr>
</tbody>
</table>

6 and Table 7 show the ratio of the RMSFE of the BGG model relative to the SW model for the same forecasting periods. Values greater than one denote that the SW model shows a better forecasting performance. To check the statistical significance of these ratios, we report the Clark and West (2006) test which is applicable to non-nested and nested models. When examining output, inflation, and FFR in the period 2001-2008, the SW model always predicts the last variable, whereas the SWBF model exhibits the best performance for output and inflation in the longer horizon. The Clark-West test shows that the forecasting accuracy of the SWBF model is significantly different for all horizons and not only for the three key macroeconomic variables, output growth rate, inflation, and FFR, but for all the common observable variables. Meanwhile, for the BGG model in Table 6, only the predictions for inflation and for the FFR are statistically different at all horizons. The forecasting accuracy of output growth rate is not statistically different between the SW and BGG as well as its components – consumption and investment – and hours. There is a statistical difference between the two models for wages only at shorter horizons.
Table 4: Root Mean Square Forecast Error of the SWBF model. RMSFE are computed as a ratio to the RMSFE in the SW model. Forecasting evaluation period: 2001Q1-2008Q4. Asterisks ***, ** and * denote, respectively, the 1%, 5% and 10% significance levels of the Clark - West test.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Output</th>
<th>Inflation</th>
<th>FFR</th>
<th>Wage</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.60 *** 1.31 *** 0.99 ***</td>
<td>0.92 *** 1.27 ***</td>
<td>1.01 *** 1.04 ***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.13 *** 1.41 *** 1.01 ***</td>
<td>0.85 *** 1.13 ***</td>
<td>0.89 *** 1.01 ***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.43 *** 1.33 *** 1.05 ***</td>
<td>0.84 *** 0.92 ***</td>
<td>1.04 *** 1.13 ***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.15 *** 1.20 *** 1.07 ***</td>
<td>0.72 *** 1.04 ***</td>
<td>0.94 *** 1.08 ***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.98 *** 0.83 *** 1.07 ***</td>
<td>0.90 *** 1.02 ***</td>
<td>1.12 *** 1.36 ***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.96 *** 0.89 *** 1.04 ***</td>
<td>1.04 *** 1.06 ***</td>
<td>0.99 *** 1.62 ***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Root Mean Square Forecast Error of the SWBF model. RMSFE are computed as a ratio to the RMSFE in the SW model. Forecasting evaluation period: 2009Q1-2013Q4. Asterisks ***, ** and * denote, respectively, the 1%, 5% and 10% significance levels of the Clark - West test.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Output</th>
<th>Inflation</th>
<th>FFR</th>
<th>Wage</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.44 *** 1.01 *** 0.99 ***</td>
<td>1.19 *** 1.12 **</td>
<td>1.08 *** 1.10 ***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.41 *** 0.99 *** 0.93 ***</td>
<td>0.93 *** 1.37 ***</td>
<td>1.38 1.08 ***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.09 *** 0.80 *** 0.83 ***</td>
<td>0.83 *** 1.07 ***</td>
<td>1.73 1.02 ***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.50 * 0.87 ** 0.79 ***</td>
<td>0.82 ** 1.37 ***</td>
<td>1.54 ** 0.94 ***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1.15 ** 0.74 ** 0.71 ***</td>
<td>0.92 * 1.08 ***</td>
<td>1.54 ** 0.94 ***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1.29 *** 0.72 *** 0.59 **</td>
<td>1.31 *** 0.92 **</td>
<td>1.31 ** 0.75 ***</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the period 2009-2013, Table 5 shows that the SWBF model outperforms the SW model for inflation and the FFR, while the SW model dominates in forecasting output and its components. According to the Clark - West test, the forecasting accuracy is statistically different between SW and SWBF models for almost all variables and at all horizon, except for investment at horizons 2 and 4. Meanwhile, Table 7 shows that the SW model is outperformed by the BGG model only for the FFR. For all the other variables the forecasting accuracy is not statistically different between the two models.

4.2 Parameters instabilities

In order to rationalize these results, we check for possible instabilities in the time dimension of the structural parameters, which might affect the analysis of forecasting. We implement the sensitivity analysis and identification test proposed by Ratto (2011) and Ratto and Iskrev (2011) in Dynare. For all the samples, the SW, SWBF, and BGG models do not present identification issues.
<table>
<thead>
<tr>
<th>Horizon</th>
<th>Output</th>
<th>Inflation</th>
<th>FFR</th>
<th>Wage</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.05</td>
<td>1.10***</td>
<td>0.92***</td>
<td>0.95***</td>
<td>1.00</td>
<td>0.97</td>
<td>0.93</td>
</tr>
<tr>
<td>2</td>
<td>0.88</td>
<td>1.02***</td>
<td>0.95***</td>
<td>0.99**</td>
<td>0.92</td>
<td>0.95</td>
<td>0.92</td>
</tr>
<tr>
<td>4</td>
<td>1.07</td>
<td>1.09***</td>
<td>1.02***</td>
<td>0.96**</td>
<td>0.89</td>
<td>0.93</td>
<td>0.95</td>
</tr>
<tr>
<td>6</td>
<td>0.92</td>
<td>1.05***</td>
<td>1.07***</td>
<td>0.90</td>
<td>0.93</td>
<td>0.85</td>
<td>0.94</td>
</tr>
<tr>
<td>8</td>
<td>0.86</td>
<td>0.85***</td>
<td>1.11***</td>
<td>0.94</td>
<td>0.90</td>
<td>1.09</td>
<td>1.06</td>
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<td>0.94***</td>
<td>1.20***</td>
<td>0.81</td>
<td>0.99</td>
<td>0.88</td>
<td>1.46</td>
</tr>
</tbody>
</table>

Table 6: Root Mean Square Forecast Error of the BGG model. RMSFE are computed as a ratio to the RMSFE in the SW model. Forecasting evaluation period: 2001Q1-2008Q4. Asterisks ***, ** and * denote, respectively, the 1%, 5% and 10% significance levels of the Clark - West test.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Output</th>
<th>Inflation</th>
<th>FFR</th>
<th>Wage</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
</tr>
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<td>0.93***</td>
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</tr>
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<td>1.04</td>
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<td>1.20</td>
</tr>
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<td>0.98</td>
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</tr>
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<td>1.34***</td>
<td>0.77</td>
<td>0.82</td>
<td>1.60</td>
</tr>
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</table>

Table 7: Root Mean Square Forecast Error of the BGG model. RMSFE are computed as a ratio to the RMSFE in the SW model. Forecasting evaluation period: 2009Q1-2013Q4. Asterisks ***, ** and * denote, respectively, the 1%, 5% and 10% significance levels of the Clark - West test.
As noted by Giacomini and Rossi (2010), in the presence of structural instability, the forecasting performance of two alternative models may itself be time-varying. All parameters and shocks of the DSGE models are recursively estimated with the first window covering the period 1984Q1-2000Q4 and the last window covering the whole estimation sample, 1984Q1-2013Q4. This procedure yields 53 different posterior densities of all parameters, each of which is computed with the Metropolis-Hasting algorithm with two chains of 150,000 draws each. As stated by Castelnuovo (2012), this methodology does not force the data to “discretize” the economy which actually occurs in the regime-switching approach. In addition, since we use Bayesian estimation techniques, we cannot apply the ESS procedure.

Our chosen methodology has at least two caveats. First, the DSGE models do not feature a zero lower bound (ZLB) constraint on the nominal interest rate, which is instead observed in the data. Hirose and Inoue (2015) investigate how and to what extent parameter estimates can be biased in DSGE models lacking this constraint. They find that when the nominal interest rate is bounded at zero for 6.4% of quarters the bias is small, while it becomes large when the probability of hitting the ZLB increases. Given our estimation sample, estimates up to 2010Q3 feature about 6.5% of quarters in which the ZLB hits the economy. Hence, according to the study of Hirose and Inoue (2015) the bias should not be significant. However, it is worth noting that the bias in parameter estimates could potentially increase afterward. Second, since estimation samples differ in their window size, the precision of estimates could be affected. We plan to use alternative techniques for parameters instability in DSGE models in future research.

Figures 2 and 3 show the evolution of the shock processes and of parameters of the SW model. While the standard deviations and the AR parameters of the government spending, monetary policy and price mark-up shocks are pretty constant over the recursive sample, other shocks show a considerable degree of parameter variation similarly to Giacomini and Rossi (2015). In particular, the volatility risk premium shock drops by more than a half after 2009, while its persistence more than doubles. In the Smets and Wouters (2007) model the risk premium shock captures the wedge between different interest rates. We can consider two measures as proxies for spreads: (i) Moody’s BAA corporate bond yield minus ten-
Figure 2: Evolution of the shock processes of the SW model. Solid lines represent the posterior mean, while dotted lines 5th and 95th posterior percentiles. Estimates are computed based on the recursive estimation sample starting from 1984Q1-2000Q4 and ending in 2013Q4.

year Treasury constant maturity rate, as in Del Negro and Schorfheide (2013); and (ii) the difference between the 3-month bank prime loan rate and the quarterly Treasury bill rate. In the data spreads skyrocketed in 2008 and then fall quite sharply afterward. For both measures of the spreads, their volatilities are lower in the sample 2009Q1-2013Q4 compared to the sample 2001Q1-2008Q4. In particular, the volatility of the bank spread decreased by more than 75% in the more recent sample. The higher persistence of risk premium shock can play a role in explaining the slow recovery from the financial crisis (e.g. Huang et al., 2014).

The higher volatilities of the investment-specific technology, TFP and wage mark-up shocks in the period 2009-2013 are likely to capture the higher volatilities of the investment and wage series. This is also in line with the lower estimated investment adjustment costs parameter shown in Figure 3. This figure shows recursive estimates of the most relevant parameters. Instabilities are particularly evident for the parameter measuring price stickiness, which has risen in the recent recursive sample. Calvo parameters for price and wage

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11. This is the financial observable used in the robustness exercises presented in Appendix C.
12. In the interest of brevity, we do not report charts on the recursive estimates of all parameters, which are available upon request.
stickiness show considerable variations in the time-varying literature. Giraitis et al. (2014) also find that the Calvo parameter for prices increased sharply in 2010. The coefficient of relative risk aversion decreased by more than 50% since 2009. Hence, the willingness of households to substitute consumption over different periods has decreased after the financial crisis, which seems to be a puzzling result. This parameter shows a pattern similar to the risk shock. The reduction in the volatility of the latter makes households more willing to substitute consumption over time. The Taylor rule has become more inertial over time and its responsiveness to inflation is pretty constant over the recursive sample. The chart showing the responsiveness to the output gap is remarkably similar to the model-implied output gap, which considerably falls from 2009 onwards.

In the SWBF model the variation in the estimation of shocks and parameters is less evident compared to that of the SW model, as shown by Figures 4 and 5. This can be explained by the richer set of structural shocks and observable variables in the former model.\footnote{Canova et al. (2015) study how parameter variation can affect the decision rules in a DSGE model presenting the dynamics of some parameters of Gertler and Karadi (2011). Contrary to us, they focus on a diagnostics to detect misspecification driven by parameter variations without estimating the model in a recursive or rolling scheme. However, similar to us (Cardani et al., 2015), they evidence an importance role of}
Figure 4: Evolution of the shock processes of the SWBF model, where the observable financial variable is net worth. Solid lines represent the posterior mean, while dotted lines 5th and 95th posterior percentiles. Estimates are computed based on the recursive estimation sample starting from 1984Q1-2000Q4 and ending in 2013Q4.

The risk premium shock seems to capture the volatility due to the dot-com bubble, while the volatility of the net worth shock drops from 2009 onwards. It is worth noting that the growth rate of net worth is more volatile in the period 2001Q1-2008Q4 than in the recent period. The volatilities of the TFP, price mark-up and wage mark-up, instead, increased in 2009-2013. As far as the parameters are concerned, instabilities are evident mainly for the interest-rate responsiveness to the output gap, whose chart is similar to the model-implied output gap, as in the SW model.

Figures 6 and 7 show recursive estimates of the shock processes and of structural parameters in the BGG model. The volatility of the risk premium shock has substantially decreased in the most recent sample, whereas the government spending shock is pretty stable over time. All the remaining shocks reveal evidence of variation. The parameters of the BGG model are more unstable than those of the SW model. It should be noted that the volatility of the financial variable used in the estimation – the credit spread – has increased from 2008. And time-variation for identification of DSGE parameters, even if it is not only related to the Gertler and Karadi (2011) set up.
recursive estimates are indeed more unstable since then.

This exercise has shown that the time variation of the parameters is crucial in our empirical analysis. For this reason, we employ the Fluctuation test as in Giacomini and Rossi (2010) and Giacomini and Rossi (2015).

The Clark and West test (2006) assesses the relative performance on average over the two samples we consider. But the relative performance can change over time, for example during the recent financial crisis. Hence the instabilities can affect the prediction of the macroeconomic series. The Fluctuation test – applied to DSGE models in Giacomini and Rossi (2015) and in Fawcett et al. (2015) – is implemented to assess the predictive ability when there are instabilities over time. As proposed by Giacomini and Rossi (2010), the Fluctuation test achieves to calculating the Clark-West test over a rolling window of size $m$, where $m$ is a user-defined bandwidth. Giacomini and Rossi (2010) propose a Fluctuation test statistic to test the null hypothesis that both models have equal forecasting ability at each point in time.$^{14}$

$^{14}$For more details about the test implemented in case of Clark and West (2006), the null hypothesis, as
Figure 6: Evolution of the shock processes of the BGG model, where the observable financial variable is credit spread. Solid lines represent the posterior mean, while dotted lines 5th and 95th posterior percentiles. Estimates are computed based on the recursive estimation sample starting from 1984Q1-2000Q4 and ending in 2013Q4.

Figure 8 and Figure 9 report the Fluctuation tests based on Clark - West test for GDP growth and CPI inflation. The four graphs show the relative performance between the SW model and the SWBF or BGG models. The visual perspective suggests the researcher that if the statistic drawn in blue crosses the upper bound (the red line) the SWBF or BGG model has a superior forecasting performance, instead the SW model is preferred if the lower bound is crossed. Meanwhile the blue line in the two bounds means that the predictivity performance is not statistically different between the two models. Figure 8 shows that for the GDP growth rate the SWBF and BGG models are not statistically different from the SW model, except for year 2006. Instead in Figure 9 we note that for CPI inflation, the SWBF and BGG models produce better predictive performances than that of the SW model, except for years 2009-2010 when the relative performances are similar.

well as the critical values obtained using a MonteCarlo experiment, see Giacomini and Rossi (2010).

15In the figures we report only the upper bound since the statistics do not cross the lower bound.
4.3 Density Forecast Evaluation

The forecast evaluation is completed with an assessment of the density forecasts to provide a realistic pattern of the actual uncertainty. This kind of analysis has recently become popular in forecasting exercises involving DSGE-based models (Herbst and Schorfheide, 2012; Kolasa et al., 2012; Del Negro and Schorfheide, 2013; Wolters, 2015). However, the evaluation of the density forecasts is less straightforward than the evaluation and the comparison of RMSFEs. As discussed in Wolters (2015), the true density is never observed. Notwithstanding this, the researcher can compare the distribution of observed data with density forecasts to investigate whether forecasts explain the actual uncertainty.

We evaluate and rank the density forecasts using the log predictive density scores (LPDS), similarly to Adolfson et al. (2007), Christoffel et al. (2010), Marcellino and Rychalovska (2014), among others.

Considering the assumption that $h$-step-ahead predictive density is normally distributed,
the LPDS for variable $i$ can be written as:

$$s_t(y^i_{t+h}) = -0.5 \left[ \log(2\pi) + \log(V^i_{t+h/t}) + (y^i_{t+h} - \bar{y}^i_{t+h/t})^2 / V^i_{t+h/t} \right], \quad (29)$$

where $\bar{y}^i_{t+h/t}$ and $V^i_{t+h/t}$ are the posterior mean and variance of $h$-step-ahead simulated forecast distribution for the variable $i$.

The average score (AS) in forecasting variable $i$ with the model $m$ is given by:

$$AS_{i,h}^m = \frac{1}{T_h} \sum_{t=T}^{T+T_h-1} s_t(y^i_{t+h}), \quad (30)$$

where $T_h$ represents the number of $h$-step-ahead forecasts. As discussed in Adolfson et al.
(2007), the predictive density of the DSGE models estimated using Bayesian techniques does not have a known analytical form. Hence, we approximate the DSGE predictive density using a multivariate normal density. However, this assumption depends on the property of normality for the distribution of any subset of observed variables. In Bayesian estimation of DSGE models, the parameter uncertainty is the only source of non-normality of the predictive density, as argued by Christoffel et al. (2010). In general, only a small fraction of the forecast error variance is given by parameter uncertainty, then the normality assumption does not represent a significant misspecification problem in computing the log predictive score.

Tables 8 and 9 present the score of the SWBF relative to SW for the two periods, 2001-2008 and 2009-2013. A positive number indicates an improvement over the SW model. Table 8 shows that for output growth, the SWBF model offers only a small improvement (less than 10%) for horizons 2, 4, and 6. For the other horizons, instead, the SW model is marginally better, with an improvement of around 5%. Similar mixed pattern can be detected also for consumption and wage. At the 8 and 12 quarter horizon, the SW model outperforms the SWBF model, while for shorter horizons the contrary happens with a maximum improvement of the SWBF model at horizon 4, equal to 40% over the SW log score. For inflation, FFR, and hours, the SWBF produces the best performance. Table 9 shows the LPDS in the period 2009-2013. For output growth the SW model clearly outperforms the SWBF model at any horizon. This result is in line with the findings of the point forecast reported in Table 5. An explanation for this is that the log scores of the main components of output, i.e. consumption and investment, are better in the SW model. For inflation, FFR, and hours, the SWBF is still the best model. This analysis also confirms the point density forecast in favor of the SWBF for inflation and FFR. No clear pattern emerges for wage.

Tables 10 and 11 present the score of the BGG relative to SW for the two periods, 2001-2008 and 2009-2013. The former table shows that for output growth, the BGG model does not offer an improvement compared to the SW model, which is generally better. Similar mixed pattern can be detected also for all the other series, but the federal fund rate. The BGG model produces the best performance at horizons 1, 2 and 6. Table 11 shows the LPDS in the period 2009-2013. As far as output growth is concerned, the SW model clearly outperforms
Table 8: Percentage improvement in the log predictive scores for the period 2001Q1-2008Q4 of the SWBF model over the SW model.

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Inflation</th>
<th>FFR</th>
<th>Wage</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
</tr>
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<td>2.77</td>
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<td>15.58</td>
<td>69.43</td>
</tr>
<tr>
<td>8</td>
<td>-2.32</td>
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<td>58.12</td>
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<tr>
<td>12</td>
<td>-5.82</td>
<td>6.51</td>
<td>11.37</td>
<td>1.26</td>
<td>0.18</td>
<td>-24.92</td>
<td>20.32</td>
</tr>
</tbody>
</table>

Table 9: Percentage improvement in the log predictive scores for the period 2009Q1-2013Q4 of the SWBF model over the SW model.

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Inflation</th>
<th>FFR</th>
<th>Wage</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
</tr>
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<tr>
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<td>6.23</td>
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<td>2.87</td>
<td>-84.37</td>
<td>2.79</td>
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<td>6</td>
<td>-19.13</td>
<td>1.21</td>
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<td>-85.17</td>
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<td>103.25</td>
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<td>8</td>
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<td>24.88</td>
<td>2.14</td>
<td>-69.50</td>
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<td>293.22</td>
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</table>

Similarly to the SWBF model, the log scores of the main components of output, i.e. consumption and investment, are better in the SW model. For inflation, wage and hours, no model clearly dominates the other. For the FFR the BGG model produces the best performance. This analysis confirms the point density forecast in favor of the BGG for the FFR.

In addition, we report a graphical representation of the Probability Integral Transform (PIT) using histograms. The PITs were developed by Rosenblatt (1952), Dawid (1984), Kling and Bessler (1989), and introduced in an economic application by Diebold et al. (1998). We can define the PIT as the transformation:

\[ p_\tau = \frac{1}{\infty} \int f(u)du, \]

where \( f(u) \) is the ex ante forecast density and \( x_\tau \) is the ex post observed data. If the density forecast is well calibrated, \( p_\tau \) should be independently and uniformly distributed on the uniform interval (0,1) as noted by Dawid (1984), Diebold et al. (1998) and Diebold et al. (1999). Moreover, at the one step ahead horizon, PITs are independently distributed, while
<table>
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<tr>
<th>Output</th>
<th>Inflation</th>
<th>FFR</th>
<th>Wage</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
</tr>
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<td>-7.57</td>
<td>-9.07</td>
<td>-16.60</td>
<td>-304.65</td>
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</table>

Table 10: Percentage improvement in the log predictive scores for the period 2001Q1-2008Q4 of the BGG model over the SW model.

<table>
<thead>
<tr>
<th>Output</th>
<th>Inflation</th>
<th>FFR</th>
<th>Wage</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
</tr>
</thead>
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<td>-393.19</td>
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<td>37.47</td>
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<td>-276.36</td>
</tr>
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<td>25.75</td>
<td>7.99</td>
<td>-58.78</td>
<td>-188.94</td>
</tr>
</tbody>
</table>

Table 11: Percentage improvement in the log predictive scores for the period 2009Q1-2013Q4 of the BGG model over the SW model.

Independence may be violated at longer horizons since multi-step-ahead forecast errors are serially correlated (see Knüppel, 2015, for more details). As described in Diebold et al. (1998), there are several graphical approaches to forecast evaluation. The most common approach is to present a visual assessment of the distribution of realized data points on the sequence of PITs represented as a histogram (as shown by Kolasa et al., 2012 and Wolters 2015, among others). Hence, the unit interval is divided in $K$ subintervals and the fraction of PITs in each of them is close to $K^{-1}$. We follow this methodology in Figures 10-14 and we set a histogram of 10 probability bands each covering 10%, i.e. $K = 10$. The horizontal line is expressed in percentage from 0 to 100 (where 100 means 1 in the uniform distribution). Bars represent the fraction of realized observations falling into deciles of density forecasts. The theoretical value of 10% for a well-calibrated model is represented by a solid line. We compare the three models for the GDP and inflation for horizon 1. If we consider the whole period 2001-2013, the three models are better calibrated than splitting the sample. Especially during the period 2009-2013, we recognize an empirical distribution different from the underline uniform.

\[16\] We draw the histogram of PITs following the setup proposed by Kolasa et al. (2012). Their codes are available at: http://jmcb.osu.edu/volume-44-2012.
Figures 13, 14 and 15. According to Kolasa et al. (2012), the DSGE models impose tight restrictions on the data, hence its misspecification should be absorbed by stochastic shocks (see Gerard and Nimark, 2008, and Del Negro and Schorfheide, 2009). Most probably, during a short sample, such as 2009-2013, the DSGE model restrictions fail to match the data. For the output growth forecasts, a large fraction of PITs falls into the 0.4 – 0.6 bin and the peak in the middle of the histograms of the output growth forecasts shows an overestimation of uncertainty. This result is similar to the ones reported in Del Negro and Schorfheide (2013), Kolasa et al. (2012) and Wolters (2015) and it indicates that the predictive distribution is too diffuse (Del Negro and Schorfheide, 2013).¹⁷

![Horizon 1: DSGE forecast for GDP QoQ](image1)

![Horizon 1: DSGE forecast for inflation](image2)

Figure 10: DSGE SW Density Forecasts: PIT Histograms for One-Quarter-Ahead Forecasts for the period 2001-2013.

¹⁷There are formal tests to check for a uniform distribution (Berkowitz, 2001). However, the results have to be treated with high caution (see Gerard and Nimark, 2008). Since the histograms have already shown a clear evidence against a uniform distribution of PITs, we do not add formal tests in our empirical analysis.
Figure 11: DSGE SWBF Density Forecasts: PIT Histograms for One-Quarter-Ahead Forecasts for the period 2001-2013.

Figure 12: DSGE BGG Density Forecasts: PIT Histograms for One-Quarter-Ahead Forecasts for the period 2001-2013.

Figure 13: DSGE SW Density Forecasts: PIT Histograms for One-Quarter-Ahead Forecasts for the period 2009-2013.
5 Conclusion

This paper focuses on the role of instabilities of the DSGE parameters in affecting the forecasting accuracy. It discusses the forecasting performance of two DSGE models with different types of financial frictions versus a standard medium scale DSGE \textit{à la} Smets and Wouters (2007). The empirical analysis is based on a forecasting horse-race among three models (SWBF, BGG and SW), evaluated along point and density forecast analysis. Point forecast analysis shows that in the sample 2001-2008 the SWBF model exhibits the best performance for output growth and inflation only in the longer horizon, while in the sample 2009-2013 the
SWBF outperforms the SW models in forecasting inflation and the federal funds rate, but not output growth. The BGG model exhibits a better forecasting performance only for inflation in longer horizon in the sample 2001-2008 and for the federal funds rate in the sample 2009-2013. Hence, there is no clear evidence of an outperformed model in terms of forecasting accuracy. To explain these results, we examine the time dimension of the estimated parameters, based on a recursive-window estimation.

We find a substantial degree of variations in the estimated parameters/shocks of the three models. Fluctuation test as in Giacomini and Rossi (2010) reveals that the empirical ranking among models changes over time. In particular, adding financial frictions to a standard medium-scale DSGE model helps improving the forecasting performance of inflation, while the forecasting performance of the models with financial frictions is not statistically different from that of the SW model for GDP growth rate, except in 2006. Density forecast analysis confirms the results of the point forecast.

This exercise turns out to be useful also for policy-making since there might be frictions which are more important only in some episodes. This would lead eventually to the appropriate design of policy instruments alleviating the severity of the financial frictions.

References


APPENDIX

A Data sources and transformations

This section discusses the sources of the observables used in the estimation and their transformation. GDP, GDP deflator inflation, the federal funds rate, civilian population (CNP160V) and civilian employment (CE160V) are downloaded from the ALFRED database of the Federal Reserve Bank of St. Louis. Private consumption expenditures and fixed private investment are extracted from the NIPA Table 1.1.5 of the Bureau of Economic Analysis. Average weekly hours worked (PRS85006023) and compensation per hour (PRS85006103) are downloaded from the Bureau of Labor Statistics. Net worth of banks is downloaded from the FRED database and it is computed as the difference between total assets of all commercial banks (TLAACBW027SBOG) and total liabilities of all commercial banks (TLBACBM027SBOG). The spread in the BGG model is computed as the annualized Moody’s Seasoned Baa Corporate Bond Yield spread over the 10-Year Treasury Note Yield at Constant Maturity, as in Del Negro and Schorfheide (2013).

Data are transformed as in Smets and Wouters (2007). In particular, GDP, consumption, investment and net worth are transformed in real per-capita terms by dividing their nominal values by the GDP deflator and the civilian population. Real wages are computed by dividing compensation per hour by the GDP deflator. As shown in the measurement equations, the observable variables of GDP, consumption, investment, wages and net worth are expressed in first differences. Hours worked are multiplied by civilian employment, expressed in per capita terms and demeaned. The inflation rate is computed as a quarter-on-quarter difference of the log of the GDP deflator. The fed funds rate is expressed in quarterly terms. Remaining variables are expressed in 100 times log. All series are seasonally adjusted. In the robustness exercise of the SWBF model in Appendix C, the spread is computed as the difference between the bank prime loan rate and the 3-month Treasury bill rate and it is expressed in quarterly terms. Data on spreads are also extracted from the ALFRED database of the Federal Reserve Bank of St. Louis.
Figure 16: Evolution of the variance decomposition of the SW model. The black line represents government and monetary policy shocks, the green line the risk premium shock, the red line the investment-specific technology shock, while the blue line the TFP, price mark-up and wage mark-up shocks. Estimates are computed based on the recursive estimation sample starting from 1984Q1-2000Q4 and ending in 2013Q4.

B Variance decomposition analysis

The variation in the estimation of shocks and parameters implies that the role of shocks in affecting macroeconomic variables is changing over time and across models’ specification. Figure 16 shows the evolution of the variance decomposition implied by the estimated SW model. The government and monetary policy shocks play an important role in affecting movements in output. The role of the risk premium shock increased following the collapse of Lehman Brothers, while the contrary happens for the investment-specific technology shock. The explanatory power of the supply shocks is lower than all the demand shocks. Inflation is mainly driven by supply shock, with risk premium shocks accounting for about 40% of its fluctuations at the end of the recursive sample. The fed funds rate is mainly explained by the investment-specific technology shock. The risk premium shock experiences three phases in accounting for movements in the interest rate: first, a limited role up to 2008. Second, in the recursive sample 2009-2012Q2 it explains about 40% of movements in the nominal interest rate. In the latest recursive sample it becomes its most important driver.
Figure 17 shows the recursive variance decomposition of the SWBF model, which features also a net worth shock. Supply shocks are the dominant factors behind movements in output. The most important demand shocks are government and monetary policy, with an increasing role of net worth shocks at the end of the recursive sample. The SWBF model features a financial accelerator mechanism through banks’ balance sheet: a contractionary shock is generally associated with a rise in the credit spread and a contraction in the quantity of credit. This in turn diminishes the productive capacity of the economy. Reifschneider et al. (2013) argue that in the recent financial crisis a significant portion of the damage to the supply side of the economy plausibly was endogenous to the weakness in aggregate demand. The different role of supply and demand shocks in the SW and SWBF models can be explained by the reduced amount of credit that contracted production in the latter model. Supply shocks are the most important drivers of inflation, with a bigger role in the latest sample compared to the SW model. It is well known that supply shocks play a major role in accounting for the variance of inflation (e.g. Smets and Wouters, 2007). Their larger role in the SWBF model can provide some intuition on the better forecasting performance of this model as far as inflation is concerned. The net worth shock is a dominant factor behind movements in the fed funds rate. This helps in explaining the better forecasts of the SWBF model for this variable. Finally, it is worth noting that the lower degree of instability of shocks and parameters in the SWBF model leads to a more stable profile of the recursive variance decomposition analysis in the SWBF model, compared to that in the SW model.

Figure 18 shows the recursive variance decomposition of the BGG model. Government and monetary policy shocks are the dominant factors behind movements in output in the first part of the recursive sample, while supply shocks prevail in the last part. The role of the net worth shock is negligible, whereas the risk premium shock together with the investment-specific technology shock explain more than 30% of output variation until 2008. The situation is reversed at the end of the recursive sample. Similarly to the SWBF model, supply shocks are the most important drivers of inflation. In the BGG model the net worth shock plays a larger role in accounting for movements in inflation at the end of the sample. The supply and net worths shock are almost equally important for explaining movements in the federal
fund rate until 2008, since when the net worth shock dominates. For all the three variables, the role of net worth shock increases from 2008 onwards.

C Evaluating forecasting accuracy with an alternative financial variable

This section shows the forecasting accuracy of the SWBF model when the observable financial variable is the credit spread instead of net worth. Section C.1 reports the results on the point forecast evaluation, while Section C.2 presents density forecasts.

C.1 Point forecast evaluation

Figure 19 shows the one quarter forecast series for output growth, investment growth, inflation, and federal funds rate (FFR) for the SWBF and the SW model for the whole forecasting period. The predictions for the two DSGE models are similar to the ones shown in Figure 1. Hence results are robust to the use of the different financial observable variable.
Figure 18: Evolution of the shock variance decomposition of the BGG model. The black line represents government and monetary policy shocks, the blue line the TFP, price mark-up and wage mark-up shocks, while the magenta line the net worth shock. Estimates are computed based on the recursive estimation sample starting from 1984Q1-2000Q4 and ending in 2013Q4.

We now compute Mean Forecast Error (MFE). Table 12 shows the MFE of the SWBF model over the period 2001-2008. Results are remarkably similar to the ones in Table 2: there is no bias for consumption; inflation, the nominal interest rate, wage and hours are underpredicted; output and investment are overpredicted. Same rationale applies to the MFE over the period 2009-2013, reported in Table 13. The only difference from Table 3 is that there is no bias for the wage variable.

<table>
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<th>Hours</th>
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Table 12: Mean Forecast Error for the sample 2001Q1-2008Q4 of the SWBF where the credit spread is the financial observable. Asterisks ***, ** and * denote, respectively, the 1%, 5% and 10% significance levels.
Figure 19: One quarter ahead forecast comparison.

<table>
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<th>Wage</th>
<th>Consumption</th>
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Table 13: Mean Forecast Error for the sample 2009Q1-2013Q4 of the SWBF where the credit spread is the financial observable. Asterisks ***, ** and * denote, respectively, the 1%, 5% and 10% significance levels.

Tables 14 and 15 show the RMSFE over the forecasting periods 2001-2008 and 2009-2013, respectively. Differently from Table 4, the forecasting accuracy for inflation is statistically better in the SWBF model for most horizons, while the other results are pretty similar between the two specifications (SWBF model estimated with net worth and the one estimated with the credit spread) also for the sample 2009-2013.

In order to provide some intuition for the results of the point forecasts, we present the recursive estimates of shocks and main parameters in Figures 20 and 21. The shock processes show a lower degree of instability compared to the estimates of the SW model and of the SWBF model (net worth). It should be noted that the volatility of the spread series is more than 90% lower than the volatility of the net worth series. The risk premium and the net
Table 14: Root Mean Square Forecast Error of the SWBF model where the credit spread is the financial observable. All RMSFE are computed as a ratio to the RMSFE in the Smets and Wouters model. Forecasting evaluation period: 2001Q1-2008Q4. Asterisks ***, ** and * denote, respectively, the 1%, 5% and 10% significance levels of the Clark - West test.

<table>
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<th>Consumption</th>
<th>Investment</th>
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<td>1.03 ***</td>
<td>1.76***</td>
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Table 15: Root Mean Square Forecast Error of the SWBF model where the credit spread is the financial observable. All RMSFE are computed as a ratio to the RMSFE in the Smets and Wouters model. Forecasting evaluation period: 2009Q1-2013Q4. Asterisks ***, ** and * denote, respectively, the 1%, 5% and 10% significance levels of the Clark - West test.

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worth shocks are less volatile than in the other two estimated models also from a quantitative point of view. The volatility of the investment-specific technology shock is pretty stable, while that of wage mark-up shock exhibits a clear increasing trend over the recursive sample. The volatility of the TFP shock shows a step in correspondence to the 2009. Recursive estimation of the parameters confirms the lower degree of instability.

Results are robust also for the analysis of the Fluctuation test, as shown by Figure 22.

C.2 Density forecast evaluation

Tables 16 and 17 support previous findings: the SWBF model outperforms the SW model in forecasting inflation, FFR and hours, while the contrary happens for output, consumption and investment.

PITs histograms confirm the pattern already described with the net worth variables. In the whole sample, 2001-2013, the SWBF is better calibrated. Instead, in the period 2009-2013, plots do not report a uniform distribution.
Figure 21: Evolution of the parameters of the SWBF model, where the observable financial
variable is the credit spread. Solid lines represent the posterior mean, while dotted lines 5th
and 95th posterior percentiles. Estimates are computed based on the recursive estimation
sample starting from 1984Q1-2000Q4 and ending in 2013Q4.

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Table 16: Percentage improvement in the log predictive scores for the period 2001Q1-2008Q4
over the SW model.

<table>
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Table 17: Percentage improvement in the log predictive scores for the period 2009Q1-2013Q4
over the SW model.
Figure 22: Fluctuation test for the SWBF model, where the observable financial variable is the credit spread.

Figure 23: DSGE SWBF Density Forecasts: PIT Histograms for One-Quarter-Ahead Forecasts for the period 2001-2013.

Figure 24: DSGE SWBF Density Forecasts: PIT Histograms for One-Quarter-Ahead Forecasts for the period 2009-2013.
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