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Authors(s)	Zou, Guang, González, Arturo, Banisoleiman, Kian, Faber, Michael Havbro
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An integrated probabilistic approach for optimum maintenance of fatigue-critical structural components

Guang Zou[1,2]; Arturo González[2]; Kian Banisoleiman[1]; Michael H. Faber[3]
[1]Lloyd's Register Global Technology Center, Lloyd's Register Group Limited, Southampton, UK
[2]School of Civil Engineering, University College Dublin, Dublin, Ireland
[3]Departement of Civil Engineering, Aalborg University, Aalborg, Denmark

Key words: structural integrity management; probabilistic approach; structural reliability; reliability-based optimization; maintenance; life cycle approach

Abstract

Inspection and maintenance are important means to validate or recover reliability of metallic structural systems, which usually degrade over time due to fatigue, corrosion and other mechanisms. These inspection and maintenance actions generally account for a large part of lifetime costs, which necessitate an efficient maintenance strategy to satisfy the requirements on reliability and costs. Most often, an optimum maintenance/repair crack size criterion is derived by probabilistic cost-benefit analysis, e.g. by minimization of expected lifetime costs, which are assessed based on cost models. This paper proposes an integrated approach to derive an optimum range for maintenance criterion using both reliability-based and cost-based optimization. It is found that an optimum repair (crack size) criterion exists which leads to the maximum lifetime fatigue reliability. A smaller repair criterion than the reliability-optimum do not lead to a higher lifetime fatigue reliability but leads to higher lifetime costs. Hence, a limit for repair criterion is defined by the reliability-optimum criterion, which can be obtained without cost models. The reliability-optimum criterion is found to be smaller than the cost-optimum criterion and thus an optimum range between the reliability-optimum and cost-optimum criterion is established.

1. Introduction

Structural systems with a substantial number of welded structural components are at high risk of loss of structural integrity. The issue can be partially addressed in the design stage by adopting a conservative approach that will increase structural resistance against fatigue. However, direct and indirect costs can become very high in excessively conservative designs, that aim to cover for the numerous sources of uncertainty affecting fatigue loads, fatigue capacity and deterioration process. In addition, there are inevitable initial cracks or flaws in welded structures after construction, which typically decrease fatigue lives of structural details and cause crack growth or even fracture. Hence, maintenance of these fatigue-prone components during operation is an important means of risk reduction to ensure structural integrity for many industries, e.g. ship structures, offshore installations, nuclear plants, etc. Cracks can be identified and repaired timely during maintenance interventions and disastrous failures can be avoided, which if happen would result in significant losses, not only financially, but also socially and environmentally (Frangopol and Soliman 2016).

The maintenance costs do not only include direct costs of manpower, materials and instrumentation, but also indirect costs, such as transportation, work preparation and planning, downtime loss, etc (Soliman, Frangopol, and Mondoro 2016). To reduce maintenance costs, it is important to develop a cost-efficient maintenance plan well in advance. The effectiveness of a maintenance plan depends on factors such as an accurate modelling of fatigue loading, a good understanding of the fatigue deterioration process, the choice of inspection and repair methods, the characterization of inspection quality and maintenance effect, and the selected inspection time and repair criterion. Some of the uncertainties associated with these factors have been reduced via probabilistic methods. For example, crack growth under wave loading has been the topic of many studies in the context of marine engineering (Moan and Ayala-Uraga 2008, Lotsberg et al. 2016, Garbatov and Soares 2001, Doshi, Roy, and Parihar 2017), and several approaches have been developed to characterize the inspection qualities of non-destructive testing (NDT) methods, taking uncertainties associated with detection and sizing of a crack into account (Ali et al. 2018, Schoefs, Clément, and Nouy 2009, Rouhan and Schoefs 2003). The effects of repair, e.g., the fatigue strength of repaired components, have also been investigated with simulation and experimental methods (Akyel, Kolstein, and Bijlaard 2017, Abdullah, Malaki, and Eskandari 2012). This literature contributes to an improved understanding and a broader knowledge of each of the factors, on which a theoretical basis for developing maintenance optimization methods can be built.

A complete maintenance plan comprises of a number of maintenance interventions in lifetime, inspection times and methods, and repair criteria and methods. These maintenance decisions determine the maintenance costs. The determination of an optimum maintenance strategy that is optimum strictly in mathematical format and optimum from the perspective of the whole life cycle, is a complex problem due to numerous challenges. I.e., the possible combinations of maintenance decisions are substantial, both structural damage states and maintenance activities are subjected to uncertainties, inspection results are unknown in the planning stage yet influential in optimum decision-making, optimum maintenance decisions are also dependant on how maintenance practices in reality are modelled and how the optimization problem is formulated, etc. Thus, appropriate approximations and assumptions are necessary for developing maintenance decision models or optimization methods. While some researchers put effort in modelling and comparing the effects of specific maintenance strategies (Zio and Compare 2013, Huynh, Grall, and Bérenguer 2017),others intend to utilize available inspection data to update structural deterioration condition and optimize maintenance plans accordingly (Dong and Frangopol 2016, Moan and Ayala-Uraga 2008, Soliman, Frangopol, and Mondoro 2016). Among the maintenance decisions, inspection times are a major concern in many published research (Lotsberg et al. 2016, Chen, Wang, and Guedes Soares 2011, Straub and Faber 2005, Doshi, Roy, and Parihar 2017, Beaurepaire et al. 2012, Valdebenito and Schuëller 2010), as the lifetime fatigue reliability and expected lifetime maintenance costs are very sensitive to the scheduled inspection times. Planning methods have been developed with respect to inspection times based on a given reliability or risk acceptance level (Dong and Frangopol 2016, Chen, Wang, and Guedes Soares 2011), and optimization methods have been proposed for both inspection times and qualities based on life cycle costs analysis (Beaurepaire et al. 2012, Valdebenito and Schuëller 2010). Repair criterion have also been optimized with the objective of minimizing total lifetime costs outside the optimization with respect to inspection times (Kim, Soliman, and Frangopol 2013, Soliman, Frangopol, and Mondoro 2016, Madsen, Torhaug, and Cramer 1991, Straub 2004, Gomes and Beck

2014) or with multi-objectives coupled with the optimization with respect to inspection times (Kim, Soliman, and Frangopol 2013, Soliman, Frangopol, and Mondoro 2016, Madsen, Torhaug, and Cramer 1991, Straub 2004, Gomes and Beck 2014). Basically, most existing studies derive an optimum repair criterion adopting the metric of cost, when the optimum repair criteria will provide the best trade-off between expected maintenance costs and expected failure costs. This paper finds that an optimum repair (crack size) criterion exists which leads to the maximum lifetime fatigue reliability. A smaller repair criterion than the reliability-optimum do not lead to a higher lifetime fatigue reliability. Hence, a limit of repair criterion is defined by the reliability-optimum criterion. An integrated approach using both reliability-based and cost-based optimization is proposed to derive a range for optimum repair criterion.

The modelling of fatigue, maintenance strategies, inspection methods, and repair effects are introduced first to then explain how to derive an optimum maintenance plan and to finally test it. More specifically, Section 2 simulates fatigue crack growth allowing for probabilistic modelling of the uncertainties related to the initial damage state, material property, modelling of structural response, etc., in a consistent way. Section 3 introduces the five maintenance strategies that will be tested. An equivalent repair criterion is defined to reflect the differences between the conditions of each strategy to carry out repair. For comparison purposes, time-based and detection-based maintenance strategies are analysed in addition to condition-based maintenance strategy. Section 4 establishes a probabilistic model for the quality of an inspection method in terms of crack detection considering inspection uncertainties. A simplified model is proposed for perfect inspection quality. Both imperfect and perfect inspection models are integrated into the optimum maintenance planning approach to assess the influence of inspection uncertainties. Section 5 reviews the methods for characterizing and modelling repair effect. A widely-used model considering that a structural detail is brought to initial damage state is adopted. Section 6 presents the proposed probabilistic maintenance optimization approach based on a physical-based probabilistic crack growth model, probabilistic models for inspection quality, decision tree analysis, reliability theory, cost modelling and life cycle analysis. The latter is employed to derive a range for optimum repair criterion. Section 7 illustrates the impact of the proposed optimum maintenance plans on a typical fatigue-prone detail, and investigates the influence of inspection uncertainty, the relationship between reliability-optimum and cost-optimum repair criteria with inspection times, and optimum plans under different strategies. Section 8 highlights the main contributions of the paper.

2. Probabilistic fatigue modelling

2.1 Crack growth relationship

The Fracture Mechanics (FM) approach is employed here for probabilistic fatigue deterioration modelling since it provides a means of modelling the crack growth explicitly and thus allows for reliability updating based on inspections of the crack growth. Fatigue failure is explained as the process of crack initiation, and crack growth until final fracture. For a welded structure, there are inevitable inclusions or initial flaws/cracks introduced during the welding process, which typically decrease fatigue resistance and shorten the crack initiation stage (Lassen and Recho 2009). Thus, it is often thought that the crack initiation stage is negligible compared with the crack growth stage and

ignored in fatigue deterioration modelling. The final fracture usually occurs very quickly for most structural components, and the crack growth stage counts for a major part of fatigue life. Crack growth is modelled via Paris' law (Paris and Erdogan 1963), formulated by Equations (1) and (2), which give the relationship between crack growth rate and the stress intensity factor.

$$\frac{da}{dN} = C\Delta K^m, \quad \Delta K_{th} \leq \Delta K \leq K_{mat} \quad (1)$$

$$\Delta K = \Delta\sigma Y(a)\sqrt{\pi a} \quad (2)$$

where a is crack size; N is number of cycles; da/dN is crack growth rate; C and m are material parameters; ΔK is stress intensity factor range; K_{mat} is material fracture toughness; ΔK_{th} is threshold value for the stress intensity factor range; $Y(a)$ is geometry function; and $\Delta\sigma$ is stress range. An equivalent constant-amplitude stress range is typically established by an S-N design curve as defined by Equation (3).

$$\begin{cases} N_F \Delta\sigma^{m_1} = \overline{a_1} & N_F \leq 10^7 \\ N_F \Delta\sigma^{m_2} = \overline{a_2} & N_F \geq 10^7 \end{cases} \quad (3)$$

where N_F is fatigue life, m_1 and m_2 are the fatigue strength exponents, and $\overline{a_1}$ and $\overline{a_2}$ are the fatigue strength coefficients. The fatigue strength exponents and coefficients are obtained based on statistical analysis of specimen fatigue strength test data.

If failure is defined by the crack depth reaching a critical size a_c, then the crack growth life N_P can be expressed by Equation (4), which is obtained by integration of Equation (1) from an initial crack size a_0 to a_c. The crack size $a(t)$ at time t can be obtained by solving Equation (5) incrementally from the beginning time ($t = 0$) to time t when the accumulated fatigue loading is $N(t)$ cycles.

$$N_P = \frac{1}{\pi^{m/2} C\Delta\sigma^m} \int_{a_0}^{a_c} \frac{da}{a^{m/2} Y(a)^m} \quad (4)$$

$$da(t) = \pi^{m/2} C\Delta\sigma^m a^{m/2}(t) Y^m[a(t)] dN(t) \quad (5)$$

2.2 Probabilistic modelling

Although the FM approach is well suited for fatigue deterioration modelling in support of maintenance planning, it is generally acknowledged that compared with the S-N approach, the modelling uncertainty associated with the FM approach is high, as a result of the large number of factors affecting the complex fatigue deterioration process is considered, and the shortage of validation data for the FM approach, compared to the S-N approach (Lassen and Recho 2013). Therefore, the results of the FM approach are sensitive to input parameters, that may lack sufficient statistical information. Probabilistic modelling based on Paris' law provides an explicit and rational approach for dealing with uncertainties in fatigue deterioration that is preferred here following (Moan and Ayala-Uraga 2008, Lotsberg et al. 2016, Chryssanthopoulos and Righiniotis 2006), although there are other approaches available in the literature such as random process (Gomes and Beck 2014), Markov process (Sheils et al. 2010, Lassen and Sørensen 2002b, a), etc.

The accuracy of using Paris law for crack growth modelling is believed to mainly depend on the accuracies of the input parameters: initial crack size a_0, material parameter C and stress range $\Delta\sigma$. In this paper, the initial crack size a_0 and the material parameter C are modelled as variables. Figure 1 provides a schematic representation of probabilistic crack growth, where f_1 is probability density function of a_0, f_2 is probability density function of crack size at the end of service life $a(T_R)$, and P_f is failure probability within the required service life T_R. Uncertainties associated with the calculation of stress range $\Delta\sigma$ are modelled as an additional variable B. The mean value and standard deviation (SD) for the variables are inputs for reliability analysis and maintenance decision-making.

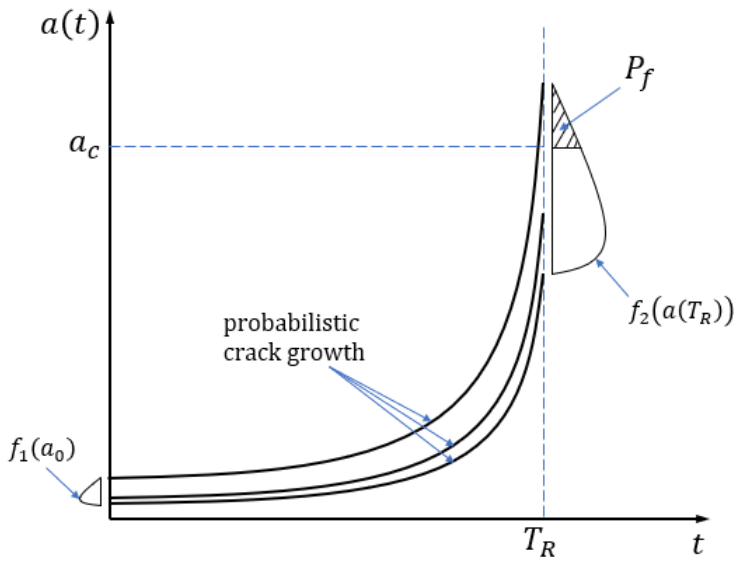

Figure 1: Schematic representation of probabilistic crack growth and failure probability

3. Maintenance strategy

Structural performance usually degrades over time under environmental loads and hazards. A maintenance strategy is a plan about when and how (the times, conditions, means and actions) to intervene a structural system to maintain or recover structural performance to a targeted level. An efficient maintenance strategy should be aimed at maximizing the potential benefits that scheduled maintenance actions would bring while minimizing the costs associated with the maintenance actions. For some non-critical structural components, reactive maintenance strategy may be feasible, i.e. a repair action is carried out after failure occurs. However, this strategy can be risky when the failure can cause serious consequences, e.g. failure of the whole system, causing pollution to the environment, danger to crew, etc. Also, the maintenance strategy may not be cost-effective if the structural system is comprised of many components with high rates of failure. Nowadays proactive maintenance strategy has been adopted extensively in many engineering fields. Maintenance interventions are scheduled and optimized with respect to the times, conditions, means and actions well in advance, and implemented at the scheduled times or under the optimized conditions before failure occurs. Proactive maintenance is also known as preventive maintenance, since components are repaired or replaced before failure occurs. Typical preventive maintenance strategies are time-based preventive maintenance and condition-based maintenance.

Table 1 lists the maintenance strategies investigated in this paper. The baseline case is the strategy MS1, in which no maintenance action is carried out during the lifetime. Time-based maintenance, labelled MS2, is used to describe maintenance activities which are carried out at specific times or after a fixed passage of service time (Sánchez-Silva et al. 2016). The times or intervals for maintenance actions can be determined by engineering experiences or expert options, or be optimized based on cost-benefit analysis (Huynh et al. 2012), etc. In the case of a condition-based maintenance strategy, maintenance actions are carried out only when a specific criterion is met, e.g., measured crack size exceeding a repair (crack size) criterion. This strategy requires measurement of crack size, and then to evaluate whether a specific repair criterion is met or not.

This paper proposes a probabilistic approach to determine the repair (crack size) criterion in a condition-based maintenance strategy (MS4) allowing for uncertainties in material property, initial crack size, modelling and inspection. The case of perfect information (provided by inspections), labelled MS3, is also examined in order to investigate the influence of inspection uncertainty on optimum maintenance decisions. Additionally, a detection-based maintenance strategy (MS5) is analysed for comparison purposes.

Table 1: Maintenance strategies under investigation and the equivalent repair criterion

	Maintenance strategies	Equivalent repair criterion $a_{r,e}$
MS1	No action	a_c
MS2	Time-based maintenance	a_0
MS3	Condition-based maintenance (perfect inspection information)	a_r
MS4	Condition-based maintenance (imperfect inspection information)	a_r
MS5	Detection-based maintenance (Repair upon detection)	$E(a_d)$

It should be noted that repair criterion is not involved under MS1, MS2 and MS5. However, for each of the aforementioned maintenance strategies, repair is carried out upon exceedance of an equivalent repair criterion $a_{r,e}$ as per Table 1. This treatment helps to model and integrate these strategies into the probabilistic optimization approach consistently and then compare the effects of these strategies directly with MS3 and MS4. The value of $a_{r,e}$ of each strategy determines the probability of repair to be actually implemented during the scheduled intervention. The value of $a_{r,e}$ for MS1, MS2 and MS5 is obtained by the definition of each strategy respectively:

- Under MS1, repair is not considered, which means that it is impossible that the repair criterion could be satisfied before failure. Therefore, the $a_{r,e}$ can be thought to be very large, e.g. equal to the critical crack size a_c.
- By the definition of MS2, repairs are carried out upon specific times, as long as the structural system has survived at the specific times. In other words, implementation of a repair under MS2 only needs to meet time requirement but no requirement on crack size. This repair criterion is so easy to meet and thus the $a_{r,e}$ is thought to be very small, e.g. equal to the initial crack size a_0.
- Under MS5, repair is carried out upon crack detection, i.e., the $a_{r,e}$ is equal to the mean detectable crack size, $E(a_d)$, of the adopted inspection method.

The $a_{r,e}$ of these strategies typically meet the relationship: $a_0 < E(a_d) < a_r < a_c$. Clearly, the larger $a_{r,e}$ is, the lower the probability of repair to be carried out and the lower the expected repair costs become. It should be mentioned that the values of $a_{r,e}$ for MS1, MS2 and MS5 (a_c, a_0 and $E(a_d)$) are fixed and are not optimization variables, while the optimal values of a_r for MS3 and MS4 are derived by the probability optimization approach. MS1, MS2 and MS5 are comparisons.

4. Probabilistic inspection modelling

Under the condition-based maintenance strategy, inspections are carried out to provide additional information on crack damage states, that together with the modelling results from Paris' law, provide basis for maintenance decision-making. Commonly-used inspection methods for fatigue crack detection are (close) visual inspection and NDT methods, e.g. liquid penetrant, ultrasonic, magnetic particle, and acoustic emission inspection methods. However, the information provided by an inspection is typically imperfect. Crack detection by NDT methods is inherently probabilistic, as there are many factors that can affect inspection results. Existing cracks may sometimes go unnoticed. Conversely, a positive indication may be false due to the absence of a crack. It is also often found that an existing crack can be detected by one inspector but missed by other inspectors. Generally, NDT results depend on the reliability of the specific instrument-human system. The following factors can influence the chance of crack detection (Georgiou 2007, Wall, Burch, and Lilley 2009):

- Crack characteristics (sizes, shape, location, etc.);
- The reliability of instrumentation;
- The environment where inspection is carried out;
- Inspection procedure;
- Human factors associated with inspector.

The reliability of the instrument-human system and confidence level on inspection results must be adequately demonstrated in terms of the level of accuracy that can be reached on the true crack characteristics. The reliability of an inspection method is often characterized by a probability of detection (PoD) curve. A PoD is defined as the probability that a given crack of a fixed size can be detected by a given inspection method (Georgiou 2007, Keprate and Ratnayake 2015). PoD curves for inspection methods are traditionally obtained by inspection experiments on structural details of a range of crack sizes. Based on inspection results, an appropriate function is assumed for the PoD curve, and parameters of the function are estimated by statistical methods. The confidence range on the PoD can be specified based on the estimated parameters. The experimental approach is usually very expensive and time-consuming as there are so many factors that can affect the probability of detection. Alternatively, simulation approaches are also developed to obtain PoD curves (Wirdelius and Persson 2012, Rentala, Mylavarapu, and Gautam 2018, Ali et al. 2018).

This paper models inspection information as imperfect (MS4 in Table 1) or perfect (MS3) in maintenance modelling and optimization to investigate the influence of inspection uncertainty on optimum maintenance decisions. Two different PoD functions are applied, as shown schematically in Figure 2. 'Inspection uncertainty' and 'Perfect inspection' mean that inspection uncertainty is considered and neglected respectively.

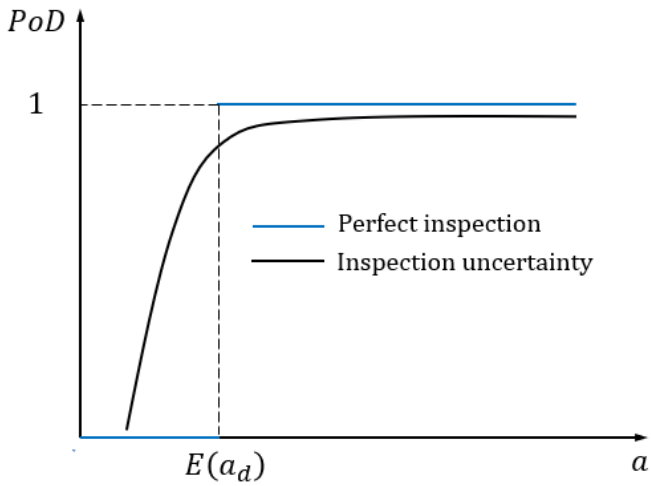

Figure 2: Probability of detection (PoD) curves

For the imperfect inspection, the exponential PoD function given by Equation (6) is employed (Chen, Wang, and Guedes Soares 2011, Lassen and Recho 2015, Moan and Song 2000, Dong and Frangopol 2016, Moan et al. 1997, Moan and Ayala-Uraga 2008, Beaurepaire et al. 2012, Valdebenito and Schuëller 2010).

$$PoD(a) = F(a) = 1 - \exp(-a/E(a_d)) \quad (6)$$

where $E(a_d)$ is the mean detectable crack size.

This function considers the uncertainties associated with the quality of an inspection method by modelling the detectable crack size, a_d, as a variable. The PoD function is also equal to the cumulative density function (CDF) of the variable a_d, and treats an inspection result as imperfect by calculating a specific probability of detection for a given crack size.

For the perfect inspection, the PoD function is defined by Equation (7).

$$PoD(a) = \begin{cases} 0 & a < E(a_d) \\ 1 & a \geq E(a_d) \end{cases} \quad (7)$$

This function assumes that cracks equal to or larger than $E(a_d)$ will be detected with a probability of 1, which indicates perfect detectability for those cracks, and cracks smaller than $E(a_d)$ will not be detected at all. Equation (7) implies that the detectable crack size, a_d, of an inspection method is a constant value $E(a_d)$.

Due to inspection uncertainty, there are probabilities of missed repair and fault repair in condition-based maintenance. A missed repair means that a crack larger than the repair criterion is not detected by the inspection method, while a fault repair means that a crack smaller than the repair criterion is detected and believed to exceed the repair criterion. It must be noted that fault repair is not considered in this paper, and that the probabilities of missed repair under different repair criteria and at different inspection times are discussed based on the illustrative example of Section 7.

5. Maintenance effect modelling

It is the repair or replacement actions that ultimately improve fatigue reliability. A condition-based maintenance strategy can improve fatigue reliability only when repair actions are carried out following negative inspection result and exceedance of a repair criterion. A model for the effect of repair action needs to be developed before the impact of maintenance strategies and maintenance actions on fatigue reliability can be evaluated and quantified. Commonly-used repair methods for cracks in structural engineering include:

- Drilling a stop hole;
- Welding;
- Welding plus post-weld treatment;
- Replacement;
- Grinding.

These methods are used to remove cracks, to decrease crack size, and to stop or slow down crack growth, by which a structural system is physically changed, as well as structural performance and fatigue reliability. The effects of repair lie in changes in crack size, deterioration rate, fatigue life etc. Fatigue strength of repaired structural details is investigated by experimental (Akyel, Kolstein, and Bijlaard 2017, Abdullah, Malaki, and Eskandari 2012, Kudryavtsev, Kleiman, and Knysh 2008) and numerical (Ayatollahi, Razavi, and Chamani 2014, Achour, Bouiadjra, and Serier 2003, Schubbe and Mall 1999) methods and some useful conclusions are drawn based on statistical analysis on the experimental or simulated data. Depending on the study areas and objectives, different repair effect models have been developed.

- (Ma and Bea 1995) studies the repair methods used for ship structures and proposes to designate different classes of S-N curve for details repaired by different methods.
- Some probabilistic maintenance modelling and optimization methods are developed using time-based safety margins, and the effect of repair on a structural detail in terms of extension of fatigue life is modelled as a variable defined by a distribution (Kim, Soliman, and Frangopol 2013, Barone, Frangopol, and Soliman 2013, Kim and Frangopol 2010, Soliman, Frangopol, and Mondoro 2016).
- On the other hand, many publications on probabilistic maintenance modelling and optimization adopt crack size-based safety margins and consider the effect of repair on a structural detail in terms of mitigation of crack damage.
- (Straub 2004, Faber, Straub, and Goyet 2003, Straub and Faber 2005, Valdebenito and Schuëller 2010, Beaurepaire et al. 2012, Gomes and Beck 2014) assumes that a detected crack is fully removed by a repair, and that the crack size is zero from the time of repair until the end of lifetime. The assumption indicates that after repair, the fatigue life of structural detail is sufficiently long, and that the failure probability caused by fatigue is zero.
- Most of maintenance planning models to date have adopted the so-called ‘as good as new’ maintenance model, which means that after repair a structural detail is brought to its initial as-built state (Garbatov and Soares 2001, Eltaief et al. 2015, Huynh, Grall, and Bérenguer 2017, Kulkarni and Achenbach 2007, Madsen, Torhaug, and Cramer 1991, Sheils et al. 2010, Zitrou, Bedford, and Daneshkhah 2013).

- Under the minimum maintenance strategy, a minimum repair is carried out and a structural system returns to its previous damage state, i.e., the structural deterioration is improved by just one stage, which is nearly as bad as the old state (Huynh et al. 2012, Badía et al. 2018, Aven and Jensen 2000). The minimum repair model is often used in maintenance planning methods based on discrete structural deterioration models.
- Some more sophisticated repair effect models have also been developed based on the idea that after repair a structural system may be brought to states between the as good as new and the as bad as old, e.g. the state after repair is a variable with a distribution (Wu, Niknam, and Kobza 2015, Van and Bérenguer 2012), the effect of repair is linked with deterioration rate of the system rather than with the deterioration state (Zhou, Xi, and Lee 2007, Zhang, Gaudoin, and Xie 2015), etc. However, the applications of these sophisticated repair effect models have been very limited.

Figure 3 illustrates the potential impact of four repair effect models following an intervention at time t_i. This paper employs the commonly-used 'as good as new' model (model 4 in Figure 3), i.e. after repair a structural detail is brought to its initial as-built state (Garbatov and Soares 2001, Eltaief et al. 2015, Huynh, Grall, and Bérenguer 2017, Kulkarni and Achenbach 2007, Madsen, Torhaug, and Cramer 1991, Sheils et al. 2010, Zitrou, Bedford, and Daneshkhah 2013). This is a relatively reasonable model, as it can model the small failure probability of a repaired structure detail. In practical, fatigue cracks are usually not easy to prevent, and periodical inspections are necessary. It's often found that cracks propagate again in original areas, and thus there is a small possibility that a repaired structural detail fails. However, (Straub 2014) has performed comparative study and concluded that the influence of maintenance model on reliability results is negligible.

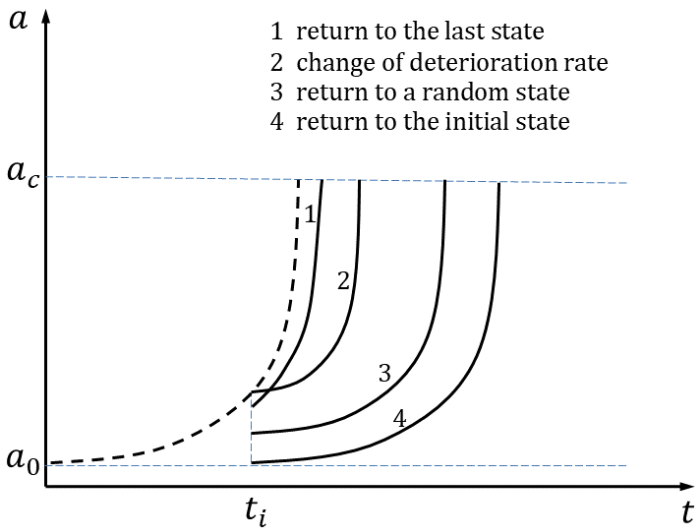

Figure 3: Illustration of repair effect models

6. Probabilistic maintenance optimization approach

6.1 Formulations for event margins

The event of failure at time t is probabilistic, since it is the outcome of a random variable, i.e., the crack size $a(t)$, that stochastically increases under cyclic fatigue loading. Structural failure is defined

as occurrence of through-thickness crack, i.e., the crack size exceeds a critical crack size a_c, which is equal to the plate thickness T. If the structural system has survived at t, and an inspection is scheduled, then the inspection result (detection or no detection) at the time is probabilistic as both the crack size $a(t)$ and the detectable crack size a_d are probabilistic. If the result is detection and the crack size has exceeded the repair criterion a_r, then a repair will be carried out. The event of repair is also probabilistic as a result of the inspection result being probabilistic. The margins for safety ($h_1(t)$), inspection ($h_2(t)$) and repair ($h_3(t)$) at a point in time *t* are formulated by Equations (8) to (10).

$h_1(t) = a_c - a(t)$ (8)
$h_2(t) = a_d - a(t)$ (9)
$h_3(t) = a_r - a(t)$ (10)

6.2 Probability of repair, failure and inspection

The number n of scheduled maintenance interventions in lifetime is pre-determined. The scheduled times for inspections, $t_1, t_2, \cdots, t_n$, are decision parameters together with the repair criterion a_r, that can adopt different values, $a_1, a_2, \cdots, a_n$, at each intervention. Herein, the probability of inspection, repair and failure and reliability index are formulated as functions of $a(t)$, a_c, a_d, n, $t_1, t_2, \cdots, t_n$ and $a_1, a_2, \cdots, a_n$, so that the design parameters $t_1, t_2, \cdots, t_n$ and $a_1, a_2, \cdots, a_n$ can be optimised based on lifetime fatigue reliability index β_l and expected lifetime costs C_l.

As the events of failure, detection and repair are all probabilistic at a specific intervention time, the probability of inspection, repair and failure are calculated based on decision tree analysis, considering all possible damage states (failure or survival), inspection results (detection and no detection, exceedance of a repair criterion or not) and repair actions (repair or 'no action') (Straub and Faber 2005, Kim, Soliman, and Frangopol 2013, Soliman, Frangopol, and Mondoro 2016, Madsen, Torhaug, and Cramer 1991). Figures 4 to 6 illustrate the decision tree analysis for MS2 (time-based maintenance strategy), MS3 (and MS4) (condition-based maintenance), and MS5 (detection-based maintenance) respectively. It can be seen from the figures that the MS3 and MS4 are more complex than the MS2 and MS5, due to the higher number of branches (or possibilities) at each intervention time.

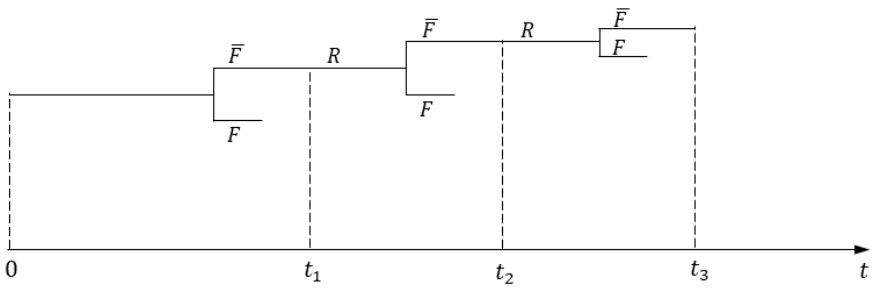

Figure 4: Decision tree analysis under the time-based maintenance strategy (F and R signifies failure and repair respectively, $\bar{F}$ signifies survival)

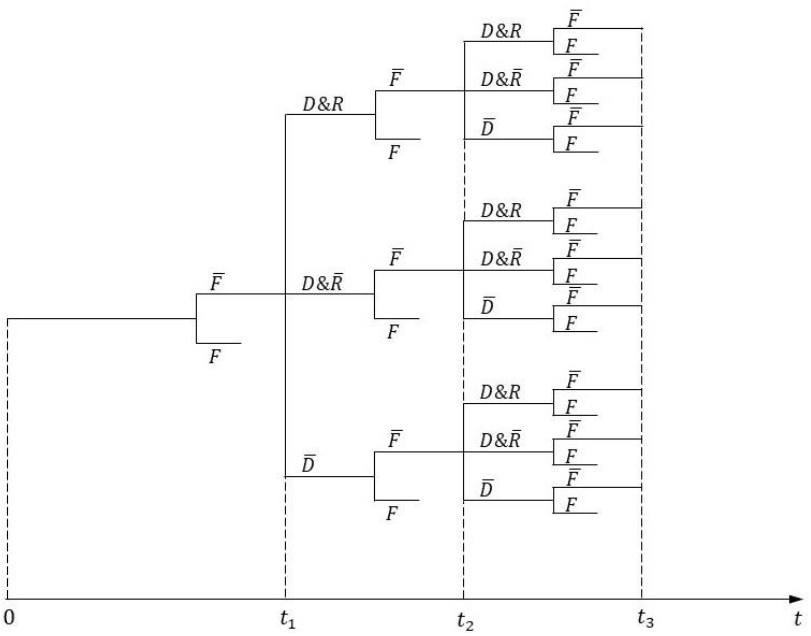

Figure 5: Decision tree analysis under the condition-based maintenance strategy (F, D and R signifies failure, detection and repair respectively, $\bar{F}$, and $\bar{D}$ signifies survival and no detection)

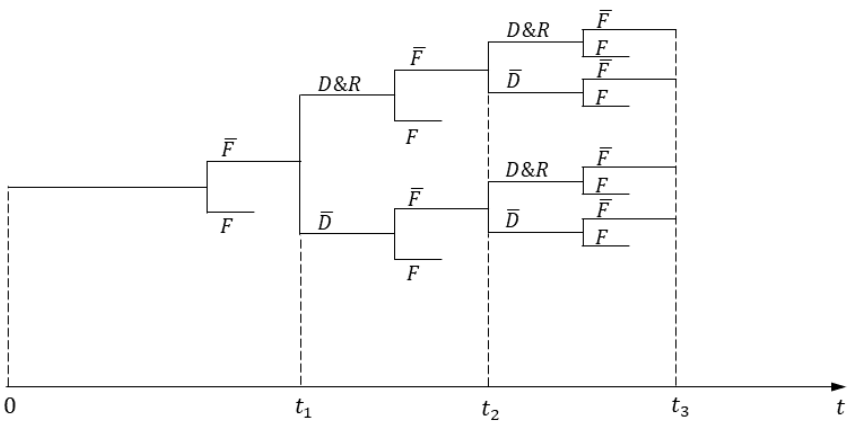

Figure 6: Decision tree analysis under the detection-based maintenance strategy (F, D and R signifies failure, detection and repair respectively, $\bar{F}$, and $\bar{D}$ signifies survival and no detection)

The probability of repair, failure and inspection under the MS3 and MS4 are formulated based on the decision tree analysis illustrated by Figure 5. The probabilities of conducting repair at the intervention time $t_1, t_2, \cdots, t_n$ are given by Equations (11)-(13).

$$pr^1(t_1) = P\left(a_{t_1} \geq a_d, a_r \leq a_{t_1} < a_c\right) \quad (11)$$
$$pr^2(t_2) = pr^1(t_1) \cdot pr^1(t_2 - t_1) + P\left(a_{t_1} < a_r, a_{t_2} \geq a_d, a_r \leq a_{t_2} < a_c\right) \quad (12)$$
$$pr^n(t_n) = pr^{n-1}(t_{n-1}) \cdot pr^1(t_n - t_{n-1}) + P\left(a_{t_{n-1}} < a_r, a_{t_n} \geq a_d, a_r \leq a_{t_n} < a_c\right) + \sum_{i=1}^{n-2} pr^i(t_i) \cdot P\left(a_{t_{n-1}-t_i} < a_r, a_{t_n-t_i} \geq a_d, a_r \leq a_{t_n-t_i} < a_c\right) \quad (13)$$

The failure probability is obtained by adding together the failure probabilities associated with all branches at an intervention time. The probabilities of failure during $0 \leq t \leq t_1$, $t_1 < t \leq t_2$, $\cdots$, $t_n < t \leq T_{SL}$ are given by Equations (14)-(16).

For $0 \leq t \leq t_1$,
$pf^0(t) = P(a_t \geq a_c)$ (14)
For $t_1 < t \leq t_2$,
$pf^1(t) = pf^0(t_1) + pr^1(t_1) \cdot pf^0(t - t_1) + P\left(a_{t_1} < a_r, a_t \geq a_c\right)$ (15)
For $t_n < t \leq T_{SL}$,
$pf^n(t) = pf^{n-1}(t_2) + pr^n(t_n) \cdot pf^0(t - t_n) + P\left(a_{t_n} < a_r, a_t \geq a_c\right) + \sum_{i=1}^{n-1} pr^i(t_i) \cdot P\left(a_{t_n - t_i} < a_r, a_{t - t_i} \geq a_c\right)$ (16)

The probability of conducting inspection at the intervention time $t_1, t_2, \cdots, t_n$ is conditional on that the structural system has survived at the intervention times as per Equations (17) to (19).

$pi^1(t_1) = 1 - pf^0(t_1)$ (17)
$pi^2(t_1) = 1 - pf^1(t_2)$ (18)
$pi^n(t_1) = 1 - pf^{n-1}(t_n)$ (19)

Formulations under the MS2 and MS5 can be obtained similarly based on Figures 4 and 6.

6.3 Fatigue reliability index

The reliability index corresponding to a failure probability is given by Equations (20) to (22).

For $0 \leq t \leq t_1$,
$\beta^0(t) = -\Phi^{-1}[pf^0(t)]$ (20)
where $\Phi^{-1}[\cdot]$ is the inverse function of standard normal cumulative density function.

For $t_1 < t \leq t_2$,
$\beta^1(t) = -\Phi^{-1}[pf^1(t)]$ (21)
For $t_n < t \leq T_{SL}$,
$\beta^n(t) = -\Phi^{-1}[pf^n(t)]$ (22)

It should be noted that structural failure probability is cumulative with time. So, the failure probability $pf^n(t)$ is the cumulative probability that a structural would fail within the time range (0, t), considering the influence of n maintenance interventions have been scheduled within the time range. Structural reliability is however decrement with time, due to cumulative failure probability. Typically, a reliability index is defined with respect to a time duration. So, the reliability index $\beta^n(t)$ is the reliability with respect to the time duration from the beginning of service time to time t, considering the influence of n maintenance interventions have been scheduled to the time duration. Lifetime fatigue reliability is the structural reliability against fatigue failure with respect to the required service life T_{SL}.

As can be seen from Figure 4-6, the branches in a decision tree increase exponentially with the

number of scheduled intervention n. Thus, the computations of failure probability and reliability index become more complex with an a larger n. However, these computations can be efficiently done by most commercial probabilistic analysis software, e.g. STRUREL, PROBAN, etc.

6.4 Expected lifetime costs

Cost-optimum maintenance planning and decision-making under uncertainty have to be performed with the aid of a probabilistic theoretical framework, which can quantify and assess the benefits and costs of future maintenance actions, taking the various sources of uncertainties affecting fatigue crack growth and inspection quality into account. Based on this framework, different maintenance strategies can be compared in a consistent way before they are implemented in order to select the most financially-efficient strategy.

The probabilistic framework uses life cycle analysis and risk assessment to formulate the expected lifetime costs C_l, which include the expected costs of maintenance interventions (inspection costs C_i, and repair costs C_r) and the expected costs of failure C_f, as given by Equation (24). It should be noted that optimum maintenance planning is addressed here for existing structural systems. The design parameters are assumed to be the same for all maintenance strategies, therefore, the design costs are also supposed to be the same and thus are not included in the C_l. Although the design process is beyond the scope of this paper, it would be possible to optimize the design parameters together with the maintenance strategy by incorporating a formulation for design costs in the C_l (Gomes and Beck 2014).

$$C_l = C_i + C_r + C_f \quad (23)$$

The expected failure costs, C_f, are defined as the product of failure probability caused by fatigue p_f and financial consequence of the failure c_{f0}, as given by Equation (24). The expected failure costs can be understood as failure risk quantified in terms of financial loss, i.e,. the potential financial loss as a result of the as-built conditions, damage development and maintenance actions of a structural system.

$$C_f = p_f \cdot c_{f0} \quad (24)$$

Maintenance actions contribute to reduction of failure probability and mitigation of failure risk, at a cost. The extent of risk mitigation and the expenses are dependent on the specific maintenance strategy, and are functions of scheduled inspection times, inspection methods, repair criteria and repair effect, for condition-based maintenance. The expected inspection costs and repair costs are given by Equation (25) and (26) respectively. Expected life time costs can be calculated by adding together the expected costs of inspection and repair associated with all maintenance interventions, as adopted by (Beaurepaire et al. 2012, Valdebenito and Schuëller 2010, Kim and Frangopol 2010). Alternatively, the expected lifetime costs can be obtained by Equation (27), where the expected costs of inspections and repairs associated with all branches in Figure 5 are added together, as adopted by (Soliman, Frangopol, and Mondoro 2016).

$C_i = \sum_{k=1}^{n} P_i^k \cdot c_{i0}^k \cdot \frac{1}{(1+r)^{t_i^k}}$ (25)

$C_r = \sum_{k=1}^{n} P_r^k \cdot c_{r0}^k \cdot \frac{1}{(1+r)^{t_r^k}}$ (26)

$C_l = \sum_{k=1}^{N_b} P(B_k) \cdot C_k + C_f$ (27)

where n is the number of interventions in the service life; c_{i0}^k and c_{r0}^k are costs for the kth inspection and repair respectively; P_i^k and P_r^k are the probability of the kth inspection and repair are carried out; t_i^k and t_r^k are the times of the kth inspection and repair; r is average annual discount rate of money; C_k is the probability of occurrence of branch k, and $P(B_k)$ is the expected total costs associated with branch k.

In life cycle cost analysis, the input parameters are n, c_{f0}, c_{i0}^k, c_{r0}^k and r. The specific maintenance plans, e.g. inspection times, adopted inspection methods, repair criteria and the repair methods, determine the probability of inspection and repair at each maintenance intervention and the lifetime failure probability, and thus determines the expected costs of inspections, repairs and failure. The exact values of c_{f0}, c_{i0}^k and c_{r0}^k are not required, as knowing their ratios is enough for deriving the cost-optimum maintenance plans. If the same inspection method and repair method were applied to all interventions, the costs for all inspections and repairs are the same, i.e., $c_{i0}^k = c_{i0}$, $c_{r0}^k = c_{r0}$. In this paper, the cost of a repair c_{r0}, and the financial costs of failure c_{f0} are defined with respect to the costs of an inspection c_{i0}, i.e., the cost ratios c_{r0}/c_{i0}=100 and c_{f0}/c_{i0}=1000. The unit of the obtained lifetime costs are the costs of an inspection.

6.5 Probabilistic maintenance optimization approach

The chart in Figure 7 and 8 shows the flow of the probabilistic maintenance optimization approach for reliability-based and cost-based optimizations. The optimization approach needs to take probabilistic fatigue crack modelling, probabilistic inspection modelling, reliability theory, decision tree analysis, cost modelling and life cycle analysis into account. The theoretical basis for the approach is a physical model for fatigue crack growth, that projects crack size over time probabilistically. Maintenance interventions are scheduled to renew the structural system upon damage condition, crack detection or just time, depending on the adopted maintenance strategy. The event margins for failure, detection and repair are formulated based on the crack size over time. The crack size and inspection result at an intervention are probabilistic and thus unknown at the planning stage, but the possible inspection results and repair actions can be obtained from decision tree analysis. The probabilities of inspection, repair and failure are formulated based on the corresponding event margin. Optimization objectives, e.g. lifetime fatigue reliability and expected lifetime costs, are formulated based on the probabilities of inspection, repair and failure, as well as cost ratios and annual discount rate of money. The reliability-based and cost-based optimum criteria and inspection times are derived for the corresponding optimization objective. The optimization problem can be solved numerically with the aid of certain optimization techniques, e.g. genetic algorithms (Kim, Soliman, and Frangopol 2013, Kim and Frangopol 2011) and subset simulation (Valdebenito and Schuëller 2010, Beaurepaire et al. 2012). Alternatively, with the increasing capacity of computational power, an exhaustive search

algorithm can be used if the number of potential solutions is not very substantial. Practical constraints can also be applied to limiting the possible solutions. For example, from the perspective of logistics and management, it may be more convenient to adopt an equal inspection interval, e.g. to carry out inspections at equal-distant times, as adopted by (Zio and Compare 2013, Huynh, Grall, and Bérenguer 2017, Scarf, Wang, and Laycock 1996, Mendes, Coit, and Ribeiro 2014, Cronvall et al. 2012, Zou et al. 2018, Tolentino and Ruiz 2014). With this constraint, the possible solutions are greatly reduced. Based on the probabilistic optimization approach, a reliability-based repair criterion $a_{r,opt}^{\beta_l}$ can be obtained, as well as a cost-optimum criterion $a_{r,opt}^{C_l}$. Next, the above approach is applied to a typical fatigue-prone structural detail of ship structures to determine the recommended range $[a_{r,opt}^{\beta_l},\ a_{r,opt}^{C_l}]$ for repair criterion.

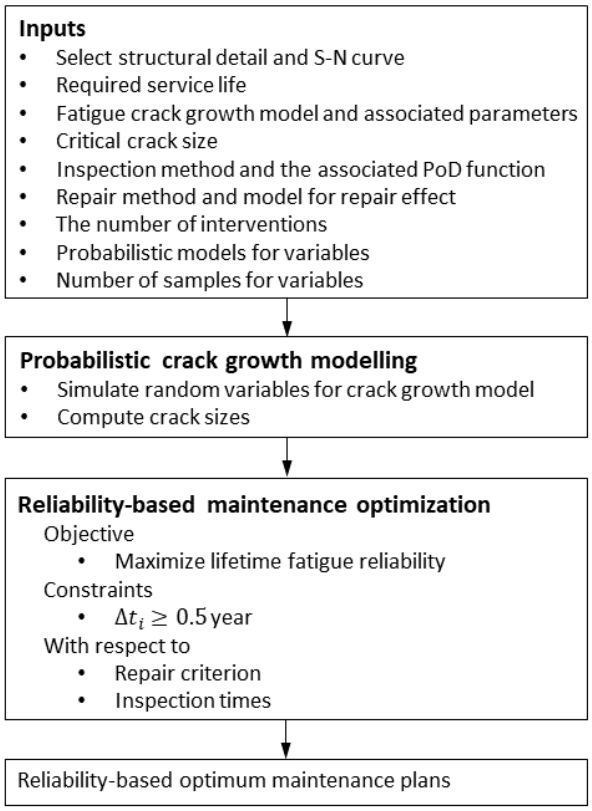

Figure 7: Illustration of probabilistic maintenance optimization approach (reliability-based)

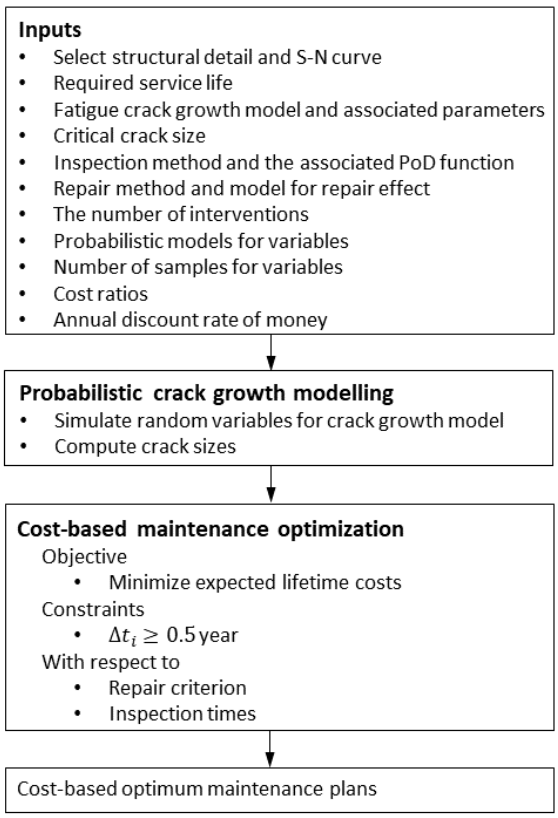

Figure 8: Illustration of probabilistic maintenance optimization approach (cost-based)

7. An illustrative example

A stiffened plate (Figure 9) of a ship structure is chosen to test the proposed probabilistic approach. Ship structures typically comprise of a substantial number of plates, stiffeners, pillars, and welding joints. One of the most common fatigue-prone details in ship structures is the welded T joint, which is also common in other steel structures, e.g. offshore installations, bridge decks, and wind turbine foundations, etc. Although the stability of the plate is greatly improved by stiffeners, fatigue

performance of the welded joints is likely to be a problem due to initial flaws introduced during welding processes. Welded joints represent weak areas in structural integrity. Under fatigue loading, failure can occur under a stress level much lower than material tensile strength and eventually lead to rupture of the entire cross-section. In order to prevent the latter, integrity and reliability of welded joints should be examined, assessed and maintained during the operational stage.

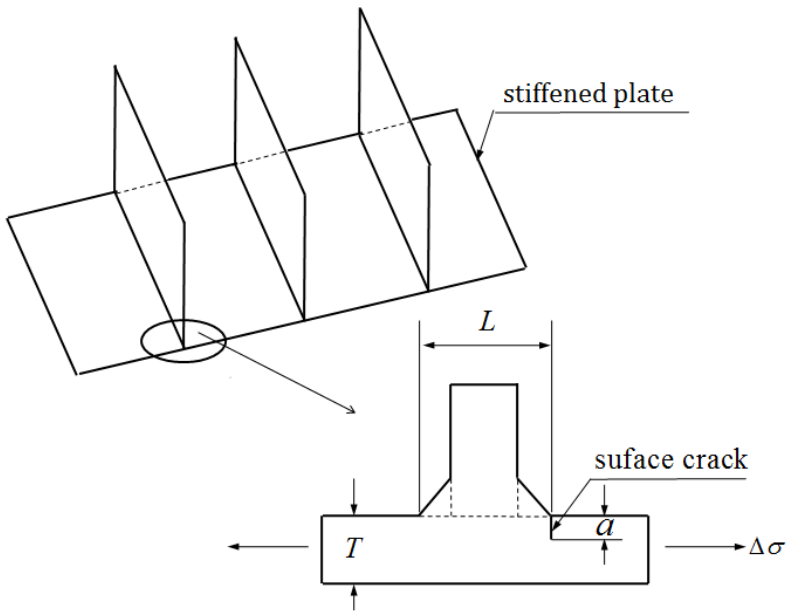

Figure 9: The studied fatigue-prone structural detail

7.1 Probabilistic model

The required service life of the ship is 20 years. The ship is trading in the sea environment, in which the frequency of wave loading is about 0.16Hz, which corresponds to approximately 5×10^6 cycles per year (Lotsberg et al. 2016, DNVGL 2015). The fatigue resistance of the joints is classified as F class and given by a two-segments S-N curve, formulated by Equation (3). Values for the parameters of the S-N curve obtain from rules of ship classification societies e.g. (DNV 2014). In consideration of inspection accessibility and failure consequences, the joints have been designed with a fatigue design factor $(FDF = 8)$, and the allowed maximal equivalent stress range is $\sigma_e = 17.28\text{MPa}$. The design plate thickness is $T = 25\text{mm}$. Failure of the structural detail is defined as occurrence of through-thickness crack. A summary for all parameters is provided in Table 2.

Table 2: Design Parameters for the structural detail

Parameter	Unit	Value
T_{SL}	Year	20
N_0	Cycle	5×10^6
$\log_{10} \overline{a_1}$	[N, mm]	11.855
$\log_{10} \overline{a_2}$	[N, mm]	15.091
T	mm	25
σ_e	MPa	17.28
m_1	-	3

m_2	-	5
a_c	mm	25
c_{r0}/c_{i0}	-	100
c_{f0}/c_{i0}	-	1000

The main sources of uncertainties associated with crack growth modelling originate from uncertainties in the initial crack size a_0, the crack growth rate C and calculation of the stress range $\Delta\sigma$. Meaningful statistical data on a_0 is difficult to obtain for specific application due to measuring and sampling limitations. Here it is assumed that a_0 follows an exponential distribution and the mean value $E(a_0) = 0.50$ (DNV 2014). The uncertainties associated with calculation of $\Delta\sigma$ originate from load description, the method used for calculation of structural response, stress concentration factor and the effect of welding notch, etc. In this paper, the uncertainties associated with calculation of $\Delta\sigma$ are modelled as an additional variable B, which follows a normal distribution with mean value $E(B) = 1$ and standard deviation (SD) $\mu(B) = 0.30$ (Lassen and Recho 2015). Although affected by many factors, the crack growth rate C is often thought to be a material property. In marine engineering, C is typically assumed to be lognormally distributed and m is equal to 3 (Lotsberg et al. 2016, DNVGL 2015). Magnetic particle inspection (MPI) is adopted herein to inspect the structural detail. Under MS4, uncertainties associated with the inspection method are considered in the maintenance planning and hence, inspection results provided by MPI are understood as stochastic and imperfect. Equation (6) gives the PoD function of MPI, where the mean detectable crack size is assumed to be $E(a_d) = 0.89$ mm based on (Dong and Frangopol 2016). The statistical distributions and descriptors for all variables are listed in Table 3.

Herein the uncertainties associated with a_0 and B are relatively high and thus the lifetime fatigue reliability would be low. Under such circumstances, the information provided by an inspection would be more valuable in terms of uncertainty reduction, and the differences in the results by reliability- and cost-based approaches are more pronounced. Monte Carlo simulations are carried out to calculate the probabilities and reliability indexes, with 5×10^6 samples for each variable. It is checked that a larger number of samples do not lead to much change in the results. An exhaustive search algorithm is used to obtain the optimal maintenance strategies. A restriction is applied that an inspection can only be carried out either at the beginning or in the mid of every year. The restriction reduces the number of feasible solutions and the optimal inspection times would be either integer year or number with 0.5 year.

Table 3: Variables and their characteristic values used in reliability analysis

Variable	Distribution	Unit	Mean (E)	SD (μ)
a_0	Exponential	mm	0.50	0.50
$\log_{10} C$	Normal	[N, mm]	-12.74	0.11
B	Normal	-	1.00	0.30
a_d	Exponential	mm	0.89	0.89

7.2 The influence of inspection uncertainty

Under MS3, inspection results are assumed to be perfect and the detectability of MPI is understood

in a deterministic way: cracks smaller than $E(a_d)$ will not be detected while cracks equal to or larger than $E(a_d)$ will be detected perfectly, i.e., as modelled by the simplified PoD function given by Equation (7). Figure 10 shows the influence of uncertainty associated with the adopted MPI by plotting the lifetime fatigue reliability indexes, β_l versus scheduled inspection time for one maintenance intervention under MS3 (dashed lines) and MS4 (solid line). Four repair criteria a_r are considered for each maintenance strategy.

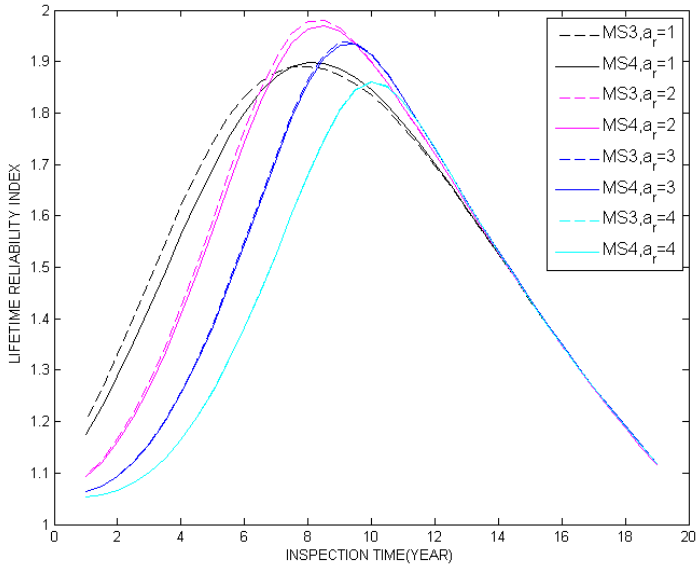

Figure 10: The influence of inspection uncertainty (lifetime fatigue reliability index against inspection time under MS3 and MS4 respectively, with different repair criteria)

The figure shows that the β_l under MS3 is almost the same as under MS4 when the repair criterion $a_r \geq 3$ mm. This is attributed to cracks larger than 3 mm having a size that will be detected with a very high probability regardless the uncertainty associated with the inspection method. In other words, when the repair criterion is set to be larger than 3 mm, a crack that is supposed to be repaired is almost unlikely to be missed by the inspection. Thus, the probability of repair is hardly affected by the inspection uncertainty, so as the β_l. The figure also shows that when the repair criterion is set to be very small, e.g. $a_r \leq 2$ mm, the β_l under MS3 is slightly larger than under MS4 if the inspection is scheduled at the earlier stage of service life, e.g. $t_1/T_{SL} < 0.4$ (less than 8 years) and almost the same as under MS4 if the inspection is scheduled at the later stage of service life, e.g. $t_1/T_{SL} > 0.6$ (more than 12 years). For a very small repair criterion a_r, e.g. $a_r \leq 2$ mm, under MS4 there is a possibility that a crack is larger than the a_r, but undetected by the inspection and then not repaired due to inspection uncertainty, i.e., missed repair, in comparison to MS3, under which the crack would be detected and repaired. The probability of missed repair $P(MR)$ under MS4 is formulated by Equation (28). It should be noted that a_d is a variable under MS4, while under MS3, it is assumed to be constant and equal to $E(a_d)$.

$$P(MR) = P\left(a_{t_i} \geq a_r \cap a_{t_i} < a_d\right) \quad (28)$$

If the inspection is scheduled at the earlier stage of service life, a missed repair due to inspection uncertainty under MS4 would lead to an increase in lifetime failure probability and accordingly a

decrease in β_l, in comparison to the case that repair would be carried out under MS3. However, the possibility of missed repair $P(MR)$ at the earlier stage of service life under MS4 is considered to be small, as the probability of exceedance of repair criterion, i.e., $P(a_{t_1} \geq a_r)$, is also small. The β_l under MS4 is just slightly lower than under MS3 if the inspection is scheduled at the earlier stage of service life. Detailed results for $t_1 = 3$ years and a_r= 2 mm can be found in Table 4.

A missed repair due to inspection uncertainty under MS4 may not lead to decrease in β_l if the inspection is scheduled at the later stage of service life. The latter can be explained by the approximately zero value of the failure probability conditional on missed repair at the later stage, as formulated by Equation (29).

$$P(F|MR) = \frac{P(F \cap MR)}{P(MR)} = \frac{P\left(a(t_c) > a_c \cap a_{t_i} \geq a_r \cap a_{t_i} < a_d\right)}{P\left(a_{t_i} \geq a_r \cap a_{t_i} < a_d\right)} \quad (29)$$

Missed repair would be repaired under MS3. The failure probability conditional on detection and repair at the later stage, as formulated by Equation (30), is also approximately zero.

$$P\big(F|(D \cap R)\big) = P(a(t_c - t_i) > a_c) \quad (30)$$

For the reasons above, the fatigue reliability under MS4 is approximately the same as that under MS3 if the inspection is scheduled at the later stage of service life. Table 4 provides specific values found for $t_1 = 15$ year and a_r= 2 mm.

In conclusion, the influence of inspection uncertainty in MPI on the β_l is marginal. When the repair criterion is set to be larger than 3 mm, missed repair is almost unlikely. Missed repair could occur when the repair criterion is set to be very small, e.g. smaller than 2 mm. The effect of a missed repair on β_l can be either marginal if an inspection is scheduled at the earlier stage of service life (i.e., very small probability of missed repair) or negligible if scheduled at the later stage of service life. Therefore, the condition-based maintenance strategy will be focused on MS3 from this point onwards.

Table 4: Probability of missed repair under MS4 with a_r=2 mm and its influence on lifetime fatigue reliability index at different inspection times

Notation	Meaning	Value	
		t_1 =3 year	t_1 =15 year
$P(a_{t_i} \geq a_r)$	Probability of exceedance of repair criterion	0.091	0.520
$P(MR)$	Probability of missed repair	0.0046	0.0089
$P(F \cap MR)$	Probability of missed repair and failure	0.0024	0
$P(F\|MR)$	Probability of failure conditional on missed repair	0.522	0
β_{l4}	Lifetime reliability index (MS4)	1.269	1.436
β_{l3}	Lifetime reliability index (MS3)	1.279	1.436

7.3 Optimum planning for one intervention

7.3.1 The optimum repair criterion for a specific inspection time

The repair criteria a_r under MS3 are optimized for a plan of inspections scheduled at specific equal-distant times. Three inspection times at the early, middle and late stage of service life are investigated, i.e. t_1 =5, 10 and 15 year (Table 5).

Table 5: The studied maintenance intervention times

t_1 (year)	5	10	15
t_1/T_{SL}	0.25	0.5	0.75

Figures 11, 12 and 13 give the lifetime fatigue reliability indexes β_l against repair (crack size) criteria a_r when inspections are scheduled at t_1/T_{SL} =0.25, 0.5 and 0.75 respectively. The figures generally show that the β_l increases with a smaller a_r up to an optimum $a_r^{\beta_l}$. However, the β_l decreases (Figures 11 and 12 when $t_1/T_{SL} = 0.25$ and 0.5) or remains the same (Figure 13 when $t_1/T_{SL} = 0.75$) if a_r is reduced beyond the optimum $a_r^{\beta_l}$. The finding indicates that a repair criterion a_r smaller than $a_r^{\beta_l}$ would lead to ineffective repair in terms of β_l. In this regard, there is a reliability-optimum repair criterion $a_r^{\beta_l}$, and it is not appropriate to say, 'the smaller the repair (crack size) criterion, the higher the lifetime fatigue reliability'. This finding is counter-intuitive, and the cause of this effect is explained in detail in the below paragraphs

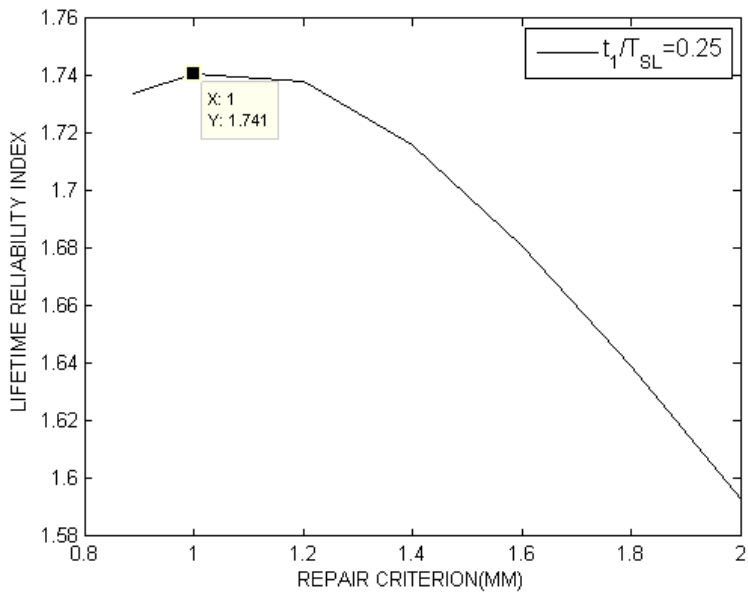

Figure 11 Lifetime fatigue reliability index against repair criterion ($t_1/T_{SL} = 0.25$)

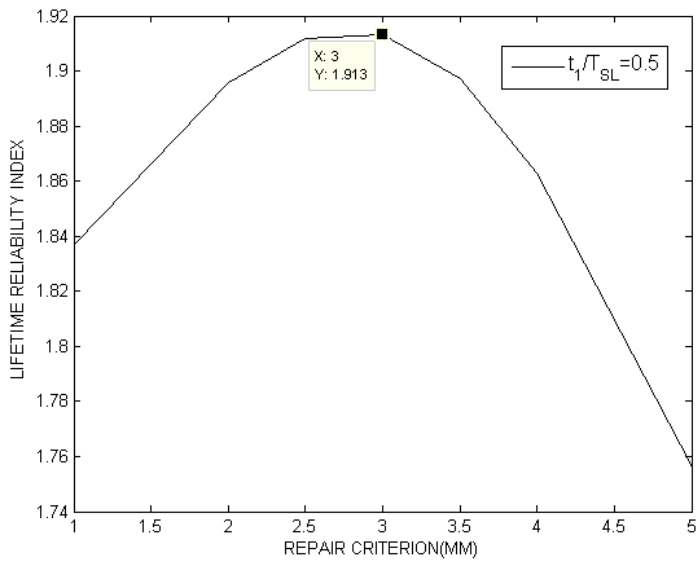

Figure 12 Lifetime fatigue reliability index against repair criterion ($t_1/T_{SL} = 0.5$)

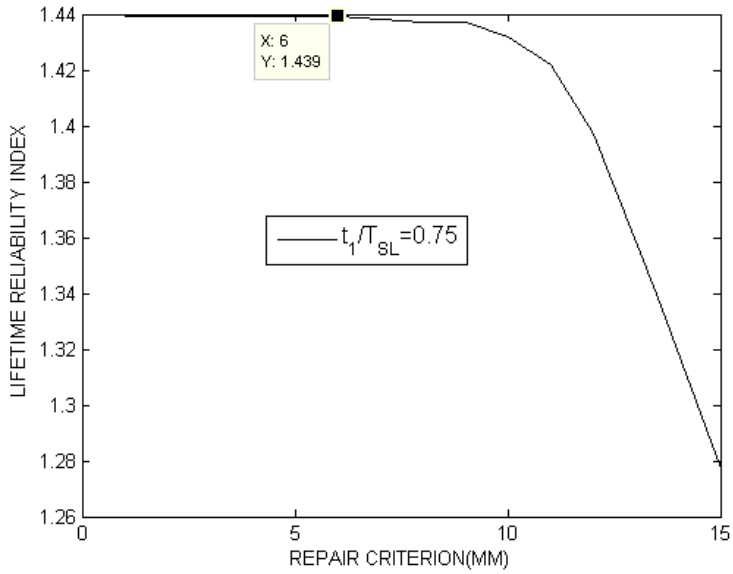

Figure 13 Lifetime fatigue reliability index against repair criterion ($t_1/T_{SL} = 0.75$)

The cause of ineffective maintenance

The maintenance effects under MS5 (detection-based maintenance) are compared to MS3 (condition-based maintenance) are examined to explain the cause of ineffective maintenance. The scheduled inspection time in both strategies is $t_1 = 10$ years. The repair (crack size) criteria under MS3 is $a_r^{\beta_l} = 3$ mm, and under MS5 is $a_{r,e} = E(a_d) = 0.89$ mm (as have discussed in Section 3 and Section 7.1). As a result, the only difference between MS3 and MS5 would be in the maintenance method for cracks falling into (0.89 mm, 3 mm) at t_1: would be detected but not repaired (denoted by $D \cap \bar{R}$) under MS3 while would be detected and repaired (denoted by $D \cap R$) under MS5. As a result, the lifetime fatigue reliability and lifetime costs would be different (as shown by Table 6). The lifetime failure probability conditional on $D \cap \bar{R}$ at t_1, $P\left(F \middle| (D \cap \bar{R})\right)$, is formulated by Equation (31), while the lifetime failure

probability conditional on $D \cap R$ at t_1, $P(F|(D \cap R))$, was formulated by Equation (30). $P(F|(D \cap \bar{R}))$ is not applicable for MS5 and $P(F|(D \cap R))$ not application for MS3, due to different maintenance methods for cracks falling into (0.89 mm, 3 mm) at t_1. As explained in Section 6.4, the lifetime costs C^l are given as relative values and thus the unit of the obtained lifetime costs in Table 6 are the costs of an inspection by MPI.

$$P(F|(D \cap \bar{R})) = \frac{P(F \cap D \cap \bar{R})}{P(D \cap \bar{R})} = \frac{P\left(a(t_c) > a_c \cap a_{t_i} > a_d \cap a_{t_i} < a_r\right)}{P\left(a_{t_i} \geq a_d \cap a_{t_i} < a_r\right)} \quad (31)$$

Table 6: Maintenance effects of two repair criteria: the reliability-optimum and a criterion leading to ineffective maintenance (t_1=10 year)

Notation	Meaning	Value	
		MS3 ($a_r^{\beta_l}$=3 mm)	MS5 ($a_{r,e}$=0.89 mm)
$P(F\|(D \cap \bar{R}))$	Failure probability conditional on detection and no repair	0.0042	n/a
$P(F\|(D \cap R))$	Failure probability conditional on detection and repair	n/a	0.021
β^l	Lifetime fatigue reliability index	1.913	1.823
C^l	Expected lifetime costs	521.6	940.4
P_{re}	Probability of repair	0.242	0.600
C_R	Expected repair costs	242.4	599.5

Table 6 shows that for $t_1 = 10$ years, $P(F|(D \cap \bar{R}))$ under MS3 is much smaller than $P(F|(D \cap R))$ under MS5. The reason is that if a crack is detected but very small (<3 mm, in this example) at $t_1 = 10$ years, this information indicates slow crack growth rate and favourable representations of uncertain input variables and lower uncertainties, and thus the failure probability would be very low (0.0042, under MS3). However, if the small crack was repaired (as would be done by MS5), the damage would be mitigated but uncertainties in the input variables would not be reduced, and the failure probability (0.021, under MS3) can be higher than leave it unrepaired (0.0042, under MS5).

For a crack falling into (0.89 mm, 3 mm) at t_i, the MS3 with $a_r^{\beta_l} = 3$ mm results in higher β_l and lower C^l (Table 6), and thus represents a better maintenance method (i.e., 'no action') than the MS5 with a repair criterion (a_r=0.89 mm). In summary,

- The cause of ineffective maintenance is that under condition-based maintenance strategy, repair actions are dependent on the repair (crack size) criterion a_r. If a_r is not optimal and set to be too small, it would trigger repair actions for cracks which have been found (by the inspection) of small sizes, without utilizing the information of 'small'. This information is positive and could have been used to prove a lower failure probability than initial predicted. So, in order to avoid ineffective maintenance, it is recommended to utilize the positive inspection information in maintenance decision by optimizing the repair criterion a_r.
- Table 6 shows β_l, is only slightly lower when a repair criterion $a_r < a_r^{\beta_l}$ is adopted. However, it

is worth to note that a lower β_l indicates higher expected failure costs. Moreover, a smaller a_r indicates a higher probability of repair and higher expected repair costs. In total, the C_l due to an ineffective repair criterion can be much higher than the optimum criterion. In Table 6, the C_l (940.4) for $a_r = 0.89$ mm nearly doubles the C_l (521.6) for $a_r^{\beta_l} = 3$ mm.

- The β_l in Figure 13 remains constant when an ineffective repair criterion $a_r <$ 6 mm is adopted (the optimal a_r is thus equal to 6 mm for $t_1 = 15$ year), instead of decreasing as in Figures 11 and 12. The cause is that for $t_1 = 15$ year (near the end of service life), both $P\left(F\middle|(D \cap \bar{R})\right)$ and $P\left(F\middle|(D \cap R)\right)$ are approximately equal to zero. Hence, both maintenance methods (repair and 'no action') would have the same effect on β_l for a crack falling into [0.89 mm, 6 mm] at $t_1 = 15$ years. However, the C_l due to an ineffective repair criterion can be much higher than the optimum criterion, as mentioned above, due to higher expected repair costs.
- In conclusion, a repair criterion smaller than the reliability-optimum will lead to unbeneficial maintenance (decrease in lifetime fatigue reliability β_l and higher costs) when considering the failure probability of a repaired structural detail (Garbatov and Soares 2001, Eltaief et al. 2015, Huynh, Grall, and Bérenguer 2017, Kulkarni and Achenbach 2007, Madsen, Torhaug, and Cramer 1991, Sheils et al. 2010, Zitrou, Bedford, and Daneshkhah 2013), i.e. $P\left(F\middle|(D \cap \bar{R})\right)$, and it is not equal to zero. If perfect maintenance model is adopted and thus $P\left(F\middle|(D \cap \bar{R})\right)$ is assumed to be equal to zero (Straub 2004, Faber, Straub, and Goyet 2003, Straub and Faber 2005, Valdebenito and Schuëller 2010, Beaurepaire et al. 2012, Gomes and Beck 2014), a smaller repair criterion than the reliability-optimum will lead to ineffective maintenance (the same reliability β_l, but higher costs).
- It should be mentioned that herein the aim is to examine the effects of repair (crack size) criterion a_r on lifetime fatigue reliability β_l and costs C^l, on the basis of given number of scheduled maintenance interventions. In Section 7.4, the case of multiple interventions is studied, in which the effects of ineffective maintenance are more pronounced, as the positive information provided by multiple inspections would be more valuable, if utilized in maintenance decision. If a_r is fixed, lifetime fatigue reliability β_l is expected to be higher with a larger number of scheduled maintenance interventions n.

The optimum planning approach based on expected lifetime costs C_l is analysed here and compared to the optimum maintenance planning approach based on lifetime fatigue reliability β_l. Figures 14, 15 and 16 give the C_l for inspections scheduled at t_1 =5, 10 and 15 year respectively, against repair criteria a_r. These figures show that there is a cost-optimum repair criterion $a_r^{C_l}$ for each specific inspection time leading to a minimum C_{min}^l.

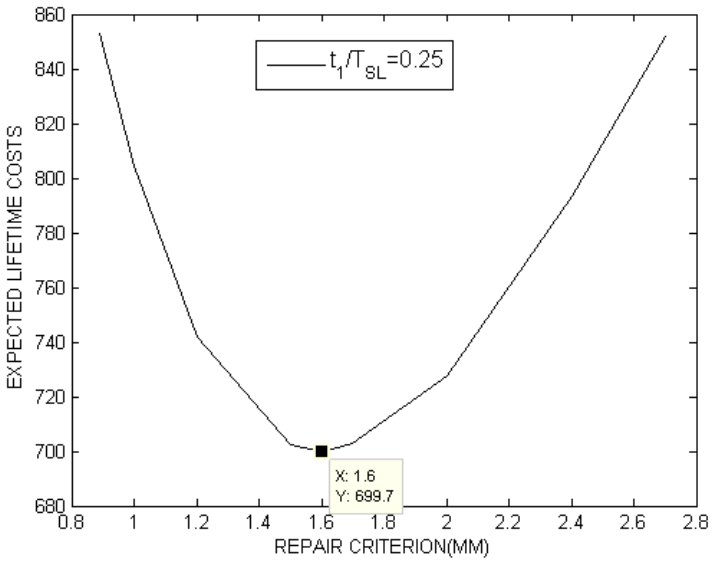

Figure 14 Expected lifetime costs against repair criterion ($t_1/T_{SL} = 0.25$)

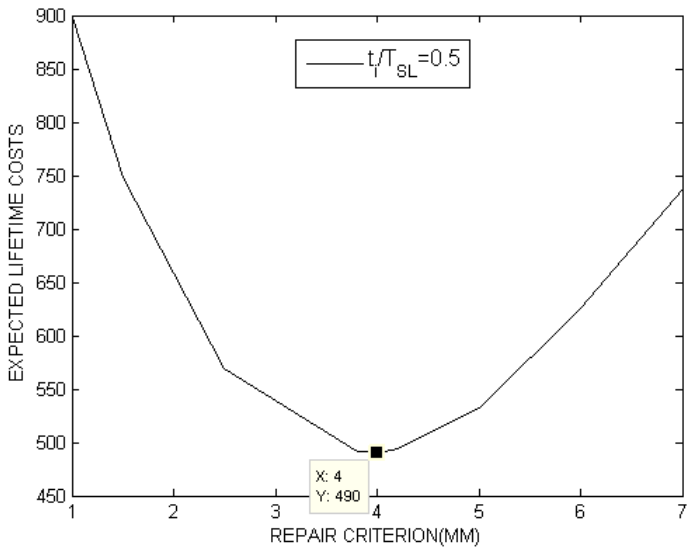

Figure 15 Expected lifetime costs against repair criterion ($t_1/T_{SL} = 0.5$)

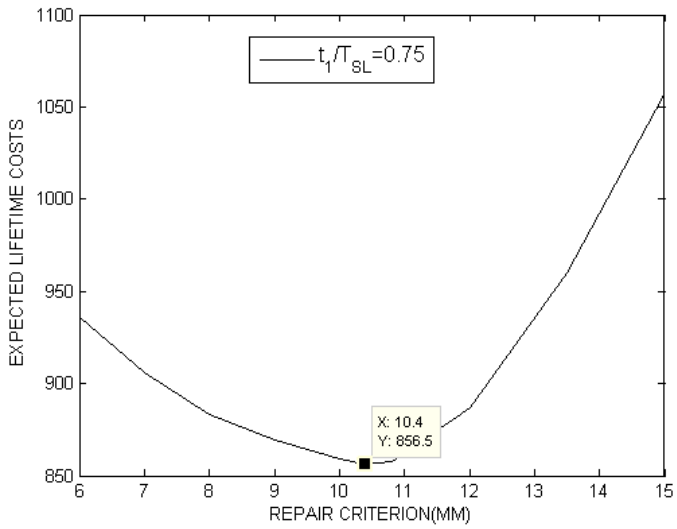

Figure 16 Expected lifetime costs against repair criterion ($t_1/T_{SL} = 0.75$)

Table 7: Optimization results with the two approaches (MS3)

	Reliability-based optimization			Cost-based optimization		
	a_r	β_{max}^l	C_l	a_r	C_{min}^l	β_l
T_1	$a_r = 1$	1.739	805.4	$a_r = 1.6$	699.7	1.681
T_2	$a_r = 3$	1.913	521.5	$a_r = 4$	490.0	1.862
T_3	$a_r = 6$	1.439	935.9	$a_r = 10.4$	856.5	1.429

Table 7 summarizes the derived optimal repair crack size criteria by reliability and cost-based optimization. Two main points are noted:

- For the three scenarios, the reliability-optimum criterion $a_r^{\beta_l}$ is smaller than the cost-optimum criterion $a_r^{C_l}$, i.e. $a_r^{\beta_l} < a_r^{C_l}$. Consequently, the optimum requirement is more difficult to meet on fatigue reliability than on lifetime costs.
- Both the optimum repair criteria $a_{r,opt}^{\beta_l}$ and $a_{r,opt}^{C_l}$ increase with the inspection time. In line with this trend, it is recommended to adopt larger repair criterion for inspections scheduled at later times, as opposed to the same repair criterion for all inspections.

The cause of insufficient maintenance

On one hand, a repair (crack size) criterion a_r should not be too small, to avoid ineffective maintenance. On the other hand, it should not be too large. A very larger a_r would make the repair criterion too strict to satisfy and the probability of repair would be very low, leading to insufficient utilization of maintenance. As a result, the lifetime fatigue reliability would be low and lifetime costs would be high. For example, when a repair criterion a_r larger than the cost-optimum $a_r^{C_l}$ is applied, i.e. $a_r > a_r^{C_l}$, according to Figure 14-16, the lifetime costs would be higher than when $a_r^{C_l}$ is applied. As $a_r^{\beta_l} < a_r^{C_l}$, then $a_r > a_r^{C_l} > a_r^{\beta_l}$, according to Figure 11-13, the lifetime fatigue reliability would be

lower than when $a_r^{C_l}$ is applied.

To further explain insufficient maintenance, the return of a maintenance strategy R_m, is defined by Equation (32). As an example, the maintenance effects of two strategies ($a_r^{C_l}$ = 4 mm and a_r =6 mm) for t_i = 10 years are shown in Table 8. The table shows that compared with $a_r^{C_l}$ = 4 mm, adopting the repair criterion $a_r = 6$ mm would result in a lower probability of repair, lower fatigue reliability, higher lifetime costs and less return of maintenance. So, insufficient maintenance refers to the case that a repair (crack size) criterion is too large, and the maximum return of maintenance would not be sufficiently reaped. In such case, more maintenance (by adopting a smaller, but larger than the reliability-optimum repair criterion,) would be more beneficial in terms of both lifetime fatigue reliability and costs.

$$R_m = \Delta C_f - C_m = C_{f0} - C_f - (C_i + C_r) \quad (32)$$

Table 8: Effects of two repair criteria: the cost-optimum and an insufficient repair criterion (t_1=10 year)

Notation	Meaning	Value	
		$a_r^{C_l}$ = 4 mm	$a_r =$ 6 mm
β	Reliability index	1.862	1.628
C_l	Lifetime costs	490.0	625.6
R_m	The maximum return of a maintenance strategy	976.7	841.4
P_{re}	Probability of repair	0.178	0.107
ΔC_f	Reduction of failure costs	1155.2	949.7
C_m	Maintenance costs	178.5	108.3

A range of optimum repair criterion

There are then two optimum repair criteria, i.e., in terms of lifetime fatigue reliability and in terms of lifetime costs. Depending on the specific target on fatigue reliability and constraint on lifetime costs, e.g. available budgets, it will possible to recommend a suitable range for the repair criterion [$a_r^{\beta_l}$, $a_r^{C_l}$].

For example, if the target was to maximize the fatigue reliability, then it is proposed to use the $a_r^{\beta_l}$. If the target was to minimize the expected lifetime costs and exact cost models are available, then $a_r^{C_l}$ is preferred. If the budget was higher than the min C_{min}^l, then a repair criterion $a_r^{\beta_l} < a_r < a_r^{C_l}$ can be adopted to increase the fatigue reliability. Table 9 summarizes the recommended range of repair criteria for maintenance interventions scheduled at t_1 =5, 10 and 15 year. The values of $a_r^{\beta_l}$ and $a_r^{C_l}$ define the ranges of the ineffective, optimum and insufficient maintenance.

It is worth to note that both the $a_r^{\beta_l}$ and $a_r^{C_l}$ increase with a longer inspection time t_1 (or a shorter

remaining service time) (Table 9). The reliability-optimum repair criterion $a_r^{\beta_l}$ depends on the stochastic nature of crack growth and the adopted inspection method, while the cost-optimum repair criterion $a_r^{C_l}$ depends on the stochastic nature of crack growth, the adopted inspection method and the cost models or ratios.

Table 9: The ranges of repair (crack size) criterion leading to ineffective, optimum and insufficient maintenances

	Ineffective maintenance $(a_d,\ a_r^{\beta_l})$	Optimum maintenance $[a_r^{\beta_l},\ a_r^{C_l}]$	Insufficient mainteance $(a_r^{C_l},\ T)$
$t_1/T_{SL} = 0.25$	(0.89, 1)	[1, 1.6]	(1.6, 25)
$t_1/T_{SL} = 0.5$	(0.89, 3)	[3, 4]	(4, 25)
$t_1/T_{SL} = 0.75$	(0.89, 6)	[6, 10.4]	(10.4, 25)

7.3.2 The optimum inspection time and repair criterion

Herein the repair (crack size) criterion a_r and inspection time t_1 are optimized for one maintenance intervention based on lifetime fatigue reliability β_l and expected lifetime costs C_l. For comparison purposes, the repair time t_r under MS2 and the inspection time t_1 under MS5 are also optimized with respect to β_l and C_l. The optimum results are shown in Table 10. Under MS3, the reliability-optimum takes place for $t_i^{\beta_l}$ = 8.5 years and $a_r^{\beta_l}$ = 2 mm, while the cost-optimum occurs for $t_i^{C_l}$ = 9 years and $a_r^{C_l}$ = 3.2 mm. The derived optimum times based on reliability and costs are approximately the same, i.e., near the middle of service life. The cost-optimum repair criterion is larger than the reliability-optimum, i.e. $a_r^{C_l} > a_r^{\beta_l}$. These results are in agreement with the conclusions of Section 7.3.1 regarding the relationship between the reliability-based and cost-optimum repair criterion for a specific inspection time.

Table 10: Optimization results with the two approaches

	Reliability-based optimization				Cost-based optimization			
	t_1 (or t_r)	a_r	β_{max}^l	C_l	t_1 (or t_r)	a_r	C_{min}^l	β_l
MS1	n/a	n/a	1.050	1470.1	n/a	n/a	1470.1	1.050
MS2	10	n/a	1.728	1398.5	10.5	n/a	1397.7	1.725
MS3	8.5	2	1.979	547.3	9	3.2	475.3	1.916
MS4	8.5	2	1.971	555.9	9	3.2	480.5	1.912
MS5	8	0.89	1.872	858.2	6.5	0.89	831.2	1.840

Table 10 shows that the lifetime fatigue reliability index β_l under MS3 with $a_r^{\beta_l}$ = 2 mm is higher than under MS2 and MS5, i.e., a condition-based maintenance strategy with a reliability-optimum repair criterion can help to achieve a higher β_l than the time-based and detection-based maintenance strategies. Results also reveal that the expected lifetime costs C_l under MS3 with $a_r^{C_l}$ = 3.2 mm are

significantly lower than under MS2 and MS5. I.e., a condition-based maintenance strategy with a cost-optimum repair criterion can help to achieve a lower C_l than the time-based and detection-based maintenance strategies.

Figures 17 and 18 give the lifetime fatigue reliability β_l and expected lifetime costs C_l respectively against intervention time (inspection or repair time) for different maintenance strategies. The cost-based approach tends to select the intervention time that will result in lower expected repair costs and lower probability of repair. It can be seen that under the condition-based and detection-based maintenance strategies (MS3 and MS5), the reliability-optimum inspection time takes place after the cost-optimum inspection time, since the probability of repair increases with the inspection time and thus $t_1^{\beta_l}$ tends to occur before after $t_1^{C_l}$. On the contrary, under the time-based maintenance strategy (MS2), the probability of repair decreases with the repair time and thus $t_1^{C_l}$ tends to occur after $t_1^{\beta_l}$, i.e., the reliability-optimum inspection time takes place slightly earlier than the cost-optimum inspection time.

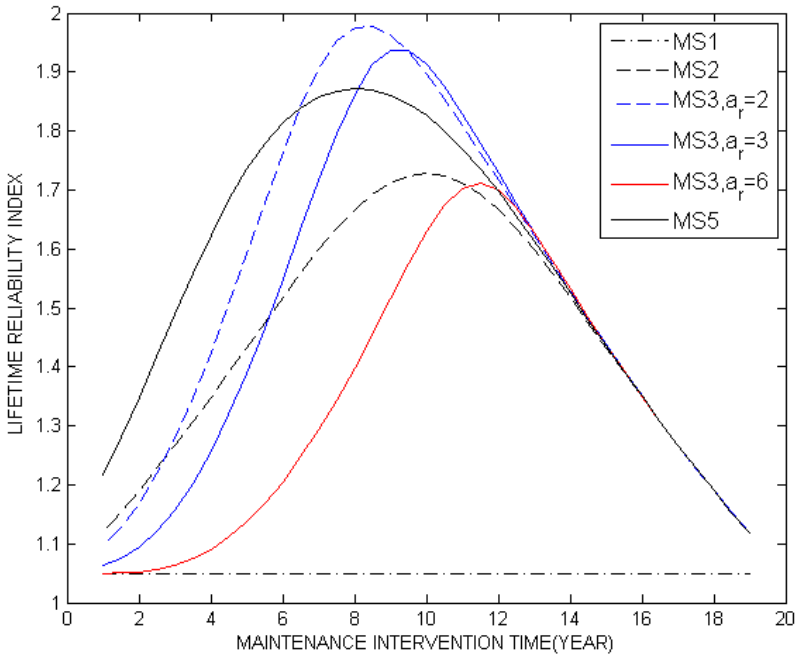

Figure 17: Lifetime fatigue reliability index against intervention time

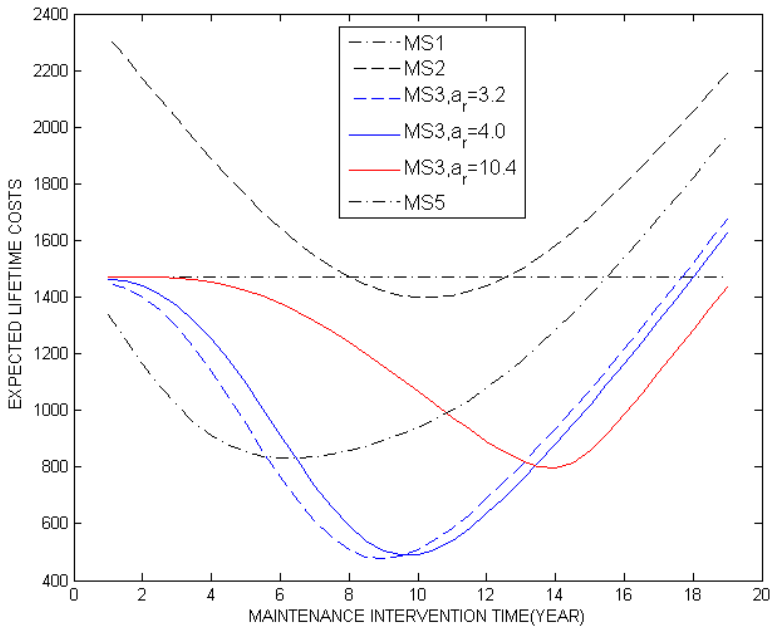

Figure18: Expected lifetime costs against intervention time

7.4 Optimum planning for multiple interventions

Tables 7 and 9 show that both the reliability-based and cost-optimum repair criteria $a_r^{\beta_l}$ and $a_r^{C_l}$ increase with the inspection time t_1. For multiple maintenance interventions, the use of the same repair criterion for all interventions in lifetime, as adopted by (Kim, Soliman, and Frangopol 2013, Soliman, Frangopol, and Mondoro 2016, Madsen, Torhaug, and Cramer 1991, Straub 2004, Gomes and Beck 2014), may not then be optimum in terms of lifetime fatigue reliability β_l and expected lifetime costs C_l. Alternatively, the implementation of larger repair criteria for inspections scheduled at later times appears to be more sensible.

In this section, two maintenance interventions are planned in lifetime and the repair criteria a_r are optimized with respect to β_l and C_l. A periodic inspection policy is applied (Zio and Compare 2013, Huynh, Grall, and Bérenguer 2017, Scarf, Wang, and Laycock 1996, Mendes, Coit, and Ribeiro 2014, Cronvall et al. 2012, Zou et al. 2018, Tolentino and Ruiz 2014) and the time intervals for interventions are $\Delta t = 6$ years. Following the approach proposed in Section 6, the derived optimum results are provided in Tables 11 and 12. Table 11 shows that when the same repair criterion is applied, i.e. $a_{r1} = a_{r2}$, the derived optimum repair criterion based on maximization of β_l is $a_r^{\beta_l}$ = 2.5 mm, which results in $\beta_{max} = 2.477$ and $C_l = 534.8$. However, when the repair criteria are not assumed to be the same, i.e. the assumption $a_{r1} = a_{r2}$ is removed, the derived optimum repair criteria based on maximization of β_l are $a_{r1}^{\beta_l}$= 2.4 mm and $a_{r2}^{\beta_l}$ = 3.6 mm, which result in $\beta_{max} = 2.488$ and $C_l = 365.8$. Table 12 shows that when the same repair criterion is applied, i.e. $a_{r1} = a_{r2}$, the derived optimum repair criterion based on minimization of C_l is $a_r^{C_l}$ = 4.8 mm, which results in $\beta = 2.261$ and $C_{l,min} = 392.3$. However, when the repair criteria are not assumed to be the same, i.e. the assumption $a_{r1} = a_{r2}$ is removed, the derived optimum repair criterion based on minimization of C_l are $a_{r1}^{C_l}$= 3.6 mm and

$a_{r2}^{C_l}$ = 7.9 mm, which result in $\beta_{max} = 2.335$ and $C_l = 287.5$. These optimum results verify that adopting different repair criteria for multiple maintenance interventions is more beneficial than adopting the same repair criterion for all interventions: the lifetime fatigue reliability β_l is higher and the expected lifetime costs C_l are much less. The optimum repair criterion, whether derived based on β_l or C_l, for a maintenance invention scheduled at later time is larger than that scheduled at earlier time, e.g. $a_{r2}^{\beta_l} > a_{r1}^{\beta_l}$ and $a_{r2}^{C_l} > a_{r1}^{C_l}$

Table 11: Comparison of reliability-optimum repair criteria for two maintenance interventions

	$a_r^{\beta_l}$	β_{max}	C_l	C_i	C_r	C_f
MS1	n/a	1.050	1470.1	n/a	n/a	1470.1
MS2	$a_{r1,} = a_{r2} = a_0$	2.262	1104.2	n/a	985.5	118.6
MS3	$a_{r1}^{\beta_l} = a_{r2}^{\beta_l} = 2.5$	2.477	534.8	2.0	466.6	66.2
	$a_{r1}^{\beta_l} = 2.4,\ a_{r2}^{\beta_l} = 3.6$	2.488	432.1	2.0	365.8	64.3
MS5	$a_{r1,} = a_{r2} = a_d$	2.392	1059.4	2.0	973.7	83.7

Table 12: Comparison of cost-optimum repair criteria for two maintenance interventions

	$a_r^{C_l}$	β	$C_{l,min}$	C_i	C_r	C_f
MS1	n/a	1.050	1470.1	n/a	n/a	1470.1
MS2	$a_{r1,} = a_{r2} = a_0$	2.262	1104.2	n/a	985.5	118.6
MS3	$a_{r1}^{C_l} = a_{r2}^{C_l} = 4.8$	2.261	392.3	2.0	271.6	118.7
	$a_{r1}^{C_l} = 3.6,\ a_{r2}^{C_l} = 7.9$	2.335	287.5	2.0	187.7	97.8
MS5	$a_{r1,} = a_{r2} = a_d$	2.392	1059.4	2.0	973.7	83.7

Tables 11 and 12 also provide the optimum results for MS2 and MS5. Table 11 shows that, although the equivalent repair criteria in MS2 and MS5 are smaller than MS3 and the probabilities of repair are higher, the resulting lifetime fatigue reliability β_l is lower than MS3 and the expected lifetime costs C_l are much higher than MS3. Again, the latter confirms that a repair criterion smaller than the reliability-optimum leads ineffective maintenance.

8 Conclusions

An efficient maintenance strategy is important for lifetime integrity, safety, reliability and cost management of structural systems with a substantial number of fatigue-prone details. This paper has proposed an integrated management method incorporating probabilistic crack growth, probabilistic inspection modelling and probabilistic planning approaches to derive an optimum range for the repair criterion in a condition-based maintenance strategy which meets requirements on lifetime fatigue reliability and expected lifetime costs simultaneously. While the repair criterion is almost exclusively

pre-determined or derived based on minimization of expected lifetime costs in existing studies on probabilistic maintenance optimization, herein it is emphasized that there is an optimum repair criterion based on maximization of lifetime fatigue reliability. A smaller repair criterion does not necessarily result in a higher lifetime fatigue reliability. A smaller repair criterion than the reliability-optimum can lead to a lower or the same lifetime fatigue reliability, but much higher lifetime costs, and thus should be avoided. The contribution of this paper can be summarized in the following points:

- An exponential probability of detection (PoD) function has been used to consider uncertainties associated with the detectability of magnetic particle inspection (MPI), in comparison with a piecewise function proposed as the PoD for perfect detectability. Both PoD functions have been employed in optimum maintenance planning. It has been found that the influence of inspection uncertainty in MPI on lifetime fatigue reliability is marginal. When the repair criterion is set to be large, e.g. $a_r > 3$ mm, a missed repair is unlikely. Missed repair could occur when a_r is very small, e.g. $a_r < 2$ mm. However, the influence of missed repair on lifetime fatigue reliability is limited if an inspection is scheduled at an early stage of service life. A missed repair can hardly affect lifetime fatigue reliability if an inspection is scheduled at a late stage of service life.
- The repair criterion in a condition-based strategy has been derived from maximization of lifetime fatigue reliability β_l, based on decision tree analysis, consideration of possible damage states, inspection results and repair actions at a specific intervention time. A smaller repair criterion than the reliability-optimum one (i.e., $a_r^{\beta_l}$) can lead to a lower or the same lifetime fatigue reliability, but much higher lifetime costs, and thus is consider as ineffective. The reliability-optimum repair criterion is dependent on the scheduled inspection time, e.g. the reliability-optimum repair criteria for inspections scheduled at later times are larger than that at earlier times.
- The repair criterion has also been derived based on the conventional approach of minimizing expected lifetime costs C_l, and the reliability-based and cost-optimum repair criterion for a condition-based maintenance strategy have been compared under the same conditions. It has been found that for a specific inspection time, the reliability-optimum criterion $a_r^{\beta_l}$ is smaller than the cost-optimum criterion $a_r^{C_l}$, which indicates that the reliability-optimum requirement is more difficult to fulfil as more repair efforts are required. When the same repair criterion is adopted, the reliability-optimum inspection time takes place at a later stage than the cost-optimum inspection time.
- The range $[a_r^{\beta_l},\ a_r^{C_l}]$ has been proposed for repair criterion. While a repair criterion smaller than $a_r^{\beta_l}$ has been considered as ineffective, a repair criterion larger than the $a_r^{C_l}$ has been considered as insufficient, given that the return of maintenance in terms of risk reduction is not fully reaped by adopting the criterion, although the return rate was higher. A suitable repair criterion can be selected from the proposed range accordingly to the specific target on fatigue reliability and constraint on expected lifetime costs, e.g. available budgets.
- The time-based and detection-based maintenance strategies have also been optimized based on maximization of β_l and minimization of C_l respectively, and the optimum strategies have been compared with the condition-based maintenance strategy. The results have shown that the condition-based strategy with a reliability-optimum repair criterion, e.g. $a_r^{\beta_l}$ = 2 mm in the

illustrative example, can help to achieve a higher β_l than the time-based and detection-based strategy at all intervention times. The time-based and detection-based strategies can be understood as condition-based strategy with very small repair criterion, and it is known that a smaller repair criterion that the reliability-optimum one can't lead to a higher β_l. A condition-based strategy with a cost-optimum repair criterion has helped to achieve a lower C_l than the time-based and detection-based strategies.

- If multiple maintenance interventions are scheduled, it is not optimum in terms of both β_l and C_l to use the same repair criterion for all maintenance interventions in lifetime. Both the reliability-based and cost-optimum repair criterion have increased with longer inspection times. Hence, the optimum repair criteria, whether based on β_l or C_l, for maintenance interventions scheduled at later times are expected to be larger than those scheduled at earlier times.

This paper has developed an integrated reliability and cost-based maintenance optimization method for one-component structural system (e.g. for one critical hotspot area) and reached some concisions on the reliability and cost-optimum maintenance (crack size) criterion, ineffective and insufficient maintenance and time-variant maintenance criteria for multiple maintenance interventions. The method and conclusions provide fundamental basis for developing optimal maintenance strategies for multi-component systems, e.g. multiple hotspots. A maintenance strategy for a one-component system can be applied to a multi-component system in which components are independent by defining a one-dimensional performance indicator of the system (Grall et al. 2002). Future work on system level maintenance planning needs to consider dependencies. These dependencies can be properly modelled and exploited to develop sampling inspection and opportunistic maintenance strategies, which are promising approaches to efficient allocation of maintenance resources among components in a multi-component structural system.

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