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Abstract—This paper develops a new technique for measuring the effect of computer usage on U.S. productivity growth. Standard National Income and Product Accounts (NIPA) measures of the computer capital stock, which are constructed by weighting past investments according to a schedule for economic depreciation (the rate at which capital loses value as it ages), are shown to be inappropriate for growth accounting because they do not capture the effect of a unit of computer capital on productivity. This is due to technological obsolescence: machines that are still productive are retired because they are no longer near the technological frontier, and anticipation of retirement affects economic depreciation. Using a model that incorporates obsolescence, alternative stocks are developed that imply a larger computer-usage effect. This effect, together with the direct effect of increased productivity in the computer-producing sector, accounted for the improvement in U.S. productivity growth over 1996–1998 relative to the previous twenty years.

I. Introduction

The 1990s saw an explosion in the application of computing technologies by U.S. businesses. Real business expenditures on computing equipment grew an average of 44% per year over 1992–1998 as plunging computer prices allowed firms to take advantage of ever more powerful hardware and, consequently, the ability to use increasingly sophisticated software.1 These investments were aimed at improving the efficiency of many core business functions such as quality control, communications, and inventory management, and the surge in computer investment did coincide with an improved productivity performance for the U.S. economy: private business output per hour grew 2.2% per year over the period 1996–1998, a rate of advance not seen late into an expansion since the 1960s. This raises the important question of whether these developments have provided a resolution to the now-famous Solow paradox that the influence of computers is seen everywhere except in the productivity statistics?

This paper addresses this question by focusing on two separate computer-related effects on aggregate productivity. First, there has been an enormous increase in the productivity of the computer-producing sector, a development that on its own contributes to increased aggregate productivity. Second, the resulting declines in computer prices have induced a huge increase in the stock of computing capital. Most of the paper is devoted to this latter (computer-usage) effect because it is here that the paper uses a new methodology. The computer-usage effect has been the subject of a number of previous studies using conventional growth-accounting methods, most notably the work of Stephen Oliner and Daniel Sichel (1994, 2000). These calculations are based on two steps—first, a stock of computer capital is calculated, and second, the effect of each unit of the stock on productivity is calculated. I develop a new growth-accounting methodology for computers that differs in its implementation of both steps.

The motivation for the new computer capital stocks developed in this paper is as follows. The National Income and Product Accounts (NIPA) capital stocks used in most growth-accounting exercises are constructed by weighting past investment according to a schedule for economic depreciation, which describes how a unit of capital loses value as it ages. However, in general, these so-called “wealth” stocks do not equal the “productive” stock appropriate for growth accounting. Take the example of a light bulb that is known to last at full power for exactly ten years. These light bulbs will lose value at a rate of about 10% per year as they approach expiration, so the wealth stock will be a weighted average of investments from the past ten years, with a weight of approximately 0.9 in on investment from years ago. In contrast, the productive stock will simply be the ten-year moving average of investment, and this will be greater than the wealth stock.

This same intuition also applies when considering the stock of computers. I show that the evidence on economic depreciation for computers suggests that, because of rapid technological change, most of the loss in value reflects the anticipation of technological obsolescence, which occurs when a machine is retired even though it retains productive capacity. In this case, the “lightbulb” logic holds, and the productive stock will be larger than the NIPA wealth stock. To illustrate this idea formally, I use an extension of Solow’s model of vintage capital, which incorporates endogenous obsolescence. The model relies on the assumption that the productive operation of computers requires additional support costs. It predicts that utilization of computers declines with age, and that computers are retired when the benefits from their operation no longer cover the support cost. The model is used to develop new (higher) estimates of productive computer capital stocks and to estimate the contribution of these stocks to aggregate productivity growth.

Because the new estimates imply that the fast-growing stock of computer capital is a more important component of capital input than previously thought, they also imply a larger contribution of computer capital to output growth.
fact, I show that higher computer usage, combined with the direct effect of increased productivity in the computer-producing sector, together accounted for all of the improvement in U.S. productivity growth over 1996–1998 relative to the previous twenty years.

The contents are as follows. Section II describes how the NIPA stocks for computing equipment are constructed, using the Solow vintage capital model to illustrate the conditions under which these wealth stocks can be identified with productive stocks. Section III looks at the evidence on computer depreciation and concludes that these conditions do not hold. Section IV develops the new theoretical framework, incorporating technological obsolescence. Sections V and VI present the empirical results on the role of computers in the recent acceleration in U.S. productivity, and section VII concludes.

II. Wealth and Productive Capital Stocks

A. The NIPA Capital Stocks

The capital stocks used in most growth-accounting exercises come from the U.S. NIPA, which are put together by the Commerce Department’s Bureau of Economic Analysis (BEA).

These stocks are constructed separately for a range of disaggregated types of equipment and structures by weighting past values for real investment according to a schedule for economic depreciation, which is the decline in the replacement value of a unit of capital (relative to the price of a new unit) that occurs as it ages. Evidence on economic depreciation is relatively hard to obtain, and the U.S. NIPAs rely heavily on cross-sectional studies of used asset prices, most notably those of Charles Hulten and Frank Wykoff (1981). When multiplied by the current investment price deflator, these stocks measure the current-dollar replacement value of the capital stock, and as such they are known as wealth stocks.

The conditions under which these real wealth stocks can be identified with the capital stock in the productive function are restrictive. For example, wealth and productive stocks are equal when all depreciation is due to physical decay and this decay occurs at a geometric rate. However, despite the restrictive conditions, it is generally assumed that wealth stocks provide a reasonable approximation to productive stocks, and so the NIPA stocks are regularly used in growth-accounting exercises.

I will argue, however, that this assumption does not hold for computers. To see why, we need to first consider how the NIPA stocks for computing equipment are calculated.

The NIPA procedures for calculating real wealth stocks for computing equipment differ from other types of equipment. The reason for this is that, since 1985, the real output of the computer industry has been measured on a “quality-adjusted” basis using hedonic price methods. That real investment in computing equipment is measured in quality-adjusted units has important implications for the calculation of wealth stocks. As Steve Oliner (1989) has demonstrated, once one is using quality-adjusted real investment data, then the construction of the real wealth stock cannot use an economic depreciation rate estimated for nonquality-adjusted units. The availability of superior machines at lower prices is one of the principal reasons that computers lose value as they age. However, once we have converted our real investment series to a constant-quality basis, to use a depreciation rate for nonquality-adjusted units would be to double-count the effect of quality improvements. Instead, Oliner (1989, 1994) proposed using the coefficient on age \( (t - v) \) from hedonic price regressions of the form

\[
\log (p_v(t)) = \beta_v + \theta \log (X_v) - \delta_e(t - v),
\]

where \( p_v(t) \) is the price at time \( t \) of a machine introduced at time \( v \), and \( X_v \) describes the features embodied in the machine. We will call \( \delta_e \) the quality-adjusted economic depreciation rate. Oliner’s depreciation schedules form the basis for the NIPA wealth stocks for computing equipment, and we will take a closer look at them in the next section.

B. Quality-Adjusted Stocks in the Solow Vintage Model

To illustrate the conditions under which the NIPA quality-adjusted real wealth stocks for computing equipment can be identified with their productive stock counterparts, we will use a slightly embellished version of Solow’s 1959 model of vintage capital.

There are two types of capital, one of which (computers) features embodied technological change and another (ordinary capital) which does not. Computers physically decay at rate \( \delta \), and the technology embodied in new computers improves each period at rate \( \gamma \), meaning that associated with each vintage of computers is a production function of the form

\[
Q_v(t) = A(t)L_v(t)^{\alpha(v)}K_v(t)^{\beta(v)}(I(v)e^{\gamma((t-\nu))})^{1-(\alpha(v)+\beta(v))},
\]

where \( I(v) \) is the number of computers purchased at time \( v \), \( L_v(t) \) and \( K_v(t) \) are the quantities of labor and other capital that work with computers of vintage \( v \) at time \( t \), and \( A(t) \) is disembodied technological change. The price of output and

---

2 See U.S. Department of Commerce (1999) for a full description of these data.
3 See Jorgenson (1973) for the general theory on the relationship between wealth and productive concepts of the capital stock. Hulten and Wykoff (1996) and Triplett (1996) are two recent papers that articulately explain the distinctions between physical decay and economic depreciation.
4 An exception is the multifactor productivity program of the Bureau of Labor Statistics (BLS), which uses productive stocks constructed according to a noneconomic “beta-decay” schedule that falls off to zero according to a specified service life. However, the BLS uses the economic depreciation rates underlying the NIPA wealth stocks to set its service life
5 Oliner (1989) labeled this a partial depreciation rate.
ordinary capital are assumed to be constant and equal to 1. The price of computers (without adjusting for the value of embodied features) changes at rate \( g \) \( (< \gamma) \). Finally, labor and capital are obtained from spot markets with the wage rate being \( w(t) \), a unit of ordinary capital renting at a price of \( r^o(t) \), and a unit of computer capital of vintage \( v \) renting at rate \( r_s(t) \).

**Defining the Aggregate Productive Stock:** The flow of profits obtained from operating computers of vintage \( v \) is

\[
\pi_v(t) = A(t) L_v(t)^{\alpha(t) \beta(t)} \left( I(v) e^\gamma e^{-\delta(t-v)} \right)^{1-\alpha(t)-\beta(t)}
- r_s(t) I(v) e^{-\delta(t-v)} - r^o(t) K_v(t) - w(t) L_v(t).
\]

Firms choose how much labor and ordinary capital should work with vintage \( v \) so as to maximize the profits generated by the vintage:

\[
L_v(t) = (I(v) e^\gamma e^{-\delta(t-v)} A(t)^{1/(1-\alpha(t)-\beta(t))}) \\
\times \left( \frac{\alpha(t)}{w(t)} \right)^{\alpha(t)/(1-\alpha(t)-\beta(t))} \\
\times \left( \frac{\beta(t)}{r^o(t)} \right)^{(1-\alpha(t))(1-\alpha(t)-\beta(t))}.
\]

\[
K_v(t) = (I(v) e^\gamma e^{-\delta(t-v)} A(t)^{1/(1-\alpha(t)-\beta(t))}) \\
\times \left( \frac{\alpha(t)}{w(t)} \right)^{\alpha(t)/(1-\alpha(t)-\beta(t))} \\
\times \left( \frac{\beta(t)}{r^o(t)} \right)^{(1-\alpha(t))(1-\alpha(t)-\beta(t))}.
\]

So, output from vintage \( v \) is

\[
Q_v(t) = (I(v) e^\gamma e^{-\delta(t-v)} A(t)^{1/(1-\alpha(t)-\beta(t))}) \\
\times \left( \frac{\alpha(t)}{w(t)} \right)^{\alpha(t)/(1-\alpha(t)-\beta(t))} \\
\times \left( \frac{\beta(t)}{r^o(t)} \right)^{(1-\alpha(t))(1-\alpha(t)-\beta(t))}.
\]

An elegant feature of this model is the fact that it can be neatly aggregated. Defining the aggregate productive stock of computing equipment as

\[
C(t) = \int_{-\infty}^{t} I(v) e^\gamma e^{-\delta(t-v)} dv
\]

and aggregating equations (4), (5), and (6) across vintages, we get

\[
L(t) = \int_{-\infty}^{t} L_v(t) dv = A(t)^{1/(1-\alpha(t)-\beta(t))} \\
\times \left( \frac{\alpha(t)}{w(t)} \right)^{(1-\beta(t))(1-\alpha(t)-\beta(t))} \\
\times \left( \frac{\beta(t)}{r^o(t)} \right)^{(1-\alpha(t))(1-\alpha(t)-\beta(t))}
\]

\[
K(t) = \int_{-\infty}^{t} K_v(t) dv = A(t)^{1/(1-\alpha(t)-\beta(t))} \\
\times \left( \frac{\alpha(t)}{w(t)} \right)^{(1-\beta(t))(1-\alpha(t)-\beta(t))} \\
\times \left( \frac{\beta(t)}{r^o(t)} \right)^{(1-\alpha(t))(1-\alpha(t)-\beta(t))}
\]

\[
Q(t) = A(t)^{1/(1-\alpha(t)-\beta(t))} \left( \frac{\alpha(t)}{w(t)} \right)^{\alpha(t)/(1-\alpha(t)-\beta(t))} \\
\times \left( \frac{\beta(t)}{r^o(t)} \right)^{(1-\alpha(t))(1-\alpha(t)-\beta(t))}.
\]

Rearranging this expression gives

\[
Q(t) = A(t) L(t)^{\alpha(t)} K(t)^{\beta(t)} C(t)^{1-\alpha(t)-\beta(t)}
\]

Aggregate output can be modelled using a Cobb-Douglas production function similar to that associated with each vintage, replacing the vintage-specific computer capital with an aggregate productive stock of computer capital, \( C(t) \).

**Economic Depreciation:** No arbitrage in the rental market implies that

\[
r_s(t) = \frac{\partial Q_v(t)}{\partial (I(v) e^{-\delta(t-v)})}
\]

and that the price of a new unit of computer capital is

\[
p_s(v) = \int_{v}^{\infty} r_s(s) e^{-(r+\delta)(t-v)} ds.
\]

Equation (6) implies \( (dr_s(s)) / (dv) = \gamma r_s(s) \): rental rates decline cross-sectionally with age at rate \( \gamma \). Differentiating the \( p_s(v) \) expression with respect to \( v \), using this cross-sectional pattern for rental rates, and rearranging gives the rental rate for new units:
Together, the information that the rental rate for new vintages changes at rate $g$ each period and that rental rates decline cross-sectionally at rate $\gamma$ with age implies that, for each specific vintage, the rental rate changes over time at rate $g - \gamma$, which is negative. The declining rental rate occurs because $L(v,t)$ and $K(v,t)$ fall over time: firms optimize profits by reallocating other factors to work with newer vintages of computers. We can use the change in rental rate over time to calculate the age-price schedule for computer capital:

$$p_v(t) = \int_t^\infty r_s(s)e^{-(r+\delta)(t-s)}ds = e^{r\delta}e^{-(\gamma+\delta)(t-v)}. \quad (13)$$

Thus, for each unit, the rate of economic depreciation is $\gamma + \delta$: computers decline in price as they age not only because of physical decay but also because the introduction of new and improved computing technologies implies falling rates of utilization.

**The Quality-Adjusted Wealth Stock:** Consider now the method used to construct the NIPA real wealth stock. By inserting the quality variable $X_v$ (such that $\theta \log (X_v) = \gamma v$) into the vintage asset price equation, we change the equation from

$$\log (p_v(t)) = gt - (\gamma + \delta)(t - v)$$

to

$$\log (p_v(t)) = (g - \gamma)t - \theta \log (X_v) - \delta(t - v).$$

Thus, adjusting for embodied features, the price index for new computers declines at rate $g - \gamma$ and, importantly, the quality-adjusted depreciation rate equals the rate of physical decay. The quality-adjusted real wealth stock for computers is

$$C_w^v(t) = \int_t^\infty I(v)e^{\gamma v}e^{-\delta(t-v)}dv = C(t). \quad (14)$$

In other words, the productive stock of computing equipment is identical to the quality-adjusted real wealth stock, a result that comes from the fact that the quality-adjusted economic depreciation rate equals the rate of physical decay. Thus, under these conditions, the NIPA real stocks for computing equipment, although intended as measures of wealth, can be used in aggregate productivity calculations. Unfortunately, though, it turns out that the evidence on computer depreciation is inconsistent with the Solow vintage model. To understand why, we need to examine the depreciation schedules underlying the NIPA stocks.

### III. Evidence on Computer Depreciation

The BEA has plenty of information on prices for new computers, and it bases its estimates of real computer investment on separate quality-adjusted price indexes for personal computers (PCs), mainframes, and other types of computing equipment. However, information on economic depreciation—how prices for computers change as the machines age—is much harder to come by. To construct its real wealth stocks of computing equipment, the BEA relies on research by Stephen Oliner (1989, 1994).

Oliner studied depreciation patterns for four categories of computing equipment: mainframes, storage devices, printers, and terminals. Figure 1 shows the quality-adjusted depreciation schedules from these studies that the BEA has used to construct the NIPA wealth stocks. Figure 2 shows the (negative of) the corresponding depreciation rates. Oliner found evidence that quality-adjusted economic depreciation rates had increased over time and so BEA applies different schedules to investment data from different vintages.

If the Solow vintage capital model is correct, these quality-adjusted economic depreciation schedules should correspond to the schedules for physical decay. However, these estimates do not appear to be measuring physical decay for computers. I will note three facts that seem inconsistent with a physical decay interpretation:

- With the exception of printers, the schedules show a marked nongeometric pattern, with depreciation rates increasing as the machines age. This contrasts with the results for other assets, for which geometric depreciation has proved a useful approximation.
- The downward shifts over time in these schedules seem inconsistent with a physical decay interpretation because one would expect that, if anything, computing equipment has probably become more reliable over time, not less.
- Most seriously, these numbers are simply too high to be physical decay rates. Table 1 shows the 1997 NIPA depreciation rates for all categories of equipment. The quality-adjusted depreciation rates from Oliner’s studies are higher than the depreciation rates for all other categories except cars. This is remarkable because, for all other types of equipment, the depreciation rates are not based on a quality-adjustment approach and so they combine the effects of both physical decay and embodied technological change. Casual observation suggests that it is unlikely that physical decay rates for computers are so much higher than for other types of equipment.

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6 Adding to the puzzle is the fact that Oliner’s studies focused on IBM equipment, which, at the time, was automatically sold with prepackaged service maintenance contracts: IBM guaranteed to repair or replace any damage due to equipment wear and tear. Thus, for the equipment in these
Together, these arguments indicate that the Solow vintage model does not appear to fit the evidence on computer depreciation. Next, we will present a simple extension to this model that will explain all three of these patterns. First, though, we need to point out an anomaly on table 1, which is the NIPA depreciation rate for PCs of (0.11); Oliner’s studies did not include PCs. BEA has acknowledged that this depreciation rate is anomalously low and intends to introduce new capital stock estimates for PCs that reflect depreciation rates closer to those used for the other computing categories.\footnote{See Moulton and Seskin (1999), p. 12.}

This section develops an extension of the Solow vintage model that can explain the evidence on computer depreciation. A new assumption is added, which is that the operation of a computer requires an additional support cost. This assumption is motivated by the fact that the basic vintage model is inconsistent with technological obsolescence as defined in the introduction. It predicts that firms will never choose to retire a machine that retains productive capacity. Rather, it suggests that the optimal strategy is simply to let the flow of income from a computer gradually erode over time. The existence of support costs motivates the

IV. Computing Support Costs and Endogenous Retirement

This section develops an extension of the Solow vintage model that can explain the evidence on computer depreciation. A new assumption is added, which is that the operation of a computer requires an additional support cost. This assumption is motivated by the fact that the basic vintage model is inconsistent with technological obsolescence as defined in the introduction. It predicts that firms will never choose to retire a machine that retains productive capacity. Rather, it suggests that the optimal strategy is simply to let the flow of income from a computer gradually erode over time. The existence of support costs motivates the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Depreciation Schedules for Computing Equipment}
\end{figure}

\begin{itemize}
\item Mainframes
\begin{itemize}
\item 1970
\item 1980
\end{itemize}
\item Storage Devices
\begin{itemize}
\item 1980
\item 1990
\end{itemize}
\item Printers
\begin{itemize}
\item 1980
\item 1990
\end{itemize}
\item Terminals
\begin{itemize}
\item 1980
\item 1990
\end{itemize}
\end{itemize}
phenomenon of technological obsolescence: once the marginal productivity of a machine falls below the support cost, the firm will choose to retire it. I show that this pattern of obsolescence can explain the evidence on economic depreciation for computers.

The support cost assumption is also motivated by empirical observation: computer systems are complex in nature and can be used successfully only in conjunction with technical support and maintenance. The explosion in demand in recent years for information technology (IT) positions such as PC network managers is a clear indication of the need to back up computer hardware investments with outlays on maintenance and support. Indeed, research by the Gartner Group (1999), a private consulting firm, shows that, as of 1998, for every $1 that firms spent on computer hardware there was another $2.30 spent on wages for IT employees and consultants. In addition, beyond these obvious support-like expenditures, the model's simple “support cost” variable should also be understood to represent the wide variety of complementary investments that go with the adoption of computer technologies, including expenditures on software and training, as well as the costs of introducing organizational change and new business practices (Brynjolfsson & Hitt, 2000).

Although the support cost formulation used here is a simple way to introduce technological obsolescence into the vintage capital model, it is not the only way to endogenize the retirement decision. For example, the putty-clay assumption of fixed ex post factor proportions, illustrated in the recent work of Gilchrist and Williams (2000), also leads
to endogenous retirement. I use the support cost formulation because the assumed lack of flexibility of the putty-clay model does not seem to capture the role of computing equipment in the production process very well. Putty-clay technology may be a reasonable assumption for an industrial plant, but it is less so for flexible technologies such as computing equipment because there is little that prevents firms from allocating less labor to work with old computing technologies after new and improved technologies are introduced. To give two examples, as personal computers grew in speed and user-friendliness, firms were able to gradually reallocate workers away from clunky mainframe-based computing systems towards newer Windows-based PC networks, and the emergence of cheap, high-quality laser printers allowed older inkjet-style printers to be used as backups. However, the important elements in the following analysis—in particular, the difference between wealth and productive stocks—would also hold true under a putty-clay model.

A. Theory

I use a very simple formulation of support costs: for each remaining computer from vintage \( v \), firms need to incur a support cost each period that is equal to a fraction, \( s \), of the original purchase price, \( p_v \). Thus, if the firm purchased the machine for $1,000 and \( s = 0.15 \), then the firm has to pay $150 per year to support it.\(^8\) The firm’s profit function can now be expressed as

\[
\pi_v(t) = A(t) L_v(t) \alpha(t) K_v(t)^{\beta(t)} I(v) e^{\gamma_v e^{-\delta(t-v)}} (1 - \alpha(t) - \beta(t)) - r_v(t) I(v) e^{-\delta(t-v)} - r_v(t) K_v(t) - w(t) L_v(t) - s p_v(v) I(v) e^{-\delta(t-v)} .
\]

(15)

How does the introduction of the support cost affect the model? First, note what has not changed. The additive support cost has no direct effect on the marginal productivity of the other factors that work with a vintage of computer capital. Thus, the first-order conditions for the allocation of labor and ordinary capital across vintages are unchanged, apart from one important new wrinkle. As before, declining utilization implies that the marginal productivity of a unit of computer capital falls over time at rate \( \gamma - g \). Now, though, instead of allowing the marginal productivity to gradually erode towards zero, once a computer reaches the age \( T \) where it cannot cover its support cost, it is considered obsolete and is retired.\(^9\) The expression for the aggregate computer capital stock is changed to

\[
C(t) = \int_{t-T}^t I(v) e^{\gamma_v e^{-\delta(t-v)}} dv,
\]

(16)

and, given this new expression, aggregate output can still be described by the aggregate Cobb-Douglas production function in equation (11). Figure 3 helps to tease out the implications of this pattern for economic depreciation. It shows the paths over time for the marginal productivity of a vintage of capital for a fixed set of values of \( r, \delta \), and \( \gamma - g \) and for two values of the support cost parameter: \( s = 0 \), in which case the model reduces to the Solow vintage model; and \( s = 0.07 \), which is shown as the horizontal line on the chart.\(^{10}\) Because firms now have to pay a support cost to operate the computer, the usual equality between the rental rate and the marginal productivity of capital needs to be amended to

\[
r_v(t) = \frac{\partial Q_v(t)}{\partial (I(v)e^{-\delta (t-v)})} - s p_v(v).
\]

(17)

For the purchase of a computer to be worthwhile, the present discounted value of these rents must still equal the purchase price:

---

\(^8\) One can certainly imagine alternative assumptions here. Support costs may increase over time, as old machines become less compatible with new software, or they may fall over time due to lower utilization. Experimentation with these alternative assumptions did not lead to much change in the empirical growth-accounting results.

\(^9\) Implicitly, I have assumed that the retired computers have zero scrap value. Adding a scrap value assumption does not change the nature of the model or the empirical results.

\(^{10}\) The parameter values for the figure are \( \gamma - g = 0.2 \), \( r = 0.03 \), and \( \delta = 0.09 \).
Thus, for a given purchase price, the marginal productivity of a unit of computer capital must be higher when there is a support cost: on the chart, the $s = 0.07$ marginal productivity schedule lies above the $s = 0$ schedule.

Consider now the path of the price of a computer as it ages. In terms of figure 3, this price is determined by the area above the support cost and below the marginal productivity curve. Importantly, as the machine ages, this area declines at a faster rate than does the marginal productivity of the computer, reaching zero at retirement age. Because this marginal productivity declines at rate $g - \gamma$ over time, this implies that the price of the computer falls over time at a faster rate than $g - \gamma - \delta$, and so the economic depreciation rate for computers is greater than $\delta + \gamma$.

The model is solved formally in appendix A. The retirement age $T$ is derived as the solution to the nonlinear equation

$$p_s(t) = \int_t^{t+T} \left( \frac{\partial Q_s(n)}{\partial (I(v))e^{-\delta(n-v)}} - sp_s(v) \right) \times e^{-r(n-t)}e^{-\delta(n-v)}dn.$$  \hspace{1cm} (18)

Although the solution to the equation will in general require numerical methods, one can show that it has the intuitive property that the faster the rate of quality-adjusted price decline for new computers, $\gamma - g$, and the higher the support cost, the shorter is the time to retirement.

Defining $\tau = t - v$, it can also be shown that the quality-adjusted economic depreciation schedule calculated from an Oliner-style study will be

$$d_s(t) = e^{-\delta T} \left[ 1 + \frac{s}{r + \delta} - \left( \frac{se^{-(r+\delta)T}}{r + \delta + \gamma - g} \right) \times \frac{\gamma - g}{r + \delta + e^{(r+\delta+\gamma-g)\tau}} \right] - e^{-(\delta+\gamma-g)\tau} \left( \frac{s}{r + \delta} \right) \left( 1 - e^{-(r+\delta)(\tau - \tau)} \right).$$  \hspace{1cm} (19)
This extension of the Solow vintage model (which we will call the obsolescence model) can explain the anomalies noted in our discussion of the evidence on computer depreciation:

- Nongeometric, quality-adjusted depreciation is an intuitive feature of the model: computers lose value at a faster pace as they approach retirement.
- The downward shifts over time in the quality-adjusted economic depreciation schedules are consistent with an increased pace of embodied technological progress, which fits with the apparent acceleration in technological change in the computer industry.
- The model explains why the estimated quality-adjusted economic depreciation rates are so high. Even if there were no physical decay, the expectation of early retirement would imply that computers still lose value as they age at a faster rate than the decline in quality-adjusted prices.

### B. Alternative Estimates of Productive Stocks

The obsolescence model tells us that the quality-adjusted depreciation rates used to construct the NIPA real wealth stocks for computing equipment will be higher than the corresponding rates of physical decay, and so—as in the lightbulb example cited in the introduction—the real wealth stocks will be smaller than the productive stocks.

The model also suggests an alternative strategy for estimating productive stocks for computing equipment. Given values for \( s \), \( \delta \), \( \gamma \), and \( g \), we can jointly simulate equations (19) and (20) to obtain both the retirement age and the schedule for quality-adjusted economic depreciation. Using the observed rate of quality-adjusted relative price decline to estimate \( \gamma - g \) and assuming a value for \( r \), we can obtain the values of \( s \) and \( \delta \) that are most consistent with the observed depreciation schedules. The estimated \( \delta \)'s can then be used to construct productive stocks.

Table 2 shows the estimated values of \( s \) and \( \delta \) obtained from this procedure for the four classes of computing equipment in Oliner’s studies.11 These values were based on the most-recent depreciation schedules for each type of equipment and were obtained from a grid search procedure to find the values giving the depreciation profiles that best fit Oliner’s schedules. The table shows that, for mainframes, storage devices, and terminals, the obsolescence model’s depreciation schedules fit far better than did any geometric alternative: root-mean-squared-errors of the predicted depreciation schedules relative to the observed schedules are far lower for the obsolescence model.

An exception to these patterns is printers, which, as seen in figure 2, show an approximately geometric shape for depreciation. It should be noted, though, that the prediction of nongeometric depreciation stemming from technological obsolescence requires the underlying pace of technological change to be fast (\( \gamma - g \) to be large), and this has not been true for printers. The rate of quality-adjusted price decline has been much lower for printers than for other types of computing equipment, and so the evidence on printer depreciation can still be interpreted as consistent with the obsolescence model.

For mainframes and terminals, the parameter combinations that fit best are those that have a physical decay rate of zero. The estimated values for the support cost parameters for mainframes and terminals (0.17 and 0.15) suggest a substantial additional expenditure, beyond the purchase price, over the lifetime of the computing equipment, but are low relative to what has been suggested by some studies, such as the Gartner Group research.

The estimated values of \( s \) and \( \delta \) imply a unique value of \( T \), which was used to fit the economic depreciation schedules. This value of \( T \) could also be used to calculate the productive stock for each type of equipment according to equation (16). We can do a little better, however. Although the model predicts that all machines of a specific vintage are retired on the same date, reality is never quite so simple: in practice, there is a distribution of retirement dates. Given a survival probability distribution, \( d(\tau) \), that declines with age, the appropriate expression for the productive stock needs to be changed from equation (16) to

\[
C(t) = \int_{-\infty}^{t} d(t - v)I(v)e^{\gamma v}e^{-\delta(t - v)}dv. \tag{21}
\]

This problem also needs to be confronted in the construction of economic depreciation schedules. If these schedules are constructed using only information on prices of assets of age \( \tau \), they will underestimate the average pace of depreciation: there is a “censoring” bias because we do not observe the price (equal to zero) for those assets that have already been retired. Hulten and Wykoff’s (1981) methodology corrects for this censoring problem by multiplying the value of machines of age \( \tau \) by the proportion of machines that remain in use up to this age. Oliner’s depreciation studies followed the same approach, and I have used his retirement distributions to construct estimates of productive stocks for computing equipment that are consistent with equation (21).

We do not have a schedule to fit for PCs. As described previously, the NIPA depreciation rate for PCs is far lower

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11 A value of \( r = 0.0675 \) was used. As explained in appendix B, this value was also used in the calibrations of the marginal productivity of capital in our growth-accounting exercises. The estimates of \( s \) and \( \delta \) were not sensitive to this choice.
than for the other categories of computing equipment. However, there is no evidence to support this assumption, and BEA intends to revise the NIPA stock for PCs to bring this category into line with the other types of computing equipment. As a result, I have chosen to treat depreciation for PCs symmetrically to mainframes, using the depreciation schedule applied by BEA for mainframes to construct a “NIPA-style” stock for PCs, and using identical schedules to derive the obsolescence model’s productive stocks for both PCs and mainframes.

Figure 4 shows the productive stocks implied by the obsolescence model and compares them with the NIPA real wealth stocks. Printers are not shown because we could not find evidence that the obsolescence model applied to this category. The low estimated rates of physical decay for the obsolescence model imply productive stocks that, in 1997 (the last year for which we have published NIPA stocks), ranged from 24% (for storage devices) to 72% (for mainframes) higher than their NIPA real wealth counterparts. The wide range in these ratios comes in part from the variation in the average age of these stocks: the NIPA stocks place far lower weights on old machines than do the alternative stocks, and the stock of mainframes contains more old investment than do the stock of storage devices. For PCs (by far the largest category in 1997), the obsolescence model implies a stock that is 44% larger than that implied by the NIPA-style stock.

We now consider the implications of these alternative productive stocks for the contribution of computer capital accumulation to aggregate output growth.
V. Calculating the Computer-Usage Effect

A. Methodology

Empirical growth accounting decomposes aggregate output growth into a weighted average of the growth in inputs and growth in total factor productivity,

\[
\frac{Q(t)}{Q(t)} = \frac{A(t)}{A(t)} - \alpha(t) \frac{L(t)}{L(t)} - \sum_{i=1}^{n} \beta_i(t) \frac{K_i(t)}{K(t)},
\]

(22)

where the weights correspond to the factor’s share in nominal income. Because labor’s share of income is an observable parameter, we can use this as a time series for \( \alpha(t) \). Although we cannot observe the actual payments of factor income to different types of capital, the standard implementation of empirical growth accounting follows Jorgenson and Griliches (1967) and uses theoretically based estimates of marginal productivities to calculate growth-accounting weights for each type of capital:

\[
\beta_i(t) = \frac{r_i(t)K_i(t)}{Q(t)}.
\]

(23)

The contribution to growth of accumulation of capital of type \( i \) is defined as

\[
\beta_i(t) \frac{K_i(t)}{K(t)}.
\]

We will present estimates of the contribution of computers to output growth using two different methods. The first is the traditional growth-accounting methodology, based on the Solow vintage model and using NIPA capital stocks for all categories. The second uses the obsolescence model for the marginal productivity and productive capital stock for computers, and the traditional methodology for all noncomputer assets.\(^{12}\) One question about this comparison is whether the second method should apply the obsolescence model to all assets, not just computers. After all, the phenomenon of capital support costs is not limited to just computers. However, as noted in our discussion of printers, for wealth and productive stocks to be notably different, we require not just the existence of support costs but also rapid technological change. In this sense, the phenomenon of technological obsolescence and its implied gap between productive and wealth stocks relate far more to computers than any other asset.

As explained in the following subsections, there are three differences between the calculation of the contribution of computer capital to growth under the Solow vintage and obsolescence models.

The Marginal Productivity of Computer Capital: The productive stock of computer capital is measured in quality-adjusted units. Thus, we need an estimate of the marginal productivity of a quality-adjusted unit. Letting \( q(t) = e^{r(t)} \) be the quality-adjusted computer price index, in the Solow vintage model, this is given by the traditional Jorgensonian rental rate:

\[
\bar{r}(t) = q(t) \left( r + \bar{\delta} - \frac{q(t)}{q_i(t)} \right).
\]

(24)

Appendix B derives the corresponding formula for the obsolescence model:

\[
\bar{r}(t) = q(t) \left[ \left( r + \bar{\delta} - \frac{q(t)}{q_i(t)} \right) + s \left( 1 - \frac{1 - e^{-rT}}{r + \bar{\delta}} \frac{q(t)}{q_i(t)} \right) \right].
\]

(25)

In addition to the terms featuring in the Jorgensonian rental rate (the net rate of return, physical decay, and capital gains), the marginal productivity of computer capital in the obsolescence model contains additional terms reflecting the fact that, to be profitable, computer investments must also compensate the firm for having to pay support costs and the need to retire the machine while still productive. These formulas show that support costs can be interpreted in two different, but compatible, ways. On the one hand, they can be viewed as costs, which require the need for a higher rate of return on the underlying investment (the computing equipment). On the other hand, one could view the whole package of outlays (hardware and support costs) as investments that make a normal, competitive rate of return.\(^{13}\)

Table 3 shows our estimates of \( \bar{r}(t) \) in 1997 for the empirical implementations of the Solow vintage and obsolescence models.

\[\begin{array}{cccccc}
\text{Mainframes} & \text{PCs} & \text{Storage} & \text{Printers} & \text{Terminals} \\
\hline
\text{Solow vintage model} & 0.61 & 0.64 & 0.31 & 0.62 & 0.39 \\
\text{Obsolescence model} & 0.67 & 0.76 & 0.20 & 0.62 & 0.33 \\
\end{array}\]

**Table 3.—The Marginal Productivity of a Unit of Quality-Adjusted Computer Capital**

\(^{12}\) Note that both methods use the same measure of output. Thus, they represent two different approaches to splitting aggregate labor productivity into a total factor productivity (TFP) effect and a capital-deepening effect, rather than two different approaches to measuring labor productivity.

\(^{13}\) Of course, this raises the controversial issue of whether these support costs should be counted in the national accounts as intangible investments, as suggested by Brynjolfsson and Hitt (2000) and Hall (2000). Given that there is no agreed way to construct such a measure, this paper uses the U.S. NIPA data as currently published.
obsolescence models. Whereas equations (24) and (25) tell us that, for a fixed value of \( \delta \), \( \tilde{r}(t) \) will be higher with a support cost and endogenous retirement, the two sets of estimates are actually quite close.\(^{14}\) This is because the models imply very different estimates of \( \delta \): the obsolescence model is consistent with (realistically) low values and the Solow model consistent with (unrealistically) high values. Essentially, because both models are calibrated off the observed high rates of economic depreciation, they agree that the marginal productivity of computer capital should be high. However, they arrive at this conclusion via different reasoning: the obsolescence model sees that firms need to be compensated for support costs and early retirement, whereas the Solow model sees that firms need to be compensated for high rates of physical decay.

\[
\text{Computer Capital Stock Growth Rates } \left( \frac{K_i(t)}{K_i(0)} \right):
\]

Perhaps surprisingly, these are almost identical under both the Solow vintage model (in which case we use the NIPA stocks) and the obsolescence model (in which case we use the alternative stocks). Although the levels of the alternative stocks are higher than the levels of the NIPA stocks, the growth rates in the 1990s are very similar.

**The Level of Computer Capital Stocks:** The final and most important difference between these two models in the calculation of the contribution to growth of computer capital accumulation is what we have already shown: that the levels of the stocks consistent with the obsolescence model are higher than the NIPA stocks consistent with the Solow vintage model. This results in a higher contribution to growth for the obsolescence model for a simple reason: although both models agree that the stock of computer capital is growing fast and has high marginal productivity, this cannot have much effect on aggregate output if this stock is too small.

**B. Results**

Our empirical implementation is for the U.S. private business sector, the output of which equals GDP minus output from government and nonprofit institutions and the imputed income from owner-occupied housing.\(^{15}\) Table 4 gives a summary for both models of the combined contributions to output growth of the five types of computer capital, and figure 5 gives a graphical illustration. The contributions generated by the Solow model are very similar to those reported by Oliner and Sichel (2000), who use the traditional growth-accounting methodology. Both models show relatively similar fluctuations over time for the contribution of computers to output growth, but the contributions from the obsolescence model are consistently about 50% higher than those from the Solow vintage model.

Both approaches agree that the contribution of computer capital accumulation picked up substantially over the latter part of the 1990s. The Solow vintage model sees this contribution (in percentage points) moving up from approximately 0.2 in the 1980s to 0.57 over 1996–1998. The obsolescence model sees this contribution going from 0.4 percentage points in the 1980s to 0.82 percentage points over 1996–1998, and both approaches imply that this contribution was accelerating rapidly in the late 1990s due to the consistent strength of real investment in computers. By 1998, the obsolescence model indicates that this contribution was worth almost a full percentage point for economic growth, 0.32 percentage points higher than for the Solow model.

These results provide an interesting counterpoint to Oliner and Sichel’s (1994) original resolution of the Solow paradox, in which they argued that, because the computer capital stock was relatively small, it could contribute only about two- or three-tenths of a percentage point per year to output growth. In an important quantitative sense, they argued that computers were really not “everywhere.” However, our estimates suggest that, by the late 1990s, the contribution of computers to output growth was more than three times as large as Oliner and Sichel had found in their earlier study. Our results imply that, although computers may not be everywhere, they are more important for productivity than the NIPA capital stocks suggest, and even the NIPA series are growing very rapidly.

### VI. Computers and Recent U.S. Productivity Developments

Our results have suggested that the substantial investments in computing technologies made by U.S. businesses in the late 1990s had an important influence on output growth, but we have not discussed the cause of this massive accumulation of computing power. The surge in computer investment has come as a direct result of rapid price declines, which themselves have been due to rapid productiv-

\(^{14}\) The values in table 3 use 1997-based prices. In other words, we set \( q(t) = 1 \) for each category.

\(^{15}\) Appendix B contains a detailed description of the empirical growth-accounting calculations.

<table>
<thead>
<tr>
<th>Year</th>
<th>Solow Vintage Model</th>
<th>Obsolescence Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970–1979</td>
<td>0.14</td>
<td>0.20</td>
</tr>
<tr>
<td>1980–1989</td>
<td>0.24</td>
<td>0.39</td>
</tr>
<tr>
<td>1990–1995</td>
<td>0.21</td>
<td>0.33</td>
</tr>
<tr>
<td>1996–1998</td>
<td>0.57</td>
<td>0.82</td>
</tr>
<tr>
<td>1996</td>
<td>0.48</td>
<td>0.67</td>
</tr>
<tr>
<td>1997</td>
<td>0.55</td>
<td>0.80</td>
</tr>
<tr>
<td>1998</td>
<td>0.66</td>
<td>0.98</td>
</tr>
</tbody>
</table>
ity growth in the production of computers themselves.\textsuperscript{16} This suggests a simple decomposition of the late 1990s acceleration in labor productivity into computer-related and non-computer-related factors. Subtracting aggregate hours growth from both sides of a standard growth-accounting equation, we get:

\[
\frac{Q(t)}{Q(t)} - \frac{H(t)}{H(t)} = \frac{A_C(t)}{A(t)} + \frac{A_{NC}(t)}{A_{NC}(t)} + \alpha(t) \left( \frac{L(t)}{L(t)} - \frac{H(t)}{H(t)} \right) + \beta(t) \left( \frac{K(t)}{K(t)} - \frac{H(t)}{H(t)} \right) + (1 - \alpha(t) - \beta(t)) \left( \frac{C(t)}{C(t)} - \frac{H(t)}{H(t)} \right).
\]

Productivity growth is a function of TFP growth (here divided into the contributions of the computer and noncomputer sectors, labeled $C$ and $NC$), of computer and noncomputer capital accumulation, and of improvements in the quality of labor input (represented as an increase in labor input relative to hours). I will focus on the two computer-related elements of productivity growth, $\frac{A_C(t)}{A(t)}$ and $\frac{A_{NC}(t)}{A_{NC}(t)}$, and represent the productivity growth due to all other factors as a residual.

We do not have sufficient information on physical capital or human capital by industry to allow for direct estimation of this series for the computer industry using a growth-accounting method. However, a back-of-the-envelope calculation that allows one to estimate the magnitude of the effect of TFP growth in computer production on aggregate productivity comes from assuming that the production functions for computer and noncomputer industries differ only in having different rates of Hicks-neutral technological progress. In this case, the differential in the growth rates of TFP between the computer and noncomputer sectors can be measured as the rate of price increase for the noncomputer sector minus the rate of price decline for computers. Moreover, for the chain-aggregated measures of output growth featured in the U.S. national accounts, aggregate TFP

\textsuperscript{16} Tevlin and Whelan (2001) document a strong statistical relationship between real business investment in computers and the price of computing equipment.
growth is very well proxied as a weighted average of TFP growth in the component sectors, in which the weights are the shares of each sector in aggregate nominal output.\textsuperscript{17}

These considerations imply that the effect of faster TFP growth in the computer sector in boosting aggregate TFP growth can be measured as the product of the share of the computer industry in nominal output times the rate of relative price decline for computers. Figure 6 shows this calculation.\textsuperscript{18} The upper panel shows that, despite enormous declines in quality-adjusted prices, the nominal output of the computer industry has fluctuated around 1.5% of business output since 1983, ticking up a bit since the mid-1990s. The middle panel shows that the pace of quality-adjusted price declines accelerated rapidly after the mid-1990s. As a result, the boost to aggregate TFP growth from the computer sector, which had fluctuated around 0.25 percentage points a year between 1978 and 1995 has picked up considerably in recent years, averaging almost 0.5 percentage points a year in 1997 and 1998.\textsuperscript{19}

\textsuperscript{17} Whelan (2001) develops a two-sector growth model that illustrates this point formally.

\textsuperscript{18} There is no official measure of the output of the computer industry. The measure of nominal computer output used here is the sum of consumption, investment, and government expenditures on computers plus exports of computers and peripherals and parts minus imports for the

\textsuperscript{19} Although this calculation is simple, more-elaborate calculations along these lines by Gordon (2000) and Oliner and Sichel (2000) produce essentially identical results.
Table 5 shows the results of this decomposition using computer capital accumulation effects from our preferred obsolescence model. Computer capital accumulation and computer sector TFP growth together account for 1.23 percentage points of the 2.15% yearly growth in business sector productivity over 1996–1998. Moreover, a remarkable 0.73 percentage points of the one-percentage-point increase in labor productivity growth over 1996–1998 can be explained by computer-related factors. In fact, the calculated acceleration of 0.26 percentage point due to other factors probably overstates the true effect of these factors because methodological changes in price measurement introduced into the GDP statistics that were not fully "back-casted" to earlier periods probably contributed around three-tenths a year to the acceleration in measured productivity in our data.20

These results show that computers have played a crucial role in the recent pickup in aggregate productivity growth, but they also contradict the position of some of the more enthusiastic believers in the benefits of technology investments. In particular, we have assumed that all capital investments earn the same net rate of return. Thus, the common belief that high-tech investments earn supernormal returns and are thus more profitable than other investments would, if correct, show up here as an improvement in productivity growth due to “All Other Factors,” which (accounting for measurement factors) we do not see.

Concerning the outlook for future productivity growth, these calculations suggest both upside and downside risks. The downside risks center around the dependence of the recent positive performance on one sector of the economy. The spectacular rates of productivity improvement in the computer sector in the late 1990s, and the associated acceleration in quality-adjusted price declines, seem unlikely to be sustainable. Given that we did not find any evidence that TFP growth picked up outside this sector, a slowdown in aggregate productivity growth would be the most likely outcome.

The upside potential has two elements. First, the improvement in productivity growth in the late 1990s does not appear to have been particularly cyclical in nature, and thus likely to be reversed: Increased utilization would show up as an increase in productivity growth due to all other factors. Second, the expansion of the Internet shows that businesses are still able to take advantage of declines in the price of computing power by finding new and productive uses for computing technologies.

VII. Conclusions

The purpose of this paper has been part methodological, part substantive. The methodological contribution has been to outline the issues surrounding capital stock measurement in the presence of embodied technological change and technological obsolescence. In particular, the paper provides a number of arguments against the use of the NIPA computer capital stocks for growth accounting and suggests an alternative approach. The substantive contribution has been to document the role that computers played in the improved productivity performance of the U.S. economy in the late 1990s: a marked pickup in the rate of computer capital deepening combined with improved productivity in the computer-producing industry accounted for almost all of this acceleration in aggregate productivity.

A final conclusion is that further empirical research in this area is clearly needed. Most of the calculations in this paper have relied on estimates of things that are difficult to measure (quality-adjusted prices for computing equipment) or studies that may themselves have become obsolete (Oliner’s depreciation schedules). Given the increasing importance of computing technologies, further empirical work on the measurement of prices and depreciation for computing equipment would be extremely useful for refining and extending the analysis in this paper.

REFERENCES


This problem has been rectified with the October 1999 benchmark revision to the NIPAs.

<table>
<thead>
<tr>
<th>Table 5.—COMPUTERS AND BUSINESS SECTOR PRODUCTIVITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Growth in labor productivity</td>
</tr>
<tr>
<td>Effect of computer capital accumulation</td>
</tr>
<tr>
<td>Effect of computer TFP growth</td>
</tr>
<tr>
<td>Total computer-related effect</td>
</tr>
<tr>
<td>All other factors</td>
</tr>
</tbody>
</table>


**APPENDIX A**

**Solution to the Obsolescence Model**

**The Marginal Productivity of Capital**

As described in the text, the computer price arbitrage formula is

\[ p_s(t) = \int_{-T}^{t} \frac{\partial Q_n(n)}{\partial (L(n)) e^{-\delta(n)}} - s p_s(v) e^{\gamma \gamma - \delta(n) v} dn. \]

Denoting the marginal productivity of a unit of capital as

\[ r_s^k(t) = \frac{\partial Q_s(t)}{\partial (L(n)) e^{-\delta(n)}}, \]

we get the following formula for the purchase price:

\[ p_s(v) = \frac{1}{1 + \frac{s}{r + \delta} (1 - e^{-\delta(n) v})} \int_{v}^{v+T} r_s^k(t) e^{-\delta(n) v} dn. \]

Now, differentiating the price of new computers with respect to \( v \), we get

\[ p_s(v) = (r + \delta + \gamma) p_s(v) - \frac{r_s^k(v)}{1 + \frac{s}{r + \delta} (1 - e^{-\delta(n) v})} + \frac{r_s^k(v + T) e^{-\delta(n) v}}{1 + \frac{s}{r + \delta} (1 - e^{-\delta(n) v})}. \]

At the time of scrappage, the computer must be just covering the support cost, implying that \( r_s^k(v + T) = sp_s(v) \). Making this substitution, using \( p_s(t) = e^{\gamma v} \), and rearranging gives us the marginal productivity of new computer capital:

\[ r_s^k(v) = \left[ (r + \delta + \gamma - g) + s \left( 1 + \frac{\gamma - g}{r + \delta} (1 - e^{-\delta(n) v}) \right) \right] e^{\gamma v}. \]

Dividing through by \( e^{\gamma v} \) to express the rental rate in quality-adjusted terms and noting that \( q(t) = e^{\gamma(t-v)} \) is the quality-adjusted computer price index, we get equation (25):

\[ \frac{\dot{r}}{r} = q_s(t) \left[ \left( r + \delta - \frac{q(t)}{q(t)} \right) + s \left( 1 - \frac{1 - e^{-\delta(n) v}}{r + \delta} \frac{q(t)}{q(t)} \right) \right]. \]

**The Retirement Age**

Because all the conditions for the allocation of other factors to each vintage are as before, the marginal productivity of a unit of computer capital still declines over time at rate \( g - \gamma \), so we can now define the retirement age \( T \) from

\[ (r + \delta + \gamma - g) + s \left( 1 + \frac{\gamma - g}{r + \delta} (1 - e^{-\delta(n) v}) \right) e^{-\gamma v} e^{\gamma v} = se^{\gamma v}. \]

Rearranging, we get

\[ e^{\gamma v} e^{\gamma v} = (r + \delta + \gamma - g) \left( 1 + \frac{1}{s (r + \delta)} \right) - \frac{\gamma - g}{r + \delta}. \]

Given values for \( r, \delta, s, \) and \( \gamma - g \), this nonlinear equation can be solved numerically to give us the retirement age, \( T \).

**Economic Depreciation**

Given a path for the marginal productivity of a unit of computer capital, we can now explain the pattern of economic depreciation implied by this path:

\[ \frac{\dot{p}_s(t)}{p_s(t)} = \frac{1}{r + \delta + \gamma - g} \int_{v}^{t} r_s^k(n) e^{-\delta(n) v} e^{\gamma v} dn - s p_s(v) \]

\[ \times \int_{v}^{t} e^{-\delta(n) v} e^{\gamma v} dn \]

To keep this calculation simple, we will break it into two, defining

\[ p^s(t) = \int_{v}^{t} r_s^k(n) e^{-\delta(n) v} e^{\gamma v} dn \]

\[ = r_s^k(v) e^{-\delta(n) v} \int_{v}^{t} e^{\gamma v} e^{-\delta(n) v} dn \]

\[ = r_s^k(v) e^{-\delta(n) v} \left( 1 - e^{-\delta(n) v} \right) e^{\gamma v} e^{-\delta(n) v} - \frac{\gamma - g}{r + \delta}. \]

Now, we use the fact that \( r_s^k(v) = se^{\gamma v} e^{\gamma v} \). Inserting this, rearranging, and defining the age of the vintage as \( \tau = t - v \), we get

\[ p_s^\tau(v) = e^{\gamma v} e^{-\delta(n) v} \left( \frac{s e^{\gamma v} e^{\gamma v}}{r + \delta + \gamma - g} \right) e^{\gamma v} e^{-\delta(n) v} = e^{\gamma v} e^{\gamma v}. \]

Finally, inserting the expression for \( e^{\gamma v} e^{\gamma v} \) into equation (19), we get

\[ p_s^\tau(v) = e^{\gamma v} e^{-\delta(n) v} \left[ 1 + \frac{s}{r + \delta} - \frac{s e^{\gamma v} e^{\gamma v}}{r + \delta + \gamma - g} \right] \times \left( \frac{\gamma - g}{r + \delta} + e^{\gamma v} e^{\gamma v} \right). \]
Thus,

\[ p_i(t) = e^\gamma (1 + \frac{s}{r + \delta} - \frac{se^{-\gamma t}}{r + \delta}) \]

\times \left( \frac{\gamma - g}{r + \delta} + e^{\gamma (r + \delta - \gamma) t} \right) \]

\[ - e^{\gamma (r + \delta - \gamma) t} \left( \frac{s}{r + \delta} (1 - e^{-(r + \delta - \gamma) t}) \right). \]

Finally, the quality-adjusted economic depreciation schedule calculated from an Oliner-style study by comparing the price of an old vintage with the price of new computers and then subtracting off the quality-improvement in the new computers is

\[ d_i(t) = e^{-\delta t} \left[ 1 + \frac{s}{r + \delta} - \frac{se^{-\gamma t}}{r + \delta} \right] \]

\times \left( \frac{\gamma - g}{r + \delta} + e^{(r + \delta - \gamma) t} \right) \]

\[ - e^{(r + \delta - \gamma) t} \left( \frac{s}{r + \delta} (1 - e^{-(r + \delta - \gamma) t}) \right). \]

**APPENDIX B**

**Details of the Empirical Growth Accounting**

*Capital Stocks*

The calculations use detailed disaggregated capital stock data: in addition to the five types of computing equipment, we use the 26 types of noncomputing equipment shown in Table 1, eleven types of nonresidential structures, and tenant-occupied housing (rental income from such housing is part of business output). For all noncomputer stocks, we use the NIPA real wealth capital stocks, altered in two ways. First, when the empirical analysis for this paper was undertaken, capital stock data were published through 1998; however, real investment data for 1998 were available. Thus, I extended each of the published capital stock series by growing them out using the 1998 investment data. Second, the depreciation rates published by Herman and Herman (1997). Second, these stocks refer to year-end values. Because the growth-accounting analysis seeks to explain year-average growth rates, year-average stocks were constructed by averaging adjacent year-end stocks. The same transformation was applied to the computer stocks for the obsolescence model.

*Rental Rates*

For all capital except computers in the obsolescence model, our empirical analysis proxied the marginal productivity of capital using the Hall-Jorgenson tax-adjusted rental rate,

\[ r_i(t) = p_i(t) \left( \frac{1 - c_i - \tau c_i}{1 - \tau} \right), \]

where \( p_i(t) \) is the price of capital of type \( i \) relative to the price of capital, \( \delta_i \) is the NIPA depreciation rate for capital of type \( i \), \( \tau \) is the marginal corporate income tax rate, \( z_i \) is the present discounted value of depreciation allowances per dollar invested, and \( c_i \) is the investment tax credit.

The real rate of return on capital, \( r \), was set equal to 6.75%: this produces a series for the “required” income flow from capital that, on average, tracks with the observed series for business sector capital income over our sample. The \( \frac{q_i(t)}{q_i(t)} \) term is calculated for each type of capital as a three-year moving average of the rate of change of the price of capital relative to the price of output. The tax terms were calculated for each type of capital using the information on tax credit rates and depreciation service lives presented in Gravelle (1994).

*Growth-Accounting Weights*

We start by imputing factor shares for each type of capital. For labor, we use \( \alpha(t) \), the labor share of income in the business sector. For capital, however, things are a bit more complicated. In theory, if we had perfect measures of the marginal productivity of each type of capital, then we could just use equation (23) to estimate the factor shares. In practice, theoretical estimates can produce a set of factor shares that do not sum to 1. There are two standard methods for dealing with this problem: one is to vary the value of \( r \) each period so that \( \sum_{t=1}^{n} r_i(t) K_i(t) \) equals total capital income, and the second method, implemented in this paper, is to define the growth-accounting weights for capital so that they sum to capital’s share of income, letting the share for capital of type \( i \) be proportional to \( r_i(t) K_i(t) \), by using the formula

\[ \beta_i(t) = (1 - \alpha(t)) \left( \frac{r_i(t) K_i(t)}{\sum_{t=1}^{n} r_i(t) K_i(t)} \right). \]

The final growth-accounting weights were constructed by averaging factor shares from adjacent years.