Title	Non-stationarity and persistence in real exchange rates
Authors(s)	Kenny, Patrick, Rodney, Thom
Publication date	1990-02
Publication information	Kenny, Patrick, and Thom Rodney. “Non-Stationarity and Persistence in Real Exchange Rates” (February, 1990).
Series	UCD Centre for Economic Research Working Paper Series, WP90/2
Publisher	University College Dublin. School of Economics
Item record/more information	http://hdl.handle.net/10197/1471
Notes	A hard copy is available in UCD Library at GEN 330.08 IR/UNI

Non-Stationarity and Persistence in Real Exchange Rates

by

Patrick Kenny
and
Rodney Thom

Working Paper No. WP90/2

February 1990

Non-Stationarity and Persistence in Real Exchange Rates.

1. Introduction.

Observed volatility of real exchange movements during the last two decades has lead to a considerable reappraisal of the standard purchasing power parity (PPP) explanation of co-movements between nominal exchange rates and the relative prices of internationally traded goods. Authors such as Dornbusch (1976) and Ainzenman (1986) maintain the traditional PPP hypothesis of a 'long-run' or 'steady-state' equilibrium value for the real exchange rate in the context of theoretical models which rationalise transitory deviations from parity in terms of commodity markets characterised by slow price adjustment interacting with flexible asset markets. For example in Dornbusch's world 'long-run equilibrium is attained [when], ..., the goods markets clear, prices are constant, and expected [nominal] exchange rate changes are zero.' (1976, p.1167) Hence long-run equilibrium is characterised by a constant or parity value for the real exchange rate so that current innovations have no implications for long-run forecasts[1].

In contrast to these sticky price or overshooting models, Roll (1979), Darby (1980) and Alder and Lehmann (1983) have presented both theoretical and empirical evidence which suggests that time series on real exchange rates contain a non-stationary

[1]Other authors such as Neary (1988) define the real exchange rate as the relative price of traded to 'non-traded' goods. We note that in most cases these authors assume long-run, and in some cases continuous, PPP between the relative prices of internationally traded goods.

component with the implication that current innovations have a permanent effect on all future values. For example using G7 data Darby concludes that 'the basic hypothesis ... that the [real exchange rate] takes a random walk ... appears to be consistent with the data. On this hypothesis there is no parity value towards which the [real exchange rate] tends in the long run.' (1980, p.469) More recently Taylor (1988) uses monthly data on five nominal exchange rates and corresponding relative prices to test the validity of PPP via cointegration techniques and is 'unable to reject the hypothesis that they tend to drift apart without bound.' (1988,p.1377) That is, the existence of a long-run equilibrium relationship between nominal exchange rates and relative prices is rejected by the data.

The significance of these results is that they preclude the existence of a 'long-run' mean or variance for the real exchange rate and are therefore in direct contrast with the predictions of both traditional PPP theory and the short-run overshooting models of Dornbusch and Ainzenman. However while it is clear that evidence supporting the existence of a unit root must influence our approach to exchange rate modelling, it is also important to assess the extent of persistence to innovations. That is, the extent to which an unforecasted change in the real exchange rate will influence long-run forecasts of the series. If, for example, the series follows a pure random walk then a unit innovation will change all future forecasts of the series by one. On the other hand if the series is an integrated process with an ARIMA(p,1,q) representation a unit innovation may have a 'large' or 'small' effect on long-run forecasts[2]. In other words the existence of a unit autoregressive root is consistent with large or small persistence defined as the long-run response to a current innovation.

More precisely, Beveridge and Nelson (1981) show that an integrated time series with an ARIMA(p,1,q) representation can be decomposed into a stationary or transitory component and a non-stationary or permanent component, with the degree of persistence to current innovations depending on the relative importance of the latter. If, for example, a unit innovation changes all future forecasts by at least one per cent then it is reasonable to conclude that the long-run behaviour of the series in dominated by the permanent component in which case economic models of the real exchange rate should seek to explain its non-stationarity rather than 'temporary deviations' from an assumed long-run equilibrium parity value. On the other hand it is possible that the underlying process generating the series may have a unit root in the autoregressive component of its ARIMA representation but that the long-run behaviour is dominated by the stationary component in which case the series will exhibit a tendency to 'return to a "trend" in the far future, but does not get all the way there'. (Cochrane 1988, p.894)

Hence the objective of this paper is to assess the importance of non-stationarity by estimating the degree of persistence in time series for real exchange rates. Section 2

---

[2]An integrated process is non-stationary in levels but stationary after differencing d times. If it is stationary in first differences the series is said to be integrated of order 1 or I(1).

outlines conditions under which we would expect the real exchange to be an integrated process. Section 3 analyses quarterly data on four real exchange rates and reports on tests of the hypothesis that each series can be modelled as a first order integrated process. Given that this hypothesis cannot be rejected Section 4 use two measures of persistence to assess the relative importance of the permanent component in an integrated series. A summary and conclusions are given in Section 5.

2. PPP and Efficient Markets.

Roll (1979), Darby (1980) and Alder and Lehmann (1983) all outline the conditions under which the real exchange will follow an integrated process. MacDonald (1988) calls these conditions the efficient markets version of PPP or EMPPP. In what follows we show that the assumption of rational expectations together with real interest parity holding are sufficient to ensure that the real exchange follows an ARIMA(p,1,q) process. That is, non-stationary in levels but stationary in first differences with the latter having an ARMA(p,q) representation. However, it is important to stress that the EMPPP model does not require instantaneous commodity price adjustment or that real interest parity holds exactly in the sense that domestic and foreign real interest rates must be equal.

To illustrate we define the logarithm of the real exchange rate as:

$$y_{jt} = s_{jt} + r_{jt} \qquad (1)$$

where s_{jt} is the logarithm the domestic currency price of one unit of currency j and r_{jt} is the ratio of foreign to domestic prices. Letting $E_t x_{t+1}$ denote the expectation of x_{t+1} held at period t we can define the difference between the domestic and foreign real interest rates at:

$$\Omega_{jt} = (i_t - i_{jt}) - (E_t\pi_{t+1} - E_t\pi_{jt+1}) \qquad (2)$$

where i is the nominal rate of interest and π denotes the inflation rate between measured as the differences between the logarithms of the price levels in t+1 and t. Substituting (1) into the uncovered arbitrage condition[3]:

$$i_t - i_{jt} = E_t(s_{jt+1} - s_{jt}) \qquad (3)$$

gives:

$$E_t(s_{jt+1} - s_{jt}) + E_t\pi_{jt+1} - E_t\pi_{t+1} = \Omega_{jt} \qquad (4)$$

Using the definition of the inflation rate π and assuming rational expectations so that:

$$E_t x_{t+1} = x_{t+1} + e_t \qquad (5)$$

where e is a white noise error gives:

$$(s_{jt+1} + r_{jt+1}) - (s_{jt} - r_{jt}) = \Omega_{jt} + v_{jt} \qquad (6)$$

where v is a composite error term and the left hand side of (6) is the first difference of the real exchange rate. Hence the real exchange rate will follow an integrated process of order one if the real interest rate differential follows a stationary process. When the real interest rate parity holds absolutely so that Ω_t is zero then the real exchange rate is a pure random walk[4]. However strict real interest rate parity is by no means necessary for the real exchange rate to be an integrated process. For example if

---

[3]Note that $s_{t+1} > s_t$ implies a depreciation of the domestic currency.

[4]With drift if the difference between the domestic and foreign interest rates if a non-zero constant.

the interest rate differential is ARMA(0,q) then the level of the real exchange rate is ARIMA(0,1,q). Hence given uncovered interest rate parity and rational expectations, the hypothesis of a unit root in the real exchange rate is consistent with a wide range of stationary processes for the real interest rate differential. Note that this does not require that the domestic and foreign real interest rates are themselves stationary. For example it is possible that these series are each integrated processes but that the difference between them is stationary. That is, they are cointegrated of order zero.

3. Tests for Non-Stationarity.

This section reports on tests that series on real exchange rates follow an integrated process of order one. Four real exchange rate series are used. In each case the US dollar is the domestic currency and the foreign currencies are the Canadian dollar, the D-Mark, Sterling and the Yen. The price series are wholesale price indices based on 1980=100 and the data are quarterly observations over 1972:1 to 1988:4 taken from the IMF <u>International Financial Statistics.</u>

Table 1 contains estimated sample autocorrelations of y_{jt} and $y_{jt}-y_{jt-1}$ for each series. The patterns displayed by these estimates are broadly consistent with those expected from time series which belong to the I(1) class. That is, autocorrelations for first differences decay more rapidly than those for the levels. Also, in all cases autocorrelations in first differences are positive at lag one and tend to be insignificantly different from zero at longer lags. Significance at lag one only for the

Table 1. Sample Autocorrelations.

Levels.

Lag	1	2	3	4	5	6	7	8	20
Canadian $	.982	.843	.754	.671	.612	.561	.502	.433	-.061
D-Mark	.936	.850	.754	.640	.521	.409	.293	.167	-.392
Sterling	.902	.798	.673	.537	.386	.283	.165	.043	-.407
Yen	.891	.763	.605	.464	.325	.219	.128	.054	-.374

First Differences.

Lag	1	2	3	4	5	6	7	8	20
Canadian $	.259	.066	.081	-.005	-.014	.063	.077	.038	.014
D-Mark	.231	.015	.129	.071	.009	.010	.069	-.137	-.125
Sterling	.115	.021	.094	.137	-.218	.079	.093	.049	-.120
Yen	.248	.102	.019	.003	-.122	-.156	-.021	.001	-.124

Table 2. Estimates For Equation (7).

$$y_t - y_{t-1} = \mu + \phi y_{t-1} + u_t + \sum_{1}^{q} \Theta_j u_{t-j}$$

	μ	ϕ	Θ_1	Θ_2	Q	AC	SC
Canadian $	-0.006	-0.056	0.292		.922	-4.083	-3.983
	(1.130)	(1.276)	(2.302)				
	-0.006	-0.059	0.294	0.032	.919	-4.054	-3.291
	(1.100)	(1.231)	(2.161)	(0.239)			
D-Mark	-0.058	-0.083	0.309		.965	-1.694	-1.594
	(1.421)	(1.524)	(2.289)				
	-0.055	-0.081	0.307	-0.022	.963	-1.664	-1.531
	(1.260)	(1.349)	(2.166)	(0.152)			
Sterling	0.048	-0.073	0.152		.281	-1.606	-1.506
	(1.311)	(1.246)	(1.118)				
	0.052	-0.079	0.157	0.033	.304	-1.577	-1.444
	(1.229)	(1.171)	(1.104)	(0.224)			
Yen	-0.328	-0.061	0.244		.596	-1.949	-1.850
	(1.090)	(1.113)	(1.853)				
	-0.435	-0.081	0.277	0.128	.546	-1.936	-1.803
	(1.204)	(1.223)	(2.004)	(0.894)			

Notes: Figures in parenthesis are t-statistics. Q is the marginal significance level of the Ljung-Box Q-statistic. AC is the Akaike Information and SC is the Schwarz Information Criterion.

Table 3. Dickey Fuller Tests.

	Φ_1	t_β	Q	Φ_2	t_β	Q
Canadian $	0.605	-1.100	.823	1.855	0.463	.889
D-Mark	0.883	-1.212	.608	0.581	-1.212	.612
Sterling	0.735	-1.093	.182	1.144	-1.444	.246
Yen	1.094	-0.864	.372	1.009	-1.125	.392

Notes: Φ_1 tests $\alpha = \beta = 0$ in $y_t - y_{t-1} = \alpha + \beta y_{t-1} + u_t$ critical values for T = 50 are: 3.94(10%) 4.86(5%) and 5.80(1%)
Φ_2 tests $\alpha = \delta = \beta = 0$ in $y_t - y_{t-1} = \alpha + \delta t + \beta y_{t-1} + u_t$ critical values for T = 50 are: 4.31(10%) 5.31(5%) and 5.94(1%).
t_β is the estimated t-ratio for β and Q is the marginal significance level of the Box-Pierce Statistic - ie. the level at which the null hypothesis of no serial correlation rejected.

Table 4. Cointegrating Regressions.

	α	β	DW	DF	ADF
Canadian $	-0.105	-1.357	0.119	-0.371	
D-Mark	-0.747	-0.783	0.118	-0.292	
Sterling	0.617	-0.782	0.162		-1.599
Yen	-5.475	-1.374	0.152	-1.436	

Notes: DW DF and ADF are test statistics on the estimated residuals from the cointegrating regression $s_{jt} = \alpha + \beta r_{jt} + u_t$ DW is used to test that the Durbin-Watson statistic is zero - ie. that the residuals are a random walk. Critical values with T=50 are: .69(10%) .78(5%) and 1(1%). DF Dickey Fuller Statistics on the same hypothesis. Critical values are 3.28(10%) 3.67(5%) and 4.32(1%). ADF includes a lagged difference in the DF regression and is used when the Q-statistic has a MSL below 10%.

differenced series is suggestive of a first order moving average process. Further, Nelson and Plosser (1982) prove that positive autocorrelation at lag one only combined with the assumption that any cyclical components of the levels are stationary is 'sufficient to imply that variation in actual ... changes is dominated by changes in the secular component rather than the cyclical component.' (p. 155)

The autocorrelations in Table 1 combined with the analysis in Section 2 suggests that we might base a more formal test for the existence of a unit root on the regression:

$$y_t - y_{t-1} = \mu + \emptyset y_{t-1} + u_t + \sum_1^q \Theta_j u_{t-j} \qquad (7)$$

where u is a random disturbance and y_t has an autoregressive unit root if $\emptyset$ is zero. Table 2 contains estimates of (7) for first and second order MA processes. Under the null hypothesis that y_t is I(1) the computed t-ratio for the estimated autoregressive coefficient $\emptyset$ does not have a student's t distribution. Further, (7) contains MA components and is therefore different from the 'standard' Dickey-Fuller unit root regression which assumes an AR(p) process for y_t. Schwert (1987) shows that appropriate critical levels for $t_\emptyset$ in (7) are sensitive to the values of the MA parameters and the order of the ARIMA model used. For example on 10,000 replications of an ARIMA(0,1,1) process with 140 observations Schwert finds .05 critical values of -2.78 for Θ_1 = 0.5 and -2.98 for Θ_1 = 0.8. The corresponding Dickey-Fuller critical value, with Θ_1 = 0, is -2.83. While the computed t-statistics are not strictly comparable to Schwert's critical values on both sample size and estimates for the MA parameters their relatively low absolute values would nonetheless lead us towards accepting the I(1) hypothesis. Further in all four cases the MA(2) model is rejected in favour of the MA(1) model on both the Akaike and Schwarz information criteria indicating that an

ARIMA(0,1,1) is an appropriate model for y_t[5]. A possible exception is the Dollar-Sterling series where the MA coefficients are insignificant indicating a possible random walk process.

As a further check on the I(1) hypothesis we carried out two additional tests. First, using the Dickey-Fuller regression:

$$y_t - y_{t-1} = \alpha + \delta t + \phi y_{t-1} + \sum_{1}^{m} \phi_j (y_{t-j} - y_{t-j-1}) \qquad (8)$$

the series y_t is a random walk without drift if $\alpha=\delta=\phi= 0$ or a random walk with drift if $\delta=\phi= 0$. Equation (8) was estimated with and without the linear trend and the appropriate test statistics are given in Table 3[6]. In all four cases the computed values for the Dickey-Fuller test statistics Φ_1 $(\delta=\beta=0)$ and Φ_2 $(\alpha=\delta=\beta=0)$ unambiguously accept the null of a unit root in the level of each series. Second, it is important to note that the above tests can be interpreted as implicit tests for the existence of a (0,-1) cointegrating vector between the nominal exchange rate and the corresponding relative price term. That is, in the stochastic PPP relationship:

$$s_t = \alpha + \beta r_t + u_t \qquad (9)$$

u_t defines the real exchange for $(\alpha,\beta) = (0,-1)$. However Taylor (1988) suggests the possibility of nominal exchange rates and relative prices moving together in the long-run, or sharing a

[5]The Akaike IC minimises log(RSS)+(2K)/T and the Schwarz IC minimises log(RSS)+(Klog(T))/T. RSS is the residual sum of squares, K is the number of regressors and T is the sample size.

[6]Critical values used in Tables 2 and 3 are based on a sample size T = 50. The actual sample size is 68 for which critical values are unavailable. Hence the values given in the notes to each Table slightly overstate the correct values. Table 2 uses Dickey and Fuller (1981, Tables IV and V) while Table 3 uses Engle and Yoo (1987, Table 2).

common equilibrium, with measurement error and/or transport costs causing deviations from one-to one-proportionately. That is s_t and r_t may be cointegrated with a cointegrating vector different from (0,-1).

Table A1 in the Appendix reports Dickey Fuller tests on nominal exchange rates and relative prices while Table 4 gives the results of cointegrating regressions normalised on the nominal exchange rate. The DF statistics in Table A1 clearly accept the hypotheses that all series are individually I(1) at the 5 percent level while the cointegrating tests in Table 4 accept non-cointegration between nominal exchange rates and relative prices. Hence both Dickey-Fuller and cointegration tests confirm the previous result that the data are inconsistent with the concept of a long-run equilibrium or parity relationship between nominal exchange rates and corresponding relative price terms.

4. Persistence Measures.

Failure to reject the hypothesis of a unit root in the autoregressive component of a time series implies that contemporaneous innovations must alter all future forecasts of that series. If, for example, the series is a pure random walk then a unit innovation changes all future values by one unit. However the existence of a unit root does not necessarily imply that the process is a pure random walk and, as a consequence, the long-run responses to a unit innovation may differ from one. More generally, any I(1) series can be decomposed into permanent and stationary components with the relative importance of the former determining the degree of persistence to current innovations. However, in order to assess the importance of the permanent component it is first necessary to model the process generating the series in question. Available procedures to accomplish this task include Unobserved Components (UC) models and ARIMA models[7]. The former assess the importance of non-stationarity by estimating the ratio of the variances of the permanent and transitory components while the latter use impulse responses for the levels of a series computed from fitted ARMA for the first differences. As Stock and Watson (1988) point out, an essential difference between these approaches is that UC models assume zero correlation between the innovations in the permanent and stationary components while ARIMA models impose perfect correlation between the same innovations. Although neither restriction is particularly attractive we note that Nelson and Plosser (1982) show that UC models cannot account for positive autocorrelation at lag one in the first differences of the series unless the zero covariance restriction between the permanent and stationary innovations is relaxed, in which case the parameters are not identified[8]. Consequently as the estimated autocorrelations reported in Table 1 reveal positive values at lag one for first differences we adopt the an ARIMA approach to modelling an I(1) series as used by Campbell and Mankiw (1987).

---

[7]See Stock and watson (1988) for a comparison of these approaches.

[8]Alternatively, this restriction could be maintained with transitory components included in the permanent part of the series. See Nelson and Plosser (1982, pp.154-155).

We also consider an alternative measure of persistence, proposed by Cochrane (1988), which is based on the ratio of the variance of $(y_t - y_{t-k})$ to k times the variance of the first differences. These measures are described as follows.

Impulse Responses.

A time series which is integrated of order one has an ARMA representation:

$$\emptyset(L)[1-L]y_t = \Theta(L)u_t \tag{10}$$

where L is the lag operator and:

$$\emptyset(L) = 1 - \emptyset_1 L - \ldots - \emptyset_p L^p$$

$$\Theta(L) = 1 + \Theta_1 L + \ldots + \Theta_q L^q$$

The moving average representation for the first difference is:

$$[1-L]y_t = A(L)u_t \tag{10}$$

where $A(L) = \emptyset(L)^{-1}\Theta(L)$. Given an I(1) process for y_t, ΣA_i^2 is finite implying that the limit of A_i is zero as i goes to infinity. Hence the level of the process has a moving average representation:

$$y_t = B(L)u_t \tag{11}$$

where $B(L) = [1-L]^{-1}A(L)$ so that

$$B_k = \Sigma A_i \qquad i = 0,\ldots k.$$

give the responses of y_{t+k} to an innovation at time t. Given that the level of the series contains an AR unit root these responses should be non-zero at all k. That is, the series will exhibit persistence to current innovations. However, persistence may be 'large' or 'small' depending on the relative importance of the permanent component. For example if the first difference of the series can be modelled by the ARMA(0,1) process:

$$y_t - y_{t-1} = \mu + u_t + \Theta u_{t-1}$$

Table 4 Impulse Responses to a Unit Innovation in Ln Real Exchange Rate.

Canadian Dollar.

(p,q)	1	2	4	8	16	30	40
(0,1)	1.27	1.27	1.27	1.27	1.27	1.27	1.27
(0,2)	1.27	1.29	1.29	1.29	1.29	1.29	1.29
(1,1)	1.27	1.33	1.35	1.35	1.35	1.35	1.35
(1,2)	1.28	1.34	1.45	1.55	1.61	1.63	1.63
(1,0)	1.26	1.34	1.36	1.36	1.36	1.36	1.36
(2,0)	1.27	1.34	1.36	1.36	1.36	1.36	1.36

D-Mark.

(p,q)	1	2	4	8	16	30	40
(0,1)	1.28	1.28	1.28	1.28	1.28	1.28	1.28
(0,2)	1.27	1.22	1.22	1.22	1.22	1.22	1.22
(1,1)	1.24	1.11	1.15	1.16	1.16	1.16	1.16
(1,2)	1.28	1.21	1.22	1.25	1.24	1.24	1.24
(1,0)	1.24	1.29	1.31	1.31	1.31	1.31	1.31
(2,0)	1.25	1.26	1.25	1.25	1.25	1.25	1.25

Sterling.

(p,q)	1	2	4	8	16	30	40
(0,1)	1.12	1.12	1.12	1.12	1.12	1.12	1.12
(0,2)	1.11	1.14	1.14	1.14	1.14	1.14	1.14
(1,1)	1.10	1.16	1.19	1.23	1.23	1.23	1.23
(1,2)	1.12	1.13	1.13	1.13	1.13	1.13	1.13
(1,0)	1.12	1.14	1.14	1.14	1.14	1.14	1.14
(2,0)	1.11	1.14	1.14	1.14	1.14	1.14	1.14

Yen.

(p,q)	1	2	4	8	16	30	40
(0,1)	1.23	1.23	1.23	1.23	1.23	1.23	1.23
(0,2)	1.23	1.25	1.25	1.25	1.25	1.25	1.25
(1,1)	1.25	1.36	1.41	1.44	1.44	1.44	1.44
(1,2)	1.25	1.37	1.41	1.43	1.44	1.44	1.44
(1,0)	1.27	1.36	1.36	1.36	1.36	1.36	1.36
(2,0)	1.25	1.36	1.41	1.43	1.43	1.43	1.43

Table 5 Estimated Variance Ratios.

Lag	2	4	8	16	20	24	28
Canadian $	1.346	1.560	1.731	1.813	1.773	1.782	1.797
	(0.328)	(0.492)	(0.732)	(0.939)	(1.146)	(1.257)	(1.364)
D-Mark	1.309	1.493	1.736	1.482	1.359	1.162	0.948
	(0.319)	(0.471)	(0.734)	(0.861)	(0.879)	(0.819)	(0.720)
Sterling	1.154	1.286	1.417	1.408	1.100	0.842	0.617
	(0.282)	(0.406)	(0.599)	(0.819)	(0.711)	(0.594)	(0.469)
Yen	1.331	1.536	1.408	1.356	1.196	1.112	0.937
	(0.325)	(0.484)	(0.596)	(0.789)	(0.773)	(0.784)	(0.712)

Notes: Figures in parentheses are estimated asymptotic standard errors computed as $V_k^1/[.75T/(k+1)]^{\frac{1}{2}}$. See Campbell and Mankiw (1987, p.873).

then a unit innovation at time t changes all future values of y by $(1+\Theta)$ with the extent of persistence depending on the value of Θ.

Although our previous results indicate the possibility of low order MA models for first differences we estimated a range of ARMA(p,q) processes and computed the impulse responses for each model. Table A2 in the Appendix gives the parameter estimates and Table 5 contains estimated impulse responses for the series level computed for each ARMA model[9]. Without exception the impulse responses indicate that a unit innovation increases the forecast for each series by at least one percent thus indicating long-run persistence to contemporaneous shocks. Further, the magnitude of the responses is similar across the models used indicating that the finding of persistence is not sensitive to the parameterisation chosen.

---

[9]The RATS V3.0 computer package was used for all computations. See VAR Econometrics (1988).

Variance Ratios.

Cochrane (1988) proposes an alternative persistence measure based on the ratio of the variance of $(y_t - y_{t-k})$ to k times the variance of the first differences. For the pure random walk:

$$y_t = y_{t-1} + u_t$$

where u is i.i.d. with mean zero and variance σ^2, the variance of the first difference is σ^2 and the variance of $(y_t - y_{t-k})$ is $k\sigma^2$ so that the variance ratio:

$$V_k = var(y_t - y_{t-k})/kvar(y_t - y_{t-1}) \qquad (12)$$

equals unity for at all k > 1. For a stationary process, on the other hand, V_k should tend to zero as k increases. Note that care has to be taken in interpreting an estimator for V_k at 'large' k. For example with a fixed sample size T V_k must approach zero as k approaches T.

To estimate Cochrane's variance ratio we use the estimator[10]:

$$V'_k = 1 + 2\sum_{j=1}^{k} [1 - j/(1 + k)]q_j \qquad (13)$$

where q_j is the estimated autocorrelation for first differences at lag k. Campbell and Mankiw (1987) report a downward bias in this estimator for the variance ratio with the mean of a V'_k being approximately (T- k)/T rather than unity for a random walk. Table 5 gives estimates of the variance ratio along with their asymptotic standard errors. The estimates of V_k are consistent with the impulse response functions in Table 4. In all four cases

[10]See Campbell and Mankiw (1987).

V_k' is greater than the level expected from a pure random walk indicating long-run persistence.

4 Conclusions.

This paper attempted to assess the extent of persistence in time series for real exchange rates. In the four cases considered the results in Section 3 fail to reject the hypotheses that the data are generated by integrated processes and that individual series on nominal exchange rates and relative prices are not cointegrated. These results are reinforced by the persistence measures used in Section 3 which indicate that unit innovations increase long-run forecasts by at least one per cent. That is by at least what would be expected from a random walk. Hence we must conclude that our analysis of time series data on nominal exchange rates and the relative prices of internationally traded goods fails to provide significant support for the purchasing power parity concept of a long-run parity value for the real exchange rate.

Finally we would wish to stress that this result is consistent with the growing literature on testing for unit roots in economic time series. Nelson and Plosser (1982) and Schwert (1987) have, along with others, found evidence of autoregressive unit roots in a wide variety of US economic time series ranging from civilian population to the dividend yield for Standard and Poor's composite index for stocks[11]. It may, of course, be

[11]We note that as to date evidence on the existence or otherwise of unit roots in European time series is, to our knowledge, unavailable.

possible that data of a higher quality and more powerful econometric techniques may eventually refute these findings. However given the available data and testing procedures we choose to interpret this evidence as a coherent organisation of officially published economic data. As such it provides a starting point which appears to be fundamentally different from that which is often imagined to approximate the economy which theory attempts to rationalise.

References.

Ainzenman, J. (1986) "Monopolistic Competition, Relative Prices and Output Adjustment in an Open Economy", NBER Working Paper No. 1787.

Alder M. and B. Lehmann (1983) "Deviations From purchasing Power Parity in the Long Run", Journal of Finance XXXVIII 1471-1487.

Beveridge, S. and C.R. Nelson (1981) "A New Approach to the Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the Business Cycle", Journal of Monetary Economics 7 151-174.

Campbell J.Y. and N.G. Mankiw (1987), "Are Output Fluctions Transitory?", Quarterly Journal of Economics 102 857-880.

Cochrane, J.H. (1988), "How Big is the Random Walk in GNP?", Journal of Political Economy 96 893-920

Darby, M. (1980) "Does Purchasing Power Parity Work?", NBER Working Paper No. 607

Dickey, D.A. and W. Fuller (1981) "The Likelihood Ratio Statistic for Autoregressive Time Series With a Unit Root", Econometrica 49 1057-1072.

Dornbusch, R. (1976) "Expectations and Exchange rate Dynamics", Journal of Political Economy 84 1161-1174.

Engle, R.F. and B.S. Yoo (1987) "Forecasting and Testing in Cointegrated Systems", Journal of Econometrics 35 143-159.

MacDonald, R. (1988) Floating Exchange Rates: Theories and Evidence Unwin Hyman (London)

Neary, J.P. (1988) "Determinants of the Real Exchange Rate", American Economic Review 78 210-215.

Nelson, C.R. and C.L. Plosser (1982) "Trends and Random Walks in Macroeconomics Time Series", Journal of Monetary Economics 10 129-162

Roll, R. (1979) "Violations of Purchasing Power Parity and Their implications for Efficient Commodity Markets", in M. Sarnat and G. Szzego (eds) International Finance and Trade Vol 1. Cambridge Mass.

Schwert, G.W. (1987) "Effects of ModelSpecification on Tests for Unit Roots in Macroeconomic Data", Journal of Monetary economics 20 73-103.

Stock, J.H. and M.W. Watson (1988) "Variable Trends in Economic time Series", Journal of Economic Perspectives Vol. 2 No 3. 147-174.

Taylor, M.P. (1988) "An Empirical Examination of Long-run purchasing Power parity using Cointegration Techniques", Applied Economics 20 1369-1381.

VAR Econometrics (1988) RATS: Users Manual.

Table A1. Dickey Fuller Tests.

	Exchange Rates			Relative Prices		
	Φ_1	$t_ß$	Q	Φ_1	$t_ß$	Q
Canadian $	1.567	-1.475	.167	3.529	0.443	.955
D-Mark	1.896	-1.423	.139	4.819	-2.224	.258
Sterling	1.901	-1.823	.514	2.829	-0.991	.486
Yen	1.029	-0.203	.522	1.125	-0.153	.952

Notes: Φ_1 tests $\alpha = ß = 0$ in $y_t - y_{t-1} = \alpha + ßy_{t-1} + u_t$ critical values for T = 50 are: 3.94(10%) 4.86(5%) and 5.80(1%)
$t_ß$ is the estimated t-ratio for ß and Q is the marginal significance level of the Ljung-Box Q-Statistic ie. the level at which the null hypothesis of no serial correlation rejected.

Table A1. Parameter Estimates.
ARMA(p,q) Models
First Differences of Real Exchange Rates.

Canadian Dollar.

(p,q)	$\emptyset_1$	$\emptyset_2$	Θ_1	Θ_2	Q	AC	SC
(0,1)			0.269*		.954	-4.117	-4.083
(0,2)			0.271*	0.017	.953	-4.087	-4.021
(1,1)	0.243		0.025		.953	-4.087	-4.020
(1,2)	0.802		-0.525	-0.155	.956	-4.060	-3.960
(1,0)	0.267*				.952	-4.117	-4.084
(2,0)	0.268*	0.002			.956	-4.086	-4.018

D-Mark.

(p,q)	$\emptyset_1$	$\emptyset_2$	Θ_1	Θ_2	Q	AC	SC
(0,1)			0.275*		.983	-1.714	-1.681
(0,2)			0.275*	-0.054	.984	-1.687	-1.621
(1,1)	-0.537*		0.780*		.979	-1.691	-1.624
(1,2)	-0.782*		1.058*	0.147	.991	-1.669	-1.568
(1,0)	0.263*				.983	-1.708	-1.674
(2,0)	0.249*	-0.049			.983	-1.678	-1.641

Table A2 Continued.

Sterling.

(p,q)	ϕ_1	ϕ_2	θ_1	θ_2	Q	AC	AS
(0,1)			0.121		.474	-1.636	-1.603
(0,2)			0.120	0.050	.479	-1.606	-1.540
(1,1)	0.565		-0.463		.564	-1.627	-1.560
(1,2)	0.575		-0.467	-0.013	.561	-1.596	-1.496
(1,0)	0.121				.461	-1.653	-1.620
(2,0)	0.114	0.011			.478	-1.623	-1.556

Yen.

(p,q)	ϕ_1	ϕ_2	θ_1	θ_2	Q	AC	AS
(0,1)			0.232*		.670	-1.973	-1.940
(0,2)			0.251*	0.104	.612	-1.955	-1.888
(1,1)	0.440		-0.188		.601	-1.956	-1.888
(1,2)	0.403		-0.152	0.016	.593	-1.924	
(1,0)	0.267*				.638	-1.983	-1.950
(2,0)	0.250*	0.055			.585	-1.956	-1.889

Notes: Q is the marginal significance level of the Ljung-Box Q-statistic. AC is the Akaike Information Criterion and AS is the Schwarz Information Criterion. * indicates significance at the 5% level.