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Modeling Long Memory in REITs

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Modeling Long Memory in REITs

Abstract

One stylized feature of financial volatility impacting the modeling process is long memory. This paper examines long memory for alternative risk measures, observed absolute and squared returns for Daily Equity REITs and compares the findings for a market equity index. The paper utilizes a variety of tests for long memory finding evidence that REIT volatility does display persistence. Trading volume is found to be strongly associated with long memory. Results suggest differences in the findings with regard to REITs in comparison to the broader equity sector.

Keywords: Long Memory, FGARCH, REITs

Modeling Long Memory in REITs

1. Introduction

The continued development and increased investor awareness of the Real Estate Investment Trust (REIT) sector has led to a dramatic increase in daily trading in the sector in recent years. SNL Financial estimates that average daily volume in Equity REITs has increased from just over 7m shares in 1996 to over 40m shares in 2005. In addition, as the sector continues to mature and develop there will be increased interest in derivative products based on the sector. At present a number of OTC (over-the-counter) products are available and the Chicago Board Options Exchange (CBOE) provides traded options on the Dow Jones Equity REIT Index. The growth in both traded and OTC derivative products based on REITs furthers interest in the dynamics of the sector at higher frequencies such as daily intervals.

A large number of studies have examined the return behavior of REITs, but very few have examined volatility in the sector, and even fewer have studied volatility using high frequency data. Two early papers on REIT volatility (Devaney, 2001 and Stevenson, 2002a) both analyzed monthly data. The analysis conducted by Devaney (2001) was primarily concerned with the sensitivity of REIT returns and volatility to interest rates and was undertaken using a GARCH-M framework. The Stevenson (2002a) paper was, in contrast, concerned with volatility spillovers both across different REIT sectors, and between REITs and the equity and fixed-income markets. Four recent papers have examined various aspects of daily REIT volatility. Winniford (2003) concentrates on seasonality in REIT volatility. The author finds strong evidence that volatility in Equity REITs varies on a seasonal basis, with observed increased volatility in April, June, September, October and November. Cotter & Stevenson (2006) utilize a multivariate GARCH model to analyze dynamics in REIT volatility. Using a relatively short and quite distinct period of study (1999-2003) they find an increasing relationship between Equity REITs and mainstream equities in terms of both returns and volatility. Similar to the Devaney (2001) paper, Bredin et al. (2007) concentrate on the specific issue of interest rate sensitivity, examining the impact of unanticipated changes in the Fed Funds Rate on REIT volatility. The results show a significant response in REIT volatility to unanticipated rate changes.

However, in contrast to much of the broader equity market evidence, no evidence of asymmetry in the response is found. The final paper to have examined daily REIT volatility is the one most similar to the current paper. Najand & Lin (2004) utilize both GARCH and GARCH-M models in their analysis, reporting that volatility shocks are persistent.

Persistence in volatility is a common empirical finding in financial economics and is studied extensively in Taylor (1986). Whereas asset returns have largely been found to contain very little autocorrelation, it has been noted in a large number of papers across different asset classes that autocorrelation in various measures of volatility does exist at significant levels and remains over a large number of lags[1]. This effect, referred to as long memory, has been documented across a large sphere of the finance literature from macroeconomic series such as GNP (Diebold & Rudebusch, 1989) and exchange rate series (Baillie et al., 1996; Andersen & Bollerslev, 1997a, 1997b) both at low and at relatively high frequencies. Moreover, long memory is documented for equities using daily data (Ding et al., 1993, Ding & Granger, 1996).

This paper examines the long memory properties of alternative risk measures, observed absolute and squared returns for REITs, and compares these to the S&P500 composite index. Specifically, the long memory property and its characteristics are explored. The long memory property occurs when volatility persistence remains at large lags and the series are fractionally integrated. Fractionally integrated series are integrated to order d where $0 < d < 1$ unlike integrated series of order 1, where $d = 1$, and non-stationary series of order 0, where $d = 0$. Fractionally integrated series have observations far apart in time that may exhibit weak but non-zero correlation. Much focus has been on the absolute returns series, $|R_t|^k$, or a squared returns series, $[R_t^2]^k$, for different power transformations, $k>0$. This property adds to the general clustering condition usually referred to in the context of squared returns persistence originally modeled in Engle's (1982) ARCH paper. There are daily cycles to the dependence structure giving rise to daily seasonality that exhibits a slow decay of the autocorrelation structure but also involves a *u*-shaped cyclical pattern (Andersen & Bollerslev, 1997a, 1997b). In addition, Ding et al. (1993) indicate that this non-linear dependence is strongest for absolute volatility with a power transformation of k=1 and as a consequence they suggest that parametric modeling of volatility should focus on absolute returns rather than the commonly used squared returns.

This paper begins by examining the autocorrelation structure of the REIT returns and volatility. It then formally tests for the long memory property and measures the magnitude of the fractional integration parameter. In terms of model building, there are several approaches from linear and non-linear perspectives that could be applied. This paper fits two long memory volatility models, Fractionally Integrated GARCH (FIGARCH) and Fractionally Integrated Exponential GARCH (FIEGARCH) that allow for asymmetry. Baillie et al. (1996) find that these models have considerable success in modeling daily equity returns and we will investigate whether these GARCH models can capture the long memory properties of daily REIT returns. The paper examines the association between trading volume and volatility in the long memory volatility models. The paper proceeds as follows. In Section 2, long memory is discussed. The section incorporates a presentation and discussion of our GARCH models that are fitted to the daily series. Details of the series and data capture follow in Section 3. Section 4 presents the empirical findings. It begins by briefly describing the indicative statistics of the volatility series, followed by a thorough analysis of their long memory characteristics. In addition, the ability of the GARCH processes to model volatility persistence is presented. Finally, a summary of the paper and some conclusions are given in section 5.

2. Long Memory

Baillie (1996) shows that long memory processes have very strong autocorrelation persistence before differencing, thereby being non-stationary, whereas the first differenced series does not demonstrate persistence and is stationary. However, the long memory property of these price series is not evident from first differencing alone, but results from analysis of the associated risk measures. In fact, financial returns themselves have only been found to exhibit short memory, with significant first order dependence that dissipates rapidly over subsequent lags. Thus the finance literature has concentrated its analysis of long memory on the volatility series and we follow this convention.

Long memory properties may be investigated by focusing on the absolute returns series denoted $\left|R_t\right|^k$, or the squared returns series denoted $[R_t^2]^k$, and on their power transformations, where k>0.[2] Davidian & Carroll (1987) find that absolute realizations are more robust in the presence of fat-tailed observations often found in financial series than are their squared counterparts. Moreover, empirical analysis of financial time series suggests that the long memory feature dominates for absolute over squared realizations (see Ding & Granger, 1996). However, squared volatility is also important because it underpins commonly used risk measures such as standard deviation and variance.

Series with a long memory property have dependency between observations of a variable for a large number of lags so that Cov[R_{t+h}, R_{t-j}, $j\geq 0$] tends to zero slowly over a large number of lags h.[3] In particular, long memory in financial time series has concentrated on volatility realizations where unexpected shocks affect the time series for a long time. Thus confirmation of long memory properties for REITS would have major implications for the associated investment strategies that need to take account of the persistence and the dependence structure of REIT volatility. However, if the dependency between observations of a variable disappears for a small number of lags, h, such as in a stationary ARMA process, then the data is described as having a short memory property and Cov[R_{t+h}, R_{t-j}, $j\geq 0$] $\rightarrow$ 0. Formally, long memory is defined for a weakly stationary process if its autocorrelation function $\rho(\cdot)$ has a hyperbolic decay structure:

$$\rho(j) \sim C_j^{2d-1} \text{ as } j \rightarrow \infty, C \neq 0, 0 < d < \frac{1}{2} \tag{1}$$

where d represents the long memory parameter, or degree of fractional integration.

In contrast, short memory, or anti-persistence is evident if $-1/2 < d < 0$.

The corresponding shape of the autocorrelation function for a long memory process is hyperbolic if there is a relatively high degree of persistence in the first lag(s) that declines rapidly initially and is followed by a slower decline over subsequent lags.

Thus the decay structure remains strong for a very large number of time periods. Previous analysis of equity returns suggest that the long memory parameter, *d*, is generally found to be between 0.3 and 0.4 (e.g. Andersen and Bollerslev, 1997a; and Taylor, 2000).

The explanations for long memory are varied. One economic rationale results from the aggregation of a cross-section of time series with different persistence levels (Andersen & Bollerslev, 1997a; Lobato & Savin, 1998). Alternatively, regime switching may induce long memory into the autocorrelation function through the impact of different news arrivals (Breidt et al., 1998). The corresponding shape of the autocorrelation function is hyperbolic, beginning with a high degree of persistence that reduces rapidly over a few lags, but that slows down considerably for subsequent lags to such an extent that the length of decay remains strong for a large number of time periods. Also, with a slight variation, it may follow a slowly declining shape incorporating cycles that correspond to, for example, daily seasonality (Andersen et al, 1997a).

We test for the existence of long memory in REITs by using an informal analysis of autocorrelation dependence of our volatility series augmented by two formal tests for the existence of the property. We are interested in two issues: 1) whether or not REITs exhibit long memory properties, and 2) how the characteristics of the dependence structure of REITs compares to the broader equity market. The first test statistic is the parametric Modified Rescale Range (*R/S*) statistic developed by Lo (1991):

$$Q_n = \frac{1}{\hat{\sigma}_n(q)}\left[\max_{1\le k\le n}\sum_{j=1}^{k}\left(z_j-\bar{z}\right)-\min_{1\le k\le n}\sum_{j=1}^{k}\left(z_j-\bar{z}\right)\right] \quad (2)$$

Where $\hat{\sigma}_n$ is the estimate of the long run variance for sample size *n*. For any series *z*, we estimate the first *k* deviations of z_j, from its sample mean, $\bar{z}$. The Modified *R/S* allows for short memory in the time series but can distinguish if long memory exists separately. In contrast, the original *R/S* statistic (Hurst, 1951) is not able to distinguish between long and short memory. Given that microstructure issues such as bid-ask bounce induces first order correlation and short memory in returns series (Andersen et al., 2001) we may have both long and short memory characteristics in the series analyzed[4]. As a by product, we can also estimate the degree of fractional integration, denoted *R/S d*, by applying this test. *R/S d* describes the degree of fractional integration and allows us to compare it to different benchmarks. For example, we test whether or not it is in the domain $0<d<\frac{1}{2}$, and whether its magnitude differs between REIT and the broad market series.

In addition, long memory is investigated by using the semi-nonparametric Geweke & Porter-Hudak (1983) log-periodogram regression approach (*GPH*) as updated for non-Gaussian volatility estimates by Deo & Hurvich (2000). This adjustment is required given the fat-tailed and skewed behavior of financial time series. We also obtain semi-nonparametric estimates of the long memory parameter denoted *GPHd*. Given that $I(\omega_j)$ stands for the sample periodogram at the j^{th} fourier frequency, $\omega_j=2\pi j/T$, $j=1, 2, \ldots, [T/2]$, the log-periodogram estimator of *GPHd* is computed by regressing the logarithm of the periodogram estimate of the spectral density against the logarithm of ω over a range of frequencies ω:

$$\log\left[I\left(\omega_j\right)\right]=\beta_0+\beta_1\log\left(\varpi_j\right)+U_j \tag{3}$$

where $j=1, 2,\ldots, m$, and $d=-1/2\beta_1$. This approach allows us to determine if the long memory property is evident in the series analyzed, and it also gives estimates of the long memory parameter. As is the case in the *R/S* approach, estimates of *d* are dependent on the choice of *m*. We estimate the test statistic by using $m=T^{4/5}$ as suggested by Andersen et al. (2001). For our sample size 788 periodogram estimates are employed.

Given that long memory is not evident in financial returns series but is found in their volatility counterparts, we need to examine volatility models and their suitability in describing the persistence patterns of volatility in the REIT and broad market series. Whilst second order dependence is a characteristic of financial returns, usually modeled by a stationary GARCH process, these specifications have been questioned as to their ability to model the long memory property as well as their Fractionally Integrated GARCH counterparts do (Baillie, 1996). For instance, while stationary

GARCH models show that the financial returns volatility series based on $[R_t^2]$ and $|R_t|$ show strong persistence, they assume that the autocorrelation function follows an exponential pattern not corresponding to a long memory process. In particular, the correlation between $[R_t^2]$ and $|R_t|$ from stationary GARCH models and their power transformations remain strong for a large number of lags, with the rate of decline following a constant pattern (Ding et al., 1993), or an exponential shape (Ding & Granger, 1996). In contrast, a number of returns series, both $[R_t^2]$ and $|R_t|$, have been found to decay in a hyperbolic manner, declining rapidly initially and very slowly later (Ding & Granger, 1996).[5]

Turning to the set of conditional volatility models applied in this study, we first use the Fractionally Integrated GARCH (FIGARCH) model introduced by Baillie et al. (1996). These incorporate the standard time-varying volatility models and estimate the short run dynamics of a GARCH process. More importantly, they also measure the long memory characteristic of the data by estimating the degree of fractional integration *d*. First, the GARCH (*p*,*q*) process for time varying volatility σ_t^2 is given as:

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2 \tag{4}$$

With $\alpha(L)$ and $\beta(L)$ being polynomials of order q and p in the lag operator. The process can be written as an ARMA *(m, p)* process in ε_t^2 where $m = \max(p, q)$:

$$\{1 - \alpha(L) - \beta(L)\}\varepsilon_t^2 = \omega + \{1 - \beta(L)\}\nu_t \tag{5}$$

For $\nu_t = \varepsilon_t^2 - \sigma_t^2$ are the innovations in the conditional variance process.

Converting this expression back into a GARCH type process gives the FIGARCH (*p,d,q*) model:

$$\phi(L)(1 - L)^d \varepsilon_t^2 = \omega + \{1 - \beta(L)\}\nu_t \tag{6}$$

Where $\phi(L)=\{1-\alpha(L)-\beta(L)\}(1-L)^{-1}$ is of order *m - 1*, and all the roots of $\phi(L)$ and $\{1-\beta(L)\}$ lie outside the unit circle.

This model can be expanded to deal with additional stylized features of financial data. For instance, Black (1976) empirically notes a leverage effect where bad news tends to drive the price of an equity down thus increasing the debt-equity ratio (its leverage) and causing the equity to be more volatile. The leverage effect has an asymmetric impact on volatility with bad news having a greater impact than positive news. This leverage effect led Nelson (1991) to introduce the (Exponential) EGARCH process with a specific variable that distinguished between good news volatility and bad news volatility. If this variable's coefficient is negative, bad news shocks have a greater impact on volatility than good news shocks. Engle and Ng (1993) provide further support for the existence of leverage effects in equity data following their introduction of a news impact curve that graphically separates the impact of good new and bad news shocks on volatility. If the effects of news are long lasting as suggested by the fractionally integrated process we should also determine if the long memory exhibits asymmetric effects. In order to allow for asymmetric effects we also apply the Exponential version of the FIGARCH model, the FIEGARCH developed by Bollerslev & Mikkelsson (1996):

$$\log(\sigma_t^2)=\omega+\phi(L)^{-1}(1-L)^{-d}\left[1-\lambda(L)\right]g(\xi_{t-1}) \tag{8}$$

Where

$$g(\xi_t)=\theta\xi_t+\gamma\left[|\xi_t|-E|_t|\right]$$

Where volatility shocks follow an asymmetric function, and all the roots of $\phi(L)$ and $\lambda(L)$ lie outside the unit circle. The function has a slope of $\theta-\gamma$ when ξ is negative (market falls) and when ξ is positive (market rises) the slope is $\theta+\gamma$.

The residuals from both the FIGARCH and FIEGARCH processes were initially assumed to be from a conditionally fat-tailed process as is commonly in financial returns. We assume that the underlying data conditionally follow a student-*t* distribution as in Baillie & DeGennaro (1990).

3. Data

The data used in this paper consists of daily logarithmic returns for the period January 1 1990 through December 30 2005 totaling 4175 observations. During this time the popularity of REITS expanded dramatically with massive growth in investor awareness and interest that focused on the return and volatility characteristics of the sector. As we are interested in the long memory of the REIT sector we compare the findings to the broad equity market, as represented by the S&P 500 Composite.

Some descriptive statistics of the respective series are outlined in Table 1, which details the first four moments of each series and presents a test for normality. Separate analysis is completed for the returns series and the two proxies of volatility, absolute and squared volatility. Starting with returns we find that the average daily returns of both series are near zero but positive for the time frame analyzed suggesting that for the mainstream equity market the 1990s boom has slightly outweighed the downturn at the start of this decade. The reverse is true for the REIT sector, where strong recent performance outweighs underperformance during the late nineties. Overall however, the average risk of REITs approaches 1% and is almost identical to the S&P. The time series behaviour of both series is given in Figure 1. Here we can see an increase in volatility at the turn of the decade possibly related to, amongst other events, the Asian crises, September 11, and the technology bubble which was associated with greater turbulence and very poor return performance. In the last couple of years the markets have settled down to some degree.

Table 1 presents evidence on higher moments of returns. We observe negative skewness in both series, suggesting that the effects of large negative returns dominate their positive counterparts. Consistent with the literature, we also find excess kurtosis suggesting that the series exhibit a fat-tailed property. Combining these findings for skewness and kurtosis, we find that all series are non-normal using the Jarque-Bera test statistic. Therefore we need to incorporate this property in our modeling approach.

Turning to the proxies of the volatility series, we find that, similar to the returns series, the REIT index exhibits volatility similar to the S&P over the sample period. Average volatility (regardless of proxy) as reported in Table 1 is similar for both indexes. Looking at the plots in Figure 1 we see the behaviour of the volatility associated with the series' since 1990. We clearly see the volatility clustering property where periods of high volatility or low volatility can remain persistent for some time before switching. This property suggests that volatility on any day is dependent on the previous day's values and we will model this phenomenon using a GARCH process that specifically incorporates long memory. The lack of independence of either absolute or squared volatility is clearly seen by the lack of normality and excess kurtosis reported in Table 1 for both series. We also observe strong positive skewness for all series that is reasonably similar across the series. Comparing the two measures of volatility, we see that the magnitude of the squared realizations dominate their absolute counterparts but that the squared values are more prone to extreme outliers regardless of which series one examines.

4. Empirical Analysis

Our main focus in this paper is to examine the long memory properties of REITs and it is to this issue that we now turn. We begin by discussing the autocorrelation plots. We then consider formal testing of long memory and the magnitude of the long memory parameter. Finally, we outline our findings from applying two time-varying long memory volatility models. First, looking at dependence using the autocorrelation function (ACF), we provide plots over 100 lags for the volatility series. These are given in Figure 2 for absolute volatility and Figure 3 for squared volatility.[6] Ding et al. (1993) suggest that, since volatility is unobservable, long memory in equity data should be examined for different power transformations of the volatility proxy series. We follow this suggestion by examining the volatility series for 5 different power transformations [*k*=0.25, 0.5, 1, 1.5, 2]. This supports the analysis of Beran (1994) in his seminal work in the area. In its strictest sense, the ACF plots in Figures 2 and 3 do not offer conclusive evidence that REITs exhibit long memory in volatility, but they are much more striking in their support for the property in the broad market index. Moreover there is strong variation in the strength of the long memory feature for the different power transformations. The feature tends to be stronger for lower *k*. These findings are consistent for squared and absolute volatility. It is noticeable that REITs appear to display less persistence in volatility than the general market. The ACF plots for the S&P indexes report enhanced long memory. It can be seen that in general the first lag for the REIT volatility ACF's tends to be of a greater magnitude but that the persistence decays at a faster rate than it does for mainstream equities.

Table 2 reports details of the initial tests for long memory using the approaches described in Section 2. There is extensive evidence of long memory in both the absolute and squared volatility series. This is consistent across all of the different power transformations, although the effect is generally enhanced as *k* declines, particularly in the case of REITs. Furthermore, the magnitude of the test statistics is generally lower for the REIT sector than for the S&P. The findings from fitting the long memory volatility models are given in Table 3. The results generally show that both the FIGARCH and FIEGARCH models provide good fits for the data, and are broadly in line with expectations and the previously reported findings. The degree of fractional integration, as measured by the d-values, is in the range of 0.3-0.4 for the FIGARCH model for both series and is consistent with previous empirical evidence. In relation to the FIEGARCH model the significant negative leverage coefficients imply asymmetry in the long memory process with negative shocks affecting volatility more than positive shocks do, both in the short term and on a persistent basis.

In the literature trading volume is seen as an important explanatory variable for time varying volatility[7]. Therefore we investigate whether trading volume is related to the long memory characteristics of the volatility series. In Figure 4 we see the large increase in trading activity in equities and this is particularly pronounced for REITs, which had a very low volume. In Figure 5 we see that the change in trading volume shows similar patterns to that of the price series, namely, there is clustering of inactive (active) trading periods followed by active (inactive) trading periods.

Taking the volume data we fit a FIEGARCH model and results are reported in Table 4 with the associated time series plots given in Figure 6.[8] We are trying to determine whether volume is an important mixing variable for long memory in volatility.

Trading volume is clearly an important explanatory variable for our conditional volatility with a strong statistical significance. A 1% change in REIT volume is associated with a 0.01% change in its volatility and this effect is approximately doubled for the S&P series. Interestingly, by including the change in volume variable we see a major revision in the volatility specification with GARCH and ARCH coefficients being considerably amended in comparison to the FIEGARCH model results excluding volume. Overall, changes (increases) in volume are strongly associated with the long memory in property found in REIT (and market) data.

5. Conclusion

This paper has examined long memory properties in the volatility of the REIT sector at daily frequencies. As the sector develops and daily trading volume increases we can expect increased interest in the daily dynamics of REITs, and of derivative instruments based on the sector. The paper shows that, as is the case in the general equity market, volatility persistence occurs in the REIT sector. However, there is evidence that long memory in REIT volatility is not of the same magnitude as that observed in the S&P 500 index. Moreover, trading volume is an important explanatory variable in modeling long memory of REIT volatility.

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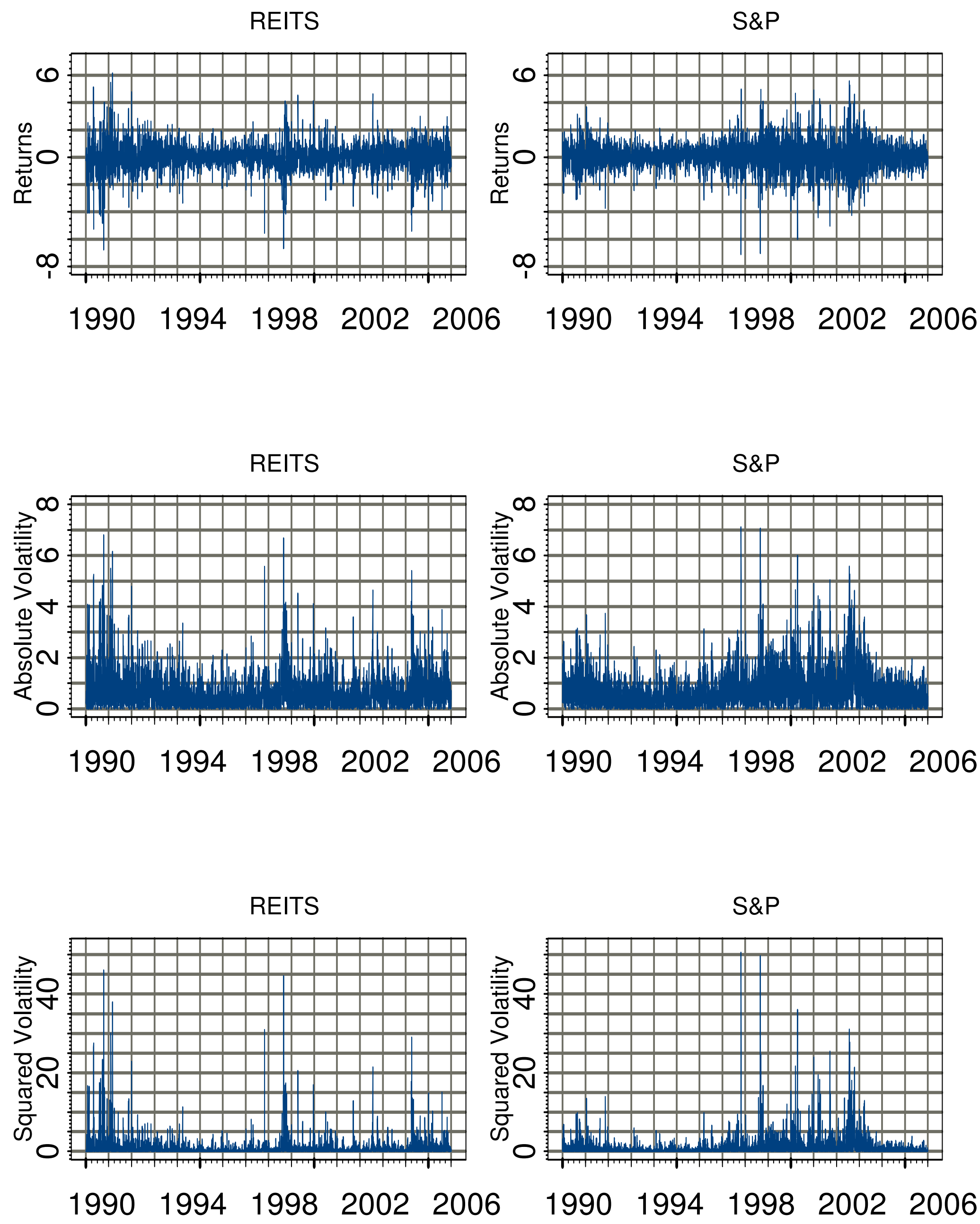
Figure 1: Time Series Plots of Daily Series



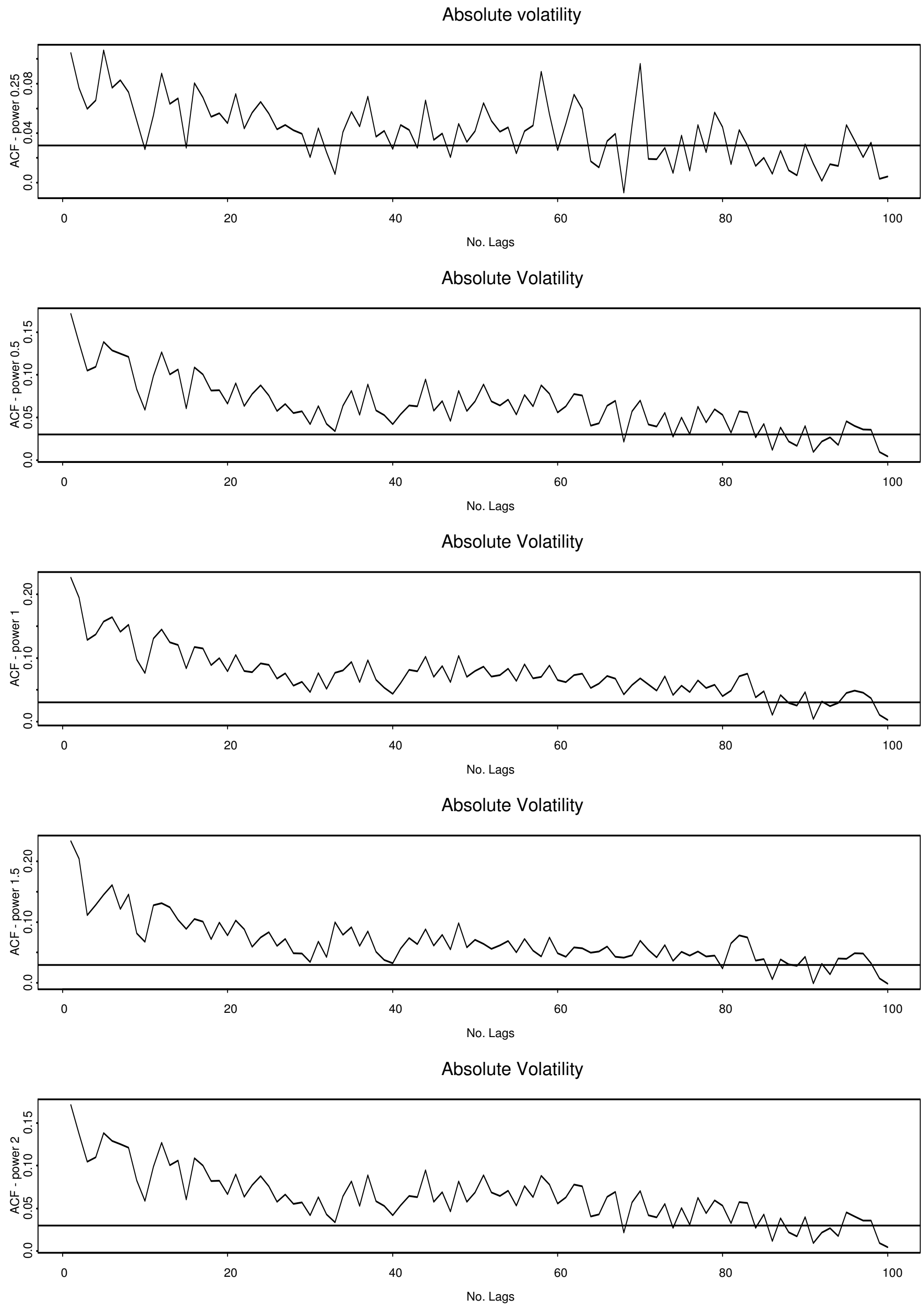
Notes: The plots show the time series behaviour of daily percentage values for the returns, absolute returns (absolute volatility) and squared returns (squared volatility) series' between 1990 and 2005 inclusive.

Table 1: Summary Statistics for Daily Series

	REITs	S&P 500
Panel A: Returns		
Mean	0.029	0.030
Std Dev	0.944	0.997
Skewness	-0.256*	-0.100*
Kurtosis	8.297*	7.011*
Normality	4925.84*	2805.12*
Panel B: Absolute Volatility		
Mean	0.659	0.701
Std Dev	0.677	0.709
Skewness	2.617*	2.244*
Kurtosis	14.55*	11.66*
Normality	27963*	16541.8*
Panel C: Squared Volatility		
Mean	0.893	0.994
Std Dev	2.405	2.432
Skewness	8.535*	8.396*
Kurtosis	109*	119*
Normality	2003006*	2387327*

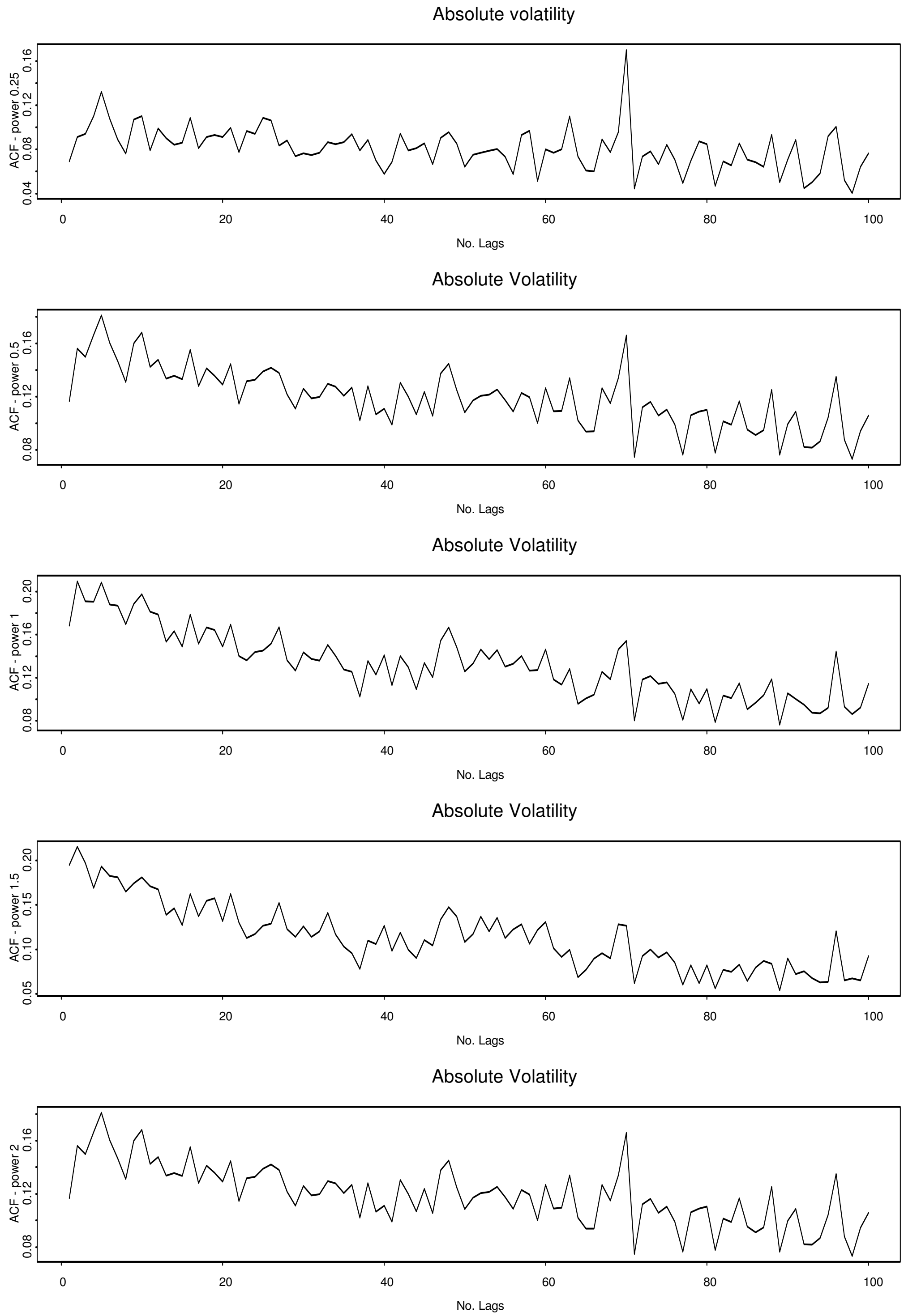
Notes: Estimates are given for returns (Panel A), for Absolute Volatility (Panel B) and Squared Volatility (Panel C). Mean and standard deviations are expressed in percentage form. Skewness and kurtosis are tested using Fisher's G and Fisher's G2 statistics respectively. Normality is tested for using the Jarque-Bera test statistic. The skewness, kurtosis and normality statistics have a value of 0 for a normal distribution. All skewness, kurtosis and normality statistics are significant at 5% significance levels indicated by *.

Figure 2a: Plots of Autocorrelation Values for REIT Daily Absolute Volatility



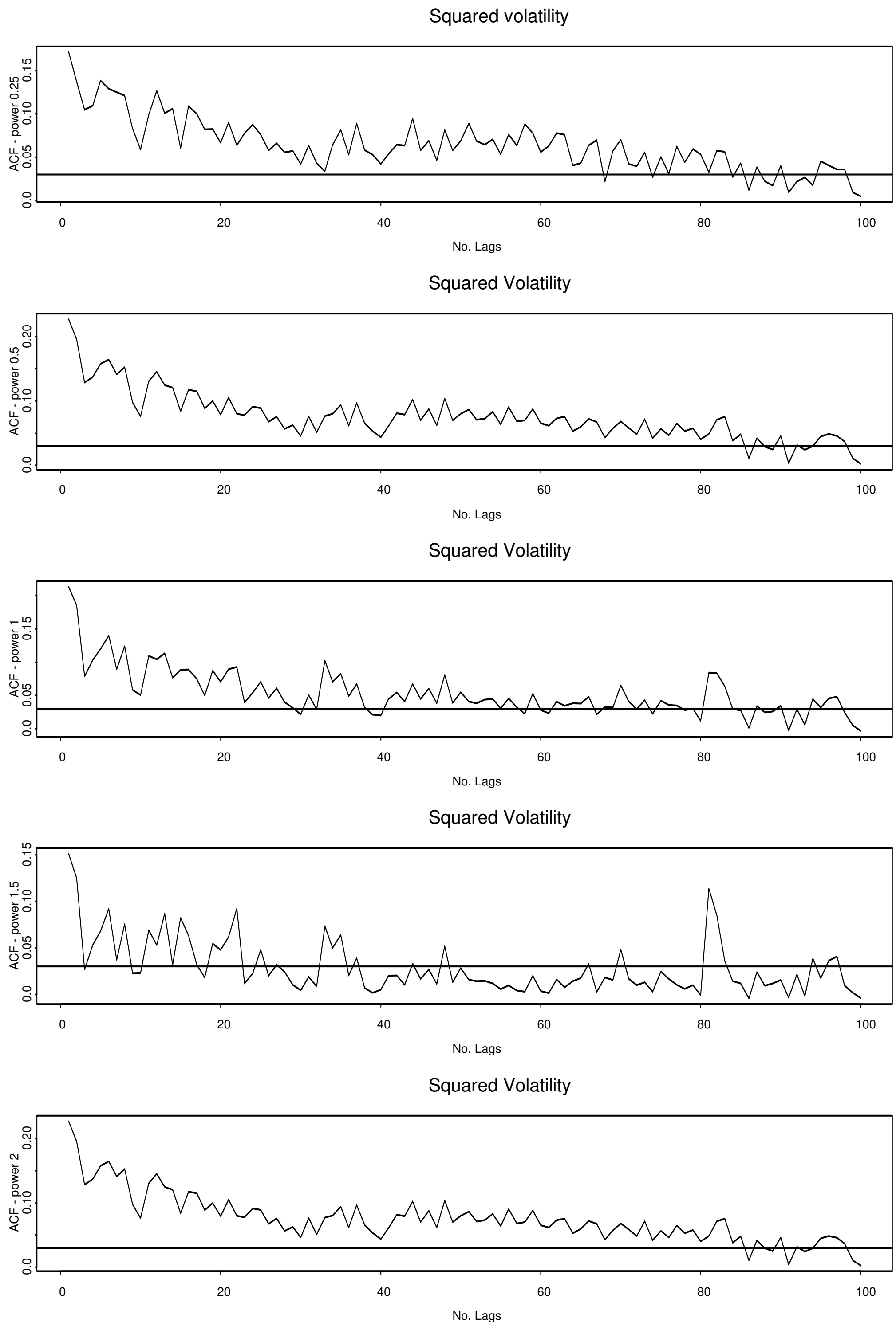
Notes: The plots show the dependence in REIT daily absolute volatility for 5 different power transformations [k=0.25, 0.5, 1, 1.5, 2] using the autocorrelation function for 100 lags. All plots include confidence bands using the 95% critical values ($\pm 1.96/\sqrt{n}$) so significance occurs at ± 0.03 and these are imposed where appropriate.

Figure 2b: Plots of Autocorrelation Values for S&P Daily Absolute Volatility



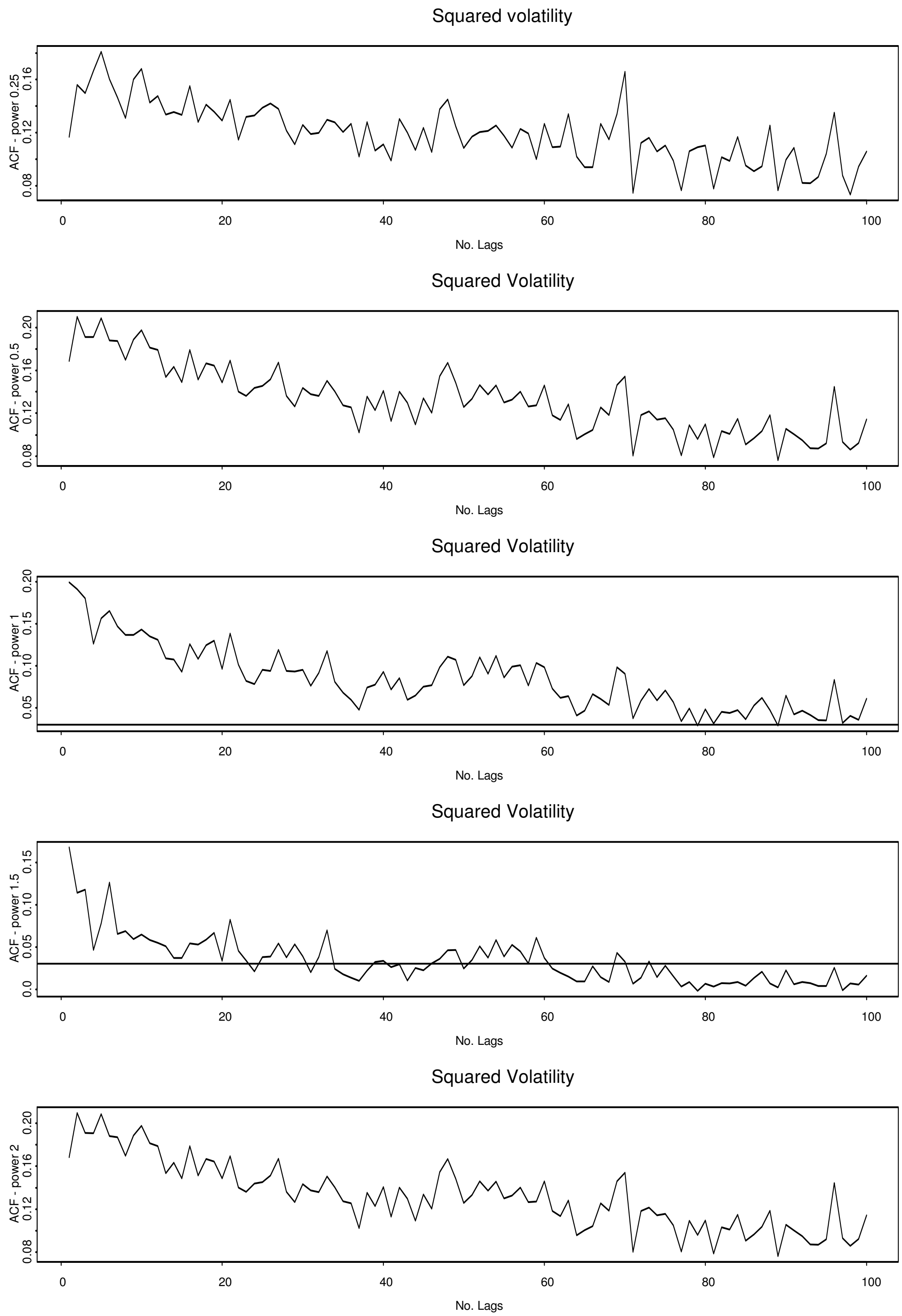
Notes: The plots show the dependence in S&P daily absolute volatility for 5 different power transformations [*k*=0.25, 0.5, 1, 1.5, 2] using the autocorrelation function for 100 lags. All plots include confidence bands using the 95% critical values ($\pm$ 1.96/√n) so significance occurs at $\pm$ 0.03 and these are imposed where appropriate.

Figure 3a: Plots of Autocorrelation Values for REIT Daily Squared Volatility



Notes: The plots show the dependence in REIT daily squared volatility for 5 different power transformations [*k*=0.25, 0.5, 1, 1.5, 2] using the autocorrelation function for 100 lags. All plots include confidence bands using the 95% critical values ($\pm 1.96/\sqrt{n}$) so significance occurs at ± 0.03 and these are imposed where appropriate.

Figure 3b: Plots of Autocorrelation Values for S&P Daily Squared Volatility



Notes: The plots show the dependence in S&P daily squared volatility for 5 different power transformations [*k*=0.25, 0.5, 1, 1.5, 2] using the autocorrelation function for 100 lags. All plots include confidence bands using the 95% critical values ($\pm 1.96/\sqrt{n}$) so significance occurs at $\pm$ 0.03 and these are imposed where appropriate.

Table 2: Long Memory Diagnostics for Daily Series

		R/S	GPH	R/S d	GPHd
Panel A: Absolute Volatility					
REIT	k = 0.25	4.1083**	3.703**	0.139796	0.2791813
	0.5	4.2062**	4.0009**	0.162991	0.3362795
	1	3.8566**	4.5319**	0.174352	0.3619588
	1.5	3.5013**	4.1893**	0.168648	0.3556661
	2	3.1706**	3.7235**	0.15553	0.312547
S&P	k = 0.25	5.7324**	5.5284**	0.12405	0.3157433
	0.5	6.0847**	5.9749**	0.138839	0.385766
	1	5.9084**	5.7912**	0.144529	0.4221734
	1.5	5.4511**	5.4709**	0.142107	0.4150753
	2	4.8572**	4.7648**	0.134813	0.3655939
Panel B: Squared Volatility					
REIT	k = 0.25	4.2062**	4.0009**	0.162991	0.3362795
	0.5	3.8566**	4.5319**	0.174352	0.3619588
	1	3.1706**	3.7235**	0.15553	0.312547
	1.5	2.5481**	2.8687**	0.129716	0.2333424
	2	2.0718*	2.2246*	0.110291	0.1758527
S&P	k = 0.25	6.0847**	5.9749**	0.138839	0.385766
	0.5	5.9084**	5.7912**	0.144529	0.4221734
	1	4.8572**	4.7648**	0.134813	0.3655939
	1.5	3.6671**	3.5083**	0.117991	0.2193724
	2	2.725**	1.7617	0.103408	0.1310806

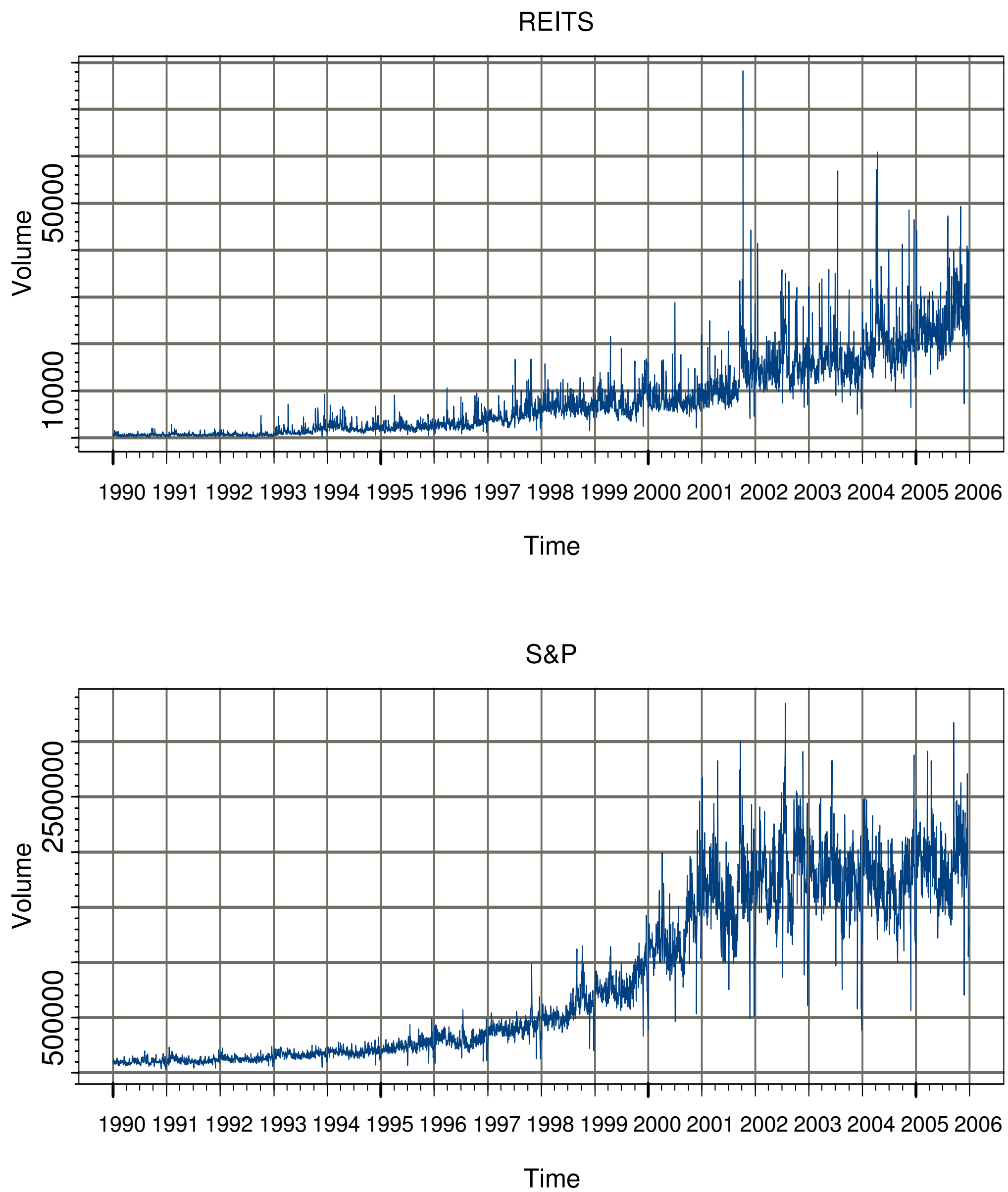
Notes: Further technical details of the long memory tests and parameter estimates are given in the text. The R/S test is the modified R/S statistic (Lo, 1991). The GPH test is the Geweke & Porter-Hudak (1983) semi-nonparametric statistic. The R/S *d* is the R/S long memory parameter. The *GPHd* is the periodogram long memory parameter. Estimates are given for Absolute Volatility (Panel A) and Squared Volatility (Panel B) with different power transformations, [*k*=0.25, 0.5, 1, 1.5, 2]. A single asterisk represents significance at the 5% level whereas two represents significance at the 1% level.

Table 3: Fractionally Integrated GARCH Models for Daily Return Series

	REITs		S&P 500	
	Coefficient	**p-value**	**Coefficient**	**p-value**
Panel A: FIGARCH (1,1)				
A	0.04641***	3.66E-06	0.02803***	2.10E-07
GARCH(1)	0.50543***	1.01E-11	0.54397***	0.00E+00
ARCH(1)	0.37822***	1.69E-08	0.18921***	8.22E-15
d	0.3175***	0.00E+00	0.39632***	0.00E+00
LM (12)	20.9*	0.05189	7.942	0.7896
Q^2 (12)	20.29*	0.06178	20.32*	0.06127
Panel B: FIEGARCH (1,1)				
A	-0.24867***	0.00E+00	-0.10651***	0.00E+00
GARCH(1)	0.13449**	2.72E-02	0.452***	7.38E-08
ARCH(1)	0.33099***	0.00E+00	0.13623***	0.00E+00
Leverage	-0.05662***	1.41E-08	-0.0983***	0.00E+00
d	0.59397***	0.00E+00	0.63067***	0.00E+00
LM (12)	17.94	0.1176	9.205	0.6853
Q^2 (12)	17.61	0.1279	9.118	0.6928

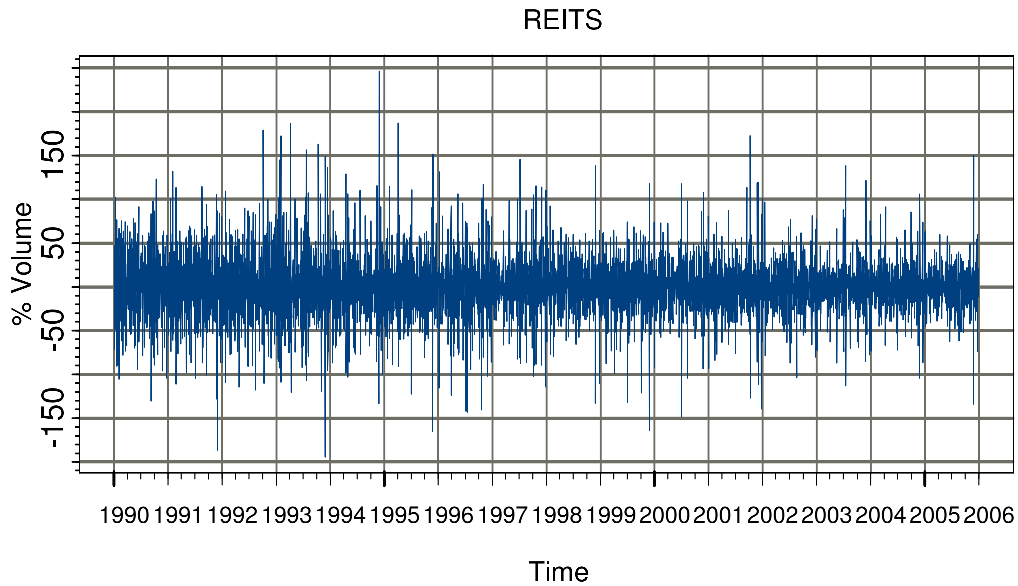
Notes: Coefficients and marginal significance levels for the FI(E)GARCH models are presented with full details of the models given the text. The respective optimal model is chosen based on Akaike's (AIC) and Schwarz's (BIC) selection criteria. A single asterisk denotes statistical significance at the 10%, two denotes statistical significance at the 5% level, while three denotes statistical significance at the 1% level. The FIEGARCH model incorporates a leverage variable that is significant for both indexes. Significant (G)ARCH effects are reported for both indexes. The long memory parameter, *d*, is tested for statistical significance from 0 and occurs in all cases. The diagnostics are supportive of a good fit for both fractionally integrated models. The diagnostics used are the $Q^2(12)$ Ljung-Box test on the squared standardised residual series and Engle's (1982) LM test for up to 12^{th} order ARCH effects on the squared standardised returns series.

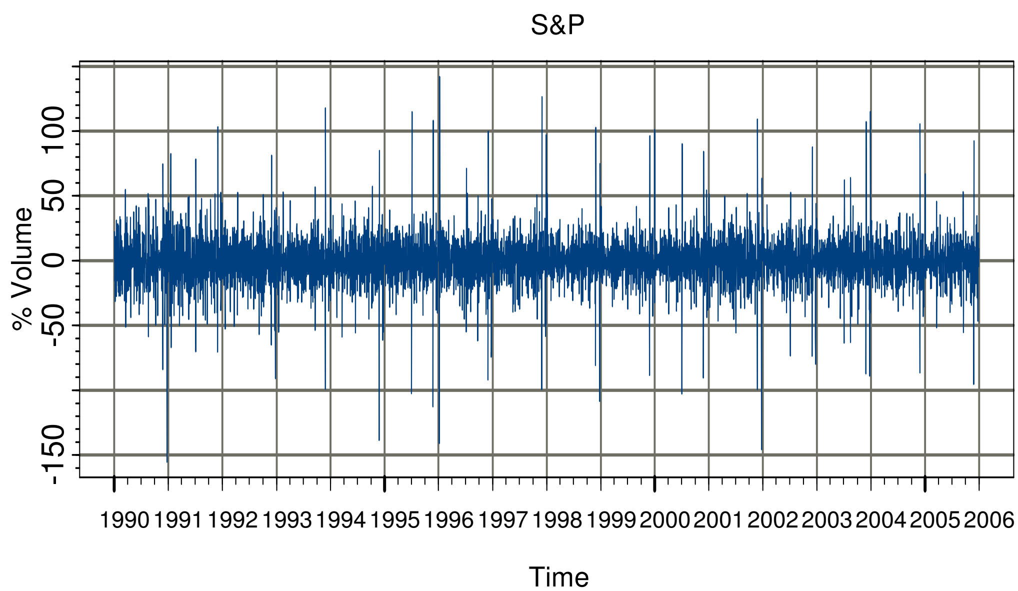
Figure 4: Time Series Plots of Daily Volume Series



Notes: The plots show the time series behaviour of daily trading volume for both indexes between 1990 and 2005 inclusively.

Figure 5: Time Series Plots of Daily Change in Volume Series





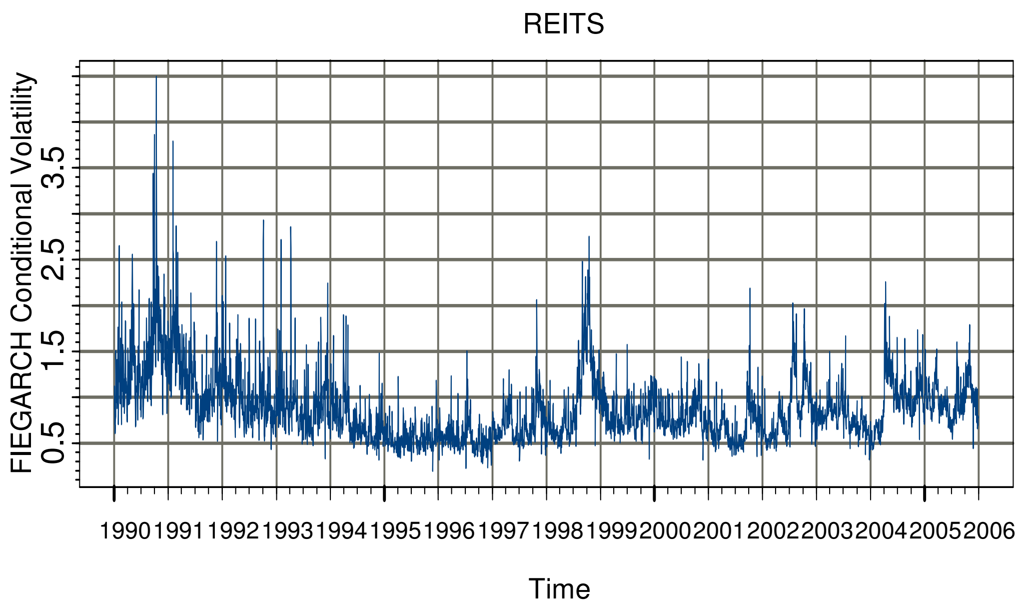
Notes: The plots show the time series behaviour of daily percentage values for the change in volume for both indexes between 1990 and 2005 inclusive.

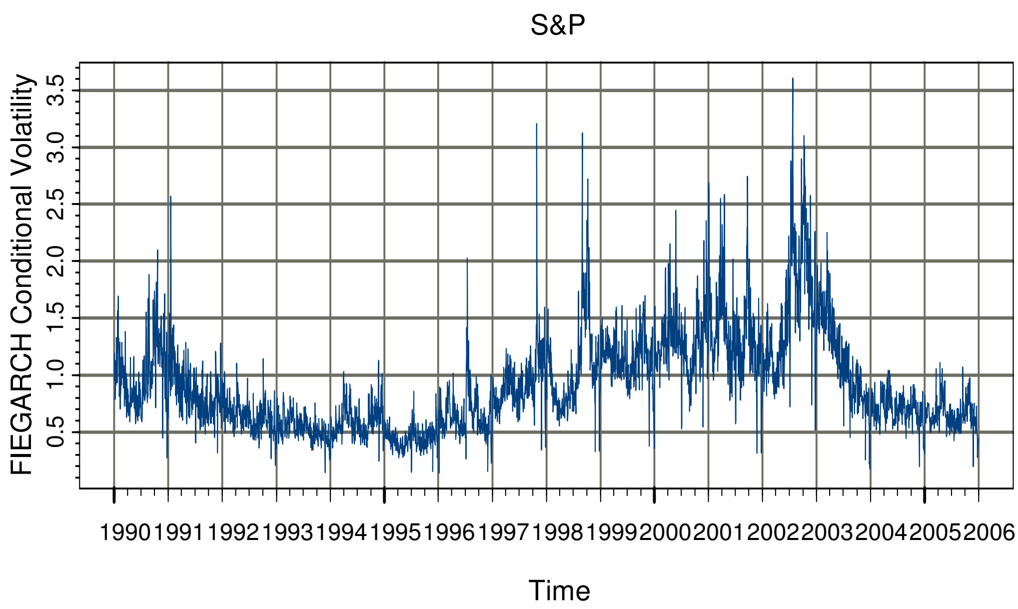
Table 4: Fractionally Integrated EGARCH Model with Volume

	REITs		S&P 500	
	Coefficient	**p-value**	**Coefficient**	**p-value**
FIEGARCH (1,1)				
A	-0.14787***	0.00E+00	-0.08081***	2.00E-15
GARCH(1)	0.17908***	9.65E-04	0.08229*	8.82E-02
ARCH(1)	0.19329***	0.00E+00	0.1001***	2.07E-14
Leverage	-0.03489***	9.13E-08	-0.06085***	1.45E-13
Volume	0.01184***	0.00E+00	0.02012***	0.00E+00
d	0.77339***	0.00E+00	0.86554***	0.00E+00
LM (12)	20.84*	0.05277	21.21*	0.04734
Q^2 (12)	20.71*	0.05477	21.95*	0.03811

Notes: Coefficients and marginal significance levels for the FIEGARCH model are presented with full details of the model given the text. The respective optimal model is chosen based on Akaike's (AIC) and Schwarz's (BIC) selection criteria. A single asterisk denotes statistical significance at the 10%, two denotes statistical significance at the 5% level, while three denotes statistical significance at the 1% level. The FIEGARCH model incorporates both volume and leverage variables. Significant (G)ARCH effects are reported for both indexes. Both volume and leverage variables are significant for both indexes. The long memory parameter, *d*, is tested for statistical significance from 0 and occurs in all cases. The diagnostics are supportive of a good fit for the fractionally integrated model. The diagnostics used are the $Q^2(12)$ Ljung-Box test on the squared standardised residual series and Engle's (1982) LM test for up to 12^{th} order ARCH effects on the squared standardised returns series.

Figure 6: Time Series Plots of FIEGARCH Daily Conditional Volatility Series





Notes: The plots show the time series behaviour of daily percentage conditional volatility for both indexes between 1990 and 2005 inclusive. Conditional volatility was obtained from fitting the FIEGARCH model with volume included as an explanatory variable.

Endnotes:

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[1] Two recent papers to have examined persistence and mean reversion in REIT and international real estate security returns are Kleiman et al. (2002) and Stevenson (2002b).

[2] As volatility is a latent unobservable variable proxies of volatility such as absolute and squared returns are examined in the literature.

[3] For an excellent treatment of long memory processes see Beran (1994).

[4] Extensions of the Hurst (1951) *R/S* statistic involve replacing the sample standard deviation of the series, *Z*, with the square root of the Newey-West estimate of the long run variance.

[5] One such example of a relatively successful application of standard GARCH models is the application of the APARCH model (see Cotter, 2005; for an example). The APARCH specification, developed by Ding et al. (1993) nests seven commonly applied GARCH models. However, the specification has an exponential decline structure that shows strong dependence but is not fully consistent with the long memory decline structure.

[6] We also examine dependence of returns formally through long memory tests and informally through ACF plots. In line with previous studies we find negligible evidence to support the presence of long memory of returns. Results are available on request.

[7] See Lamoureux & Lastrapes (1990). They find that trading volume reflects the dependence in information flows to the market that feeds directly into price volatility.

[8] We avoid fitting the FIGARCH specification as our exogenous variable, change in trading volume, is not always positive as can be seen from the time series plot and would result in negative conditional variance values. Also we have already documented asymmetric effects in the long memory of volatility.