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<th>The observational equivalence of the Ricardian and Heckscher-Ohlin explanations of trade patterns</th>
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<td><strong>Authors(s)</strong></td>
<td>Neary, J. Peter</td>
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<tr>
<td><strong>Publication date</strong></td>
<td>1984-04</td>
</tr>
<tr>
<td><strong>Series</strong></td>
<td>UCD Centre for Economic Research Working Paper Series; No. 22</td>
</tr>
<tr>
<td><strong>Publisher</strong></td>
<td>University College Dublin. School of Economics</td>
</tr>
<tr>
<td><strong>Item record/more information</strong></td>
<td><a href="http://hdl.handle.net/10197/1411">http://hdl.handle.net/10197/1411</a></td>
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<td><strong>Notes</strong></td>
<td>A hard copy is available in UCD Library at GEN 330.08 IR/UNI</td>
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THE OBSERVATIONAL EQUIVALENCE OF THE RICARDIAN
AND HECKSCHER-OHLIN EXPLANATIONS OF TRADE PATTERNS

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Working Paper No. 22

April 1984

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Research is given at the end of this paper.
This paper reviews the issue, first considered by J.L. Ford, of the observational (as opposed to the logical) equivalence of the Ricardian and Heckscher-Ohlin explanations of trade patterns. A necessary and sufficient condition for local equivalence is derived and interpreted: loosely speaking, capital intensity and elasticities of substitution must be positively associated across sectors. However, it is shown that global equivalence may be prevented either by factor intensity reversals or by labour productivity reversals. Finally, it is noted that conditions which guarantee equivalence appear to be empirically plausible.
THE OBSERVATIONAL EQUIVALENCE OF THE RICARDIAN AND
HECKSCHER-OHLIN EXPLANATIONS OF TRADE PATTERNS

The Ricardian and Heckscher-Ohlin theories are usually seen as alternative explanations of the pattern of trade. The former emphasises inter-country differences in technology (specifically, in labour productivity), the latter differences in factor endowments as the basis for comparative advantage and therefore as the fundamental determinant of trade patterns. However, in a series of papers, J.L. Ford has argued that under certain conditions the two theories are in fact equivalent. This claim has generated considerable controversy, of which the exchange between Lloyd and Ford in this issue is the latest example, and in the present note I attempt to give a self-contained exposition of the issues involved.

At the outset, it should be stressed that what is at stake is not the logical but the observational equivalence of the two theories. Under strict Ricardian assumptions, comparative advantage (meaning inter-country differences in autarky prices) and therefore trade patterns are determined by exogenous differences between countries in relative labour productivities. By contrast, Ford and the other participants in this debate have assumed throughout that all the assumptions needed to guarantee both the Heckscher-Ohlin and the factor-price equalisation theorems hold. In particular, the available technology in a given industry is assumed to be identical in both countries. Hence, the techniques in use in an industry, and so its observed labour productivity, can differ between countries only as a result of inter-country differences in autarky factor prices. In the light of this, differences in labour productivities may be correlated with differences in autarky commodity prices, but they cannot cause them.
The simplest context in which to illustrate "equivalence", as the term is used by Ford, is that of the textbook two-country Heckscher-Ohlin world, in which two commodities (labelled X and Y) are produced competitively using two factors (labelled labour, L, and capital, K), with identical technology in both countries. To begin with, we assume that there are no factor-intensity reversals; i.e., that one sector is relatively capital-intensive at all relevant factor-price ratios. Since all the conditions for the Heckscher-Ohlin theorem apply, the country in which capital is relatively abundant will have a lower autarky relative price of the more capital-intensive commodity, and so will export that commodity if the two countries engage in free trade. Equivalence now requires that an observer who looked only at relative labour productivities in autarky in the two countries and who assumed (incorrectly) that these were the basis for comparative advantage, would correctly predict the pattern of trade. Such an observer would predict that each country would export the commodity with the higher labour productivity in autarky. This gives the following definition:

**DEFINITION:** Under the assumptions given, the Ricardian and Heckscher-Ohlin explanations of trade patterns are equivalent if and only if the Heckscher-Ohlin theorem holds and the country with the lower value of $p$ in autarky also has the higher value of $\Pi$ in autarky.

Here $p$ denotes the relative price of commodity $X$ (i.e., $p = P_X/P_Y$) and $\Pi$ denotes the ratio of labour productivity in $X$ to that in $Y$; i.e., $\Pi = a_{LY}/a_{LX}$, where $a_{Lj}$ is the labour requirement per unit of output, and therefore the inverse of labour productivity, in sector $j$.

Using the approach of Jones (1965) it is now straightforward to derive a necessary and sufficient condition for local equivalence; i.e., for small differences between countries in the autarky relative price of $X$ to be negatively associated with small differences in the autarky value of the productivity ratio $\Pi$:

**LOCAL EQUIVALENCE THEOREM:** A necessary and sufficient condition for local equivalence in the sense defined is that the product of the elasticity
of substitution and the share of capital in the capital-intensive sector exceed the corresponding product in the other sector.

**Proof:** (1) Competition requires that price equal unit cost in both sectors. Differentiating the zero-profit conditions and rearranging gives the standard relationship between changes in relative factor prices and changes in relative commodity prices:

\[
\hat{P} = \theta \hat{\omega}
\]  

(1)

In this equation, \(\omega\) is the factor price ratio \(\omega = w/r\) and \(\theta\) is a measure of relative factor intensity, which is positive if and only if sector \(Y\) is more capital-intensive:

\[
\theta = \theta_{kY} - \theta_{kX}
\]  

(2)

where \(\theta_{ij}\) is the share of payments to factor \(i\) in the value of output of sector \(j\).

Next, from Jones (1965), the change in labour productivity in each sector is related to the change in the wage-rental ratio by a simple multiple of that sector's elasticity of factor substitution:

\[
- \hat{\alpha}_{Lj} = \theta_{k_j} \sigma_j \hat{\omega}, \quad j = X, Y
\]  

(3)

Hence:

\[
\hat{\pi} = \hat{\alpha}_{LY} - \hat{\alpha}_{LX} = -\theta^* \hat{\omega},
\]  

(4)

where:

\[
\theta^* = \theta_{kY} \sigma_Y - \theta_{kX} \sigma_X.
\]  

(5)

Combining (1) and (4) yields:

\[
\hat{P} = -\frac{\theta}{\theta^*} \hat{\pi}.
\]  

(6)
Hence, a necessary and sufficient for the required inverse relationship between \( p \) and \( \pi \) is that \( \theta \) and \( \theta^* \) have the same sign, which proves the proposition. For example, if sector \( Y \) is relatively capital-intensive, \( \theta \) is positive and equivalence requires that:

\[
\theta_{KX} \sigma_X < \theta_{KY} \sigma_Y
\]  

(7)

Q.E.D.

As a corollary, we may note that a sufficient condition for equivalence is that the relatively capital-intensive sector have the higher elasticity of substitution.

An intuitive explanation for this result may be given as follows. Before trade takes place, the country in which labour is relatively scarce has the higher wage-rental ratio. This induces substitution away from labour towards capital in both sectors, and so labour productivity in autarky is higher in both sectors in that country than in the corresponding sector abroad. However, it is relatively higher in the sector with the higher elasticity of substitution. If the Ricardian explanation of trade is to hold for this country, it must export this good (i.e., the one with the relatively higher labour productivity). But, since the Heckscher-Ohlin theorem holds by hypothesis, the country, being relatively capital-abundant, will export the relatively capital-intensive good. Hence, for the two theories to be observationally equivalent requires that there be a positive association between capital intensity and elasticities of substitution. The precise form which this association must take is given by equation (6).

II

Since the proof of the proposition relies on differential calculus, it is clear that it can only hold locally, and this point is stressed by Lloyd (1984): Sufficiently large changes in factor prices may lead to factor intensity reversals which change the sign of \( \theta \). It might be argued in response that this is not relevant to the issue of equivalence,
since as is well known the Heckscher-Ohlin theorem itself ceases to hold if factor-intensity reversals occur. However, the situation is more complicated than this, since a change in factor prices may also lead to a labour productivity reversal, i.e., a change in the sign of $\theta^*$. 

These points are illustrated in Figures 1 and 2.[2] Reading from the left, the first two panels of Figure 1 give the familiar Samuelson-Harrod-Johnson depiction of the relationships between commodity prices, factor prices and factor intensities. As drawn, the diagram assumes a single factor-intensity reversal and a higher elasticity of substitution in sector $X$ than in sector $Y$. Hence, for wage-rental ratios above $\omega_A$, sector $X$ is relatively capital-intensive and so $p$ and $\omega$ are negatively related. Moreover, from the corollary to the proposition in the last section, it also follows that $\Pi$, the ratio of labour productivities in sectors $X$ and $Y$, is negatively related to $\omega$ in this range, as the third panel of Figure 1 illustrates. For wage-rental ratios above $\omega_A$, therefore, there is a negative relationship between $p$ and $\Pi$, as the segment $MA'$ of the curve in Figure 2 illustrates.

By contrast, for wage-rental ratios below $\omega_A$, sector $Y$ is relatively capital-intensive and so $p$ is an increasing function of $\omega$. Hence, if $\theta^*$ does not change sign, the relationship between $p$ and $\Pi$ becomes an increasing one, corresponding to the segment $A'B'$ in Figure 2. Finally, the diagrams assume that at some still lower wage-rental ratio, $\omega_B$, $\theta^*$ also changes sign.[3] Since $p$ is still an increasing function of $\omega$ whereas $\Pi$ is now a decreasing function of $\omega$, the implied relationship between $p$ and $\Pi$ is once again decreasing, corresponding to the segment $B'N$ in Figure 2.

Whether or not equivalence holds in this world now depends on the endowment ratios of the two countries. Three possibilities may be distinguished:

(i) Both countries' endowment ratios above $k_A$: In this case, the Heckscher-Ohlin theorem holds, provided the countries' endowment ratios are sufficiently close that each produces both goods after trade is opened up. Each country's export good is also the
good which has the higher productivity ratio in autarky. Hence, equivalence is satisfied in this range.

(ii) One country’s endowment ratio above $k^A$, the other below: In this case, the Heckscher-Ohlin theorem cannot hold, because of a factor-intensity reversal. However, it is still possible (though not inevitable) that each country will export the good with the higher labour productivity in autarky, so the Ricardian explanation of trade patterns may appear to hold.

(iii) Both countries’ endowment ratios below $k^A$: As in case (i), the Heckscher-Ohlin theorem now holds. However, equivalence is not inevitable. Many configurations are possible, depending on whether the autarky factor price ratio in each country lies above or below $\omega_B$. If it is below $\omega_B$ in both countries, then the relevant segment of the curve in Figure 2 is B’N and equivalence holds. However, if one is above and one is below $\omega_B$, equivalence may not hold. And if both are above $\omega_B$ (though, of course, below $\omega_A$), the relevant segment is A’B’ and equivalence does not hold: each country will export the good with the lower labour productivity in autarky.

III

The last section has illustrated that many different outcomes are possible, even with only a single factor-intensity reversal and a single labour productivity reversal. Multiple reversals of both types are quite possible in principle, so making equivalence even less likely. However, it is reasonable to ask what light empirical evidence throws on this issue. This has been investigated by Falvey (1981) who reviewed several sets of econometric results and found strong (though not overwhelming) evidence for a positive association between $\theta$ and $\theta^*$. Advocates of equivalence can therefore derive comfort from the fact that conditions which ensure it appear to be consistent with the available empirical evidence.
Finally, what is the significance of all this? Although I have stressed that this controversy is concerned with observational equivalence, it is only relevant if pre-trade relative labour productivities can be observed. For, if not, then under the assumptions made free trade will equalise factor prices and therefore labour productivities in the same sector in each country; and so there is no basis for testing the Ricardian theory. The obvious facts that the world has more than one factor of production and that labour productivities in a given industry are not equalised by trade merely emphasise that neither of these simple theories can be expected to provide a complete explanation of trade patterns in the real world. My own view of the practical value of simple models such as these is rather that they highlight mechanisms which may be expected to operate, alongside many other mechanisms, in more realistic environments. A variety of distinct simple models should therefore be seen as a welcome feature, since each throws light on different questions, rather than as a drawback to be overcome by embedding one model within another or within some grand all-encompassing model.[4] From this perspective, equivalence in the sense of Ford is an interesting theoretical property, but to organise my thinking in the hope of understanding the real world better, my motto remains "vive la difference"!
FOOTNOTES

[1] This proof has been independently derived on at least three occasions: in a referee report by the present author on an early version of Ford (1982), by Falvey (1981) and in the first version of Deardorff (1984).

[2] These diagrams are not drawn to scale. Examples of Figure 2 drawn for actual parameter values are given in an appendix to this note, available on request from the author.

[3] Note that $\Theta$ and $\Theta^*$ must change sign at different wage-rental ratios (and so there must be a finite segment such as A'B') unless the elasticities of substitution in the two sectors are equal (in which case the curves $k_X$ and $k_Y$ would either be parallel to or coincident with one another).

REFERENCES


Figure 1: Illustrative Relationships between Commodity Prices, Factor Prices, Factor Intensities and Labour Productivities
Figure 2: Relationship between Relative Commodity Prices and Relative Labour Productivities implied by Figure 1