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Strategic and Rent Extracting Tariffs in the Presence of Persuasive Advertising

by

Dermot Leahy
Department of Economics
University College Dublin

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UNIVERSITY COLLEGE DUBLIN

Department of Economics

BELFIELD, DUBLIN 4, IRELAND
STRATEGIC AND RENT EXTRACTING TARIFFS
IN THE PRESENCE OF PERSUASIVE ADVERTISING

Dermot Leahy
University College Dublin

November 1993

Abstract

This paper examines positive and normative implications of tariffs in the presence of persuasive advertising. It demonstrates that protection affects imports, the domestic consumer price and the terms of trade directly and through its effect on the level of advertising. If protection reduces advertising a Metzler paradox can occur. A tariff can be used to induce foreigners to allow entry. Jointly optimal tariffs and advertising taxes and the optimal tariff for the situation when it is the only available policy instrument are derived under both monopoly and Cournot oligopoly.

Keywords: Persuasive advertising, tariffs, oligopoly, entry.

Correspondence to: Dermot Leahy, Department of Economics, University College Dublin, Belfield, Dublin 4 Ireland.

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STRATEGIC AND RENT-EXTRACTING TARIFFS
IN THE PRESENCE OF PERSUASIVE ADVERTISING

1 Introduction

Consider a country importing an advertising-intensive commodity from an imperfectly competitive foreign firm. I will be examining the case in which the foreign firm is engaging in "persuasive advertising"\textsuperscript{1}, that is advertising which affects consumer tastes and enhances the households' marginal valuation of the good. An example might be a developing country importing soft drinks, cigarettes or beer. Does the presence of persuasive advertising strengthen the case for intervention? There is reason to believe that it might. If trade restrictions are introduced they are likely to have a significant effect on the optimal choice of advertising and other promotional expenses aimed at that particular market, which in turn will affect sales, price and demand elasticity. When the industry in question is "advertising intensive" and if this advertising is principally "persuasive", protection that reduces the amount spent on marketing is likely to bring about a lowering of the perceived quality of the imported brand and a

\textsuperscript{1} Television advertising probably fits this description best. See Resnik and Stern (1977) for an analysis of the low informational content of television advertising in the United States.

\textsuperscript{2} I do not discuss advertising that provides information and directly enhances efficiency, nor do I discuss advertising that acts as a signal of quality. For models in which advertising is informative see for instance Butters (1977) and Grossman and Shapiro (1984). For advertising that acts as a signal of quality see Nelson (1974), Milgrom and Roberts (1986) and Hertzendorf (1993). Schmalensee (1986) and Tirole (1988) provide overviews of the literature.
reduction in the firm's market power. This could be expected to further reduce the import price and strengthen the favourable terms of trade effect of protection\(^3\).

It is already well known that if a foreign firm has monopoly power in the home market and is earning significant rents, then a tariff or other form of protection can allow the home country to capture some of this rent in the form of government revenue or increased home firm profits. In practice protection can be expected to affect a whole range of variables including capacity choice, R&D, product development and advertising. While the role of R&D and capacity choice have been examined in strategic trade models, the role of advertising has so far received little attention.\(^4\)

The purpose of this paper is to examine the positive and normative implications of tariffs in a situation in which firms are engaging in persuasive advertising. On the positive side an important result of this paper is the possibility of a Metzler paradox: a tariff will in some instances lead to a fall in the home market price of a good. This occurs when the tariff-induced fall in advertising so undermines the foreign firm's market power that its price is reduced despite the increase in marginal costs. In that case the home country government not only extracts rent from the foreign firm in the form of tariff revenue but also ensures

\(^3\) Advertising that provides relative price information tends to reduce the market price (see for instance Benham 1972)), while persuasive advertising can be expected to lower price elasticity and raise price.

\(^4\) See Spencer and Brander (1983) for R & D in an international subsidy game and Venables (1990) for strategic trade models with international capacity choice. Leahy (1992) examines rent extraction in the presence of advertising.
that its consumers enjoy the good at a lower price. At first sight, at least, this would appear to strengthen the case for activist trade policy. In fact when the normative side is examined in detail I show that the presence of persuasive advertising can strengthen or weaken the case for a tariff depending, among other things, on whether the good is over valued or under valued by consumers, and whether or not advertising directly raises welfare.

The paper builds on the work of Katrak (1977), Svedberg (1979) and Brander and Spencer (1981, 1984) all of whom examine rent-extracting tariffs employed against a foreign monopolist. Brander and Spencer (1984) are particularly concerned with the role of the curvature of the demand function in determining the optimal tariff and they show that if it is very convex an import subsidy may be optimal. They also extend earlier work by introducing a domestic firm. Leahy (1992) considers the role of a rent-extracting tariff employed against a foreign monopolist that engages in advertising. In that paper however, attention is restricted to the special case of linear demand.

Following some preliminaries, section 2 addresses positive questions concerning a rent extracting tariff employed against a foreign monopolist selling into the home market. It is shown that protection affects imports in two ways: firstly there is the direct effect that operates through the increase in the marginal

---


6 Brander and Spencer (1981) consider a related issue, the role of a tariff in facilitating entry by a domestic firm.
costs faced by foreigners: secondly there is an indirect effect since protection influences the level of advertising and through this sales. It is demonstrated that comparative static results will often turn on the curvature of the inverse demand curve and on the effect of advertising changes on the slope of the demand function.

In section 3 of the paper I consider normative issues. The optimal tariff is calculated for two alternative specifications of the welfare function. The first of these welfare functions follows Dixit and Norman (1978) and makes the assumption that there is a reference level of advertising at which the inverse demand function equals the "true" marginal social valuation function. Welfare depends both on consumption of imports and on the divergence between actual and reference levels of advertising. When this approach is adopted persuasive advertising only affects welfare indirectly, via changes in outputs. In contrast to this approach I also consider a less paternalistic approach in which it is assumed that the true marginal social valuation curve always coincides with the inverse demand. This second approach implies that welfare is directly increasing in advertising. I then consider the possibility that the home government can employ an advertising tax and a tariff against the foreign monopolist. The jointly optimal tariff and advertising tax combination are examined under these circumstances.

In section 4 I introduce a domestically owned firm which competes with the foreign firm on the home market. Assuming that only the foreign firm advertises, a model in which the firms play a Nash game in advertising and quantities is considered. It is shown that a tariff increases the price charged by the domestic
firm and usually encourages an increase in home output. A Metzler paradox is once again a possibility, a necessary condition for this being that the tariff reduces advertising. This oligopoly model is used to derive both the jointly optimal tariff and advertising tax and the optimal tariff for the situation when it is the only available policy instrument, for both specifications of the welfare function.

In section 5 I consider a model in which advertising is used as a barrier to entry by the foreign firm. It is shown that a tariff can be used to induce the foreign firm to abandon entry deterrence. The model used in this section has some resemblance to that used by Brander and Spencer (1981). However in their model (in which there is no advertising) it is assumed the foreign firm is a Stackelberg leader in quantities. In the model presented here firms play Cournot in the final stage of a game in which advertising is a precommitted variable. Finally section 6 is a short conclusion.

2 Foreign monopoly

I begin with the simplest case, that of a foreign monopolist supplying a good to the domestic market. Assume an aggregate household with the following quasi-linear utility function:

\[ U = u(y,m) + n_v, \]  \hspace{1cm} (2.1)

where \( y \) represents the imports of the monopolised good, \( m \) represents the level of real advertising on good \( y \) and \( n_v \) the consumption of a competitively produced numeraire good. The numeraire good is produced in the home country. Let \( q \) be the consumer price of the import. Utility maximisation implies that:
\[ U_y = q(y,m). \]  

I assume \( q_m > 0 \), that is the inverse demand is shifted to the right by an increase in advertising. In order to ensure an interior solution it is necessary to assume that \( q(y,m) \) is strictly concave in advertising so that \( q_{mm} < 0 \). As the inverse demand curve is shifted by the advertising increase its slope may also change. The effect of advertising on the position of the demand curve can be decomposed into the upward shift which is captured by \( q_m \) and the change in the slope which is captured by \( q_{m\nu} \). Define \( \varepsilon = y q_{m\nu}/q_m \), the elasticity of \( q_m \) with respect to sales. If this is negative then at a higher level of sales the demand curve is shifted less by an increase in \( m \). I will assume that \( \varepsilon \leq 0 \), a small increase in advertising will not make the demand curve flatter. This is intended to capture the positive effect of persuasive advertising on brand loyalty.\(^7\)

Assume that the monopolist faces a constant marginal production cost denoted by \( c \) and a constant marginal cost of advertising denoted by \( \mu \). The home government will impose a tariff of \( t \) and may also impose an advertising tax \( \theta \) on advertising spending carried out in the home country. In practice one would expect the firm to do some of its advertising in the foreign country and some in the home country (into which it is exporting). As I show below however, the results obtained when a tariff is the only policy instrument are independent of where the advertising expenditure takes place.

\(^7\) Lambin (1976) suggests that increases in advertising lower brand specific price elasticities. This cannot occur at constant \( y \), if \( \varepsilon \) and therefore \( q_{m\nu} \) is positive.
The variable profit function of the monopolist is:

$$\pi^* = (q - c - t) y - (\mu + \theta) m,$$  \hspace{1cm}(2.3)

for its sales to the home market (note that foreign variables will often be starred). If the firm sells on more than one market then it is assumed that the overall profit function is separable and \( m \) is its advertising specifically for the home market. The firm chooses output and advertising to maximise profits. This implies the first-order conditions:

(i) \[ \pi^*_{y} = q + q_{y} y - c - t = 0, \]  \hspace{1cm}(2.4)

(ii) \[ \pi^*_{m} = y q_{m} - \mu - \theta = 0, \]

for optimal choice of output and advertising respectively. The second-order conditions are: \( \pi^*_{yy} = 2 q_{y} + q_{yy} y < 0, \) and \( \pi^*_{yy} \pi^*_{mm} - \pi^*_{ym} \pi^*_{my} > 0. \) The cross partial derivative \( \pi^*_{ym} = q_{m} (1 + \varepsilon), \) which shows the impact on marginal profits of an increase in advertising, is ambiguous in sign though it should usually be positive. If \( \pi^*_{ym} \) is negative then if the firm intends to advertise more it will want to sell less. In that case the advertising increase leads to a sharp fall in the elasticity of demand allowing the firm to sell less at a much higher price. It will prove convenient to define \( K = \pi^*_{mm} - (\pi^*_{ym})^{2}/\pi^*_{yy} \) which is negative from the second order conditions.

I turn now to the comparative statics of the equilibrium. Proceed by totally differentiating the equations in (2.4) to obtain the following comparative static derivatives:

(i) \[ \frac{dy}{dt} = \frac{\pi^*_{mm}}{\pi^*_{yy}} K < 0, \]  \hspace{1cm}(2.5)

(ii) \[ \frac{dm}{d\theta} = \frac{1}{K} < 0, \]
(iii) \( \frac{dy}{d\theta} = \frac{dm}{dt} = -\frac{\pi^{*}_{ym}}{\pi^{*}_{yy}}K. \)

The sign of \( \frac{dy}{d\theta} = \frac{dm}{dt} \) depends on the effect of advertising on marginal profits. If \( \pi^{*}_{ym} \) is negative then an increase in the advertising tax, though it reduces advertising causes sales to rise. The intuition is that the demand curve moves inwards but becomes so much flatter that sales actually rise. In addition \( \pi^{*}_{ym} \) negative implies that the firm will respond to a tariff increase by intensifying its promotional activity to increase the perceived quality of the good.

It is useful to rewrite (2.5 (i)) in terms of (2.5 (iii)):

\[
\frac{dy}{dt} = \left(\frac{1}{\pi^{*}_{yy}}\right)(1 - \pi^{*}_{ym} \frac{dm}{dt}).
\] (2.6)

From (2.6) it is now clear that the level of imports is affected in two ways by a tariff: directly and through induced changes in \( m \). The expression \( 1 - \pi^{*}_{ym} \frac{dm}{dt} \) is larger than unity, while \( \frac{1}{\pi^{*}_{yy}} \) represents the impact of a tariff on the level of imports at a constant level of advertising. Hence:

**Proposition 1:** For \( \varepsilon \neq -1 \), the supply response is larger in the presence of endogenous advertising than in its absence.

This result is an example of the le Chatelier-Samuelson principle.

In order to analyse the role of the curvature of demand in the determination of comparative static results it proves useful to define: \( R \equiv y_{q_y}/q_y \), the elasticity of the slope with respect to sales, This is a measure of the concavity of demand.
A positive $R$ indicates concave demand. The impact of a tariff on the domestic price can be obtained by using (2.6) in the total derivative of (2.2). It is:

$$dq/dt = q[1 + (1 + R - \varepsilon)q_m dm/dt].$$  \hspace{1cm} (2.7)

The effect of the tariff at a given level of advertising is represented by $q_t = q/\pi_{yy} = 1/(2+R)$, the ratio of the slope of the inverse demand function to the slope of the marginal revenue curve. Clearly this is positive from the second-order condition and less than unity when $(1 + R)$ is positive. Provided that $\pi_{ym}$ is positive (which is necessary and sufficient to ensure that $dm/dt$ is negative), and $(1 + R) > 0$, the term in square brackets must be less than unity. This implies that the domestic price of the import does not rise as much when advertising is endogenous as it does when it is exogenously fixed. The intuition is that the tariff reduces output leading to an upward movement along the inverse demand curve but it also induces a fall in advertising spending which shifts this demand curve inwards. As a result the quantity imported and the import price $q^*$ both fall by more than they would if advertising was fixed.\textsuperscript{10}

It can be shown that $dq/dt$ may be negative, that is to say a Metzler paradox is possible. The condition for this is: $J = \pi^*_{mm} - \beta(\pi^*_{ym})^2/\pi^*_{yy} > 0$,\textsuperscript{11} where $\beta = $

\textsuperscript{8} If $R$ is more negative than - 1 then the inverse demand function is steeper than the marginal revenue curve, since $(\pi_{yy} - q)/q = 1 + R$.

\textsuperscript{9} This is shown in Brander and Spencer (1984).

\textsuperscript{10} If $\pi^*_{ym} < 0$, then the tariff increases advertising, the demand function shifts to the right, and the domestic price of the import rises even more. Note that if $\pi^*_{ym} < 0$, then $-\varepsilon > 1$, and this implies that $1 + R - \varepsilon > 0$.

\textsuperscript{11} Note that the impact of a tariff on the domestic price can be written as $dq/dt = q_J/K$ which is clearly negative if and only if $J$ is positive.
(2+R)/(1+ε). Since K is negative in order to ensure an interior solution, J can only be positive if β exceeds unity. For β to be less than unity either: (a) (1+R-ε) < 0, when \( \pi^*_{ym} \) is positive or (b) \( \pi^*_{ym} < 0 \).

**Proposition 2:** In the monopoly case, when advertising is endogenous and provided that \( \beta > 1 \), a tariff causes the price to rise by less than when the level of advertising is fixed. The consumer price of the import will rise iff \( J < 0 \).

A necessary condition for a Metzler paradox is that \( dm/dt < 0 \), or equivalently that \( \pi^*_{ym} \) is positive (see figure 1). The paradoxical price movement is more likely the larger is R and the larger is q_m. I turn now to the impact of the tariff on the terms of trade facing the home country. The import price will be represented by \( q^* = q - t \). As has already been shown by Brander and Spencer (1984), in the no-advertising case the tariff will lead to an improvement in the terms of trade if the marginal revenue curve is more steeply sloped than the demand curve. Using (2.7) it can be shown that the effect of the tariff on the home terms of trade when advertising is endogenous, is:

\[
\frac{dq^*}{dt} = q^*_t \left[ 1 - q_m \left( \frac{1 + R - \epsilon}{1 + R} \right) \frac{dm}{dt} \right]
\]

where \( q^*_t = -(1+R)/(2+R) \), represents the effect of the tariff on the terms of trade at constant advertising. The sign of \( dq^*/dt \) depends on the two elasticities \( \epsilon \) and \( R \) and the relationship between them. There are four main possibilities (see table
1 in the appendix) but perhaps the most plausible case is where $(1+\varepsilon) > 0$, and $(1+R) > 0$. In that case advertising falls in the tariff and the terms of trade improve by more when advertising is endogenous than when it is fixed.\textsuperscript{12}

An advertising tax or any other shock that increases the cost of advertising affects the price in the following way:

$$
\frac{dq}{d\theta} = \frac{dq^*/d\theta}{q_m \cdot dm/d\theta} = \frac{1}{1+R-\varepsilon+(2+R)} q_m \cdot dm/d\theta,
$$

(2.9)

Note that $q_m \cdot dm/d\theta$ must be negative. It is clear that a necessary and sufficient condition for an advertising tax increase to lead to a fall in the price of the monopolised good and an improvement in the terms of trade is $(1+R-\varepsilon) > 0$. Problems are most likely when demand is very convex and $\varepsilon$ is not very negative see figure 2. Notice that if $\pi^*_m$ is negative then $dq/d\theta$ must be negative. In that case the induced fall in advertising leads to a large increase in the elasticity of demand and this tends to work towards a lower price.

3 Optimal policy under monopoly

I now turn to the problem of welfare maximisation. I will first consider a situation in which the government is restricted to using a tariff and only later examine a case in which the government chooses both a tariff and advertising tax.

3.1 Tariff only

In this subsection I will consider situations in which the home government's only policy instrument is a tariff. Following Dixit and Norman (1978) I shall assume

\textsuperscript{12} An interesting special case which satisfies these restrictions is that of linear separable demand. Suppose that the inverse demand is of the form: (2.2') $q = \phi(m)$ - by (where $b$ is a positive constant). Then $dq^*/dt = (\phi' dm/dt - 1)/2 < q^*_i = -1/2$. 

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that there is a reference level of advertising, \( m = \bar{A} \), which could be zero, that sets
the inverse demand curve for the import equal to the government's marginal social
valuation curve for that good. Welfare is evaluated at the actual level of
consumption of the goods and the reference level of advertising. A change in the
actual level of advertising has no direct effect on welfare, but affects it indirectly
through induced changes in the consumption mix. The domestic welfare function
is therefore:

\[
w = U(y,\bar{A},n_e) = u(y,\bar{A}) + n_e, \quad (3.1)
\]

The model can be embedded in a quasi-general equilibrium\(^{13}\) framework
by assuming that the home country exports the numeraire good in exchange for
the good produced by the monopolist. The balanced trade condition is:

\[
n_p - n_e = q^*y - h\mu m, \quad (3.2)
\]

where \( n_e \) is the production of the numeraire good, and where \( h \) is the proportion
of advertising carried out in the home country. If \( h = 1 \) the foreign monopolist does
all its advertising expenditure in the home country and if \( h = 0 \) it does it all
abroad. Assume that the home country has an endowment of a composite factor
of production which I will call "labour" \( e \), and that the numeraire good uses one
unit of labour to produce one unit of output. Letting \( \mu \) be the labour coefficient of
advertising gives the following full employment condition:

\[
e = n_p + h\mu m^*. \quad (3.3)
\]

Using (3.2) and (3.3) in (3.1) it is possible to write the welfare function in
the following way:

\(^{13}\) It could be said that this not true general equilibrium as it ignores such
issues as income effects and factor price effects.
\[ w = u(y, \bar{A}) - qy + ty + e, \]  
(3.4)

which is independent of \( h \). If a tariff is the only available instrument it is immaterial where the advertising is done because the resources are valued at their true social cost. The first two terms on the right-hand side of (3.4) represent the social surplus, net of tax revenue, that is obtained from consumption of the imported good. The third term (\( ty \)) is government revenue from the tariff and the last term \( e \) the economy's total labour income is constant.

Total differentiation of the welfare function yields:

\[ dw = -y[dq/dt] + tdy + [u_y - q]dy, \]  
(3.5)

where \( u_y = \partial u(y, \bar{A})/\partial y \): the domestic shadow price of the import. The first two terms on the right hand side of (3.5) represent the familiar terms of trade and volume of trade effects. The last term occurs because there is in general a gap between the shadow price of the monopolised good and its market price. This gap, \( u_y - q \), will be negative when the actual level of advertising exceeds the reference level \( \bar{A} \).

Assuming an interior solution the optimal tariff is:

\[ t^* = q - u_y + y(dq^*/dt)/(dy/dt). \]  
(3.6)

The sign of the optimal tariff will depend on the gap between the market and shadow price of the import and the response of the terms of trade to the tariff. The change in the terms of trade will depend on the curvature of the demand function and \( \epsilon \), (which will determine the sign of \( dm/dt \)). Using (2.6) and (2.8) in (3.6) it is possible to obtain a more explicit expression for the optimal tariff:
\[ t^o = q - y^o q, (1 + R) \sigma, \]  
(3.6')

where:

\[ \sigma = 1 + \epsilon \frac{(2 + R)}{(1 + R)} \left[ \frac{q_m dm/dt}{1 - \tau^* y_m dm/dt} \right] \]

First note that if \( \epsilon = 0 \), then \( \sigma = 1 \) and the sign of the final term on the right-hand side of (3.6) depends only on the curvature of demand. Second note that if (1+R) and (1+\epsilon) have the same sign (so that (1+R) and dm/dt have opposite signs), then \( \sigma \geq 1 \). Third, a necessary but not a sufficient condition for \( \sigma \) to be negative and so reverse the sign of the final term in (3.6) is for (1+R) and (1+\epsilon) to have opposite signs. Loosely speaking, concave demand works towards a positive optimal tariff unless advertising increases sufficiently in the tariff and convex demand works against an optimal tariff unless advertising falls decisively in the tariff.

**Proposition 3:** If \( \sigma > 0 \) and: (i) if \( 1 + R \geq 0 \), and \( q - v, \geq 0 \), with at least one inequality strict, then \( t^o > 0 \); (ii) if \( 1 + R \leq 0 \), and \( q - v, \leq 0 \), with at least one inequality strict, then \( t^o < 0 \); and (iii) if \( 1 + R \) and \( q - v, \) have opposite sign then the sign of the optimal tariff is ambiguous.

It can therefore be seen that the sign of the optimal tariff depends in a complex way on three main factors: (i) The convexity/concavity of the inverse demand function, which determines the relative slope of the inverse demand function and the marginal revenue. When \( \sigma \) is positive convex demand works
against a positive optimal tariff. (ii) The gap between the market price and the shadow price of the import: the wider is this the larger is the optimal tariff. (iii) The sign of $\frac{\text{dm}}{\text{dt}}$ also plays a part in determining the sign of the optimal tariff.$^{14}$

3.2 A non-paternalistic welfare specification

I now consider the optimality of a tariff under an alternative specification of the welfare function. Dixit and Norman's approach to modelling welfare change in the presence of advertising and endogenous tastes has been criticised for its exclusion of actual advertising from the welfare function.$^{15}$ Their approach can be regarded as "paternalistic" in that the government is assumed to know better than the consumer. It is therefore worth comparing the results obtained so far with those obtained under a non-paternalistic specification of the welfare function. I will replace the welfare function given in (3.1) with the utility function of the aggregate household given in (2.1). The government's welfare maximisation problem is now:

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$^{14}$ In order to concentrate on the role played by the curvature of the inverse demand function and the gap between the actual and reference levels of advertising abstracting from the effects of $\varepsilon$ consider the following separable inverse demand function: $q = \phi(m) + \psi(y)$. With this demand specification it can be checked that $\sigma = 1$, independently of the curvature of the demand function. The results outlined in proposition 3 are then obtained.

$^{15}$ Fisher and McGowan (1979) argue that "if the amount of advertising enters the utility function the natural criterion for welfare comparison is the unrestricted utility function so defined". When this "non-paternalistic" approach is adopted the welfare function is increasing in advertising. See Dixit and Norman (1979) for reply to the criticism of Fisher and McGowan and Leahy (1992) for a further discussion of the differences between the two approaches.
\[
\max_t w = U(y_m, n_e) - u(y_m) + n_e
\] (3.7)

Using (3.2) and (3.3) in (3.7) and totally differentiating yields:

\[
dw = u_m dm - y(dq - dt) + tdy. \tag{3.8}
\]

Unlike in the Dixit-Norman case there is now no divergence between the shadow price of the import and its market price. The term in \((u_y - q)\), present in (3.4) is absent from (3.8). Instead there is a new term in \(u_m dm\), which captures the direct welfare-increasing effect of advertising.

As before I assume that there is an interior optimal tariff. From (3.8) it is possible to obtain an expression for this:

\[
t^* = u_m(\pi^*_m/\pi^*_{mm}) - y^0q_y(1 + R)\sigma. \tag{3.9}
\]

The sign of (3.9) is clearly ambiguous. The first term is usually negative while the second is usually positive. If the separable demand function in note 14 above is imposed, then the first term on the right-hand side of (3.9) is negative while the sign of the second depends on that of \((1 + R)\).

It is clear from (3.6) and (3.9) that an import subsidy may be optimal under both "paternalistic" and "non-paternalistic" specifications of the welfare function. However the conditions required for an optimal subsidy differ between the two cases. Under the Dixit and Norman specification a negative optimal tariff cannot occur in the plausible case in which \(\pi^*_m\) and \((1 + R)\) are positive provided consumers do not "over value" the import. While under the alternative non-paternalistic specification where the import subsidy causes an increase in

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advertising which in itself raises welfare an optimal import subsidy is possible even though consumers never undervalue the good. Note that whereas in the Dixit and Norman welfare approach a negative $\pi^*_m$ tends to work against an optimal tariff (for $(1+R) > 0$ at least) this may not be the case under the non-paternalistic specification of the welfare function. A negative $\pi^*_m$ implies that higher tariffs are associated with higher levels of advertising which under the non-paternalistic specification of the welfare function, raises welfare directly.

3.3 Maximum price policy

It has been shown by de Meza (1979) that the first-best policy for the home government faced by a foreign monopoly which only chooses output (or price) and has constant marginal costs, is a maximum price set at the foreign firm’s marginal cost. This policy will not always raise welfare under persuasive advertising. The effect of a maximum price set at marginal cost will be to render advertising useless from the point of view of the foreign monopolist. This means that the inverse demand curve under the maximum price will be to the left of that under free trade. The home consumer will be able to purchase the good at a lower price but if $\bar{A}$ is large enough the lost surplus as measured beneath the "true" marginal social valuation curve may outweigh the benefits of the lower price.\footnote{If $\bar{A}$ is zero then the de Meza maximum price is always the best policy.} If the non-paternalistic welfare function is used then the lost utility from lower advertising must be weighed against the benefits of the lower price.
3.4 Advertising tax and tariff

When the monopolist buys its advertising in the home country it may face a home advertising tax. I now wish to consider an optimal tariff imposed in a situation in which an optimal advertising tax is also being used. I will adopt a welfare function like that in (3.4) extended to include advertising tax revenue $\theta m$:

$$w = u(y, A) - qy + ty + \theta m + e,$$

(3.4')

The first-order condition for the optimal choice of the tariff and advertising tax are:

$$\frac{dw}{dt} = (u_y + t - q)(dy/dt) - y(\frac{dq^*/dt}{dt}) + \theta(dm/dt) = 0,$$

(3.10)

$$\frac{dw}{d\theta} = (u_y + t - q)(dy/d\theta) - y(dq/d\theta) + m + \theta dm/d\theta = 0.$$

These equations can be solved simultaneously for the joint optimal tariff and advertising tax. The overall optimal tariff is found to be:

$$t^* = [q - u_y] - yq_y(1 + R) - m\pi^*_{ym},$$

(3.11)

The overall optimal tariff depends positively on (i) the gap between the market and shadow prices and, (ii) the concavity of the inverse demand function. In contrast to the tariff-only case the tariff depends negatively on $\pi^*_{ym}$. If this is positive a higher tariff reduces advertising and so reduces the revenue from the advertising tax. The advertising tax is already adjusting the level of advertising optimally. The overall optimal advertising tax is:

$$\theta^* = - y[q_{mm}m + q_{my}y] > 0,$$

(3.12)

From equation (3.12) the following result is obtained:

**Proposition 4:** When the Dixit-Norman welfare specification is used and the government imposes an optimal tariff on imports produced by a foreign monopolist, the optimal advertising tax is positive.
Note that this result is independent of the gap between market and shadow prices\textsuperscript{17}.

In this model tariffs and advertising taxes are doing two things, extracting rent and closing the gap between private and social valuation of the good. The tariff acts directly on the gap between market and social valuation leaving the advertising tax with a pure rent-extracting role. This becomes somewhat more transparent when the demand function is separable and linear in \( y \) as in (2.2') (see note 12 above). The optimal tariff can then be written as:

\[
t^\infty = \phi(m^\infty) - \phi(\bar{A}) + by^\infty - \phi'(m^\infty)m^\infty. \quad (3.11')
\]

Using this it is possible to obtain the following result:

**Proposition 5:** When a linear demand specification \( q = \phi(m) - by \) is imposed, the tariff will be lower in the presence of an optimal advertising tax than in the absence of any advertising tax.

To prove this note that under the linear demand assumption the tariff-only optimum described in (3.6) can be rewritten as:

\[
t^\circ = \phi(m^\circ) - \phi(\bar{A}) + by^\circ, \quad (3.6')
\]

Proceed by subtracting \( t^\circ \) in (3.6') from \( t^\infty \) in (3.11') to give:

\[
t^\infty - t^\circ = \phi(m^\infty) - \phi(m^\circ) - (2/3)\phi'm^\infty. \quad (3.13)
\]

For the left hand side of this to be non-negative, \( m^\infty \) must exceed \( m^\circ \) but with a

\textsuperscript{17} This result is more general than that obtained in Leahy (1992) which only considered the special case of linear separable demands.
higher tariff and a higher advertising tax (from (3.12)) this is clearly impossible since the level of advertising is monotonically declining in both t and θ. Therefore it must be the case that t^∞ - t^ is strictly negative.

4 Oligopoly

Now consider an industry in which there are two firms competing on the home market. One of the firms, which I will call the home firm, is located in the domestic market, the other called the foreign firm is located abroad (though it may do its advertising in the home country). A very general specification of advertising with both firms advertising does not yield transparent results so I will adopt the following assumptions: (i) only the foreign firm engages in advertising\(^\text{18}\) and (ii) the utility function of the aggregate consumer takes the following form:

\[
U = u(x + a(m)y) + n_u,
\]

(4.1)

where a(m) > 0, is the index of perceived quality and a'(m) > 0. Increases in advertising expenditure lead to an increase in the index of perceived quality which will cause the marginal utility curve for the import to shift to the right\(^\text{19}\). I am assuming vertical product differentiation in the market for the imperfectly competitive good. Advertising increases the home consumer’s perception of the quality of the foreign good. An alternative way to view the effect of an increase in

\(^{18}\) One way this assumption could be justified is by postulating the existence of a threshold level of advertising, below which advertising is ineffective, and by assuming that the domestic firm is unable to profitably reach this level. The existence of such threshold levels are empirically well established, (see Sutton (1991)).

\(^{19}\) See Sutton (1991) for a similar approach to modelling advertising in the demand function.
advertising is to note that at constant $c$ it leads to a reduction of the marginal cost of producing output in quality units. When seen in this way advertising has some similarity to process R&D. However it differs from process R&D in two important respects. (i) Unlike R&D it will in general affect the slope of the demand function. (ii) Process R&D will only reduce marginal production costs while advertising will in addition reduce the marginal trade costs in quality units. For simplicity I will assume that the $a(m)$ function takes a simple linear form: $a(m) = a + m$, (where $a > 0$, a constant, is the index of perceived quality in the absence of advertising$^{20}$). Let $x$ denote the home firm’s output and $y$, as before, represent the output of the foreign firm that is sold on the home market. The inverse demand functions obtained from (4.1) are:

\[ p = u'(x + Y), \]  
\[ q = a(m)u'(x + Y), \]  

where $Y = a(m)y$ is foreign output in quality units, and $p$ is the price of the domestically produced good$^{21}$. As before in order to ensure an interior solution it is necessary to assume that $q(m,x,y)$ is concave in $m$:

\[ q_{mm} = yp'(2 + \alpha^*R) < 0, \]  

where $\alpha^* = Y/(x + Y)$, the foreign market share in quality units. Similarly the home market share in quality units is $\alpha = x/(x + Y)$. $R$ is now the elasticity of the slope of demand with respect to sales in perceived-quality units: $R = (x+Y)p''/p'$. 

---

$^{20}$ If $a$ is equal to unity then the goods are homogeneous in the absence of foreign advertising.

$^{21}$ Note that the equilibrium relative price of the import in terms of the domestic variety is $q/p = a$, the index of perceived quality.
A decision must now be made about the appropriate move order of the game. If advertising is regarded as an investment in brand image that decays only slowly then it is reasonable to think of expenditure on advertising as being similar to expenditure on plant capacity or R&D and to model it as a precommitted strategic variable that can be used to induce the rival to produce less or deter entry. In that case it is appropriate to use a multi-stage game framework in which advertising is chosen before output. Alternatively, if the effect of advertising on brand loyalty and perceived quality is rather short-lived then it is more appropriate to adopt a Nash-game framework in which advertising and output are chosen simultaneously. In this section I use the latter approach.

The domestic firm chooses \( x \) to maximise the variable profit function:

\[
\pi = x(p - c). \tag{4.4}
\]

The first-order condition is then:

\[
\pi_x = p + p_x x - c = 0, \tag{4.5}
\]

with second-order condition: \( \pi_{xx} = p'(2 + \alpha R) < 0 \). The foreign firm chooses \( y \) and \( m \) simultaneously to maximise the profit function in (2.3) and its two first-order conditions are given in (2.4). The firms move simultaneously and the outcome is a Nash equilibrium in quantities and advertising.

To analyse the comparative static properties of the equilibrium totally differentiate the first-order conditions. These can be rearranged in matrix form as follows:
\[
\begin{bmatrix}
\pi_{xx} & \pi_{xy} & \pi_{xm} \\
\pi^*_{yx} & \pi^*_{yy} & \pi^*_{ym} \\
\pi^*_{mx} & \pi^*_{my} & \pi^*_{mm}
\end{bmatrix}
\begin{bmatrix}
dx \\
dy \\
dm
\end{bmatrix}
= \begin{bmatrix}
0 \\
dt \\
d\theta
\end{bmatrix}.
\]

A necessary condition for the local stability of the equilibrium is that the trace and \( \Gamma \), the determinant of the coefficient matrix are negative\(^{22}\). I will also assume that the coefficient matrix has a dominant diagonal in the Hadamard sense\(^{23}\).

4.1 The positive effects of a tariff

I first examine the impact of a change in the tariff. This gives rise to the following comparative static derivatives obtained from (4.6):

\[
\begin{align*}
(i) \quad \frac{dy}{dt} &= \frac{(\pi_{xx} \pi^*_{mm} - \pi_{xm} \pi^*_{mx})}{\Gamma}, \\
(ii) \quad \frac{dx}{dt} &= \frac{(y/a)\pi_{xy} q_m}{\Gamma}, \\
(iii) \quad \frac{dm}{dt} &= - \frac{(\pi_{xx} \pi^*_{my} - \pi_{xy} \pi^*_{mx})}{\Gamma},
\end{align*}
\]

The term in parentheses in (4.7(i)) is positive from the diagonal dominance of the coefficient matrix in (4.6). This implies that foreign output must fall in the

\(^{22}\) I am concerned with stability with respect to the usual myopic adjustment rule (see Hahn (1962) and Seade (1980)).

\(^{23}\) This can be explained as follows: Let \( a_{ij} \) be the ijth element of the coefficient matrix then:

\[|a_{ij}| > \sum_{j \neq i} |a_{ij}|\]

This is a sufficient condition for stability (see Dixit (1986) and Takayama (1985).
Home output increases in the tariff if and only if \( \pi_{xy} \) is negative. This is intuitive: an increase in the tariff moves the foreign reaction function inwards and leads to an increase in home output if the home reaction function has a negative slope\(^{24} \). Assuming that \( \pi_{xy} \pi^*_{mx} \) is positive\(^{25} \) a sufficient condition for the tariff to reduce advertising is for \( \pi^*_{my} \) to be positive. The intuition is that an increase in the tariff reduces \( y \) and if \( \pi^*_{my} \) is positive this causes the marginal profitability of advertising to fall so leading to a fall in its equilibrium level.

It will prove useful to obtain an expression for the effect of the tariff on the level of foreign output in quality units. This is:

\[
\frac{dY}{dt} = \frac{ady}{dt} + \frac{ydm}{dt} = - \pi_{xy} y q_m / \Gamma < 0.
\]  

(4.8)

Using this it is possible to rewrite (4.7(ii)) as \( \frac{dx}{dt} = - \frac{[(1+\alpha R)/(2+\alpha R)]dY}{dt} \). It is clear that the tariff only affects \( x \) through its effect on \( Y \). The tariff leads to a movement along the home reaction function in \((x, Y)\) space.

I now wish to consider the effect of the tariff on prices. Totally differentiate (4.2) and make use of (4.7) to obtain:

---

\(^{24}\) Here \( \pi_{xy} = ap'(1 + \alpha R) \), is negative if home output is a strategic substitute for foreign output from the point of view of the home firm and positive if they are strategic complements from the home firm's point of view (see Bulow et al (1985)). If \( \pi_{xy} \) is negative an increase in foreign output will reduce the marginal profits of the home firm. Graphically the home reaction function is negatively sloped. When \( \pi_{xy} \) is positive an increase in foreign output raises home marginal profits: the reaction function is positively sloped.

\(^{25}\) Where \( \pi_{xy} \pi^*_{mx} = Y(p')^2(1 + R + \alpha \alpha^* R^2) \).
\[
\frac{dp}{dt} - \frac{p'}{(2 + \alpha R)} \frac{dY}{dt} > 0
\]

(4.9)

It is easy to see that a tariff will increase the price of the domestically produced good independently of whether the home firm views its output as a strategic substitute or a strategic complement for that of the foreign firm. Note that when outputs are strategic substitutes home sales and price both rise guaranteeing that home profits also rise.

As in the monopoly model the market price charged by the foreign firm could increase or decrease in the tariff:

\[
\frac{dq}{dt} - p \frac{dm}{dt} + \frac{ap'}{(2 + \alpha R)} \frac{dY}{dt} < 0
\]

(4.10)

The second term on the right-hand will be positive as already shown. This can be thought of as an upward movement along the inverse demand function. The first term on the right-hand side of (4.10) has the same sign as dm/dt. If the tariff works to reduce advertising this will tend to put downward pressure on the consumer price of the imported good. This can be represented as an inward shift of the demand function. So, as in the monopoly case, a necessary condition for a Metzler paradox to occur is that the tariff reduces advertising.\(^{26}\)

---

\(^{26}\) It is not difficult to show that a Metzler paradox can occur without violating the stability of the equilibrium. For instance suppose \((1 + \alpha R)\) is zero so that the cross effects on home marginal profits vanish. The determinant of the coefficient matrix is then \(\Gamma = (ap')^2(2 + \alpha R)K\). Following some manipulation it can be shown that:

\[
\frac{dq}{dt} = qJ/K,
\]

where the numerator can be positive while \(K\) remains negative.
4.2 The positive effects of an advertising tax

It follows from diagonal dominance in the coefficient matrix that an advertising tax or any other increase in the marginal cost of advertising, must lead to a fall in the level of advertising. Its effect on imports, $y$, is ambiguous though its effect on imports in quality units is unambiguously negative:

(i) \[
\frac{dy}{d\theta} = - \frac{(\pi_{xx} \pi^*_ym - \pi^*_{yym})}{\Gamma},
\]

(ii) \[
\frac{dY}{d\theta} = \pi_{xx} (y \pi^*_{yym} - a \pi^*_{ym}/\Gamma < 0.
\]

The impact of the tax on home sales is:

\[
\frac{dx}{d\theta} = - \frac{[(1 + \alpha R)/(2 + \alpha R)]dY/d\theta}
\]

which has a sign opposite to that of $(1 + \alpha R)$.

The home price is increasing in the advertising tax in this model:

\[
\frac{dp}{d\theta} = \frac{p'}{(2 + \alpha R)} dY > 0
\]

The consumer price of the import is:

\[
\frac{dq}{d\theta} = p \frac{dm}{d\theta} + \frac{ap'}{(2 + \alpha R)} dY > 0
\]

As in the monopoly case convexity of demand can cause this to be positive. However if $(2+R)$ is positive an increase in the advertising tax must reduce the consumer price of the import. This can be shown by using expressions for $dy/d\theta$, and $dm/d\theta$ in (4.14) to obtain\(^{27}\):

\(^{27}\) Note that in the strategic substitutes case $(2+R)$ must be positive but that it can be negative without violating stability.
\[ \frac{dq}{d\theta} = \frac{(ap')^2(2 + R)p - Yp'}{\Gamma} \quad (4.14') \]

4.3 Welfare

I turn now to the welfare implications of the tariff in the Nash game case. I will adopt the Dixit and Norman approach to measuring welfare change in the presence of endogenous tastes. Let the welfare function be:

\[ w(x, y, \bar{A}) = u(x + (\bar{a} + \bar{A})y) + n_c. \quad (4.15) \]

As before, \( \bar{A} \) is the government's reference level of advertising for \( y \). Welfare is evaluated at the actual level of consumption of \( x \), \( y \) and \( n_c \) and the reference level of advertising \( \bar{A} \). I will assume that the domestically produced good is only sold on the home market so that the balanced trade condition is given by (3.2). The full employment condition must be modified to take account of the production of good \( x \). It is now:

\[ e = n_p + hm + cx \quad (4.16) \]

Proceed by eliminating \( n_c \) in (4.15) using (3.2) and (4.16). Then totally differentiate the resulting expression with respect to \( t \), and solve for the optimal tariff:

\[ t^* = q - u_y - (u_x - c)(dx/dt)/(dy/dt) + y(dq*/dt)/(dy/dt). \quad (4.17) \]

where \( u_x \equiv \partial u(x + (\bar{a} + \bar{A})y)/\partial x \), is the shadow price of the home produced good and \( u_y \equiv \partial u(x + (\bar{a} + \bar{A})y)/\partial y \) is, as before, the domestic shadow price of the import. The first term on the right-hand side of (4.17) is positive if \( \bar{A} < m \), while the second
term which occurs because there is a gap between the marginal social valuation of the domestically produced imperfectly competitive good and its marginal cost will also be positive for $\tilde{A} < m$ if and only if home output is a strategic substitute for that of its rival.\textsuperscript{28} The third term will be positive if the tariff improves the terms of trade. It can be shown that a positive optimal tariff is more likely if the tariff reduces the level of advertising.

Suppose the government has the possibility of employing both an advertising tax and a tariff\textsuperscript{29}. Straightforward calculations lead to expressions for the jointly optimal tariff and advertising tax. The optimal advertising tax is:

\begin{equation}
\theta^* = y \frac{(v_x - c - Yp')(1 + \alpha R) - Yp'(3 + R)(1 + m/a)}{(2 + \alpha R)}
\end{equation}

\textbf{Proposition 6:} A sufficient condition for the optimal advertising tax to be positive in the oligopoly case is that outputs are strategic substitutes and that $v_x - c > 0$.

For the case of strategic substitutes and $v_x - c > 0$ the first term in the numerator will be positive. The second term must be positive, since $(3 + R)$ is positive from stability considerations. It was shown above that the optimal advertising tax under monopoly is always positive but it is possible for the optimal advertising tax

\begin{flushright}
\textsuperscript{28} Note that if $q - \upsilon_y$ is positive then $v_x - p$ is also positive. This is clear because the foreign price is increasing and the home price decreasing in advertising at constant outputs. Since $(p-c)$ must be positive for the home firm to produce at all, $\tilde{A} < m$ is a sufficient condition for $(v_x - c)$ to be positive.

\textsuperscript{29} For the remainder of this section I assume that the foreign firm does all its advertising in the home country.
\end{flushright}
to be negative under oligopoly; a necessary condition for this (assuming \( u_x > c \)) is that the home firm views its output as a strategic complement to that of its foreign rival. The overall optimal tariff can be shown to be:

\[
t^{\infty} - q = v_y + a \frac{(v_x - c)(1 + \alpha R) - Yp'(2 + R) + (m\Gamma'\alpha p')(dm/dt)}{2 + \alpha R}
\] (4.19)

The optimal tariff will once again depend on (i) the gap between \( m \) and \( \bar{m} \), (ii) the curvature of the demand function, in particular the sign of \((1+\alpha R)\), and, (iii) how advertising responds to the tariff\(^{30}\).

If \( m \) exceeds \( \bar{m} \) then \( q - v_y \) and \( v_x - c \) are both positive. A positive \( q - v_y \) implies that the domestic consumers overvalue the import, strengthening the case for a tariff. With \( v_x - c \) positive welfare will be increasing in home output. This strengthens the case for a positive optimal tariff if and only if home output is a strategic substitute for that of the foreign firm. The second term in the numerator captures various effects of the tariff on the terms of trade and it contributes to a positive tariff if \((2+R)\) is positive. The final term will be negative if the tariff reduces foreign advertising. This is due to the fact that an increase in the tariff which leads to a fall in advertising reduces the rent that can be extracted by the advertising tax. It is clearly possible for the optimal tariff to be negative even if \( m > \bar{m} \). This is more likely in the case of strategic complements and for \( dm/dt < 0 \).

\(^{30}\) Which will depend, among other things on \( \pi_{ym}^* \).
4.4 The non-paternalistic overall optimum

I now wish to consider the optimality of a tariff and advertising tax under the alternative non-paternalistic specification of the welfare function discussed earlier. Assume that the welfare function is the utility function of the aggregate household given in (4.1). Then proceeding as before it is straightforward to obtain:

\[ t^\infty = a \frac{(p - c)(1 + \alpha R) - Yp'(2 + R) + (m\Gamma/\alpha p')(dm/dt)}{(2 + \alpha R)} \quad (4.20) \]

\[ \theta^\infty = -U_m + \gamma \frac{(p - c - Yp')(1 + \alpha R) - Yp'(3 + R)(1 + m/a)}{(2 + \alpha R)} \]

Note that there is now no divergence between market prices and the shadow prices. The expression for the overall optimal tariff under the non-paternalistic welfare takes the same form and has the same sign as that under the Dixit-Norman welfare function when \( \bar{A} \) is the post-intervention level of advertising. The expression for the optimal advertising tax now includes the term \(-U_m = -(\gamma/a)q < 0\), which is the negative of the marginal benefit to the consumer of an increase in advertising at constant consumption levels and it weakens the case for an advertising tax. It is now possible for the optimal advertising tax to be negative even if outputs are strategic substitutes.

5 Strategic Entry Deterrence.

So far I have assumed that the foreign firm chooses the level of advertising at the same time as it selects its output. Sutton (1991) has examined a model in which advertising outlays are interpreted as a fixed cost incurred prior to the final
(output or price) stage of the game thus making them a sunk cost\textsuperscript{31}. In this section I consider the possibility that the foreign firm will attempt to use its advertising expenditure to deter entry of the domestic firm into the market. In order to do this I will have to make some adjustments to the analysis so far. In particular it will now be assumed that the foreign firm choose its level of advertising before the output stage of the game. In addition assume that the home firm faces some fixed start-up cost $F$.

The order of play is now as follows. In the first stage the home government chooses its tariff or advertising tax. In the second stage the foreign firm, which is assumed to be the incumbent in the market, decides on its level of advertising in anticipation of the outputs subsequently produced by the two firms. In the third stage the home firm observing the level of foreign advertising, decides whether to enter the market or to stay out. If it enters it must bear the entry cost $F$. It will be willing to do this if the anticipated variable profits from entry are at least equal to $F$. In the final stage of the game the two firms play Cournot if entry occurs, while if the home firm stays out the foreign firm remains the monopoly producer. The equilibrium of the game is subgame perfect\textsuperscript{32}. The home profit function is now:

$$
\pi = x[p(x,y) - c] - F. \quad (5.1)
$$

\textsuperscript{31} Sutton distinguishes between "classical" advertising outlays such as television, radio and press, and promotional outlays, expenditures entering directly into variable cost incurred in the final "market" stage of the game.

\textsuperscript{32} Brander and Spencer (1981) also considered a rent-extracting tariff in the presence of potential home firm entry. In their model quantities are the only strategic variables and the foreign firm has a first-mover advantage.
while that of the foreign firm is given in (2.3).

Following standard practice in analysing games of this type I begin at the last stage and work back to the beginning. The fourth stage equilibrium in which entry occurs is described by the following pair of first-order conditions:

(i) \[ \pi_x = p + p_x x - c = 0, \]  
\[ (5.2) \]

(ii) \[ \pi_y^* = q + q_y y - c - t = 0, \]

These first-order conditions are the reaction functions in the absence of entry costs. Advertising can be used to shift the fourth-stage equilibrium point in output space and reduce the variable profits of the home firm. When \( F \) is positive there will be a minimum level of advertising which drives home profits to zero. I will represent this "limit advertising" level by \( m^g \). If the home firm observes a level of advertising at or above \( m^g \) its best reply is not to enter.

The fourth-stage equilibrium will also be directly disturbed by changes in the tariff. The tariff shifts the foreign reaction function inwards while a change in \( m \) will in general affect both reaction curves. I continue to assume the aggregate household's utility function takes the form given in (4.1).

The fourth-stage comparative static derivatives are:

(i) \[ x_t = - \frac{\pi_x}{D}, \]  
\[ \text{sign } x_t = \text{sign } - \frac{\pi_{xy}}{D} \]  
\[ (5.5) \]

(ii) \[ y_t = - \frac{\pi_{xy}}{D} < 0. \]

for the tariff and:
(i) \[ x_m = -\frac{(c^*/a)x_i}{x}, \tag{5.6} \]

(ii) \[ y_m = -(y/a) + (c^*/a) y_i. \]

for advertising. Here \( c^* = c + t \) is foreign marginal cost inclusive of the tariff.\(^{33}\)

In the third stage of the game the home firm enters if the anticipated variable profit arising from the resulting Cournot subgame exceeds the entry cost. In the second stage the foreign firm chooses \( m \) to maximise its profits taking into account the effect that this has on the outcome of the third and fourth stages of the game. The foreign firm compares the profitability of the entry deterrence outcome with that of accommodating entry. If entry is deterred then the foreign firm chooses the monopoly output \( y^B(m^B(t),t) \)\(^{34}\) given this level of advertising and the tariff. Alternatively the foreign firm can accommodate entry and choose \( m = m^A \) to maximise its total profits given the reaction of the home firm. I will assume that initially at least \( m^A < m^B \).\(^{35}\) The foreign firm compares:

(i) \[ \pi^{x_A}(x^A,y^A,m^A,t), \tag{5.7} \]

with (ii) \[ \pi^{x_E}(0,y^B,m^B,t), \]

where \( x^A = x(m^A(t),t) \) and \( y^A = y(m^A(t),t) \).

\(^{33}\) The expressions for \( x_m \) and \( y_m \) are obtained by using the foreign firm's fourth-stage first-order condition in \( \pi_{y_m}^* \).

\(^{34}\) Note that this level of output is greater than \( y^B \) the level of output that the foreign firm threatens to play in the event of home entry at \( m^B \).

\(^{35}\) I will also assume that \( m^B \) is greater than the level of advertising that would be chosen under unthreatened monopoly. If this was not the case entry would be blockaded and the model would simplify to that analysed in the monopoly section of the paper.
5.1 Entry deterrence

Suppose that the foreign firm finds entry deterrence at \( m^B \) and \( y^E \) to be its optimal strategy. The foreign firm is just deterring entry with the advertising level \( m^B \) in so far as a small decrease in advertising or a small increase in the tariff will permit the home firm to enter profitably. Now a small increase in the tariff that leaves the foreign entry deterrence strategy optimal will require a compensating increase in \( m \) in order to keep home profits under entry at zero. The foreign firm chooses \( m \) such that:

\[
\pi(x(m,t),y(m,t),m,F) = 0. \tag{5.8}
\]

Total differentiation of (5.8) and use of the envelope theorem yields:

\[
\frac{dm^B}{dt} = -\frac{y_t}{y_m + (\pi_y/\pi_f)} \tag{5.9}
\]

Making use of (5.6), \( \pi_y = ap'x \) and \( \pi_m = yp'x \) gives:

\[
dm^B/dt = a/c^* > 0. \tag{5.10}
\]

I turn now to the question of how the tariff affects the level of foreign output \( y^E \) under the strategy of entry deterrence:

\[
\frac{dy^E}{dt} = \frac{\partial y^E}{\partial t} + \frac{dm^B}{dt} \frac{\partial y^E}{\partial m} = -\frac{y}{c^*} \tag{5.11}
\]

This gives the following result for a small tariff increase that does not alter the foreign firm's entry-deterrence strategy:

**Proposition 7:** A small tariff that does not alter the foreign firm’s entry deterrence
strategy reduces imports and causes the level of advertising to rise. It leaves foreign output in quality units unchanged.

The effect of the tariff on the limit output in quality units is:

$$\frac{dY^E}{dt} = - \frac{d(a_BY^E)}{dt} - y^E\left(\frac{dm^B}{dt}\right) + a^B\left(\frac{dy^E}{dt}\right) = 0.$$ 

A small tariff that leaves foreign entry deterrence optimal will cause the home terms of trade to deteriorate and the domestic consumer price of the import to rise. The change in the consumer price of the good is:

$$\frac{dq^E}{dt} = q_y\frac{dy^E}{dt} + q_m\frac{dm^B}{dt}$$  

Making use of (5.10),(5.11) and the foreign fourth-stage first-order condition for the monopoly subgame, yields the following expression for the impact of the tariff on the terms of trade:

$$dq^*Y^E/dt = - ap'Y/c^* = a^2p'(dy^E/dt) > 0.$$  

When the Dixit-Norman approach to welfare evaluation is used the net effect on welfare of a small increase in the tariff is ambiguous. It is more likely to be positive the larger is $q - \upsilon_y$. However under the non-paternalistic specification the following result is obtained:

$$\frac{dw}{dt} = U_m\frac{dm^B}{dt} - \frac{dq^*E}{dt} + t\frac{dy^E}{dt} - y^E(1 + \eta) \geq 0.$$  

35
where \( \eta = (t/y^x)(dy^x/dt) = -1/(1 + c/t) \) is the elasticity of supply of imports with respect to the tariff which cannot be more negative than -1. It benefits the home government to raise the tariff at least to the level that leaves the foreign firm indifferent between entry deterrence and accommodation.

5.2 The foreign firm accommodates entry

If instead the foreign firm decides to accommodate entry it will choose its level of advertising to solve the following optimisation problem:

\[
\frac{d\pi^*}{dm} = \pi^* x_m + \pi^*_m = 0. \tag{5.15}
\]

The envelope theorem having been invoked to eliminate a term in \( \pi^*_y \). The first term on the right hand side of (5.15) represents the strategic effect of advertising. \( \text{For the case of strategic substitutes this represents its effect in reducing the level of home output and thus raising home profits} \)\(^{36}\). The second term is the direct effect of advertising on foreign profits: for the case of strategic substitutes this is negative indicating that at constant outputs the marginal profitability of advertising is negative. Making use of the foreign fourth-stage first-order condition yields:

\[
\frac{d\pi^*}{dm} = B \frac{yc^*}{a} - \mu = 0 \tag{5.16}
\]

where \( B = q_2(x_m/q_m) + 1 \)

5.3 A tariff induced regime change

\(^{36}\) For the case of strategic complements this term is negative. The firm wishes to keep its advertising down in order to keep the rival output down.
How does an increase in the tariff affect the profits from entry accommodation? The maximised level of profit under accommodation is:

\[
\pi^{A}(t) = y(m(t), t)\{q(m(t), x(m(t), t), y(m(t), t) - c^*)\} - \mu m(t). \tag{5.17}
\]

To obtain an expression for the effect of an increase in the tariff on foreign profits totally differentiate (5.17) and make use of the foreign second- and fourth-stage first-order conditions to get:

\[
d\pi^{A}/dt = -\mu(\bar{a} + m^A)/(c + t) < 0. \tag{5.18}
\]

The profits of the foreign firm fall. The change in foreign profits under entry deterrence can be shown to be:\(^{37}\)

\[
d\pi^{E}/dt = -\mu(\bar{a} + m^B)/(c + t) < 0. \tag{5.19}
\]

**Proposition 8:** A tariff can be used to persuade the foreign firm to abandon entry deterrence in favour of accommodation.

Proof: Suppose the foreign profits from entry deterrence are initially infinitesimally higher than those under accommodation then by (5.18) and (5.19) a small increase in the tariff will cause a regime shift.

\[
d\pi^{A}/dt - d\pi^{E}/dt = -\mu(m^A - m^B)/c^* > 0. \tag{5.20}
\]

The profit fall under accommodation is less than that under entry deterrence. Furthermore an increase in the tariff under accommodation will reduce the level of advertising and so the rate of profit fall is declining under accommodation while

---

\(^{37}\) To show this note that:

\[
d\pi^{E}/dt = [q_m m^B/dt - 1]y^2 - \mu dm^B/dt. \] The term in parentheses is zero. Due to the concavity of the advertising response function this loss of profits gets more serious the larger is \(m^B\).
it is rising under the entry deterrence regime.

From the foreign fourth-stage first-order condition:

\[ q = c + t - (\bar{a} + m)^2 p'y. \]  \hspace{1cm} (5.21)

Therefore:

\[ q^E - q^A = - \{(\bar{a} + m^B)^2 p'(Y^E)y^E - (\bar{a} + m^A)^2 p'(x^A + Y^A)y^A)\} + t^E - t^A. \]  \hspace{1cm} (5.22)

I am concerned with the change in market price of the import in the neighbourhood of the regime switchover point. Unfortunately the term in chain brackets is ambiguous in sign as it depends on a comparison between the slope of the inverse demand at two different places. In the special case of linear inverse demands \((p' = -b, a constant)\) it can be shown that a small tariff increase which brings about a regime shift from entry deterrence to accommodation will improve the terms of trade facing the home country and lead to a fall in the domestic price of the import. Given that \(b\) and \(c\) are constants and that \(y^E > y^A\), and \(m^B > m^A\), it can be shown that a Metzler paradox will ensue. A small tariff increase can cause a big fall in output and advertising and with \(t^E - t^A\) arbitrarily close to zero, \(q^E - q^A\) will be positive.

6. Concluding Remarks

This paper focuses on the positive and normative aspects of tariffs in situations in which firms choose advertising as well as output. The simplest case I examined was a model in which the home government faces a foreign monopolist. In addition firm rivalry was modelled both as a Nash game in advertising and quantity and a multi-stage game in which the foreign firm chose advertising before
both firms selected their outputs.

It was demonstrated that protection affects imports, the domestic consumer price and the terms of trade directly, through an increase in marginal costs, and through its effect on the level of advertising. A surprising positive finding was the possibility of a Metzler paradox. If the tariff reduces advertising then it can in some cases bring about a fall in the domestic price of the import. This is possible in both the monopoly case and in both the oligopoly games.

Jointly optimal tariffs and advertising taxes and the optimal tariff for the situation when it is the only available policy instrument were derived for both the monopoly and Cournot oligopoly cases. The optimal tariff was shown to depend in a complex way on the curvature of the demand function and on the effect of advertising on marginal profits. In particular in the oligopoly case a concave inverse demand implies that outputs are strategic substitutes and this usually contributes to a positive optimal tariff. Optimal policy was examined for two different specifications of the welfare function and in both the monopolistic and oligopolistic market structures. When the welfare function takes the Dixit and Norman form the optimal tariff was shown to depend on the gap between the actual and reference level of advertising while under a non-paternalistic specification it depended on the marginal utility of advertising.

It was also shown that a tariff can be used to persuade the foreign firm to abandon a strategy of entry deterrence and that this is likely to bring about a fall
in the domestic price of the import as the level of advertising is sharply reduced.

Although I have concentrated on the issue of persuasive advertising, it is possible to extend the analysis to consider product development. If product development means an increase in the quality of the good, then this quality improvement can be modelled as raising the level of consumer utility and national welfare directly, just as advertising does under the non-paternalistic welfare function considered in this paper.
REFERENCES


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Table 1:
Effects of a tariff on the terms of trade. See equation (2.8) and associated discussion.
In this example the tariff induced fall in advertising causes the inverse demand function to shift inwards from aa' to bb'. Although marginal costs rise by t per unit and imports fall from $y_1$ to $y_2$, it is possible for the post-intervention domestic consumer price $q_2$ to fall below its initial level $q_1$. 

**Figure 1: Metzler Paradox**
Figure 2: A price increase resulting from a fall in demand
As mentioned in the text and shown above an increase in the marginal cost of advertising can cause a perverse price change. A price increase is guaranteed if the inverse demand is very convex, (i.e. if \((1+R) < 0\)) and the demand curve shifts parallel downwards from aa' to bb'. The marginal revenue curve must move inwards from \(r_1\) to \(r_2\), and in this case the price will rise from \(q_1\) to \(q_2\).