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FISCAL POLICY IN A SMALL OPEN ECONOMY WITH
UNEMPLOYMENT AND CAPITAL ACCUMULATION

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I would like to thank J. P. Neary for helpful discussions on an earlier draft of this paper. Sole responsibility for remaining shortcomings rests with the author.

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ABSTRACT

This paper presents a two-sector model of a small open economy with wage rigidities and capital accumulation. The short- and medium-run effects of government expenditure policies are analysed and the results are contrasted with those of previous models which either ignore investment or restrict it to the case of inter-sectoral reallocation of capital.

I. INTRODUCTION

Theoretical studies of the small open economy have tended to deal either with a short-run period in which the stock of capital in each sector of the economy is fixed while employment is variable, or with a long-run period in which capital stocks can change but full employment is assumed to prevail.

Several recent contributions, however, have analysed the behaviour of economies in which sectoral capital stocks can change whilst wage rigidities prevent the adjustment to full employment, adopting the position that medium term resource allocation will be substantially affected by rigidities stemming from the current distribution of income between capital and labour. Amongst these have been papers by Helpman (1976), Brecher (1978) and Kouri (1979, 1982).

The only explicit analysis of the medium-run effects of fiscal policy is contained in Helpman. He employs the Heckscher-Ohlin model of international trade theory which postulates that changes in sectoral capital stocks occur through the movement of physical capital between sectors.

This assumption of intersectoral capital mobility is often considered by trade theorists as the 'long-run' analogue to short-run models with sector-specific capital (e.g. Mayer (1974), Mussa (1974)), but the present paper constructs an alternative model of long-run resource allocation, in which new investment,

financed by the savings of the economy, occurs in the expanding sector, while the declining sector contracts through a slowing down in the rate of replacement of depreciating capital equipment.

The next section presents this model, and is followed by an analysis of the effects of fiscal policies. A final section contrasts these results with those emerging from the Heckscher-Ohlin model.

II. THE MODEL

Two goods, one traded internationally and the other one non-traded, are assumed to be produced with constant returns-to-scale technologies and with physical capital that is sector-specific, so that unit costs, which reflect market prices, are given by:

$$(1) \quad P_n = a_{ln} \cdot w + a_{kn} \cdot r_n$$

$$(2) \quad P_t = a_{lt} \cdot w + a_{kt} \cdot r_t$$

The price of traded goods, P_t , is constant given the small open economy assumption, a fixed exchange rate and constant world prices. The wage rate, w , is equalised across sectors because of labour mobility whilst the rental rates on capital can differ because capital is sector specific. Wage rigidities are assumed to exist and to be unresponsive to the level of unemployment in the time period under consideration.

$$(3) \quad w = \phi(P_n, P_t)$$

If real wages are rigid, the wage restriction is homogeneous of degree one.

Each sector faces rising marginal costs of adjusting its capital stock, as in standard investment theory, so that optimal adjustment occurs over time. When these costs are a function of gross investment relative to the existing capital stock as in Lucas (1967), and when the investment goods are imported or produced by tradable-goods industries, the investment functions take the form:¹

$$(4) \quad I_n = f \left(\frac{q_n}{P_t} - 1 \right) \cdot K_n$$

$$(5) \quad I_t = f \left(\frac{q_t}{P_t} - 1 \right) \cdot K_t$$

where q_n and q_t are the nominal prices of equities for each sector and where $I_i = \dot{K}_i + \delta \cdot K_i$, δ being the rate of depreciation of capital.

It is assumed that $f(0) = \delta$, so that $\dot{K}_i = 0$, when $q_i = P_t$; $i = n, t$.²

These micro-foundations are consistent with Tobin's theory of investment.³ Investment responds to the difference between the market valuation of the titles to capital and the replacement cost of capital goods.

There are three financial assets in the economy: money and the equities associated with the stock of capital in each sector. Money is the only internationally-traded financial asset.⁴ The equities are assumed to be perfect substitutes for each other so that their rates of return, which are the value marginal products of capital in each sector divided by the price of the sector's equity, must be equalised:

$$(6) \quad \frac{r_n}{q_n} = \frac{r_t}{q_t}$$

The demand for money as a proportion of total wealth depends negatively on the interest rate prevailing in the economy. This

may be written:

$$(7) \quad M = \alpha \left(\frac{r_t}{q_t} \right) \cdot (q_n \cdot K_n + q_t \cdot K_t)^5$$

Savings are assumed to be related to the discrepancy between a target wealth stock (A^t) and current real asset holdings, as in Dornbusch (1975, 1976). Measuring real variables in terms of traded-goods and denoting such variables by "-", we have:

$$\text{Savings} = \gamma \left\{ \tilde{A}^t - (\tilde{M} + \tilde{q}_n K_n + \tilde{q}_t K_t) \right\}$$

On life-cycle principles the target stock of wealth is a function of disposable labour income. For simplicity a linear relationship is assumed.

$$\tilde{A}_t = \theta \left[\tilde{w} K_n \ell_n \left(\frac{w}{p_n} \right) + \tilde{w} K_t \ell_t \left(\frac{w}{p_t} \right) - \tilde{T} \right]$$

where K_i is the capital stock and $\ell_i \left(\frac{w}{p_i} \right)$ the labour-capital ratio in sector i , and $\tilde{T}$ is the level of real lump-sum taxation.⁶

Consumption expenditure is net disposable income less savings, i.e.

$$Z = (r_n - \delta) K_n + (r_t - \delta) K_t + (1 - \gamma \theta) \left[w K_n \ell_n + w K_t \ell_t - T \right] + \gamma (M + q_n K_n + q_t K_t)$$

The equilibrium condition for the non traded goods market, which is satisfied at each point in time is:

$$(8) \quad \tilde{P}_n \cdot Y_n = \tilde{P}_n D_n(\tilde{P}_n; Z) + \tilde{P}_n \cdot G_n$$

where Y_n is the output of the non-tradables sector, $D_n(\quad)$ is private demand for non-tradables, which depends on relative prices and on total consumption expenditure, and G_n is government expenditure on non-tradables.

The government budget is assumed to be balanced throughout the analysis by adjusting taxation to levels of government expenditure:

$$(9) \quad \tilde{T} = G_t + \tilde{P}_n \cdot G_n$$

Finally, savings (net asset accumulation) must equal net investment plus the government deficit plus the balance of trade surplus (inflow of reserves). Given (9) this implies that savings less net investment must equal the balance of trade surplus.⁷

$$(10) \quad \gamma \left\{ \tilde{A}^t - (\tilde{M} + \tilde{q}_n K_n + \tilde{q}_t K_t) \right\} - \tilde{q}_n \dot{K}_n = \tilde{q}_t \dot{K}_t = \dot{M}$$

III. STRUCTURE AND STABILITY OF THE MODEL

In the short-run K_n, K_t and M are fixed and the model determines $P_n, w, r_n, r_t, q_n, q_t, \bar{T}, M, \dot{K}_n$ and $\dot{K}_t$. In the long-run stationary equilibrium the first seven of these variables, along with long-run values of M, K_n and K_t are determined by the model along the requirement that $\dot{M} = \dot{K}_n = \dot{K}_t = 0$.

Some properties of this long-run equilibrium can be seen immediately. From (4) and (5) $\tilde{q}_n$ and $\tilde{q}_t$ will equal one, so from (6) we see that, as in the Heckscher-Ohlin model, the value marginal products of capitals are equalised across sectors.⁸ This implies that in stationary equilibrium prices and wages are determined independently of demand conditions, so that a long-run non-substitution theorem applies, and factor input ratios in each sector, as in the Helpman model, will be constant across steady states. This allows us to depict the long-run solution graphically. Substituting (7) into (8) and setting $\dot{M} = 0$ yields the long-run non-traded goods market equilibrium curve, NN, which, as shown in the appendix where the mathematical results are gathered, is upward sloping where K_t is plotted on the horizontal and K_n on the vertical axis. The balance of payments equilibrium curve, BB, is obtained by substituting (7) into (10) and setting $\dot{M} = 0$. The necessary and sufficient condition for stability of the model is that, when the non-tradables sector is relatively labour (capital) intensive, the BB curve have a greater (lesser) slope than the NN curve.

As discussed in the appendix, the economic requirement for this to occur is that the marginal propensity to consume non-tradables be low (high).

Henceforth the stability conditions are assumed to be satisfied.

IV. EFFECTS OF GOVERNMENT EXPENDITURE ON NON-TRADABLES

Consider a balanced-budget expansion in government spending on non-traded goods. The price of these goods and the return on capital in that sector rises, while the return on capital in the traded-goods sector will either remain constant or fall depending on whether nominal or real wages are rigid in the short-run.

The effects on equity prices can be seen from equations (6) and (7). With nominal wages rigid, the price of the non-trading sector's equity jumps forcing up the rate of interest so as to equilibrate the money market. Since the rate of interest on the tradable sector's equity must also rise because equities are perfectly substitutable, q_t falls with the result that investment falls in this sector and rises in the non-tradable sector.⁹ The increased government expenditure increases the value of assets in the short-run under these circumstances, by engineering a fall in real wages.

The situation is a little more complex when real wages are rigid, since the return on capital in each sector moves in opposite directions. This causes q_n to rise and q_t to fall, so that the effects on each sector's investment are the same as before, but the direction of movement of the interest rate depends directly on whether the total value of capital rises or falls. This in turn depends on whether the increased value of the output of non-tradables is greater or less than the increase in the total labour bill resulting from money wage increases.¹⁰

Since one of the innovations of the model is to introduce wealth effects, it is of interest to see the effect that the changes in equity prices have on the response of commodity prices to government expenditure. It is shown in the appendix, where the mathematical results are gathered, that if the impact effect lowers the total value of capital in the economy as may occur when real wages are rigid, then the short-run response of the price of non-tradables to government spending is lower, since the reduction in the value of financial wealth lowers consumption.

In any case, the short-run effects on the non-tradable sector are that overall demand is expanded, prices rise, investment is stimulated, the real product wage $(\frac{w}{p_n})$ falls and supply rises.

Now consider the tradable-goods sector. The short-run effect on output depends on whether nominal or real wages are rigid. When the former prevails then the real product wage $(\tilde{w})$ is constant, so output changes only over time as the capital stock changes. Private demand for tradables rises because of the income and substitution effects while the wealth effect will depend, as before, on whether the rise in p_n increases or decreases the total return to capital in the economy. If this increases, as it will for example under nominal wage rigidity, then the increase in the value of assets and in the total level of investment combined with the fall in disposable labour income leads to a trade balance deficit.

On the other hand, if the total value of assets falls, as it may under real wage rigidity, then the effect on savings is ambiguous while investment falls, so that the possibility of the trade balance moving into surplus cannot be completely ruled out. Exclusively short-run models ignore investment and wealth effects and thus preclude this possibility.

The short-run effects on total employment can be seen with the aid of the following equation:

$$(11) \quad L = L_t \left(\frac{P_t}{w} \right) + L_n \left(\frac{P_n}{w} \right)$$

where L is total employment, and L_i is employment in sector i .

Using (3), we see that:

$$\frac{dL}{dG_n} = L_n' \cdot \frac{1}{w} \cdot \frac{dP_n}{dG_n}, \text{ when } \phi_1 = 0,$$

where L_i' is the partial derivative of the functions above, which is negative.

In other words, total employment rises under nominal wage rigidity, since the non-trading sector expands and tradable-goods output is unchanged.

Under real-wage rigidity (3) is homogeneous of degree one. The condition for total employment to rise in this case is that:

$$\frac{\epsilon(L_n, \frac{P_n}{w})}{\epsilon(L_t, \frac{P_t}{w})} \cdot \frac{L_n}{L_t} \cdot \frac{\phi_2}{\phi_1} \cdot \frac{P_t}{P_n} > 1$$

where $\epsilon(L_i, \frac{P_i}{w})$ is the elasticity of labour demand with respect to $\frac{P_i}{w}$. This condition is more easily interpreted if it is assumed that the share of non-traded goods in production and consumption is the same, i.e. that:

$$\frac{\phi_1 \frac{P_n}{w}}{\phi_2 \frac{P_t}{w}} = \frac{P_n Y_n}{P_t Y_t}$$

in which case the condition becomes:

$$\epsilon(L_n, \frac{P_n}{w}) \cdot \frac{\frac{L_n}{L}}{\frac{P_n Y_n}{Y}} > \epsilon(L_t, \frac{P_t}{w}) \cdot \frac{\frac{L_t}{L}}{\frac{P_t Y_t}{Y}}$$

where $Y = P_n Y_n + P_t Y_t$.

This condition is satisfied if non-tradables are sufficiently labour-intensive relative to tradables, and/or if the elasticity of demand for labour in the non-tradables sector is sufficiently high relative to that for the tradable sector.

If this condition is not met, then an increase in government spending on non-traded goods when real wages are rigid will lead to a contraction in total employment in the short run.

Similar conditions are derived by Helpman (1977), Rødseth (1979) and Calmfors and Viotti (1982).

Since the dynamics of a system with three state variables are largely intractable, the results of the stability analysis are relied upon to ensure that the economy moves towards the long-run equilibrium position.¹¹

The long-run comparative static results apply when $\dot{K}_n = \dot{K}_t = \dot{M} = 0$. As mentioned previously, this means that equity prices and value marginal products of capital are equalised across sectors, and r , p_n and w return to their initial positions.

Figure 1 depicts the long-run response of the economy to a balanced budget increase in government spending on non-tradables, in the case where this sector is relatively labour-intensive. The increased taxation shifts the BB curve upwards to the left since, for any given K_t , K_n must rise, raising labour income and causing desired wealth to rise more than the value of assets so as to counteract the tendency of a rise in taxation to generate an excess supply of assets.

The increase in spending induced by the balanced-budget expansion also shifts the NN curve upwards to the left.

The long run-multipliers (evaluated for $P_n = P_t = 1$) are:

$$\frac{dK_n}{dG_n} = \frac{1}{\Delta} \{ [\theta w \ell_t - (1 + \alpha)] + D_{n2} [\theta(r - \delta) + (1 + \alpha)] \}$$

$$\frac{dK_t}{dG_n} = \frac{1}{\Delta} \{ (1 - D_{n2})(\theta r + 1 + \alpha) + \theta \delta D_{n2} \} < 0$$

$$\text{where } \Delta = (r + w l_n) \left[\theta w l_t - (1 + \alpha) \right] + D_{n2} w (l_n - l_t) \left[1 + \alpha + \theta (r - \delta) \right] < 0.$$

The negativity of the determinant, Δ , is guaranteed by the stability condition. The tradable-goods sector contracts in all circumstances, while the non-traded sector will contract if tradables are labour-intensive and appears likely to expand if it itself is the labour-intensive sector. The more labour-intensive it is, the more likely it is to expand.

V. EFFECTS OF GOVERNMENT EXPENDITURE ON TRADABLES

Consider the effects of a balanced budget increase in government spending on tradables. The effects on P_n , r_n , r_t , w , q_n and q_t can be found from equations (1) - (3) and (6) - (8) which do not contain the variable G_t , so that the only effects on these variables in the short-run arise as a result of the increase in taxation. These effects are that r_n , q_n and thus also investment in the non-tradable sector fall, r_t either remains constant or rises depending on whether nominal or real wages are rigid, while q_t , and thus investment in the tradable sector, rises due to the fall in interest rates. The increased taxation, by restricting demand, also drives down the price of non-traded goods.

These short-run results, apart from the wealth and investment effects, are also to be found in Helpman (1977) and Rødseth (1979).

If the value of assets rises in response to the fall in non-traded goods prices saving falls and investment is stimulated so that the balance of trade moves into deficit. In the opposite case however, which occurs for example when a nominal wage rigidity causes the fall in non-traded goods prices to drive up real wages, the effect on disposable labour income is ambiguous, whilst saving is stimulated directly by the fall in asset values and investment is reduced, so that the counter-intuitive result of a balance of trade surplus cannot be ruled out.¹²

Under the circumstances which guarantee that an increase in government spending on non-tradables produces a trade deficit, the increased taxation associated with a balanced-budget expansion in spending on tradables works towards producing a trade surplus.

Figure 2 depicts the long-run response of the economy to this policy, where non-tradables are labour intensive. The increase in taxation, as before, shifts the BB curve to the left while the decreased demand for non-tradables, resulting from the increase in taxation, shifts the NN curve to the right.

It is clear that under these circumstances, both sectors contract, and employment falls. It can also be seen, by comparing the short and long-run responses, that the dynamic path of the economy involves cycling of the tradable sector's capital stock.

The long-run multipliers are:

$$\frac{dK_n}{dG_t} = \frac{1}{\Delta} \{ D_{n2} \theta(r - \delta) + (1 + \alpha) \} < 0$$

$$\text{and } \frac{dK_t}{dG_t} = \frac{1}{\Delta} \{ \theta [r(1 - D_{n2}) + w l_n + \delta D_{n2}] - D_{n2}(1 + \alpha) \}$$

A balanced-budget increase in government spending on tradables will induce a long-run decline in the non-tradable goods sector, whilst the effect on the tradable sector will be negative if non-tradables are labour intensive and may be positive, with this

sector holding on to some of its short-term increased investment, if tradables are labour-intensive. This becomes a possibility because the opportunity to invest in the non-traded sector is limited by the long-term decline in demand for that sector's output, so that savings may be channelled towards the tradable sector.

CONCLUDING COMMENTS

The model presented in this paper differs from the popular Heckscher-Ohlin model in recognising that medium-term resource allocation is more often carried out through channelling new investment towards the expanding sector and allowing the contracting sector to decline through depreciation of capital, rather than through the intersectoral reallocation of existing capital equipment. The long-run stock of capital in the economy must therefore be determined by savings behaviour, which the Heckscher-Ohlin model ignores.

The analyses have one important similarity. Since the returns to capital across sectors are equalised in the long-run, a non-substitution theorem applies under conditions of wage rigidities and a fixed exchange rate, so that sectors will expand or contract across steady states at constant capital-labour ratios.

It is clear that when the total capital stock is fixed, as in Helpman (1976), one sector can expand only at the expense of the other. The employment effects of fiscal policy will depend, therefore, only on relative factor intensities.

In the present paper this direct link between the size of sectors is replaced by a less direct link through the savings mechanism. Thus the medium-run results presented here are quite different from those of Helpman.

The perspective of this paper, by focussing on savings and investment, also sheds light on aspects of the effects of government policies on the balance of trade which cannot be analysed in exclusively short-run models.

FOOTNOTES

1. The investment functions can be derived from the following model. Assume a quadratic form for the adjustment costs, as in Lucas, and let installation be carried out by the capital-producing (tradable) sector. The firm's revenue results from sales of output and equities, so the firm maximises:

$$\pi_i = P_i F_i(K_i, L_i) + q_i \dot{K}_i - P_t \left(I_i + \frac{b l_i^2}{K_i} \right) - wL - (r_i - \delta)K_i$$

where $I_i = \dot{K}_i + \delta K_i$, $i = n, t$.

One of the first order conditions is that $\frac{\delta \pi_i}{\delta \dot{K}_i} = 0$

which implies $I_i = \frac{1}{2b} \left(\frac{q_i}{P_t} - 1 \right) \cdot K_i$

2. With neo-classical production and Lucas-type adjustment costs the optimal growth rate of a firm that maximises its market value is constant. A stationary long-run solution exists only if the shadow value of capital is such that this optimal growth rate of the firm is zero. See Kouri (1982) for an explicit derivation of this value.
3. See Bailey and Scarth (1980).

4. The introduction of imperfect capital mobility will not affect the qualitative results, as the domestic holdings of foreign bonds will vary proportionately across steady states with the size of the domestic capital stock, whilst foreign holdings of domestic equities is constant across steady states because the foreign interest rate and portfolio size are assumed constant and the domestic interest rate, as shown in the text, returns in the long run to its initial value. These complications are omitted because of the considerable simplification in dynamic structure that this allows.

The case of perfect capital mobility cannot be accommodated because this would introduce rigidities into all factor prices.

5. The usual dependence of money demand on income is suppressed for convenience. Some justification comes from the fact that long-run wealth is a linear function of disposable income, so that the long-run money demand function in the model has a not-unreasonable income elasticity of unity.
6. As Kouri (1976) points out, this approach is identical, under fixed exchange rates, to specifying a consumption function dependent on disposable labour income and the stock of financial wealth.

7. There is no independent traded-goods market condition since equations (8) and (10) imply:

$Y_t - q(\dot{K} + \delta K) - D_T(\quad) - G_T$, equals the balance of trade surplus.

8. The model satisfies the requirements for this laid out in Mussa (1978) in the section on 'Capital Formation with Increasing Marginal Costs'. Unlike Mussa's, the model in this paper reflects a long-run non-substitution theorem because of the presence of the wage restriction.

9. This result contrasts with that of simpler models where investment in a sector depends on the difference between the value marginal product of capital and an international bond rate of return.

10. As Jones (1965) notes, the change in costs resulting from a small change in factor prices is the same whether or not factor proportions are altered.

11. Wissen (1982) studies the dynamics of a two-sector small open economy with capital accumulation and a slowly adjusting labour market.

12. This result perhaps warrants further comment. Short-run models ignore the government budget constraint and investment and wealth effects so that the goods market equilibrium condition is:

$$\text{SAVINGS} - \text{INVESTMENT} = \text{TRADE SURPLUS} + \text{GOVERNMENT DEFICIT}.$$

An increase in government spending on tradables, *ceteris paribus*, has no effect on savings or investment and thus generates an equivalent reduction in the trade surplus. The present paper does not, of course, ignore these longer-run considerations and so does not generate this familiar result.

REFERENCES

- Bailey R.E. and Scarth W.M. (1980) "Adjustment Costs and Aggregate Demand Theory". *Economica*.
- Brecher R. (1978) "Money, Employment and Trade Balance Adjustment with Rigid Wages". *Oxford Economic Papers*.
- Calmfors L. and Viotti S. (1982) "Wage Indexation, The Scandinavian Model and Macroeconomic Stability in the Open Economy". *Oxford Economic Papers*.
- Dornbusch R. (1975) "A Portfolio Balance Model of the Open Economy". *Journal of Monetary Economics*.
- Dornbusch R. (1976) "Capital Mobility, Flexible Exchange Rates and Macroeconomic Equilibrium" in E. Claassen and P. Salin (eds) *Recent Developments in International Monetary Economics* (North Holland).
- Helpman E. (1976) "Macroeconomic Policy in a Model of International Trade with a Wage Restriction". *International Economic Review*.
- Helpman E. (1977) "Non-traded Goods and Macroeconomic Policy under a Fixed Exchange Rate". *Quarterly Journal of Economics*.
- Jones R. (1965) "The Structure of Simple General Equilibrium Models". *Journal of Political Economy*.
- Kouri P. (1976) "The Exchange Rate and the Balance of Payments in the Short-run and in the Long-run: A Monetary Approach". *Scandinavian Journal of Economics*.

- Kouri P. (1982) "Profitability and Growth". Scandinavian Journal of Economics.
- Lucas R. (1967) "Adjustment Costs and the Theory of Supply". Journal of Political Economy.
- Mayer W. (1974) "Short-run and Long-run equilibrium for a Small Open Economy". Journal of Political Economy.
- Mussa M. (1974) "Tariffs and the Distribution of Income: The Importance of Factor Specificity, Substitutability and Intensity in the Short and Long Run". Journal of Political Economy.
- Mussa M. (1978) "Dynamic Adjustment in the Heckscher-Ohlin-Samuelson Model". Journal of Political Economy.
- Rødseth A. (1979) "Macroeconomic Policy in a Small Open Economy". Scandinavian Journal of Economics.
- Wissén P. (1982) "Growth Models for Open Economies with Non-Shiftable, Malleable Capital and Non-traded Goods". Seminar Paper No. 205. Institute for International Economic Studies, Stockholm.

APPENDIX

For convenience all results are evaluated for $P_n = P_t = 1$.

The Slope of the BB curve

Setting $\dot{K}_n = \dot{K}_t = \dot{M} = 0$, $\tilde{q}_n = \tilde{q}_t = 1$, $r_n = r_t = r$, and differentiating equation (10) we find

$$\left. \frac{dK_n}{dK_t} \right|_{BB} = \frac{(1 + \alpha) - \theta w l_t}{\theta w l_n - (1 + \alpha)}$$

This slope is of ambiguous sign.

The Slope of the NN curve

Setting $\dot{M} = 0$, $\tilde{q}_n = \tilde{q}_t = 1$, $r_n = r_t = r$ and differentiating equation (8), we find

$$\left. \frac{dK_n}{dK_t} \right|_{NN} = \frac{D_{n2}(r - \delta + w l_t)}{(r + w l_n)(1 - D_{n2}) + \delta D_{n2}} > 0$$

Stability Analysis

Taking the partial derivatives of the seven static equations of the model and inserting them in the dynamic equations (4), (5) and (10), linearised around the long-run equilibrium $(\bar{K}_n, \bar{K}_t)$ we have a system of the form:

$$\begin{bmatrix} \dot{K}_n \\ \dot{K}_t \\ \dot{M} \end{bmatrix} = \begin{bmatrix} & & \\ & H & \\ & & \end{bmatrix} \begin{bmatrix} K_n - \bar{K}_n \\ K_t - \bar{K}_t \\ M - \bar{M} \end{bmatrix}$$

The Routh-Hurwitz theorem enables us to find necessary and sufficient conditions for the stability of this system in terms of the properties of the co-efficient matrix H .

For the model under discussion these conditions are:

$$J < 0,$$

$$\text{and } (r + w\ell_n) \left[(1 + \alpha) - \theta w\ell_t \right] + D_{n2}(w\ell_t - w\ell_n) \left[(1 + \alpha) + \theta(r - \delta) \right]$$

$$> 0$$

$$\text{where } J = \{-wK_n \ell_n' (\phi_1 - w) + G_n\} \{1 - D_{n2}(1 - \gamma\theta)\} \cdot C$$

$$+ D_{n2} \cdot C - Y_n \cdot C + D_{n2}(1 - \gamma\theta) \{K_n \ell_n \phi_1 + K_t \ell_t \phi_1 + wK_t \ell_t' \phi_1\} \cdot C$$

$$+ D_{n2} rK \left[\alpha' (r + \gamma) - \alpha \right] \{K_n \ell_n \phi_1 + K_t \ell_t \phi_1 - Y_n\},$$

$$C \equiv r(\alpha K - rK\alpha'),$$

and ℓ_n' , ℓ_t' , α' and ϕ_1 are derivatives.

ℓ_n' , ℓ_t' , $\alpha' < 0$; $\phi_1 > 0$.

The negativity of J is also required for the short-run multipliers to work in plausible directions, so this is assumed without further comment.

When $l_n > l_t$, i.e. when non-tradables are relatively labour intensive, the second stability condition requires

$$\frac{Y_n/K_n}{(Y-T)/K} > D_{n2} \frac{wl_n - wl_t}{\frac{(Y_L - T) - wl_t}{K}}$$

where Y_L is labour income. This requires the marginal propensity to consume non-tradables to be relatively low. (For $T = 0$, the condition is that the income elasticity of demand be less than unity). In graphical terms, the requirement is that the BB curve must slope upwards more steeply than NN.

When tradables are labour intensive the condition is:

$$D_{n2} \left[\frac{w l_t - w l_n}{w l_t - \frac{(Y_L - T)}{K}} \right] > \frac{Y_n / K_n}{(Y - T) / K} , \quad \text{which means}$$

that the marginal propensity to consume non-tradables must be high. Graphically this requires that NN be more steeply sloped than BB.

The Short Run Effects of Government Expenditure on Non-Tradables

$$\frac{dP_n}{dG_n} = \frac{-\{1 - D_{n2}(1 - \gamma\theta)\} C}{J} > 0$$

$$\frac{dr_n}{dG_n} = \frac{1}{K_n} \{Y_n - L_n \phi_1\} \frac{dP_n}{dG_n} > 0$$

$$\frac{dr_t}{dG_n} = -\frac{1}{K_t} \{L_t \phi_1\} \frac{dP_n}{dG_n} \leq 0$$

$$\frac{dq_n}{dG_n} = \frac{1}{C \cdot K_n} \left[(Y_n - L_n \phi_1) (\alpha K_t - r K \alpha') + K_n L_t \phi_1 \alpha \right] \frac{dP_n}{dG_n} > 0$$

$$\frac{dq_t}{dG_n} = \frac{1}{C \cdot K_t} \left[r K \alpha' \phi_1 L_t - \alpha K_t (Y_n - L_n \phi_1) - \alpha K_n L_t \phi_1 \right] \frac{dP_n}{dG_n} < 0$$

The effect on assets: $\frac{dA}{dG_n} = \frac{1}{C} \left[(Y_n - L_n \phi_1 - L_t \phi_1) (-rK\alpha') \right] \frac{dP_n}{dG_n}$

$$\begin{matrix} > \\ < \end{matrix} 0 \text{ as } Y_n \begin{matrix} > \\ < \end{matrix} L_n \cdot \phi_1 + L_t \cdot \phi_1$$

The effect on interest rates: $\frac{d(\frac{r_n}{q_n})}{dG_n} = \left\{ \frac{r\alpha}{C} \right\} \left[Y_n - L_n \phi_1 - L_t \phi_1 \right] \frac{dP_n}{dG_n}$

$$\begin{matrix} > \\ < \end{matrix} 0 \text{ as } Y_n \begin{matrix} > \\ < \end{matrix} L_n \phi_1 + L_t \phi_1$$

The effect on disposable labour income (DLY): $\frac{d(DLY)}{dG_n} =$

$$\frac{1}{J} \left[(Y_n - L_n \phi_1 - L_t \phi_1) \{ C(1 - D_{n2}) + D_{n2} rK\alpha' \gamma \} - C \{ D_{n1} + wK_t \ell' \phi_1 \} \right] \quad (?)$$

The effect on saving (S): $\frac{dS}{dG_n} =$

$$\frac{Y}{J} \left[(Y_n - L_n \phi_1 - L_t \phi_1) \{ \theta C(1 - D_{n2}) - rK\alpha'(1 - D_{n2}) \} - \theta C \{ D_{n1} + wK_t \ell' \phi_1 \} \right] \quad (?)$$

The effect on net investment (I): $\frac{dI}{dG_n} = f' \cdot \frac{dA}{dG_n}$

The Short-Run Effects of Tax-Financed Government Expenditure
on Tradables

$$\frac{dP_n}{dG_t} = \frac{D_{n2}(1-\gamma\theta)C}{J} < 0$$

$$\frac{dr_n}{dG_t} = \frac{1}{K_n} \{Y_n - L_n\phi_1\} \frac{dP_n}{dG_t} < 0$$

$$\frac{dr_t}{dG_t} = -\frac{1}{K_t} \{L_t\phi_1\} \frac{dP_n}{dG_t} \geq 0$$

$$\frac{dq_n}{dG_t} = \frac{1}{C \cdot K_n} \left[(Y_n - L_n\phi_1)(\alpha K_t - rK\alpha') + K_n L_t\phi_1\alpha \right] \frac{dP_n}{dG_t} < 0$$

$$\frac{dq_t}{dG_t} = \frac{1}{C \cdot K_t} \left[rK\alpha'\phi_1 L_t - \alpha K_t(Y_n - L_n\phi_1) - \alpha K_n L_t\phi_1 \right] \frac{dP_n}{dG_t} > 0$$

$$\frac{dA}{dG_t} = \frac{1}{C} \left[(Y_n - L_n\phi_1 - L_t\phi_1)(-rK\alpha') \right] \frac{dP_n}{dG_t}$$

$$\begin{matrix} < \\ > \end{matrix} 0 \text{ as } Y_n \begin{matrix} \geq \\ < \end{matrix} L_n\phi_1 + L_t\phi_1$$

$$\frac{d\left(\frac{r_n}{q_n}\right)}{dG_t} = \left\{ \frac{r\alpha}{C} \right\} \left[Y_n - L_n\phi_1 - L_t\phi_1 \right] \frac{dP_n}{dG_t}$$

$$\begin{matrix} < \\ > \end{matrix} 0 \text{ as } Y_n \begin{matrix} \geq \\ < \end{matrix} L_n\phi_1 + L_t\phi_1$$

$$\frac{d(DLY)}{dG_t} = \frac{1}{J} \left[(Y_n - L_n \phi_1 - L_t \phi_1) D_{n2} rK \{ \alpha' (r + \gamma) - \alpha \} \right. \\ \left. + C \{ w K_n \ell_n' (P_n \phi_1 - w) - G_n - D_{n1} + N \} \right]$$

(?)

$$\frac{dS}{dG_t} = \frac{Y}{J} \left[\theta C \{ w K_n \ell_n' (P_n \phi_1 - w) - G_n - D_{n1} \} \right. \\ \left. + C \cdot Y_n + (Y_n - L_n \phi_1 - L_t \phi_1) D_{n2} rK (\theta \alpha' r + \alpha' - \alpha \theta) \right]$$

(?)

FIG. 1.

FIG. 2