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Title	Design charts for the stability analysis of unsaturated soil slopes
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Publication date	2010-01
Publication information	Gavin, Kenneth, and Jianfeng Xue. "Design Charts for the Stability Analysis of Unsaturated Soil Slopes" 28, no. 1 (January, 2010).
Publisher	Springer
Item record/more information	http://hdl.handle.net/10197/3101
Publisher's statement	The final publication is available at springerlink.com
Publisher's version (DOI)	10.1007/s10706-009-9282-z

Downloaded 2023-04-07T10:52:29Z

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Design Charts for the Stability Analysis of Unsaturated Soil Slopes

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Received: 16 April 2009 / Accepted: 6 November 2009
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Abstract Simple limit equilibrium analyses can be performed to determine the Factor of Safety (FOS) against slope failure of unsaturated soil slopes. However, many of the input parameters needed for these analyses are highly variable, and the FOS value obtained is critically dependent on assumptions made by the designer. This paper describes a suite of reliability analyses on unsaturated soil slopes performed using an invariant reliability model. The results are presented in design charts from which a designer can choose the FOS value required to ensure a given target reliability index for a slope. The approach ensures that despite the variability of input parameters the slope will have a probability of failure of 2.23% or less.

Keywords Slope stability · Reliability analyses · Design charts

Abbreviations

C Total cohesion
COV Coefficient of variation
E(X) Mean value of vector *X*
FOS Factor of Safety

P_f Probability of failure
X Vector of variables
 $\bar{X}$ Vector of reduced variables
W The weight of a slice of the slope
C' Effective cohesion of soil
g(X) Limit state function defined with vector [*X*]
g($\bar{X}$) Limit state function defined with reduced variables
h Wetting front depth
r Radius in polar coordinate
u_a Pore air pressure
u_w Pore water pressure
 α Slope angle
 β Reliability index
 β_{HL} Reliability index defined with Hasofer–Lind method
 γ Unit weight of soil
 ϕ' Friction angle of soil
 ϕ^b Angle indicating the rate of increase in shear strength relative to the matric suction
 θ Angle in polar coordinates
 $\sigma(X)$ Standard deviation of vector *X*
 σ_n Total normal stress on the failure plane
 τ Shear strength of unsaturated soils
 ∂ Partial differential

1 Introduction

Rainfall induced slope failures are caused by the development of a wetting front (downward percolating

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water) which causes a reduction in near surface suctions and thereby reduces the soil strength. The rate of development and depth of a wetting front is dependent on the initial soil state (initial water content/suction) and the amount of the applied rainfall which infiltrates into the slope. The initial soil state largely depends on the soil type, water table level and antecedent rainfall. The amount of infiltration depends on additional factors such as the slope angle, vegetation type, rainfall intensity and rainfall duration. It is clear that to perform even a simple analysis of the stability of a slope; the designer must first make a number of assumptions regarding the current soil state and likely future infiltration. A number of these assumptions are not straightforward and differ from other geotechnical design situations in that they are not based on knowledge of soil characteristics and super-structure loading and therefore involve expertise in areas beyond those normally developed by geotechnical engineers. Given changing climatic conditions the quantification of the effects of future rainfall events which may be heavier and of longer duration is an area of particular uncertainty.

Analysis of the stability of unsaturated slopes is a complex problem and a range of approaches from simple limit equilibrium solutions to advanced finite element method analyses have been proposed. The more advanced approaches, presented by (Ng and Shi 1998; Gasmo et al. 2000; Cho and Lee 2001) although more rigorous, require input data such as parameters derived from the Soil Water Characteristic Curves (SWCC) which are, in some cases difficult to obtain.

Probabilistic analysis offers a rational way for engineers to include uncertainty in their designs. Uncertainty may be caused by natural variability evident in soil properties, or in the range of likely rainfall events (duration and intensity) experienced by a slope. This paper describes a framework whereby the FOS value required from a deterministic analysis can be chosen from design charts. The design charts summarise the results of reliability analyses which account for uncertainty in the design values of parameters used in the analysis. As such they provide a simple and rational method of ensuring a meaningful FOS for a given slope.

2 Background

A broad range of stability analyses are used in practice ranging from deterministic approaches where single

design values are assumed for the variables, to probabilistic approaches where variability is considered.

2.1 Deterministic Methods

Fredlund et al. (1978) expanded the Mohr–Coulomb model to incorporate negative porewater pressure (matric suction) effects:

$$\tau = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \quad (1)$$

$$= C + (\sigma_n - u_a) \tan \phi'$$

where τ = shear strength of unsaturated soils, c' is the effective cohesion, σ_n is the total normal stress on the failure plane, u_a is the pore-air pressure on the failure plane, ϕ' is the angle of internal friction associated with the net normal stress state variable $\sigma_n - u$, u_w is the pore-water pressure on the failure plane, $u_a - u_w$ is the matric suction on the failure plane, and ϕ^b is the angle indicating the rate of increase in shear strength relative to the matric suction. τ , c' , σ_n , u_a and u_w are typically given in kPa, whilst ϕ' and ϕ^b are given in degrees. By combining the effects of c' and the contribution of matric suction $(u_a - u_w) \tan \phi^b$ into a single parameter (C), (Fredlund and Rahardjo 1993) note that Eq. 1 takes the form of the conventional Mohr–Coulomb model, and thus can be implemented easily into existing slope stability software developed to analyse saturated soil slopes. Using the method of slices, they suggest that the variation of suction with depth in an unsaturated soil slope can be modelled by dividing the soil into discrete layers, and varying C within these layers. They note that the drawbacks of this approach are that C must be computed manually, remain constant throughout the analysis and be set by the user.

Fourie et al. (1999) note that in most slope failures caused by infiltration, the failure plane forms parallel to the existing slope surface. For this reason they suggest using an infinite slope model in which the FOS is given by:

$$\text{FOS} = \frac{C + \gamma h \cos^2 \alpha \tan \phi}{\gamma h \cos \alpha \sin \alpha} \quad (2)$$

in which γ is the unit weight of soil (kN/m^3), h is the wetting front depth in metres and α is the slope angle in degrees.

They suggest that Eq. 2 can provide useful information on the contribution of suction to the stability of an unsaturated soil slope. This is illustrated in Fig. 1

Fig. 1 Effect of total cohesion and wetting front depth on slope stability

where the stability of an embankment with a slope angle of 40° , soil friction angle of 30° and a wetting front depth which varies from 0.25 to 1.5 m is presented. The effect of increasing the total cohesion (C) from 0 to 12 kPa is considered. When C is zero, the slope angle exceeds the friction angle of the soil, and the FOS is below 1. Assuming the effective cohesion of the soil is zero, the role of even a small suction in maintaining the stability of steep slopes is evident. For a given total cohesion, the FOS value of a slope reduces as the wetting front depth increases, for example when $C = 4$ kPa, the slope will fail when the wetting front depth reaches 1 m. The limitations of this approach are that the soil properties are assumed to be constant in the zone above the wetting front.

The drawbacks associated with the method of slices and infinite slope method can be overcome by performing a Finite Element Analysis (FEA). Using a constitutive model, which accounts for the effect of suction on soil strength, and allows full discretisation of the slope, thus permitting the soil properties to vary in the wetted zone, a coupled seepage and stability calculation can be performed. Although these FEA approaches are possible, the complex soil properties needed for the constitutive models are difficult to measure accurately and are rarely available for routine design situations.

2.2 Probabilistic Methods

There are many uncertainties associated with the choice of variables to use in the analysis of the stability of a soil slope. All soils are heterogeneous, and therefore soil properties vary with depth and location. Additional uncertainties associated with the analysis of unsaturated slope stability problems include; the variation of total cohesion when suctions reduce as a result of changes in water content during infiltration and difficulties in assessing the rate at which wetting front development occurs. The depth to which infiltrating water will penetrate into a slope, which represents the depth from ground surface to the slip surface, depends on many parameters, a number of which are highly variable. These include the initial suction profile, the geometry of the slope, permeability of the soil, rainfall intensity and duration amongst other factors. Traditional deterministic design methods, in which the soil properties are assigned fixed (conservative) values, cannot adequately model the dynamic process of wetting front development, and its effect on the disturbing forces causing slope failure, and the resistance (soil strength) preventing failure. For this reason (Zhang et al. 2005; Babu and Murthy 2005) and others propose the use of probabilistic design approaches.

In an attempt to improve commercial software, which typically performs uncoupled analysis, (Zhang et al. 2005) describe the development of a coupled hydro-mechanical model and slope stability analysis implemented using the finite element approach. Using the FE method they determined the FOS of a slope where the porosity, saturated permeability parameters derived from the SWCC and the shear strength values used in the assessment were varied.

Babu and Murthy (2005) carried out a probabilistic analysis of a slope using a planar slip surface model, with c' , ϕ' , γ , ϕ^b , K (hydraulic conductivity), the wetting front depth and the matric suction as variables. The reliability index (β) was calculated using a First Order Second Moment (FOSM) approach. In this method, the FOS is determined at a design point (normally the mean values of these variables). By considering the variation of these variables, the reliability of the slope is obtained. A sensitivity analysis was performed which identified matric suction, K and ϕ^b as the critical variables controlling slope stability.

2.3 Invariant Reliability Analysis

Whilst the probabilistic approaches described are a significant advance on traditional deterministic methods, they do not fully utilise the power of probabilistic analysis. Due to its simplicity the FOSM approach is widely used. However, there are several disadvantages associated with FOSM, the most significant being the lack of invariance of the approach for non-linear performance functions (USACE 1997).

Hasofer and Lind (1974) propose a method to determine an invariant estimate of β . By transforming the random variables (x) into non-dimensional form using the mean (E) and standard deviation (σ), an invariant estimate of β is determined:

$$\bar{x}_i = \frac{x_i - E[x_i]}{\sigma[x_i]} \quad (i = 1, 2, \dots, n). \quad (3)$$

When the limit-state (performance) function which compares the Capacity and Demand is given by:

$$g(\bar{X}) = \text{Capacity} - \text{Demand}. \quad (4)$$

In the non-dimensional (reduced variable) space the limit-state function can be rewritten for a number of variables which affect both Capacity and Demand as:

$$g(\bar{X}) = g(\bar{x}_1, \bar{x}_2, \bar{x}_3, \dots, \bar{x}_n). \quad (5)$$

The limit-state surface $[g(\bar{X})] = 0$ is the boundary between safe and unsafe regions (see Fig. 2). The reliability index estimated using the Hasofer–Lind approach (β_{HL}) is the minimum distance from the origin of the reduced variable space to the limit-state surface (design point). Many workers have used a cosine directional search approach to determine this distance, however (Val et al. 1996) demonstrate that because of the highly non-linear form of the performance functions considered, there is a tendency for this approach to identify local minima. In such circumstance the entire limit-state surface is not examined and the method may fail to locate the true reliability index. They show that by transforming the normalised variables from rectangular to polar coordinates this problem is overcome. Although the problem could also be addressed by using a full population, iterative search technique such as the Monte–Carlo or Genetic Algorithm methods, (Xue and Gavin 2007) adopted the technique of transform-

Fig. 2 Definition reliability index in polar coordinates

ing the variables to polar coordinates as it facilitated the formulation of the complex objective function and allowed them to develop a model which simultaneously determined the critical slip surface and reliability index of slopes using Bishop's simplified method applied to non-circular failure surfaces. The variables considered in the analyses included the soil properties (c' , ϕ' and γ) and the coordinates of the slip surface. The reliability index β_{HL} , is given by (see Fig. 2):

$$\beta_{HL} = \min_{X' \in \Psi} (r) \quad (6)$$

in which $r = \{(X')^T X'\}^{1/2}$.

This constrained optimisation problem was solved through the use of the Genetic Algorithm (GA) method.

Gavin and Xue (2009) extended this model to consider planar slip surfaces which develop during the downward migration of a wetting front in unsaturated soil slopes. The variables which describe the soil properties were C , ϕ , γ and h . At the ultimate limit-state (FOS = 1) Eq. 2 becomes:

$$g(X) = C + \gamma h \cos^2 \alpha \tan \phi - \gamma h \cos \alpha \sin \alpha. \quad (7)$$

The coordinates of the design point in Fig. 2 can be described using the radial distance (r) and the polar angle (θ):

$$\begin{aligned} C &= E(C) + r \cos \theta_1 \cos \theta_2 \cos \theta_3 \sigma(C) \\ \phi &= E(\phi) + r \cos \theta_1 \cos \theta_2 \sin \theta_3 \sigma(\phi) \\ \gamma &= E(\gamma) + r \cos \theta_1 \sin \theta_2 \sigma(\gamma) \\ h &= E(h) + r \sin \theta_1 \sigma(h). \end{aligned} \quad (8)$$

To avoid mathematical complexities when substituting expressions describing the variables in Eq. 8, into Eq. 2, it is convenient to group some of the variables:

$$\begin{aligned} Y_1 &= \gamma h \\ Y_2 &= Y_1 \cos^2 \alpha \tan \phi. \end{aligned} \quad (9)$$

The limit-state or performance function can now be rewritten as:

$$g(X) = C + Y_2 - Y_1 \cos \alpha \sin \alpha. \quad (10)$$

Assuming C , ϕ , γ and h are normally distributed and un-correlated, according to the Taylor series of multi-variables we have:

$$\begin{aligned} E(Y_1) &= E(\gamma)E(h) \\ \sigma(Y_1) &= \left[(E(h)\sigma(\gamma))^2 + (E(\gamma)\sigma(h))^2 \right]^{1/2} \\ E(Y_2) &= E(Y_1) \cos^2 \alpha \tan(E(\phi)) \\ \sigma(Y_2) &= \left[\left[\cos^2 \alpha \tan(E(\phi)) \sigma(Y_1) \right]^2 \right. \\ &\quad \left. + \left[E(Y_1) \sigma(\tan \phi) \cos^2 \alpha \right]^2 \right]^{1/2} \end{aligned} \quad (11)$$

and the reduced variables C , Y_1 and Y_2 can be expressed as:

$$\begin{aligned} C &= E(C) - r\omega_1\sigma(C) \\ Y_1 &= E(Y_1) - r\omega_2\sigma(Y_1) \\ Y_2 &= E(Y_2) - r\omega_3\sigma(Y_2) \end{aligned} \quad (12)$$

where: $\omega_1 = \cos\theta_1 \cos\theta_2$, $\omega_2 = \cos\theta_1 \sin\theta_2$ and $\omega_3 = \sin\theta_1$.

Substituting Eq. 12 into Eq. 10 and rewriting we have:

$$r = \frac{E(C) + E(Y_2) - E(Y_1) \cos \alpha \sin \alpha}{\omega_1 \sigma(C) + \omega_3 \sigma(Y_2) - \omega_2 \sigma(Y_1) \cos \alpha \sin \alpha}. \quad (13)$$

Because of the assumption that the variables are normally distributed, we must set lower bound values for these variables:

$$\begin{aligned} C &= E(C) - r\omega_1\sigma(C) \geq 0 \\ Y_1 &= E(Y_1) - r\omega_2\sigma(Y_1) \geq 0 \\ Y_2 &= E(Y_2) - r\omega_3\sigma(Y_2) \geq 0. \end{aligned} \quad (14)$$

Fig. 3 Comparison of GASSA, Monte-Carlo and FOSM methods

This constrained optimisation problem was solved through the use of a computationally efficient Genetic Algorithm method Gavin and Xue (2009), known as Genetic Algorithm for Slope Stability Analysis (GASSA). The reliability index β , computed using GASSA, Monte-Carlo analysis and FOSM are compared in Fig. 3. The stability of embankment slope, considered in Fig. 1, was calculated using the three approaches with the wetting front depth increasing from 0.02 to 1 m below ground level. Whilst all methods produce similar results for shallow failure surfaces (wetting front depth < 0.2 m), below this depth the Monte-Carlo and GASSA methods produce comparable results, however, the FOSM method predicts significantly higher β values, with the difference increasing with wetting front depth. Since higher β values suggest larger FOS, it is clear that the FOSM approach does not determine the minimum (critical) FOS and is therefore unconservative. The similarity between the β values computed using the full-population Monte-Carlo analysis and the computationally more efficient GASSA model for the whole

range of wetting front depths considered are in keeping with similar calibration exercises performed on saturated slope profiles by (Xue 2007)

3 Hybrid Design approach

Probabilistic design approaches such as that described in the previous section are not yet in widespread use in geotechnical engineering practice. Deterministic approaches are taught widely in the majority of undergraduate engineering design courses and this coupled with their simplicity and perceived transparency means such methods remain the industry standard. In deterministic design, the target FOS is normally set as a minimum of value, e.g. 1.3. The soil properties are usually chosen based on characteristic values as defined in (Eurocode 7 2004), often referred to as moderately conservative parameters. In practice this means adopting design values which are below the actual mean value of the available measurements of soil properties such as shear strength. The design values are often chosen without regard for the actual variation of soil properties, which means that although the FOS value is fixed, the reliability index will vary, reducing as the variability of the soil properties increase. In these situations the adoption of conservative input parameters does not in fact guarantee a safe design. An alternative approach would be to set a fixed minimum target reliability index (β_T), which would represent an acceptable probability of failure for a slope. For a given slope a rational design approach which does not involve a full probabilistic analysis would be to determine the FOS required to obtain the set target reliability index. The FOS required to meet this β_T value will naturally increase as uncertainties regarding the input parameters for the slope stability analysis increase.

Considering Eq. 2, routine laboratory tests are used to obtain values of ϕ and γ . In common soils these values are relatively well defined are therefore mean design values can be assigned with some confidence. Because of significant recent interest in the behaviour of partly saturated soils, major developments in laboratory and field measurement complimented by modelling have been achieved. It is relatively straightforward to measure the total cohesion C , in the laboratory or field (Springman et al. 2003). An estimate of the wetting front depth h , can

be obtained using simple empirical methods (Gavin and Xue 2008) or more accurate complex numerical analyses (Ng and Shi 1998). By making a number of such determinations the variability of the critical input parameters can be determined.

3.1 Target Reliability Index (β_T)

The use of reliability theory to select a FOS consistent with a target reliability index has been discussed by (Benjamin and Cornell 1970; Whitman 1984) in his seminal paper on the application of reliability analysis in geotechnical engineering. The choice of β_T is normally based on (Benjamin and Cornell 1970); (1) historical data, (2) mathematical modelling based on probability theory or (3) quantification of expert systems (Paikowsky et al. 2004). Although for most geotechnical problems the choice of β_T is not straightforward (Chowdhury and Flentje 2003), suggest that for most slopes (outside of those in urban areas) β_T can be set at 2.0, which corresponds to a probability of failure of 2.23%. Although this reliability index would classify the performance as poor in accordance with (USACE 1999), it is consistent with β_T which is implied by conventional practice in geotechnical engineering (Whitman 1984) and will be adopted in this paper. A framework is proposed in which a designer can use a set of design charts, which have been developed based on the invariant reliability model, to choose a target FOS value to ensure a consistent reliability index regardless of the uncertainty surrounding the input parameters. To minimise the number of variables which need to be considered in the design charts, the critical input parameters which most strongly affect the reliability index are determined in the following section.

3.2 Sensitivity Analysis of Input Parameters

The influence of the absolute depth of the wetting front (h) and the degree of uncertainty with respect to the estimation of this value, i.e. the coefficient of variation (COV = standard deviation/mean value), on the FOS required to maintain the target reliability index (β_T) of a 30° slope is illustrated in Fig. 4. As would be expected from the analyses of (Fourie et al. 1999) and others (see Fig. 1), as the wetting front depth increases from 0.7 to 1.5 m, the mean cohesion $E(C)$ required to maintain the target reliability index

Fig. 4 Effect of uncertainty with respect to wetting front depth on FOS

($\beta_T = 2$) for a given $COV(h)$ value increased. It is evident from Fig. 4, that for the range of the wetting front depths considered, as the level of uncertainty regarding the evaluation of the wetting front depth increased, the mean cohesion and the FOS value required to achieve β_T increased. For example for a given wetting front depth, the FOS value required increased from 1.36 to 1.68 when the $COV(h)$ increased from 0.05 to 0.15. The analyses suggest that for a given value of $E(C)$, the FOS required to achieve β_T is influenced significantly by the level of uncertainty with respect to the estimate of the wetting front depth.

To study the influence of uncertainty with respect to the mean value of cohesion $COV(C)$ on the FOS required to maintain the target reliability index, analyses were performed on three slopes, with slope angles (α) of 30°, 35° and 45°. The soil properties adopted in the analyses were $E(\phi) = 34^\circ$ and $COV(\phi) = 0.1$. The $COV(h)$ was varied from 0.05 in Fig. 5a to 0.15 in Fig. 5b to examine the effect of $COV(C)$ on the FOS value required to achieve a target reliability index. The results shown in Fig. 5 reveal that:

1. The FOS required to achieve the target reliability index increased gradually until the $COV(C)$ exceeded 0.3, after which a significant increase in FOS was required to meet the target reliability index.

Fig. 5 Relationship between $COV(C)$ and the required FOS. **a** Coefficient of variation of wetting front depth = 0.05. **b** Coefficient of variation of wetting front depth = 0.15

2. The rate of increase of FOS depended on the slope angle and the $COV(h)$. Considering Fig. 5a, only a small increase in FOS was required to ensure stability of a 30° slope, with $COV(h) = 0.05$,

when the $COV(C)$ increased from 0.1 to 0.3. In contrast, when the $COV(h)$ increased to 0.15 (see Fig. 5b), the rate of increase of FOS was much more significant.

3. In all cases, the largest increase in FOS required, occurred for slopes where α was highest (i.e. the difference between the slope angle and the mean friction angle was greatest).

The effect of varying the water content of a soil on the total cohesion (C) can be established using simple in situ or laboratory tests (Springman et al. 2003; Doherty et al. 2007) whilst the wetting front depth can be estimated using either simple empirical models or Finite Element Analyses. Once estimates of the two parameters C and h can be determined the FOS value of the slope can be determined using Eq. 2 or a design chart such as Fig. 1.

The sensitivity analyses performed using GASSA in Figs. 4 and 5 suggests the uncertainty associated with the determination of C and h , significantly influences the FOS value required to achieve a target reliability index. It is necessary to confirm this observed behaviour using more established numerical techniques such as Monte–Carlo analyses. In order to perform this calibration exercises the FOS required to maintain a β_T value of 2.0 was calculated for a 30° slope, with a fixed wetting front depth of 0.7 m, soil unit weight of 20 kN/m^3 ($COV = 0.05$). A Monte–Carlo analysis was performed on a slope with a slope angle of 35° , soil friction angle $\phi = 30^\circ$, $COV(\phi) = 0.1$, mean wetting front depth = 0.9 m, and mean cohesion = 4 kPa (see Fig. 6a). The FOS value was fixed at 1.3 and the probability of failure (P_f , which is the probability that the demand will equal or exceed the capacity) for a range of $COV(C)$ between 0.1 and 0.4, and the $COV(h)$ between 0.05 and 0.15 was calculated. The data shown in Fig. 6b show that for all $COV(C)$ values considered, P_f increased approximately linearly as $COV(h)$ increased from 0.01 to 0.15. Thereafter, P_f increased sharply. This response is entirely comparable with the predictions of GASSA.

4 Design Charts

The sensitivity analyses performed in the previous section suggests that uncertainty associated with the determination of C and h significantly influences the

FOS value required to achieve a target reliability index. By considering different values for the COV of C and h , design charts can be constructed for a range of slopes. In the charts the target reliability index is fixed at 2, and the FOS required to obtain this value was investigated for different values of COV of the total cohesion and wetting front depth. Four values of $COV(C)$, 0.1, 0.2, 0.3 and 0.4 were considered. Since in the sensitivity analyses the FOS was found to increase almost linearly with $COV(h)$, for values between 0.05 and 0.15, only the upper and lower values in this range were included. Slope angles of 25° , 30° , 35° , 40° and 45° were assumed and for each of these slopes analyses were performed for soil friction angles of 26° , 30° and 40° to give a range of values typically encountered in design practice. The coefficient of variation of the soil friction angle was assumed to be 0.1, and a mean soil unit weight of 20 kN/m^3 was assigned to the soil with a COV of 0.05. The FOS values required for the conditions considered are shown in Fig. 7. It should be noted that the results for three or four slope angles are plotted on each design chart merely for brevity and this analysis approach could be applied to a wider range of slope angles and soil properties. The FOS values for all slope angles at a given $COV(C)$ are not directly comparable as the mean values for cohesion were not equal. The values of the parameters ϕ and α were chosen such that linear interpolation could be performed to consider intermediate values not specifically shown in Fig. 7.

5 Application of the Design Charts

A deterministic analysis of the stability of an unsaturated soil slope, with a slope angle of 40° was presented in Fig. 1. In this analysis, the soil friction angle and unit weight were taken as 30° and 20 kN/m^3 respectively. When the total cohesion was assumed to be 8 kPa, and the wetting front depth was 1 m, the FOS value calculated using Eq. 2 was 1.45. Although this value is significantly above the value of 1.3 which is normally assumed to be the minimum allowable, the actual safety of the slope depends on the assumptions made by the designer in their choice of design values for the variables.

We could consider determine a more meaningful value of FOS if we quantify the variability of the

Fig. 6 **a** Slope layout for Monte–Carlo analyses.
b Sensitivity analyses calibrated using Monte–Carlo analysis

parameters used in the analysis. Assuming the soil properties and wetting front depth given above are the mean values obtained from a number of measurements or estimates of ϕ , γ , C and h , and the COV of ϕ and γ were 0.1 and 0.05 respectively, the uncertainty with respect to the measured or estimated values of C and h can be investigated using the design charts given in Fig. 7.

If we consider in the first instance, good confidence in the values of C and h , and assign COV values of 0.1 and 0.05 respectively, then a target value for $\text{FOS} = 1.25$ is obtained from Fig. 7c. This is lower than the value (1.45) achieved using just the mean values of the variables and the minimum allowable value (1.3). The slope therefore has a low probability of failure and a reliability index significantly higher than 2.0 . An alternative view of its stability can be considered in Fig. 8, where for the FOS value of 1.25 (associated with a target reliability

index of 2), suggest the safety of the slope is adequate until the wetting front depth reaches 1.4 m below ground level (i.e. much deeper than the wetting front depth of 1 m identified in the deterministic analysis).

However, if there was greater uncertainty about the estimate of cohesion and the variability of this parameter $\text{COV}(C)$ increased to 0.3 , then from Fig. 7c the FOS value required to meet the target reliability index increases to 1.75 . This is larger than the value of 1.45 achieved in the deterministic analysis and this indicates that the reliability index is less than 2 . According to the probabilistic analysis, the slope performance would become unacceptable if the wetting front depth reach 0.75 m below ground level. Whilst the reduction in FOS is fundamentally caused by the increased variability of the parameters used in the stability analysis, the relatively large increase in FOS required to meet the target reliability index, from 1.25 to 1.75 is exacerbated because for

Fig. 7 Hybrid stability analysis design charts

Fig. 8 Application of design charts

the slope considered the slope angle is 10° higher than the soil friction angle. We see from Fig. 7c, when the difference between the slope angle and soil friction angle is within 5° , the FOS required is relatively insensitive to increases in $\text{COV}(C)$ until the latter value exceeds 0.3. Although it is clear from charts 7a to f, that even when the slope angle is lower than the mean friction angle, the FOS value required to obtain target reliability index, exceeds the value of 1.3 often used in practice.

6 Conclusions

A hybrid method of assessing the stability of unsaturated soil slopes is presented. The method utilises the benefits of the rational treatment of the variability of input parameters provided by reliability based design and the simplicity afforded by deterministic approaches. The FOS value required to ensure a target reliability index of 2 is achieved can be chosen from a design chart which accounts for the COV of the total cohesion and wetting front depth. If the FOS value achieved using the deterministic approach with design values for the variables chosen at the mean value exceeds the FOS determined from the design chart, the probability of failure of the slope will be lower than 2.23%.

The hybrid design approach incorporates a reliability model in which an invariant measure of the

reliability index, defined using the Hasofer–Lind method, is determined. The approach assumes no knowledge of probabilistic analysis and requires the designer only to determine the mean value and the COV of the variables. The key benefit of the approach is its simplicity in that it provides a mechanism for considering parameter variability in routine design. Its usefulness will of course depend on the amount of information available to describe parameter variability at a given site and some of the assumptions inherent in the reliability model.

Whilst simple deterministic analyses of a slope identified the effect of total cohesion and wetting front depth on the FOS achieved, the sensitivity analyses performed using the reliability model identified the critical effect of uncertainty with respect to the design values of both C and h to the value of FOS required to achieve the target reliability index. Such information will allow the designer to concentrate on the determining a representative value of these materials and suggests that effort spent investigating the likely variability of these parameters, for example performing additional laboratory tests to determine C or additional numerical analyses to determine the sensitivity of the wetting front depth to the choice of variables used in its determination would be beneficial.

Acknowledgments This project is funded by *Iarnród Éireann*. The authors wish to thank Mr. Brian Garvey, former Chief Civil Engineer with *Iarnród Éireann* for technical and financial assistance received. The second author was the recipient of a Geotechnical Trust Fund award from the Geotechnical Society of Ireland. We wish to thank Paul Doherty (PhD student at University College Dublin) for performing the Monte–Carlo simulations presented in this paper.

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